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Sudharman Jayaweera

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

SCHOOL OF ENGINEERING
UNIVERSITY OF NEW MEXICO

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Sudharman K. Jayaweera and Ding Li
Department of Electrical and Computer Engineering
The University of New Mexico
Albuquerque, NM 87131
e-mail: {jayaweera, liding }@ece.unm.edu

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Abstract

In this technical report we outline a control framework for a utility-maintained central power plant in a future smart-grid integrated with renewable distributed generators at customer premises having independent time-dependent household load demands. In traditional electric grid planning, the uncertainty that the utility has to deal with arises mainly due to the random consumption behavior of households. However, in future smart-grids there is to be a second source of uncertainty due to the inherent intermittent nature of integrated renewable distributed generations such as solar, wind and tidal resources at customer premises that would also be integrated to the electric grid. Different demand response and demand side management schemes have been proposed to affect customer load demands, but not much work has been focused on mitigating the uncertainty due to this ever-increasing penetration of renewable generation. Difficulty comes from the fact that renewable generation can be heavily affected by weather\climate conditions. One approach to address this is to develop sophisticated prediction models of the natural environment which, however, could proven to be be even harder than general weather forecast since renewable generation can be a function of many weather condition factors, not to mention other factors related to renewable generation facilities. Thus it is important for the utility to be able to adjust its power output in real time, taking in to account all the uncertainties mentioned above. Towards this end, we first propose a finite state non-stationary Markov chain model to represent household load demands. The proposed model is tested against real measured data to justify its validity. Next, we propose a quadratic stochastic reference tracking scheme for the utility generation control. Based on the state space representation of an assumed synchronous generator, we propose control rules corresponding to different assumptions on reference signals. These assumptions corresponds to different levels of knowledge available to the utility, which are known deterministic reference signal, stochastic reference signal with known moments and stochastic reference signal with known reference system dynamics. Simulation results are presented for tracking performance analysis and comparison.

Keywords

Smart grid control, load demand modeling, stochastic reference tracking, renewable distributed generation.

1 Introduction

The Smart-grid is supposed to be intelligent, efficient, resilient and green [2], enhancing every facet of the electric system, including generation, transmission, distribution and consumption that will transform the current grid to one that functions more cooperatively, responsively and organically. As renewable generation, which is mostly based on solar, wind and tidal resources, grows at a rapid pace, renewable distributed generation (RDG) becomes a necessary and desirable component of a cleaner energy future. However, there are many technical challenges to increased penetration of RDG, such as voltage rise effects, power quality and power grid protection when they are to be integrated in to the traditional power grid (to form a smart-grid). The most challenging aspect of integrating renewable distributed generators (renewables) is dealing with their inherent intermittent generation profile. Historically, in the equation of supply and demand, operators have primarily had to deal with the demand variable. With more integrated renewable distributed generators coming online, however, operators need more efficient and effective control schemes to balance variables on both sides of the equation [13].

Different approaches have been proposed to overcome the intermittent nature of renewable generation. In [16], the proposed robust unit commitment solution methodology helps the power system operators in optimal day-ahead planning with indeterminate information about the wind generation. A particle swarm optimization based scenario generation and reduction algorithm is used in [16] for modeling the uncertainties. The stochastic unit commitment problem is solved by using a parameter-free self-adaptive particle swarm optimization algorithm. An alternative approach for dealing with the intermittent nature of renewables, pursued in, for example [6] and [11], is to develop sophisticated wind and cloud prediction models so that operators can be more accurate in predicting renewable production in advance. However, currently even a single day ahead wind forecast can be dramatically erroneous. A third approach is to build renewable resources in geographically diverse locations. Geographic diversity allows for the generation imbalance of different renewable resources to "net" against each other over the system. Other approaches include building more balancing reserves from quick ramping thermal units.

We believe that in future smart-grid technology one of the main responsibilities of the utility companies will be to maintain the grid stability and ensure grid reliability in the face of integrated RDG's which will be driven by the diverse individual self-objectives of the customers. Allowing for distributed RDG's to flexibly interact with the power-grid, as opposed to being passive energy consumers as today, will be the key to achieving expected efficiencies of the smart-grid technology. However, this interaction as customers reverse their roles from consumers to energy generators and vice versa in real-time coupled with the inherent intermittent nature of their RDG's would pose a significant challenge to the utility companies in maintaining grid stability and reliability. The purpose of this technical report is to formulate the problem of grid control to ensure its stability and reliability in the face of integrated RDG's that are driven by their own self-objectives.

An important difference between conventional and renewable generators is that the former is fully controllable although the latter is at the mercy of the nature's randomness. Thus, in this technical report we propose a stochastic control scheme for conventional central power generator with suitable modeling of RDG's and power consumption behavior of individual households. We first present and compare different models for renewable generation and power consumption modeling of individual households. Then we propose a linear quadratic stochastic reference tracking control problem to make the conventional generation output to track the reference signal while keeping the designated quadratic cost as small as possible. We discuss the optimal control law for three scenarios according to different assumptions about the reference signal: 1) deterministic reference; 2) stochastic reference with known second order moments; 3) stochastic reference as the output of a known dynamic system driven by white noise.

The rest of this report is organized as follows: In Section 2 we formulate the control problem for a smart-grid that integrates a single conventional central plant with multiple distributed customers equipped with renewable generators. Next, models for renewable distributed generation, power consumption of individual households (customers) and conventional plant are proposed in Section 3. To arrive at a tractable control problem, for the conventional plant in Section 3 we assume a synchronous generator that can be modeled by a third order system. Following a linearization approach we next obtain a linear model which is then used to formulate and solve a stochastic linear quadratic tracking control problem for the conventional generation to follow a desired reference signal in the face of integrated renewable distributed generators in Section 4. We specifically discuss three different scenarios in Section 4 with different assumptions on the form of the reference signal to be tracked, and derive the optimal control laws for each of these scenarios. In section 4, we also present simulation results of the control performance and finally conclude this report in Section 5.

