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Risk Classes And The Required Rate Of Return On The Common Stock Of A Firm.

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Master of Business Administration

RISK CLASSES AND THE REQUIRED RATE OF RETURN ON THE COMMON STOCK OF A FIRM

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THESIS

Submitted in Partial Fulfillment of the
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ABSTRACT

This paper proposes a method of examining the assumption that there are homogeneous risk classes of firms which will have the same required rate of return on the common stock, the risk classes being defined as those firms whose expected average income is subject to the same degree of uncertainty. This assumption, proposed by Franco Modigliani and Merton H. Miller in their famous work on the cost of capital to a firm, has tended to be neglected in the empirical studies, and in particular the tests of Alexander Barges, which have been made during the recent years. The method proposed will separate firms belonging to a given industry into two risk classes on the basis of the uncertainty of the expected earnings. The hypothesis that there will be a significant difference in the required rate of return is submitted to statistical tests and the results of the tests are then examined.

Further tests are made on the relation leverage has on the required rate of return by common stockholders and some of the assumptions underlying Modigliani-Miller's proposition of the required rate of return and leverage.

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I. INTRODUCTION

A. Topic

The topic of this paper is the assignment of firms to risk classes by the degree of uncertainty of the average expected earnings as proposed by Modigliani-Miller in their paper on the cost of capital.[1]

B. Purpose of the Study

It is the purpose of this study to examine an industry for the existence of risk classes within the industry using the coefficient of variation of past earnings as a measure of uncertainty of the future earnings. The relationship between the rate of return to the common stockholder and leverage will then be examined to determine if either the Modigliani-Miller or Traditional proposition would be appropriate for this industry.

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[1] Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," The American Economic Review, Vol. 48, June, 1958, p. 261

II. PROBLEM ANALYSIS

A. Introduction to the Problem

The cost of capital has long been a subject of interest to managerial economists and financial managers. This topic, which has appeared in textbooks of finance, managerial economics, marketing, and accounting, is pointed out as being a criterion for evaluating all decisions for allocations of funds. The cost of capital is the prime factor in evaluating the economic feasibility of making such decisions as investment in long term assets, replacement of equipment, retiring debentures, investing in inventory stock, and the obtaining of external capital.[1,2,3,4] There are many subjective factors which will have a bearing on the final decision, but as far as putting a label of profitability on the decision, the cost of capital is the

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[1] John A. Howard, Marketing Theory, Allyn and Bacon, Inc., Boston, 1965.

[2] J. Robert Lindsay and Arnold W. Sametz, Financial Management, An Analytical Approach, Richard D. Irwin, Inc., Homewood, Ill., Rev. Ed., 1967, pp. 193.

[3] Earnest W. Walker, Essentials of Financial Management, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965.

[4] James A. Fremgen, Managerial Cost Analysis, Richard D. Irwin, Inc., Homewood, Ill., 1966.

standard by which the investment must be evaluated.

In many of the texts used in business administration there was little theory given behind the derivation of the cost of capital value. The specific costs in obtaining debt or equity funds did not determine the cost of capital for the investment, because of the risk involved when debt funds were added to the capital structure of the firm. The cost of equity funds was noted as being influenced to some extent by the debt to equity ratio of the firm and thus the average cost of capital would reflect the dependence of both debt and equity costs.

In most finance texts prior to the appearance of Modigliani-Miller's article, there was simply a method of calculating the average weighted cost of capital. A hypothetical illustration of trading on the equity along with a graph depicting how the cost of capital declined by introducing debt into the capital structure of the firm was usually a part of the discussion on the cost of capital. This relationship, which will be referred to as the Traditional hypothesis, states: As moderate amounts of debt funds are added to the firms capital structure, the cost of capital will decrease to a minimum point, then increase thereafter. This minimum point of the cost of capital is

referred to as the optimum cost of capital and occurs at the corresponding optimal financial structure of the firm. The reasoning behind this proposition can be outlined in the following manner. When debt is first introduced into the capital structure of the firm, stockholders and future investors will not view the moderate amounts of debt as increasing the risk on the return to the common stock. This is true as their holdings are not diluted, with the capital obtained being employed at a greater rate of return than the interest rate, resulting in a greater return to the stockholder. Only after the amount of debt in the capital structure has increased to where the fixed interest charges will make the return to the stockholder subject to greater risk, will the investor require a greater rate of return on the equity funds. The lending institutions, viewing the increased likelihood that the firm will be unable to meet the interest payment or debt repayment, will increase the interest rate on the debt funds available to the firm.

