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Neutrosophic Crisp Set Theory

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Abstract. The purpose of this paper is to introduce new types of neutrosophic crisp sets with three types 1, 2, 3. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Also, we introduce and study the neutrosophic crisp point and neutrosophic crisp relations. Possible applications to database are touched upon.

Keywords: Neutrosophic Set, Neutrosophic Crisp Sets; Neutrosophic Crisp Relations; Generalized Neutrosophic Sets; Intuitionistic Neutrosophic Sets.

1 Introduction

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The fundamental concepts of neutrosofic set, introduced by Smarandache in [16, 17, 18] and Salama et al. in [4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 19, 20, 21], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 4, 23] such as a neutrosophic crisp set theory. In this paper we introduce new types of neutrosophic crisp set. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Also, we introduce and study the neutrosophic crisp points and relation between two new neutrosophic crisp notions. Finally, we introduce and study the notion of neutrosophic crisp relations.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [16, 17, 18], and Salama et al. [7, 11, 12, 20]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where \(0,1\) is nonstandard unit interval.

\textbf{Definition 2.1 [7]}

A neutrosophic crisp set (NCS for short) \(A = \{A_1, A_2, A_3\}\) can be identified to an ordered triple \(\{A_1, A_2, A_3\}\) are subsets on \(X\) and every crisp set in \(X\) is obviously a NCS having the form \(\{A_1, A_2, A_3\}\).

Salama et al. constructed the tools for developed neutrosophic crisp set, and introduced the NCS \(\phi_N, X_N\) in \(X\) as follows:

\(\phi_N\) may be defined as four types:

\(i)\) Type1: \(\phi_N = \{\phi, \phi, X\}\), or

\(ii)\) Type2: \(\phi_N = \{\phi, X, X\}\), or

\(iii)\) Type3: \(\phi_N = \{\phi, X, \phi\}\), or

\(iv)\) Type4: \(\phi_N = \{\phi, \phi, \phi\}\)

\(i)\) \(X_N\) may be defined as four types

\(i)\) Type1: \(X_N = \{X, X, \phi\}\),

\(ii)\) Type2: \(X_N = \{X, X, \phi\}\),

\(iii)\) Type3: \(X_N = \{X, X, \phi\}\),

\(iv)\) Type4: \(X_N = \{X, X, X\}\),

\textbf{Definition 2.2 [6, 7]}

Let \(A = \{A_1, A_2, A_3\}\) a NCS on \(X\), then the complement of the set \(A (A^c\), for short) may be defined as three kinds

\(c_1\) Type1: \(A^c = \{A_1^c, A_2^c, A_3^c\}\),

\(c_2\) Type2: \(A^c = \{A_1, A_2, A_3\}\)
be a non-empty set, and NCSS \( A \) and \( B \) in the form \( A = \{A_1, A_2, A_3\}, B = \{B_1, B_2, B_3\} \), then we may consider two possible definitions for subsets \( (A \subseteq B) \)

- Type 1: \( A \subseteq B \) \iff \( A_1 \subseteq B_1, A_2 \subseteq B_2, A_3 \subseteq B_3 \)
- Type 2: \( A \subseteq B \) \iff \( A_1 \subseteq B_1, A_2 \supseteq B_2, A_3 \supseteq B_3 \)

\[ \text{Definition 2.5 [6, 7]} \]

Let \( X \) be a non-empty set, and NCSS \( A \) and \( B \) in the form \( A = \{A_1, A_2, A_3\}, B = \{B_1, B_2, B_3\} \) are NCSS Then

1) \( A \cap B \) may be defined as two types:
   i) Type 1: \( A \cap B = \{A_1 \cap B_1, A_2 \cap B_2, A_3 \cap B_3\} \)
   ii) Type 2: \( A \cap B = \{A_1 \cap B_1, A_2 \cap B_2, A_3 \cap B_3\} \)

2) \( A \cup B \) may be defined as two types:
   i) Type 1: \( A \cup B = \{A_1 \cup B_1, A_2 \cup B_2, A_3 \cup B_3\} \)
   ii) Type 2: \( A \cup B = \{A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3\} \)

\[ \text{Definition 2.3 [6, 7]} \]

Let \( X \) be a non-empty set, and NCSS \( A \) and \( B \) in the form \( A = \{A_1, A_2, A_3\}, B = \{B_1, B_2, B_3\} \), then we may consider two possible definitions for subsets \( (A \subseteq B) \)

1) Type 1: \( A \subseteq B \) \iff \( A_1 \subseteq B_1, A_2 \subseteq B_2, A_3 \subseteq B_3 \)
2) Type 2: \( A \subseteq B \) \iff \( A_1 \subseteq B_1, A_2 \supseteq B_2, A_3 \supseteq B_3 \)

\[ \text{Definition 3.3} \]