2 A Smart-grid Control Model with Integrated Multiple Renewable Distributed Generators

We assume that there is a single utility-maintained, central conventional generator which support load demands from all customers in the assumed electricity market. The smart-grid is then formed by integrating this central generator with multiple renewable distributed generators (such as solar panels) at distributed customer premises to form a single networked-system as shown in Fig. 1. The conventional generator is assumed to be controllable and has a stable power output. The purpose of integrating renewable generation to the utilitys own electric grid is that each household can at least partly support their load demands by consuming the renewable power generated locally generated renewable power, thus reducing the total load that needs to be supported by the central conventional generator. Compared to conventional electricity generation, the distributed renewable generation is clean and free of long distance power transmission requirements. However, as mentioned earlier, renewable generation is highly environment-dependent (such as insolation or cloud movement), thus is intermittent and difficult to predict. In practice, it may be the case that the customers would also be equipped with storage devices, such as PHEVs, to maintain at least short-term stable power supply from the renewables in the face of uncertainty related to their electricity generation. This is an important issue that our current work is focused on and thus we do not discuss customers equipped with storage units in this technical report. Instead, we assume that the renewable generation is to be the quantity of interest.

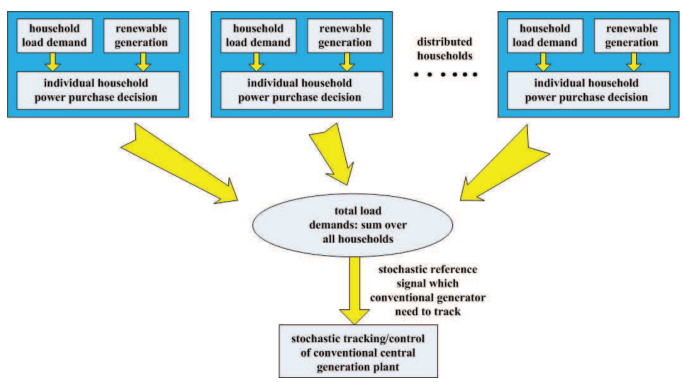

Figure 1: A smart-grid made of a single utility-maintained central conventional plant integrated with multiple renewable distributed generators at distributed customer premises.

As far as the interaction between the grid and a particular customer is concerned, the final demand of interest is then the difference between the customer's renewable generation and its consumption needs. This difference between its own renewable generation and power load demand is termed the *net power demand*: A positive net power demand means that the customer is in the *buying-mode* (buying electrical power from the grid), while a negative net power demand indicates that it is in the *selling-mode* in which the customer sells excess electrical power to the grid.

Clearly, both renewable generation and power load demand of a customer are better modeled as time-varying and possibly random, rendering the net power demand to be also time-varying and possibly random. In addition, of course the net power demands from different customers are to be also varying. Hence, the conventional generator maintained by the utility is to play an important role in keeping balance between the generation and demand in the whole electricity market by filling the gap between the total load demand and total renewable generation, which is also the sum of all net power demands.

In the following, we adopt an abstract framework to model the objective of the utility-maintained central plant concerning the grid: i.e. we will assume that its objective is to ensure that the overall power output to follow a given reference signal in the presence of integrated RDG's and their net power demands. Note that, this framework is rich enough to model many different grid stability and reliability objectives without having to worry about physical processes involved.

3 Mathematical Models for Renewable Distributed Generation, Power Consumption of Individual Households and Conventional Generators

To define a tractable control problem for the smart-grid maintenance we need to mathematically model the RDG, the household/customer load demand and the central plant dynamics. Of course, to obtain realistic predictions and performance these models have to be realistic and conform to the measured data. In the following, we provide overviews of existing models for these as well as the models adopted in our work.

3.1 Renewable Distributed Generation Modeling

The intermittence of renewable generation mainly comes from the uncertainty of environment, such as variations of wind speed, solar irradiation and cloud movement. Modeling these weather factors by themselves are very difficult and are out of the scope of this paper although there are previous work that has suggested dynamic system models for renewable generation [3, 14]. Note that, in our framework what we are mainly interested in is the renewable generation output, rather than the system model of renewable generators. This RDG output can reasonably be modeled as a random process. What type of random distribution is appropriate model the RDG output would obviously depend on what type of renewable generation we are interested in. In this report, we mainly confine ourselves to discussing wind generation. The wind speed distributions are often characterized by Weibull distributions [6, 11]. Historical hourly data for the wind farm site collected over a significant time period, however, are normally required to obtain the shaping parameters of the suitable Weibull distribution. In general, depending on these shaping parameters the distribution can have various shapes. Interestingly, the wind speed probability distributions obtained for three diverse geographic locations in Canada, were shown to be close to a normal distribution in [11]. Following this, in this report, we will assume that the RDG output is Gaussian distributed.

3.2 Load Demand Modeling

Two approaches are widely adopted in literature for load demand modeling. The first approach is component-based load modeling approach, which reconstructing the expected daily electrical loads of a household based on appliance sets, occupancy patterns, and statistical data. For example, in [19, 7, 20, 4], the authors constructed such electric load profiles from individual appliance profiles. By considering availability and proclivity functions, they predict whether someone is available (at home and awake) and their tendency to use an appliance at any given time. These functions were applied to predict individual appliance events, which were then aggregated into a load profile. The second approach is termed the measurement-based load modeling approach. In [15], the authors used this approach to create electrical profiles to examine demand side management strategies for Finland. They used a different bottom-up approach based on statistical consumption data, and not detailed occupant behavior. Electrical data from hundreds of apartments in Finland formed the basis for the statistics used to fabricate these hourly demand profiles. In [18], a methodology of measurement-based load modeling for transient stability analysis is proposed and Genetic Algorithms (GA) is used to estimate load model parameters.

Note that, in practice it can be expected that a customer load demand will be highly correlated in time. In many cases, there may also be spatial correlations among distributed load demands. However, both above approaches fail to sufficiently emphasize and capture such correlation properties of a customer load demand profile. Hence, in this work we propose a disturbed finite state Markov chain model for the household load demand. The model is tested against real data measured by the Electric Reliability Council of Texas in order to verify its applicability in practice.

According to the real data measured from the power market [1], load demand profile shows high correlation over consecutive time instants. The difficulty in modeling the load demand is due to the time dependent randomness of power consumption activities. Thus the stochastic process that needs to describe the load demand profile is not stationary. Hence, in the following we propose a non-stationary, finite states Markov chain to model a household load demand. We will assume that the load demands take values among a certain number of discrete levels (states in Markov chain). At each time step, the probability of the occurrence of certain amount of load demand is only determined by the state and transition matrix in the previous time step.

Based on the data pool of the Electric Reliability Council of Texas [1], in which both forecasted and actual power load were recorded every 15 minutes for nearly 200 different locations, we consider a Markov chain of 96 time steps corresponding to all 15 minutes intervals in a day. The entire range from minimum to maximum load demand are uniformly divided into consecutive intervals, and the mean and variance of the data samples in each interval are calculated. The model is then conducted as follows: First, we uniformly divide the range of load demand into certain number of intervals, the mean value of each interval is adopted as the state of the Markov chain. All data samples in a interval is represented by the state value of that interval. Second, we calculate the transition matrix for each step based on the statistics of transition behavior between states of consecutive intervals. In our work, we derive a Maximum-likelihood estimator (MLE) for the transition matrix at each time step from the real data.

Figure 2: Mean of real household load and load demand profile generated by a 6-state Markov chain model.

Figures 2 and 3 show the mean and standard deviations over time of actual household demand data and the Markov model generated profile, respectively. In these figures we have assumed that the household load demand can be represented by a 6-states Markov chain. Note that, the most suitable model parameters can also be determined by using real data to estimate them, though we do not discuss them in this report. Indeed, Figure 4 shows the variations of average errors in mean and standard deviation as a function of the number of states assumed in the Markov chain model. It can be seen that both errors decrease as number of states increase. In general, we may expect this to be true since larger number of states allows for smaller quantization steps in demand so that the quantization error is reduced. However, the trade-off would be the resulting computational complexity.

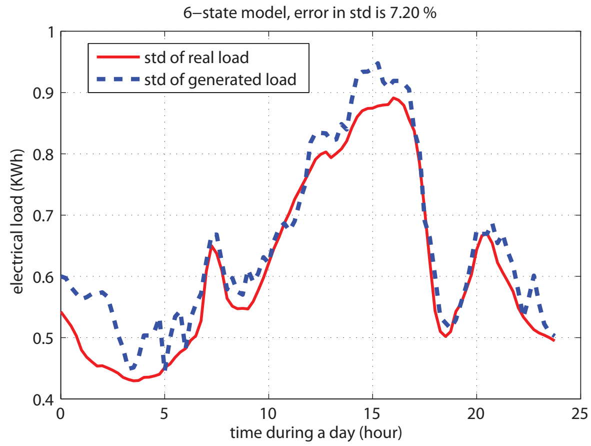

Figure 3: Standard deviation of real household load and load demand profile generated by a 6-state Markov chain model.

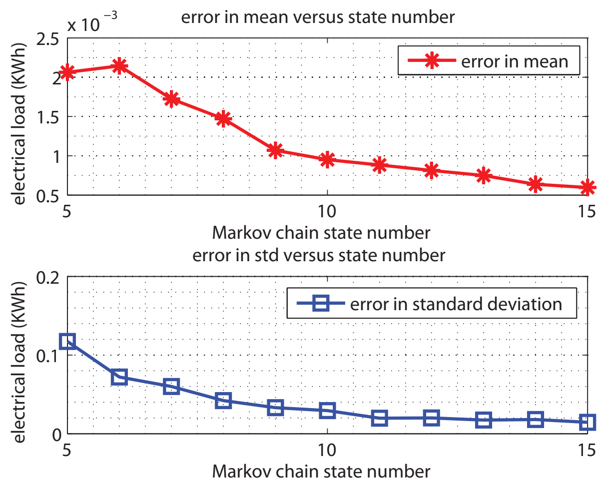

Figure 4: Dependance of average mean and standard deviation of household load demand on the number of the states assumed in the Markov chain model.

3.3 Synchronous Generator Modeling and a State-space Representation for the Conventional Power Plant

In this sub-section, we derive a state-space representation for a synchronous generator based conventional central plant. Based on the system model developed, we will design a stochastic control framework to make the central plant track the load demands from households in the next section .

There are different types of generators that are used in conventional power plant facilities including squirrel-cage induction generator, doubly-fed induction generator, self-excited induction generator and synchronous generator. Much work has been done on dynamic system modeling of these generators [8, 17, 10, 5]. In the following, we assume the widely adopted synchronous generator as the conventional power plant model and derive a state space representation of the corresponding dynamical system.