1) \( A \) \subseteq \( A' \) \iff \( A_1 \subseteq A'_1, A_2 \subseteq A'_2, A_3 \subseteq A'_3 \)

2) \( \text{(Generalized Neutrosophic Set [8]): Let } X \text{ be a non-empty fixed set. A generalized neutrosophic (GNS for short) set } A \text{ is an object having the form } A = \{x, \mu_A(x), \sigma_A(x), v_A(x)\} \text{ where } \mu_A(x), \sigma_A(x) \text{ and } v_A(x) \text{ which represent the degree of membership function (namely } \mu_A(x) \text{), the degree of indeterminacy (namely } \sigma_A(x) \text{), and the degree of non-membership (namely } v_A(x) \text{) respectively of each element } x \in X \text{ to the set } A \text{ where } 0 \leq \mu_A(x), \sigma_A(x), v_A(x) \leq 1^+ \text{ and } 0 \leq \mu_A(x) + \sigma_A(x) + v_A(x) \leq 3^+. \)

3) \( \text{(Intuitionistic Neutrosophic Set [22]): Let } X \text{ be a non-empty fixed set. An intuitionistic neutrosophic set } A \text{ (INS for short) is an object having the form } A = \{\mu_A(x), \sigma_A(x), v_A(x)\} \text{ where } \mu_A(x), \sigma_A(x) \text{ and } v_A(x) \text{ which represent the degree of membership function (namely } \mu_A(x) \text{), the degree of indeterminacy (namely } \sigma_A(x) \text{), and the degree of non-membership (namely } v_A(x) \text{) respectively of each element } x \in X \text{ to the set } A \text{ where } 0 \leq \mu_A(x), \sigma_A(x), v_A(x) \leq 1^+ \text{ and the functions satisfy the condition } \mu_A(x) \cap \sigma_A(x) \leq 0.5, \mu_A(x) \cap v_A(x) \leq 0.5, \sigma_A(x) \cap v_A(x) \leq 0.5, \text{ and } 0 \leq \mu_A(x) + \sigma_A(x) + v_A(x) \leq 2^+. \) A neutrosophic crisp set with three types the object } A = \{A_1, A_2, A_3\} \text{ can be identified to an ordered triple } \{A_1, A_2, A_3\} \text{ is a subset on } X, \text{ and every crisp set in } X \text{ is obviously a NCSS having the form } \{A_1, A_2, A_3\} \).

Remark 3.1

1) The neutrosophic set not to be generalized neutrosophic set in general.
2) The generalized neutrosophic set in general not intuitionistic NS but the intuitionistic NS is generalized NS.

Intuitionistic NS \rightarrow Generalized NS \rightarrow NS
Corollary 3.1
Let $X$ non-empty fixed set and $A = \langle \mu_A(x), \sigma_A(x), v_A(x) \rangle$ be INS on $X$. Then:
1) Type1- $A^c$ of INS be a GNS.
2) Type2- $A^c$ of INS be a INS.
3) Type3- $A^c$ of INS be a GNS.

Proof
Since $A$ INS then $\mu_A(x), \sigma_A(x), v_A(x)$ and $\mu_A(x) \wedge \sigma_A(x) \leq 0.5, v_A(x) \wedge \mu_A(x) \leq 0.5$
$v_A(x) \wedge \sigma_A(x) \leq 0.5$ implies
$\mu^c_A(x), \sigma^c_A(x), v^c_A(x) \leq 0.5$ then is not to be Type1-$A^c$
INS. On the other hand the Type 2- $A^c$,
$A^c = \langle v_A(x), \sigma_A(x), \mu_A(x) \rangle$ be INS and Type3- $A^c$,
$A^c = \langle v_A(x), \sigma_A(x), \mu_A(x) \rangle$ and $\sigma^c_A (x) \leq 0.5$ implies to
$A^c = \langle v_A(x), \sigma^c_A (x), \mu_A(x) \rangle$ GNS and not to be INS

Example 3.1
Let $X = \{a, b, c\}$, and $A, B, C$ are neutrosophic sets on $X$, $A = (0.7, 0.9, 0.8) \setminus a, (0.6, 0.7, 0.6) \setminus b, (0.9, 0.7, 0.8 \setminus c)$.
$B = (0.7, 0.9, 0.5) \setminus a, (0.6, 0.4, 0.5) \setminus b, (0.9, 0.5, 0.8 \setminus c)$ By the Definition 3.3 no.3 $\mu_A(x) \wedge \sigma_A(x) \wedge v_A(x) \geq 0.5$, $A$ be not GNS and INS,$B = (0.7, 0.9, 0.5) \setminus a, (0.6, 0.4, 0.5) \setminus b, (0.9, 0.5, 0.8 \setminus c)$ not INS, where $\sigma_A (b) = 0.4 < 0.5$. Since
$\mu_B(x) \wedge \sigma_B(x) \wedge v_B(x) \leq 0.5$ then $B$ is a GNS but not INS.
$A^c = \langle (0.3, 0.1, 0.2) \setminus a, (0.4, 0.3, 0.4) \setminus b, (0.1, 0.3, 0.2 \setminus c)$
Be a GNS, but not INS.
$B^c = (0.3, 0.1, 1.0) \setminus a, (0.4, 0.6, 0.5) \setminus b, (0.1, 0.5, 0.2 \setminus c)$
Be a GNS, but not INS, $C$ be INS and GNS,
$C^c = (0.3, 0.1, 1.0) \setminus a, (0.4, 0.2, 0.5) \setminus b, (0.1, 0.5, 0.2 \setminus c)$
Be a GNS but not INS.