Depending on required model precision and affordable complexity, synchronous generator dynamics are modeled by first-order, third-order or fifth-order systems [21]. In this paper, we adopt the commonly used third-order nonlinear model. Let us introduce the following variables where all parameters above defined in per unit values:

z_d, z_q and $z_d^{'}$ are the augmented reactance of the line and transformer reactances are added with them, δ is the rotor angle with respect to the machine terminals, ω is the relative speed of the rotor in rad/s, $v_q^{'}$ is the transient internal voltage of armature, E_{FD} is the equivalent electromotive force (EMF) in the excitation coil, i_d, i_q are the direct and quadrature axis stator currents, J and D are the rotor inertia and the damping factor, P is the terminal active power per phase, $T_{do}^{'}$ is the direct-axis transient time constant, T_e is the output electric torque, T_m is the input mechanical torque, v_t is the generator terminal voltage, V is the infinite bus voltage, z_d is the direct axis reactance, $z_d^{'}$ is the direct axis transient reactance and z_q is the quadrature axis reactance.

With above definitions, a third order nonlinear model for a synchronous generator can be described by the following equations [9]: $\dot{\delta} = \omega$, $\dot{\omega} = \frac{1}{J}(T_m - T_e - T_D)$, and $\dot{v}_q^{'} = \frac{1}{T_{do}^{'}}\left(E_{FD} - v_q^{'} - \left(z_d - z_d^{'}\right) i_d\right)$, where $i_d = \frac{v_q^{'} - v_t \cos\delta}{z_d^{'}}$, $i_q = \frac{v_t \sin\delta}{z_q}$ and $\dot{v}_q^{'} = \frac{1}{T_{do}^{'}}\left(E_{FD} - v_q^{'} - \left(z_d - z_d^{'}\right) i_d\right)$. For a single synchronous generator, it is assumed that the field voltage, rotor angle and the electrical power can be measured. Thus, by defining the system state $\mathbf{x}_c$ and input $\mathbf{u}_c$ as

$$\mathbf{x}_c = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \delta \\ \omega \\ v_q^{'} \end{bmatrix} \quad \text{and} \quad \mathbf{u}_c = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} E_{FD} \\ T_m \end{bmatrix},$$

a nonlinear state space model of a synchronous generator is can be written as $\dot{\mathbf{x}}_c = A_c^{'}\mathbf{x}_c + B_c^{'}\mathbf{u}_c + f(\mathbf{x}_c)$, where

$$A_c^{'} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{D}{J} & 0 \\ 0 & 0 & -\frac{1}{T_{do}^{'}}\left(\frac{z_d}{z_d^{'}}\right) \end{bmatrix}, \quad B_c^{'} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{J} \\ \frac{1}{T_{do}^{'}} & 0 \end{bmatrix}$$

and

$$f(\mathbf{x}_c) = \begin{bmatrix} 0 \\ -\frac{1}{J}\left(\frac{v_t}{z_d^{'}} x_3 \sin x_1 + \frac{v_t^2}{2}\left(\frac{1}{z_q} - \frac{1}{z_d^{'}}\right)\sin(2x_1)\right) \\ \frac{1}{T_{do}^{'}}\left(\frac{z_d - z_d^{'}}{z_d^{'}}\right) v_t \cos x_1 \end{bmatrix}$$

To simplify the control problem to follow, we may linearize the above non-linear dynamics in the vicinity of an operating point 'o'. Following the procedure in [9], indeed it can be shown that the corresponding linear dynamic equation is $\dot{\mathbf{x}}_c = A_c\mathbf{x}_c + B_c\mathbf{u}_c$, with

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = A_c\mathbf{x}_c + B_c\mathbf{u}_c = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{K_1}{J} & -\frac{D}{J} & -\frac{K_2}{J} \\ -\frac{K_4}{T_{do}^{'}} & 0 & -\frac{1}{K_3 T_{do}^{'}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{J} \\ \frac{1}{T_{do}^{'}} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{1}$$

where $K_1 = \frac{v_t}{z_d^{'}} x_{3o} \cos x_{1o} + \frac{v_t^2}{2}\left(\frac{1}{z_q} - \frac{1}{z_d^{'}}\right)\sin(2x_{1o})$, $K_2 = \frac{v_t}{z_d^{'}} \sin x_{1o}$, $K_3 = \frac{1}{1 + \frac{z_d - z_d^{'}}{z_d^{'}}} = \frac{z_d^{'}}{z_d}$ and $K_4 = \left(z_d - z_d^{'}\right)\frac{v_t \sin x_{1o}}{z_d^{'}}$.

Since the active power generated by the synchronous generator is the most important quantity for our control framework, we take it to be the output of the system. Thus, the system output equation is

$$y_c = \begin{bmatrix} K_1 & 0 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

where $K_1 = \frac{v_t}{z_d} x_{3o} \cos x_{1o} + \frac{v_t^2}{2}\left(\frac{1}{z_q} - \frac{1}{z_d}\right)\sin(2x_{1o})$ and $K_2 = \frac{v_t}{z_d}\sin x_{1o}$ as above.

Next, we may discretize the above original continuous system (1) to obtain the following discrete-time system representation for the synchronous generator central plant:

$$\begin{aligned} \mathbf{x}[k+1] &= A\mathbf{x}[i] + B\mathbf{u}[i] \\ y[i] &= C\mathbf{x}[i] \end{aligned} \tag{2}$$

where $\mathbf{x}[i] = \mathbf{x}_c(kh)$, $\mathbf{u}[i] = \mathbf{u}_c(kh)$, $y[i] = y_c(kh)$, $A = e^{A_c h}$, $B = \int_0^h e^{A_c \tau} B_c d\tau$ and $C = C_c$.

It is worth mentioning that for a central power plant which consists of multiple synchronous generators, the same dynamic equations as above still hold for the augmented system, if we were to assume that these generators are decoupled.

4 Stochastic Tracking Control of a Conventional Generator in an RDG-integrated Smart-grid

As mentioned in the previous section, the goal of the control design for the central generator is for its active power to fill up the difference between the total load demand and total renewable generation from all households. This difference acts as a reference which the conventional output need to track. Denote by $y_r[i]$ the total power that households buy from the grid which then is the amount of power the conventional generator need to provide. The tracking control problem diagram is shown in Fig. 5.

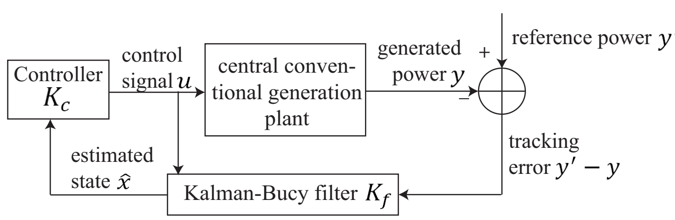

Figure 5: Tracking control of a utility-maintained central plant in an RDG-integrated smart-grid.

From the point of view of controller design for the central plant, then, the relevant quantity is the reference signal y_r. From its definition, the assumed models for the load demand and the RDG's directly determine the assumptions to be made on y_r. In the following, to derive optimal control policies we start with a simple scenario with relatively strong assumptions on this reference signal y_r, and then proceed to more general and complex situations. Note that stronger assumptions on the reference signal means that more information about household load demands and renewable generators is required to implement the control policy for the central plant.

4.1 Optimal Tracking of a Deterministically Known Net Load

In the first scenario, we assume that the reference signal is deterministic and known by the central plant: i.e. the difference between the total load demand and total renewable generation from all households is a deterministic quantity known to the central plant. Though this is not true in practice as it is, this model is still of importance because it can be applied to a situation in which the central plant uses deterministic estimations of renewable generation and household load demands for planning its electricity generation. Moreover, the solution to this scenario is the basis of control policies in more complicated scenarios.

For the time invariant linear system (2), we may consider a time variant quadratic objective function U to measure the tracking control performance over control interval $[i_0, i_1]$, as below:

$$
\begin{aligned}
U &= \textstyle\sum_{i=i_0}^{i_1} \frac{1}{i_1-i_0}\left[\left(y[i]-y_r[i]\right)^T R_1[i]\left(y[i]-y_r[i]\right)+u^T[i]R_2[i]u[i]\right] && (3)\\
&= \textstyle\sum_{i=i_0}^{i_1} \frac{1}{i_1-i_0}\left[\left(Cx[i]-y_r[i]\right)^T R_1[i]\left(Cx[i]-y_r[i]\right)+u^T[i]R_2[i]u[i]\right] && (4)
\end{aligned}
$$

where $R_1[i]$ is a positive semi-definite matrix and $R_2[i]$ is a positive definite matrix. The term $(y-y_r)^T R_1 (y-y_r)$ in the above cost function is a measure of the control accuracy, and the term $\mathbf{u}^T R_2 \mathbf{u}$ is a measure of the control effort. The initial condition is given by $\mathbf{x}[i_0]$ and the final condition $\mathbf{x}[i_1]$ is free with i_1 fixed. In other words, we require that the squared error, where error $e[i]$ is defined as $e[i] = y[i] - y_r[i]$, to be as small as possible with minimum control effort.

From standard linear quadratic control theory [12], the optimal control law for the above problem is then given by $\mathbf{u}[i] = -F[i]\mathbf{x}[i]$, $i = i_0, i_0+1, \ldots, i_1-1$, where $F[i] = \left\{R_2[i] + B^T\left[C^T R_1[i+1]C + P[i+1]\right]B\right\}^{-1} B^T \ldots$ $\left[C^T R_1[i+1]C + P[i+1]\right]A$. Here the inverse always exists and the sequence of matrices $P[i]$, $i = i_0, i_0+1, \ldots, i_1$ satisfies the matrix difference equation $P[i] = A^T\left[C^T R_1[i+1]C + P[i+1]\right][A - BF[i]]$, $i = i_0, i_0+1, \ldots, i_1-1$, with the terminal condition $P[i_1] = C^T R_1[i_1]C$. However, since in practice the state is usually not directly available for feedback control, we need to reconstruct the state based on the output observation. Thus, the optimal feedback control law is given by $\mathbf{u}[i] = -F[i]\hat{\mathbf{x}}[i]$, where the observer is of the form $\hat{\mathbf{x}}[i+1] = A\hat{\mathbf{x}}[i] + B\hat{\mathbf{u}}[i] + K[i]\left(y[i] - C\hat{\mathbf{x}}[i]\right)$. It should be noted that since there are no system disturbance and observation noise, a certain arbitrariness remains in the choice of the gain matrix $K[i]$.

4.2 Optimal Tracking of a Stochastic Net Load Reference Signal with Known Moments

In general, both load demands and renewable generation are stochastic in nature due to random behavior of household energy consumption and the intermittence of renewable distributed generation. However, as mentioned in section 3, these random quantities can be well modeled by different distributions (e.g. Weibull distribution for wind velocity and Markov chain model for household load demand modeling). As a result, the moments of the reference signal that need to be tracked by the central plant can be calculated. Thus, in this section, we assume that the reference signal is a stochastic signal with its first and second order moments known to the central plant. In practice these can be estimated and adapted as necessary.