Definition 3.2
A NCS-Type1 $\phi_{N_1}$ in $X$ as follows:
1) $\phi_{N_1}$ may be defined as three types:
   i) Type1: $\phi_{N_1} = \langle \phi, \phi, X \rangle$, or
evenly $\phi_{N_1} = \langle \phi, X, \phi \rangle$, or
   iii) Type3: $\phi_{N_1} = \phi, \phi, \phi$.
2) $X_{N_1}$ may be defined as one type

Definitio 3.3
A NCS-Type2, $\phi_{N_2}$, $X_{N_2}$ in $X$ as follows:
1) $\phi_{N_2}$ may be defined as two types:
   i) Type1: $\phi_{N_2} = \langle \phi, \phi, X \rangle$, or
evenly $\phi_{N_2} = \langle \phi, X, \phi \rangle$, or
   ii) Type2: $\phi_{N_2} = \langle \phi, X, \phi \rangle$
2) $X_{N_2}$ may be defined as one type

Corollary 3.2
In general
1) Every NCS-Type 1, 2, 3 are NCS.
2) Every NCS-Type 1 not to be NCS-Type2, 3.
3) Every NCS-Type 2 not to be NCS-Type1, 3.
4) Every NCS-Type 3 not to be NCS-Type2, 1, 2.
5) Every crisp set be NCS.
The following Venn diagram represents the relation between NCSs

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\[ B = \{(a, b, c), \{d, e\}\} \] be a NCT-Type 1 but not NCS-Type 2, 3. \( C = \{(a, b, c), \{d, e, f\}\} \) be a NCS-Type 3 but not NCS-Type 1, 2.

**Definition 3.5**

Let \( X \) be a non-empty set, \( A = \{A_1, A_2, A_3\} \)

1) If \( A \) be a NCS-Type 1 on \( X \), then the complement of the set \( A \) \( (A^c, \text{ for short}) \) maybe defined as one  

kind of complement Type 1: \( A^c = \{A_1, A_2, A_3\} \).

2) If \( A \) be a NCS-Type 2 on \( X \), then the complement of the set \( A \) \( (A^c, \text{ for short}) \) maybe defined as one kind of complement Type 2: \( A^c = \{A_1, A_2, A_3\} \).

3) If \( A \) be NCS-Type 3 on \( X \), then the complement of the set \( A \) \( (A^c, \text{ for short}) \) maybe defined as one kind of complement of the set defined as three kinds of complements

\[ (C_i) \text{ Type 1}: A^c = \left\{A^c_1, A^c_2, A^c_3\right\}, \]

\[ (C_i) \text{ Type 2}: A^c = \{A_1, A_2, A_3\} \]

\[ (C_i) \text{ Type 3}: A^c = \{A_1, A_2, A_3\} \]

**Example 3.3**

Let \( X = \{a, b, c, d, e, f\} \), \( A = \{a, b, c, d\}, \{e\}, \{f\}\) be a NCS-Type 2, \( B = \{a, b, c\}, \{\phi\}, \{d, e\}\) be a NCS-Type 1, \( C = \{a, b, c, d\}, \{e\}, \{f\}\) NCS-Type 3, then the complement \( A = \{\{a, b, c, d\}, \{e\}, \{f\}\}\), \( A^c = \{(\phi\}, \{e\}, \{a, b, c, d\}\}\) NCS-Type 2, the complement of \( B = \{a, b, c\}, \{\phi\}, \{d, e\}\), \( B^c = \{(d, e\)), \{\phi\}, \{a, b, c\}\}\) NCS-Type 1. The complement of \( C = \{a, b\}, \{c, d\}, \{e\}, \{f\}\) may be defined as three types:

Type 1: \( C^c = \{\{a, b, c, d\}, \{e\}, \{a, b, c, d\}\}\).

Type 2: \( C^c = \{\{e, f\}, \{a, b, c, d\}\}\).

Type 3: \( C^c = \{\{e, f\}, \{a, b\}\}\).

**Proposition 3.1**

Let \( A_j : j \in J \) be arbitrary family of neutrosophic crisp subsets on \( X \), then  

1) \( \cap A_j \) may be defined two types as:

i) Type 1: \( \cap A_j = \left(\cap A_j \cap A_{j_2} \cap A_{j_3}\right) \) or

ii) Type 2: \( \cap A_j = \left(\cap A_j \cup A_{j_2} \cup A_{j_3}\right) \).