Denote by $\bar{y_r}$ and $\mathbf{M}_y$ the mean process and second order moment matrix (slightly different from covariance matrix) of y_r. In the case of a stochastic reference tracking control, the objective function needs to be modified so that the performance index is optimized in the sense of expectation over all realizations:

$$
\begin{aligned}
U &= \frac{1}{i_1-i_0} E\left\{\sum\left[\left(y-y_r\right)^T R_1\left(y-y_r\right)+\mathbf{u}^T R_2 \mathbf{u}\right]\right\} && (5)\\
&= \frac{1}{i_1-i_0}\sum\left[\underbrace{\operatorname{tr}\left(R_1\mathbf{M}_y\right)-\left(\bar{y_r}\right)^T R_1 \bar{y_r}}_{\substack{\text{deterministic terms}\\ \text{independent of control input}}}+\underbrace{\left(y-\bar{y_r}\right)^T R_1\left(y-\bar{y_r}\right)}_{\text{control accuracy measure}}+\underbrace{\mathbf{u}^T R_2 \mathbf{u}}_{\text{control effort measure}}\right] && (6)
\end{aligned}
$$

It can be observed then that the optimal control law for this mean-square optimization problem is the same as that of a standard deterministic tracking problem as in the previous section in which the known deterministic reference signal is the mean process of the stochastic reference signal. The corresponding deterministic tracking objective can be written as $U' = \frac{1}{i_1-i_0}\sum\left[\left(y-\bar{y_r}\right)^T R_1\left(y-\bar{y_r}\right)+\mathbf{u}^T R_2 \mathbf{u}\right]$, and we indeed have that $\arg\min_{\mathbf{u}} U = \arg\min_{\mathbf{u}} U'$.

4.3 Optimal Tracking of a Stochastic Net Load Reference Signal with Known Reference System Dynamics

Since one may generate a stochastic process by a linear differential equation driven by white noise so that its power spectral density approximates arbitrarily closely the power spectral density of the original stochastic process, more generally we can model the random reference signal process assumed above as the output of a dynamic system driven by white noise [12]. Moreover, in the following we may further take into account effects of system and observation noise in our control design, by considering the following system model with disturbance:

$$\begin{aligned}\mathbf{x}[i+1] &= A\mathbf{x}[i]+B\mathbf{u}[i]+w_1[i],\\ z[i] &= C[i]\mathbf{x}[i],\\ y[i] &= z[i]+w_2[i],\end{aligned}$$

where $z[i]$ is the system output and $y[i]$ is the noisy output measurement. The process noise w_1 and measurement noise w_2 are assumed to be zero mean white noise processes with auto-covariance matrices $\mathbf{C}_{w_1}(i,i+\tau)\delta(\tau)$ and $\mathbf{C}_{w_2}(i,i+\tau)\delta(\tau)$, respectively. The cross covariance matrix of w_1 and w_2 is $\mathbf{C}_{w_{12}}(i,i+\tau)\delta(\tau)$. Let the initial state $\mathbf{x}[0]$ be a random vector with mean $\bar{\mathbf{x}}_0$ and variance matrix $\mathbf{Q}_0$. The control interval is $i\in[i_0,i_1]$. Next, we assume that the reference signal is generated by the following dynamical system:

$$\begin{aligned}\mathbf{x}_r[i+1] &= A_r\mathbf{x}_r[i]+w_{r1}[i]\\ z_r[i] &= C_r[i]\mathbf{x}_{r1}[i]\\ y_r[i] &= z_r[i]+w_{r2}[i]\end{aligned}$$

where z_r and y_r are defined similar to the original system above, $\mathbf{x}_r[0]$ is a random vector with mean $\bar{\mathbf{x}}_{r0}$ and covariance matrix $\bar{\mathbf{Q}}_{r0}$, and w_{r1} and w_{r2} are white noises with covariance matrices $\mathbf{C}_{r1}[i]$, $\mathbf{C}_{r2}[i]$ and $\mathbf{C}_{r12}[i]$. With this reference dynamical system model, the original problem can then be formulated as an optimal linear output tracking problem with incomplete and noisy observation, in which the objective is to minimize $U=\sum_{i=i_0}^{i_1}\frac{1}{i_1-i_0}\left[(z[i]-z_r[i])^T R_1[i]\,(z[i]-z_r[i])+\mathbf{u}^T[i]R_2[i]\mathbf{u}[i]\right]$.

By considering an augmented system with the augmented state $\tilde{\mathbf{x}}[i]=[\mathbf{x}[i],\mathbf{x}_r[i]]^T$, where

$$\underbrace{\begin{bmatrix}\mathbf{x}[i+1]\\ \mathbf{x}_r[i+1]\end{bmatrix}}_{\tilde{\mathbf{x}}_r[i+1]} = \underbrace{\begin{bmatrix}A & \mathbf{0}\\ \mathbf{0} & A_r\end{bmatrix}}_{\tilde{A}}\underbrace{\begin{bmatrix}\mathbf{x}[i]\\ \mathbf{x}_r[i]\end{bmatrix}}_{\tilde{\mathbf{x}}[i]}+\underbrace{\begin{bmatrix}B\\ \mathbf{0}\end{bmatrix}}_{\tilde{B}}\mathbf{u}[i]+\underbrace{\begin{bmatrix}\mathbf{w}_1[i]\\ \mathbf{w}_{r1}[i]\end{bmatrix}}_{\tilde{\mathbf{w}}_1[i]} \tag{7}$$