2) \( \cup A_j \) may be defined two types as:

i) Type 1: \( \cup A_j = \left(\cup A_j \cup A_{j_2} \cup A_{j_3}\right) \) or

ii) Type 2: \( \cup A_j = \left(\cup A_j \cap A_{j_2} \cap A_{j_3}\right) \).

**Definition 3.6**

(a) If \( B = \{B_1, B_2, B_3\}\) be a NCS in \( Y \), then the preimage of \( B \) under \( f \), denoted by \( f^{-1}(B) \), is a NCS in \( X \) defined by \( f^{-1}(B) = \{f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3)\}\).

(b) If \( A = \{A_1, A_2, A_3\}\) be a NCS in \( X \), then the image of \( A \) under \( f \), denoted by \( f(A) \) is a NCS in \( Y \) defined by \( f(A) = \{f(A_1), f(A_2), f(A_3)\} \).

Here we introduce the properties of images and preimages some of which we shall frequently use in the following.

**Corollary 3.3**

Let \( A, \{A_j : j \in J\} \) be a family of NCS in \( X \) and \( B, \{B_j : j \in K\} \) NCS in \( Y \), and \( f : X \rightarrow Y \) a function. Then  

(a) \( A_1 \subset A_2 \Rightarrow f(A_1) \subset f(A_2) \).

(b) \( A \subset f^{-1}(f(A)) \) and if \( f \) is injective, then \( A = f^{-1}(f(A)) \).

(c) \( f^{-1}(f(B)) \in B \) and if \( f \) is surjective, then \( f^{-1}(f(B)) = B \).

(d) \( f^{-1}(\cap B) = \cap f^{-1}(B) \).

(e) \( f(\cup A_j) \subseteq f(A_j) \subseteq \cap f(A_j) \) and if \( f \) is injective, then \( f(\cup A_j) = \cap f(A_j) \).

(f) \( f^{-1}(Y_j) = f^{-1}(\phi_j) = \phi_j \).

(g) \( f(\phi_j) = f(X_j) = \phi_j \) if \( f \) is subjective.

**Proof**  

Obvious

**4 Neutrosophic Crisp Points**

One can easily define a nature neutrosophic crisp set in \( X \), called "neutrosophic crisp point" in \( X \), corresponding to an element \( X \):  

**Definition 4.1**

Let \( A = \{A_1, A_2, A_3\} \) be a neutrosophic crisp set on a set \( X \), then \( p = \{p_1, p_2, p_3\} \in X \) is called a neutrosophic crisp point on \( A \).
A NCP $p = \{p_1, p_2, p_3\}$, is said to belong to a neutrosophic crisp set $A = \{A_1, A_2, A_3\}$, of $X$, denoted by $p \in A$, if may be defined by two types
Type 1: $\{p_1\} \subseteq A_1, \{p_2\} \subseteq A_2$ and $\{p_3\} \subseteq A_3$ or
Type 2: $\{p_1\} \subseteq A_1, \{p_2\} \supseteq A_2$ and $\{p_3\} \subseteq A_3$

**Theorem 4.1**

Let $A = \{A_1, A_2, A_3\}$ and $B = \{B_1, B_2, B_3\}$ be neutrosophic crisp subsets of $X$. Then $A \subseteq B$ iff $p \in A$ implies $p \in B$ for any neutrosophic crisp point $p$ in $X$.

**Proof**

Let $A \subseteq B$ and $p \in A$, Type 1: $\{p_1\} \subseteq A_1, \{p_2\} \subseteq A_2$ and $\{p_3\} \subseteq A_3$ or Type 2: $\{p_1\} \subseteq A_1, \{p_2\} \supseteq A_2$ and $\{p_3\} \subseteq A_3$. Thus $p \in B$. Conversely, take any point in $X$. Let $p_1 \in A_1$ and $p_2 \in A_2$ and $p_3 \in A_3$. Then $p$ is a neutrosophic crisp point in $X$. and $p \in A$. By the hypothesis $p \in B$. Thus $p_1 \in B_1$ or Type1: $\{p_1\} \subseteq B_1, \{p_2\} \subseteq B_2$ and $\{p_3\} \subseteq B_3$ or Type 2: $\{p_1\} \subseteq B_1, \{p_2\} \supseteq B_2$ and $\{p_3\} \subseteq B_3$. Hence $A \subseteq B$.

**Theorem 4.2**

Let $A = \{A_1, A_2, A_3\}$ be a neutrosophic crisp subset of $X$. Then $A = \cup \{p : p \in A\}$.