$$\tilde{z}[i] = z[i]-z_r[i]=\underbrace{[C,C_r]}_{D}\begin{bmatrix}\mathbf{x}[i]\\ \mathbf{x}_r[i]\end{bmatrix} \tag{8}$$

$$\underbrace{\begin{bmatrix}z[i]\\ z_r[i]\end{bmatrix}}_{\tilde{y}_r[i]} = \underbrace{\begin{bmatrix}C & \mathbf{0}\\ \mathbf{0} & C_r\end{bmatrix}}_{\tilde{C}}\underbrace{\begin{bmatrix}\mathbf{x}[i]\\ \mathbf{x}_r[i]\end{bmatrix}}_{\tilde{\mathbf{x}}[i]}+\underbrace{\begin{bmatrix}\mathbf{w}_2[i]\\ \mathbf{w}_{r2}[i]\end{bmatrix}}_{\tilde{\mathbf{w}}_2[i]} \tag{9}$$

the control objective can equivalently be written as $\tilde{U}=E\left\{\tilde{z}^T R_1\tilde{z}+\mathbf{u}^T R_2\mathbf{u}\right\}$, where the system state is incomplete, noisy and need to be reconstructed based on the measurements. Note that, we have converted the original tracking problem to an output regulator problem with the augmented system

$$\begin{aligned}\tilde{\mathbf{x}}[i+1] &= \tilde{A}\tilde{\mathbf{x}}[i]+\tilde{B}\mathbf{u}[i]+\tilde{\mathbf{w}}_1[i]\\ \tilde{z}[i] &= D\tilde{\mathbf{x}}[i]\\ \tilde{y}[i] &= \tilde{C}\tilde{\mathbf{x}}[i]+\tilde{\mathbf{w}}_2[i],\end{aligned}$$

in which $\tilde{z}[i]$ is the controlled variable and $\tilde{y}[i]$ is the noisy observation. The initial state and covariance matrix are given by $\tilde{\mathbf{x}}[i_0] = \tilde{\mathbf{x}}_0 = [\mathbf{x}_0, \mathbf{x}_{r0}]^T$ and $\tilde{\mathbf{Q}}_0$. We assume that the initial state is uncorrelated with both process noise and measurement noise. Thus, $[\tilde{\mathbf{w}}_1[i], \tilde{\mathbf{w}}_2[i]]$ is a joint white noise vector process, and

$$\tilde{\mathbf{Q}}_0 = E\left\{(\tilde{\mathbf{x}}_0 - \bar{\tilde{\mathbf{x}}}_0)(\tilde{\mathbf{x}}_0 - \bar{\tilde{\mathbf{x}}}_0)^T\right\} = \begin{bmatrix} \mathbf{Q}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{r0} \end{bmatrix} \tag{10}$$

$$\tilde{\mathbf{C}}_1[i] = \begin{bmatrix} \mathbf{C}_{w_1}[i] & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{r1}[i], \end{bmatrix} \quad \tilde{\mathbf{C}}_2[i] = \begin{bmatrix} \mathbf{C}_{w_2}[i] & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{r2}[i] \end{bmatrix}, \quad \tilde{\mathbf{C}}_{12}[i] = \begin{bmatrix} \mathbf{C}_{w_{12}}[i] & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{r12}[i] \end{bmatrix} \tag{11}$$

Since the system state is not directly available for feedback control, we need to reconstruct the state based on the noisy measurement. To solve this problem, we need to find a functional $\mathbf{u}[i] = f(\tilde{y}[i_0], \tilde{y}[i_0+1], \ldots, \tilde{y}[i_1-1], i)$ such that the performance index $\tilde{U} = E\left\{\tilde{z}^T R_1 \tilde{z} + \mathbf{u}^T R_2 \mathbf{u}\right\}$ is minimized. According to the separation principal [12], the solution of the stochastic linear optimal output feedback control problem is given by $\mathbf{u}[i] = -F[i]\hat{\tilde{x}}[i]$, $i = i_0, i_0+1, \ldots, i_1-1$, where feedback gain is given by $F[i] = \left\{R_2 + \tilde{B}^T \left[D^T R_1[i+1] D^T + P[i+1]\right] \tilde{B}\right\}^{-1} \tilde{B}^T \times \left[D^T R_1[i+1] D^T + P[i+1]\right] \tilde{A}$. The sequence of matrices $P[i]$ satisfies the matrix difference equation $P[i] = \tilde{A}^T \times \left[D^T R_1[i+1] D^T + P[i+1]\right] \left[\tilde{A} - \tilde{B} F[i]\right]$, with terminal condition $P[i_1] = R_1$. Furthermore, $\hat{\tilde{x}}[i]$ is the minimum mean squared linear estimator of $\tilde{x}[i]$ given $\tilde{y}[i]$, $i = i_0, i_0+1, \ldots, i_1-1$. By assuming the nonsingular case ($\mathbf{C}_{w_2}[i] > 0$), $\hat{\tilde{x}}[i]$ can be obtained as the output of the optimal observer [12]: $\hat{\tilde{\mathbf{x}}}[i+1] = \tilde{A}\hat{\tilde{\mathbf{x}}}[i] + \tilde{B}\mathbf{u}[i] + K[i]\left[\tilde{y}[i] - \tilde{C}\hat{\tilde{\mathbf{x}}}[i]\right]$.