**Proof**

Obvious

**Proposition 4.1**

Let $\{A_j : j \in J\}$ is a family of NCSs in $X$. Then 
\[(a_1)\ p = \{p_1, p_2, p_3\} \in \cap_{j \in J} A_j \text{ iff } p \in A_j \text{ for each } j \in J.\]
\[(a_2)\ p \in \bigcup_{j \in J} A_j \text{ iff } \exists j \in J \text{ such that } p \in A_j.\]

**Proposition 4.2**

Let $A = \{A_1, A_2, A_3\}$ and $B = \{B_1, B_2, B_3\}$ be two neutrosophic crisp sets in $X$. Then $A \subseteq B$ iff for each $p$ we have $p \in A \Leftrightarrow p \in B$ and for each $p$ we have $p \in A \Rightarrow p \in B$; if $A = B$ for each $p$ we have $p \in A \Leftrightarrow p \in B$ and for each $p$ we have $p \in A \Rightarrow p \in B$.

**Proposition 4.3**

Let $A = \{A_1, A_2, A_3\}$ be a neutrosophic crisp set in $X$. Then $A = \cup \{p_1 : p_1 \in A_1\}, \{p_2 : p_2 \in A_2\}, \{p_3 : p_3 \in A_3\}$.

**Definition 4.2**

Let $f : X \to Y$ be a function and $p$ be a neutrosophic crisp point in $X$. Then the image of $p$ under $f$, denoted by $f(p)$, is defined by $f(p) = \{b_1, q_2, q_3\}$, where $q_1 = f(p_1), q_2 = f(p_2)$ and $q_3 = f(p_3)$. It is easy to see that $f(p)$ is indeed a NCP in $Y$, namely $f(p) = q$, where $q = f(p)$, and it is exactly the same meaning of the image of a NCP under the function $f$.

**Definition 4.3**

Let $X$ be a nonempty set and $p \in X$. Then the neutrosophic crisp point $p_N$ defined by $p_N = \{p, \phi, \{p\}^c\}$ is called a neutrosophic crisp point (NCP for short) in $X$, where NCP is a triple (only element in $X$), empty set, (the complement of the same element in $X$). Neutrosophic crisp points in $X$ can sometimes be inconvenient when express neutrosophic crisp set in $X$ in terms of neutrosophic crisp points. This situation will occur if $A = \{A_1, A_2, A_3\}$ NCS-Type1, $p \notin A_1$. Therefore we shall define "vanishing" neutrosophic crisp points as follows:

**Definition 4.4**

Let $X$ be a nonempty set and $p \in X$ a fixed element in $X$. Then the neutrosophic crisp set $p_N = \{\phi, \{p\}, \{p\}^c\}$ is called vanishing "neutrosophic crisp point (VCNP for short) in $X$, where VNCP is a triple (empty set, (only element in $X$), (the complement of the same element in $X$)).

**Example 4.1**

Let $X = \{a, b, c, d\}$ and $p = b \in X$. Then $p_N = \{b, \phi, \{a, c, d\}\}$, $p_N = \{\phi, \{b\}, \{a, c, d\}\}$, $P = \{\{b\}, \{a, d\}\}$.

Now we shall present some types of inclusion of a neutrosophic crisp point to a neutrosophic crisp set:

**Definition 4.5**

Let $p_N = \{\{p\}, \phi, \{p\}^c\}$ is a NCP in $X$ and $A = \{A_1, A_2, A_3\}$ a neutrosophic crisp set in $X$.
(a) $p_N$ is said to be contained in $A$ ($p_N \in A$ for short) iff $p \in A_1$.

(b) $p_{NN}$ be VNCP in X and $A = \{A_1, A_2, A_3\}$ a neutrosophic crisp set in X. Then $p_{NN}$ is said to be contained in $A$ ($p_{NN} \in A$ for short) iff $p \not\in A_1$.

**Remark 4.2**

$p_N$ and $p_{NN}$ are NCS-Type1

**Proposition 4.4**

Let $\{A_j : j \in J\}$ is a family of NCSs in X. Then

(a) $p_N \in \bigcap_{j \in J} A_j$ iff $p_N \in A_j$ for each $j \in J$.

(b) $p_{NN} \in \bigcap_{j \in J} A_j$ iff $p_{NN} \in A_j$ for each $j \in J$.

**Proof**

Straightforward.

**Proposition 4.5**

Let $A = \{A_1, A_2, A_3\}$ and $B = \{B_1, B_2, B_3\}$ be two neutrosophic crisp sets in X. Then $A \subseteq B$ iff for each $p_N$ we have $p_N \in A \iff p_N \in B$ and for each $p_{NN}$ we have $p_{NN} \in A \implies p_{NN} \in B$. $A = B$ iff for each $p_N$ we have $p_N \in A \Rightarrow p_{NN} \in B$ and for each $p_{NN}$ we have $p_{NN} \in A \iff p_{NN} \in B$.

**Proof**

Obvious.