Denote by $e[i]$ the reconstruction error $e[i] = \tilde{\mathbf{x}} - \hat{\tilde{\mathbf{x}}}$, so that the optimal observer minimize the mean squared reconstruction error $E\{e^T[i] W[i] e[i]\} = \mathrm{tr}[Q[i] W[i]]$ for any predefined positive definite matrices $W[i]$. The optimal gain matrices K_i can be obtained from the following recurrence relations:

$$K[i] = \left[\tilde{A} Q[i] \tilde{C}^T + \tilde{V}_{12}[i]\right] \left[\tilde{V}_2[i] + \tilde{C} Q[i] \tilde{C}^T\right]^{-1} \tag{12}$$

$$Q[i+1] = \left[\tilde{A} - K[i]\tilde{C}\right] Q[i] \tilde{A}^T + \tilde{V}_1[i] - K[i] \tilde{V}_{12}^T[i] \tag{13}$$

where $\mathbf{Q} = E\left\{(\tilde{\mathbf{x}} - \hat{\tilde{\mathbf{x}}})(\tilde{\mathbf{x}} - \hat{\tilde{\mathbf{x}}})^T\right\}$ is the covariance matrix of the reconstruction error with the initial value $\mathbf{Q}[i_0] = \tilde{\mathbf{Q}}_0$. The initial condition for the observer state is $\hat{\tilde{\mathbf{x}}} = \bar{\tilde{\mathbf{x}}}_0$. Based on the optimal analytical controller designed above, the minimal value of the objective function $\tilde{U}$ can be shown to be

$$\bar{\tilde{\mathbf{x}}}_0^T P[i] \bar{\tilde{\mathbf{x}}}_0 + \sum_{i=i_0}^{i_1-1} \mathrm{tr}\left\{D^T R_1[i+1] D Q[i+1] + \left[P[i+1] + D^T R_1[i+1] D\right] K[i] \left[\tilde{C} Q[i] \tilde{C}^T + \tilde{\mathbf{C}}_2[i]\right] K^T[i]\right\} + \mathrm{tr}[R_1 Q[i_1]] \tag{14}$$

As an example scenario, we may consider a synchronous generator having the following continues time system parameters:

$$A_c = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & -1 \\ -1 & 0 & -1 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad C_c = \begin{bmatrix} 1 & 0 & 1.5 \end{bmatrix} \tag{15}$$

The assumed parameters for the quadratic cost function are as follows: $K = 10$, $Q = 1$, $F = 1$, R is identity matrix. The deterministic and known output reference signal is taken to be $y_r[i] = \sin 0.2\pi i$. The initial state is set as $\mathbf{x} = [\frac{\pi}{2}, \frac{\pi}{3}, 1]^T$. In this simulation, the model for the reference wind power plant is assumed to be the same as the above conventional central plant. In practice, for real synchronous generators, these matrix parameters can be calculated accordingly.

Figure 6 shows the simulated tracking performance. As expected, it can be observed that the tracking error converges to zero very fast. It should be noted that the optimal controller designed above considers wind power as the only source of renewable generation. When we incorporate other types of renewable generators, the system model could be different since not all renewable generations can be modeled as dynamical systems (such as photovoltaics).

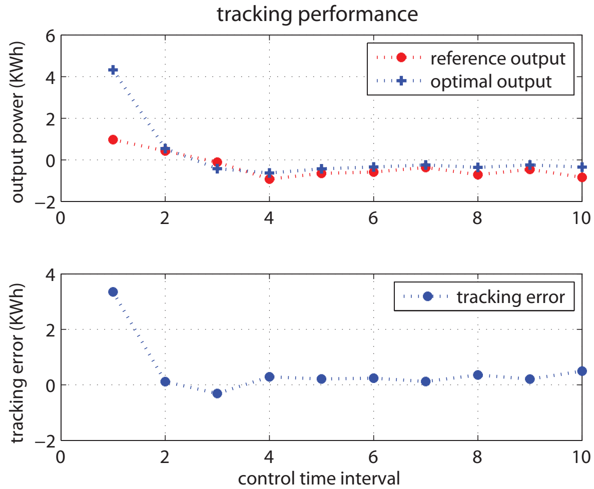

Figure 6: Tracking performance of a utility-maintained central plant in an RDG-integrated smart-grid based on reference dynamics: The reference signal is one realization of the stochastic output of reference dynamical system driven by white noise. a) system output and the reference signal. b) tracking error $e[i]$

5 Conclusion

In this report we developed a model and a stochastic control framework for a utility-maintained central power plant in an RDG-integrated future smart-grid. We first presented different models for renewable generation and household load demand. A finite state non-stationary Markov chain model was constructed for load demand modeling of individual customer premisses based on measured data. A model parameter estimation method has been incorporated so that the model can be tuned based on the available real data. Then we formulated and solved a linear quadratic stochastic reference signal tracking control problem to make the conventional generation output to track a reference signal determined by the difference between the total load demands and the total RDG while keeping a defined quadratic cost as small as possible. We discussed the optimal control law for three scenarios according to different assumptions on the reference signal to be tracked by the central plant: 1) deterministically known reference net load, 2) stochastic net load reference with known mean process and second order moments, and 3) stochastic net load reference modeled as the output of a known dynamical system driven by white noise. Since the system states are usually not directly available for feedback control, we also presented the full state linear optimal estimator to reconstruct the state from noise corrupted measurement. We showed that for the output feedback control problem, the separation principle guarantees that the optimal observer and controller can be designed separately, still preserving the optimality of the original problem. To the best of our knowledge this provides the first attempt at looking at the control problem faced by a utility in maintaining the grid performance in the face of customer-owned renewable distributed generators directly integrated into the grid and independent customer household load demands.

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