**Proposition 4.6**

Let $A' = \{A_1, A_2, A_3\}$ be a neutrosophic crisp set in X. Then $A' = \bigcup \{p_N : p_N \in A'\}$.

**Proof**

It is sufficient to show the following equalities:

$A_1 = \bigcup \{p : p \in A_1\}$, $A_1 = \phi$, $A_3 = \bigcap \{p^c : p \in A\}$, $A_3 = \{p : p \in A\}$

which are fairly obvious.

**Definition 4.6**

Let $f : X \to Y$ be a function and $p_N$ be a neutrosophic crisp point in X. Then the image of $p_N$ under $f$, denoted by $f(p_N)$ is defined by $f(p_N) = \{q : f(p_N) \in Y\}$ where $q = f(p)$.

Let $p_{NN}$ be a VNCP in X. Then the image of $p_{NN}$ under $f$, denoted by $f(p_{NN})$, is defined by $f(p_{NN}) = \{\phi, q, q^c\}$ where $q = f(p)$.

It is easy to see that $f(p_N)$ is indeed a NCP in Y, namely $f(p_N) = q_N$ where $q = f(p)$, and it is exactly the same meaning of the image of a NCP under he function $f$. $f(p_{NN})$, is also a VNCP in Y, namely $f(p_{NN}) = q_{NN}$, where $q = f(p)$.

**Proposition 4.7**

States that any NCS A in X can be written in the form $A = A_1 \cup A_2 \cup A_3$, where $A = \bigcup \{p_N : p_N \in A\}$.

$A = \{A_1, A_2, A_3\}$ and $A = \bigcup \{p_N : p_N \in A\}$. It is easy to show that, if $A = \{A_1, A_2, A_3\}$, then $A = \{A_1, A_2, A_3\}$.

**Proposition 4.8**

Let $f : X \to Y$ be a function and $A = \{A_1, A_2, A_3\}$ be a neutrosophic crisp set in X. Then we have $f(A) = f(f(A)) \cup f(A) \cup f(A)$.

**Proof**

This is obvious from $A = A_1 \cup A_2 \cup A_3$.

**Proposition 4.9**

Let $A = \{A_1, A_2, A_3\}$ and $B = \{B_1, B_2, B_3\}$ be two neutrosophic crisp sets in X. Then

a) $A \subseteq B$ iff for each $p_N$ we have $p_N \in A \iff p_{NN} \in B$ and for each $p_{NN}$ we have $p_{NN} \in A \Rightarrow p_{NN} \in B$.

b) $A = B$ iff for each $p_N$ we have $p_N \in A \Rightarrow p_{NN} \in B$ and for each $p_{NN}$ we have $p_{NN} \in A \iff p_{NN} \in B$.

**Proof**

Obvious.

**Proposition 4.10**

Let $A = \{A_1, A_2, A_3\}$ be a neutrosophic crisp set in X. Then $A = \bigcup \{p_N : p_N \in A\} \cup \bigcup \{p_{NN} : p_{NN} \in A\}$.
It is sufficient to show the following equalities:
\[ A_1 = \left( \bigcup \{ p : p \in A \} \right) \cup \left( \bigcup \{ \phi : p_{NN} \in A \} \right) \] 
and \[ A_2 = \left( \bigcap \{ p \} : p \in A \right) \cap \left( \bigcap \{ \phi \} : p_{NN} \in A \right), \] 
which are fairly obvious.

\textbf{Definition 4.7}
Let \( f : X \to Y \) be a function.

(a) Let \( p_N \) be a neutrosophic crisp point in \( X \). Then the image of \( p_N \) under \( f \), denoted by \( f(p_N) \), is defined by \( f(p_N) = \{ q \in Y : q = f(p_N) \} \), where \( q = f(p_N) \).

(b) Let \( p_{NN} \) be a VNCP in \( X \). Then the image of \( p_{NN} \) under \( f \), denoted by \( f(p_{NN}) \), is defined by \( f(p_{NN}) = \{ \phi, q_{NN} \} \), where \( q = f(p_{NN}) \). It is easy to see that \( f(p_{NN}) \) is indeed a NCP in \( Y \), namely \( f(p_{NN}) = q_{NN} \), where \( q = f(p_{NN}) \), and it is exactly the same meaning of the image of a NCP under the function \( f \). The image of a NCP under the function \( f \) is also a VNCP in \( Y \), namely \( f(p_{NN}) = q_{NN} \), where \( q = f(p_{NN}) \).

\textbf{Proposition 4.11}
Any NCS \( A \) in \( X \) can be written in the form \( A = \bigcup \{ p_N : p_N \in A \} \cup \bigcup \{ \phi : p_{NN} \in A \} \), where \( A = \bigcup \{ p_N : p_N \in A \} \) and \( A_N = \bigcup \{ \phi : p_{NN} \in A \} \). It is easy to show that, if \( A = \{ A_1, A_2, A_3 \} \), then \( A = \{ x, A_1, \phi, A_3 \} \) and \( A = \{ x, \phi, A_2, A_3 \} \).

\textbf{Proposition 4.12}
Let \( f : X \to Y \) be a function and \( A = \{ A_1, A_2, A_3 \} \) be a neutrosophic crisp set in \( X \). Then we have \( f(A) = f(A_N) \cup f(A) \cup f(A) \).

\textbf{Proof}
This is obvious from \( A = A_N \cup A_N \cup A_N \).

5 Neutrosophic Crisp Set Relations
Here we give the definition relation on neutrosophic crisp sets and study of its properties.

Let \( X, Y \) and \( Z \) be three crisp nonempty sets.

\textbf{Definition 5.1}
Let \( X \) and \( Y \) be two non-empty crisp sets and NCSS \( A \) and \( B \) in the form \( A = \{ A_1, A_2, A_3 \} \) on \( X \), \( B = \{ B_1, B_2, B_3 \} \) on \( Y \). Then

i) The product of two neutrosophic crisp sets \( A \) and \( B \) is a neutrosophic crisp set \( A \times B \) given by \( A \times B = \{ A_1 \times B_1, A_2 \times B_2, A_3 \times B_3 \} \) on \( X \times Y \).

ii) We will call a neutrosophic crisp relation \( R \subseteq A \times B \) on the direct product \( X \times Y \).

The collection of all neutrosophic crisp relations on \( X \times Y \) is denoted as \( \mathcal{NCR}(X \times Y) \).

\textbf{Definition 5.2}
Let \( R \) be a neutrosophic crisp relation on \( X \times Y \), then the inverse of \( R \) is denoted by \( R^{-1} \) where \( R^{-1} \subseteq B \times A \) on \( Y \times X \).

\textbf{Example 5.1}
Let \( X = \{ a, b, c, d \} \), \( A = \{ \{a, b\}, \{c\}, \{d\} \} \) and \( B = \{ \{a\}, \{c\}, \{d, b\} \} \) then the product of two neutrosophic crisp sets given by \( A \times B = \{ \{a, b\} \times \{a\}, \{c, c\} \times \{d, d, d\}, \{d\} \times \{d\} \} \) and \( B \times A = \{ \{a\} \times \{a, b\}, \{c\} \times \{c, c\}, \{d\} \times \{d, d\} \} \), and \( R_1 = \{ \{a, a\}, \{a, c\}, \{a, d\} \} \subseteq A \times B \) on \( X \times Y \).

\textbf{Example 5.2}
Let \( X = \{ a, b, c, d, e, f \} \), \( A = \{ \{a, b, c, d\}, \{e\}, \{f\} \} \), \( D = \{ \{a\}, \{c\}, \{f, d\} \} \) be a NCS-Type 2, \( B = \{ \{a, b, c\}, \{\phi\}, \{d\} \} \) be a NCS-Type 1, \( C = \{ \{a, b\}, \{c, d\}, \{e, f\} \} \) be a NCS-Type 3. Then \( A \times D = \{ \{a, a\}, \{b, a\}, \{b, b\}, \{b, c\}, \{b, c\}, \{d, a\}, \{d, b\}, \{e, c\}, \{e, c\}, \{f, f\}, \{f, d\}, \{f, f\}, \{f, d\}, \{f, d\}, \{f, f\}, \{f, d\}, \{f, d\} \} \) and \( D \times C = \{ \{a, a\}, \{b, a\}, \{b, b\}, \{b, c\}, \{b, c\}, \{d, a\}, \{d, b\}, \{e, c\}, \{e, c\}, \{f, f\}, \{f, d\}, \{f, f\}, \{f, d\}, \{f, f\}, \{f, d\}, \{f, d\}, \{f, d\}, \{f, f\}, \{f, d\}, \{f, d\} \} \) We can construct many types of relations on products.

We can define the operations of neutrosophic crisp relation.

\textbf{Definition 5.3}
Let \( R \) and \( S \) be two neutrosophic crisp relations between \( X \) and \( Y \) for every \( (x, y) \in X \times Y \) and NCSS \( A \) and \( B \) in the form \( A = \{ A_1, A_2, A_3 \} \) on \( X \), \( B = \{ B_1, B_2, B_3 \} \) on \( Y \). Then we can defined the following operations

i) \( R \subseteq S \) may be defined as two types
a) Type1: \( R \subseteq S \) \( \iff \) \( A_1 \subseteq B_1, A_2 \subseteq B_2, A_3 \supseteq B_3 \)
b) Type 2: \( R \subseteq S \) \( \iff \) \( A_1 \subseteq B_1, A_2 \subseteq B_2, A_3 \supseteq B_3 \)

\( B \subseteq A \) \( \subseteq B \)

ii) \( R \cup S \) may be defined as two types
a) Type 1: \( R \cup S \) \( \iff \) \( A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \)
b) Type2: 
\[ R \cup S = \{A_{1R} \cup B_{1S}, A_{2R} \cup B_{2S}, A_{3R} \cup B_{3S}\} \]

iii) \( R \cap S \) may be defined as two types

a) Type1: \( R \cap S = \{A_{1R} \cap B_{1S}, A_{2R} \cap B_{2S}, A_{3R} \cap B_{3S}\} \)

b) Type2: 
\[ R \cap S = \{A_{1R} \cap B_{1S}, A_{2R} \cap B_{2S}, A_{3R} \cap B_{3S}\} \]

**Theorem 5.1**

Let \( R, S \) and \( Q \) be three neutrosophic crisp relations between \( X \) and \( Y \) for every \((x, y) \in X \times Y\), then

i) \( R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1} \)

ii) \( (R \cup S)^{-1} \Rightarrow R^{-1} \cup S^{-1} \)

iii) \( (R \cap S)^{-1} \Rightarrow R^{-1} \cap S^{-1} \)

iv) \( (R^{-1})^\circ = R \)

v) \( R \cap (S \cup Q) = (R \cap S) \cup (R \cap Q) \)

vi) \( R \cup (S \cap Q) = (R \cup S) \cap (R \cup Q) \)

vii) If \( S \subseteq R, Q \subseteq R \), then \( S \cup Q \subseteq R \)

**Proof**

Clear

**Definition 5.4**

The neutrosophic crisp relation \( I \in NCR(X \times X) \), the neutrosophic crisp relation of identity may be defined as two types

i) Type1: \( I = [\{A \times A\}, \{A \times A\}, \phi \} \)

ii) Type2: \( I = [\{A \times A\}, \{A \times A\}, \phi \} \)

Now we define two composite relations of neutrosophic crisp sets.

**Definition 5.5**

Let \( R \) be a neutrosophic crisp relation in \( X \times Y \), and \( S \) be a neutrosophic crisp relation in \( Y \times Z \). Then the composition of \( R \) and \( S \), \( R \circ S \) be a neutrosophic crisp relation in \( X \times Z \) as a definition may be defined as two types

i) Type 1:
\[ R \circ S \leftrightarrow (R \circ S)(x, z) = \cup \{ \{A_{1R} \times B_{1S}\}, \{A_{2R} \times B_{2S}\} \} \]

\[ \{A_{2R} \times B_{2S}\} \cap (A_{2R} \times B_{2S}) \}

\[ \{A_{1R} \times B_{1S}\} \cap (A_{1R} \times B_{1S}) \}

\[ R \circ S \leftrightarrow \cap \{ \{A_{1R} \times B_{1S}\}, \{A_{2R} \times B_{2S}\} \} \]

\[ \{A_{2R} \times B_{2S}\} \cap (A_{2R} \times B_{2S}) \}

\[ \{A_{1R} \times B_{1S}\} \cap (A_{1R} \times B_{1S}) \}

**Example 3.3**

Let \( X = \{a, b, c, d\} \), \( A = \{a, b, \phi, \phi \} \) and \( B = \{a, \phi, \phi, \phi \} \) the product of two events given by \( A \times B = \{\{a, a\}, \{a, b\}, \{b, c\}, \{b, d\}\} \) and \( B \times A = \{\{a, a\}, \{a, b\}, \{b, c\}, \{b, d\}\} \)

\[ R_1 = \{(\{a, a\}, \{a, c\}, \{d, d\}\}, \{a \times B \} \}

\[ R_2 = \{(\{a, b\}, \{a, c\}, \{d, d\}, \{d, b\}\} \}

\[ R_3 = \cup \{(\{a, a\}, \{a, b\}), \{a, c\}, \{d, d\}\}) \}

\[ \{(\{a, a\}, \{a, b\}), \{a, c\}, \{d, d\}\}) \}

\[ I_{41} = \{(a, a), (a, b), (b, a), (b, a), \{\phi\}, \{\phi\} \}

\[ I_{42} = \{(\{a, a\}, (a, b), (b, a), \{\phi\}, \{\phi\} \}

**Theorem 5.2**

Let \( R \) be a neutrosophic crisp relation in \( X \times Y \), and \( S \) be a neutrosophic crisp relation in \( Y \times Z \) then \( (R \circ S)^{-1} = S^{-1} \circ R^{-1} \).

**Proof**

Let \( R \subseteq A \times B \) on \( X \times Y \) then \( R^{-1} \subseteq B \times A \).

\( S \subseteq B \times D \) on \( Y \times Z \) then \( S^{-1} \subseteq D \times B \), from Definition 5.4 and similarly we can \( I_{(R \circ S)^{-1}}(x, z) = I_{S^{-1}}(x, z) \) and \( I_{R^{-1}}(x, z) \)
then \( (R \circ S)^{-1} = S^{-1} \circ R^{-1} \)

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