The foregoing proposition had gone virtually unchallenged until the appearance of the article by Modigliani and Miller. Their article was a direct challenge to the traditional proposition, and could offer an analytical proof of their propositions subject to a number of simplifying assumptions.

There are three separate propositions in their article. The third proposition, derived from the first two propositions, pertains to the minimum acceptable rate of return on investments made by the firm. Because of the dependence of this proposition on the first two propositions, only the first two will be noted. These first two propositions are as follows:

> Proposition I. The average cost of capital to any firm is completely independent of its capital structure and is equal to the capitalization rate [p_k] of a pure equity stream of its [risk] class.
>
> Proposition II. The expected yield of a share of stock is equal to the appropriate capitalization rate [p_k] for a pure equity stream in the [risk] class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between p_k and r [the interest rate].[5]

The assumptions which underly the Modigliani-Miller propositions are crucial in the development of their analytical relations, as is true for any theory. Since the assumptions are of great importance when a hypothesis is to be emperically tested, the assumptions of Modigliani-Miller are outlined below in somewhat the same form as presented by Robichek and Meyers.[6]

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[5] Modigliani and Miller, loc. cit., pp. 268 and 271.

[6] Alexander A. Robichek and Stewart C. Myers, Optimal Financing Decisions, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963, p. 23.

1. There are homogeneous risk classes to which firms can be assigned in which the average expected earnings for every firm is regarded by investors as being subject to the same degree of uncertainty.
2. The average expected earnings is a constant and is the expected value of the random variable of the expected earnings in all future years.
3. All present and prospective investors have projected the same average expected earnings for any given firm.
4. There are perfect security markets in which stocks are traded.
5. There are no taxes on corporate income. The effect of taxes was held to be too small to noticeably effect their propositions. (This assumption was later revised in a following article which will not be considered in this paper).[7]

As long as Modigliani-Miller operate within the framework of their assumptions, there is little question that the conclusions they reach follow from the premises and can

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[7] Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment: A Correction," The American Economic Review, Vol. 53, October, 1963.

be accepted for what their model actually is - An abstraction of reality. The real question is of the value of their model when the vicissitude of the environment in which the financial manager must operate is considered. The idealization of their assumptions has brought many valid criticisms when an attempt is made to use their model as an operational tool for management.[8,9] Perhaps the greatest value of their work lies in its awakening the students of finance of the need for an accepted theory of the cost of capital that will be able to withstand the rigors of empirical tests and become a useful tool for the financial manager. Modigliani-Miller have noted that the propositions have been thoroughly analyzed and debated, the next step is the verification or rejection of the propositions and assumptions through empirical tests.[10] It is through this means that further relations and deductions can be grounded which will increase the knowledge of the theory of the cost of capital.

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[8]Robert Carson, "A Note on the Cost of Capital," Western Economic Journal, June, 1967.

[9]David Durand, "The Cost of Capital, Corporation Finance, and the Theory of Investment: Comment," The American Economic Review, Vol. 49, June, 1959.

[10]Ibid.

B. The Problem

The problem leading to the development of this paper was found when reviewing two of the empirical tests of the Modigliani-Miller propositions. Notably Modigliani-Miller's own empirical study contained in their article and the empirical study in Alexander Barges' book.[11,12] Barges' book has been heralded in the financial literature as the most complete work on the Modigliani-Miller propositions. In neither of these works do the investigators believe it is necessary to examine the basic assumption of a homogeneous risk class in any empirical manner. While they assume that by the use of a single industry their sample will be of a homogeneous risk class, they do not attempt to substantiate this assumption. In both Modigliani-Miller's and Barges' works it was noted that the consequences of including firms not of the same risk class could render the empirical tests invalid or at least there would be doubt about the conclusions that might be drawn. Modigliani-Miller's assumption

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[11] Modigliani and Miller, "The Cost of Capital, Corporation, Finance, and the Theory of Investment," pp. 275-282.

[12] Alexander Barges, The Effect of Capital Structure on the Cost of Capital, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963.

of their samples as homogeneous risk classes can be understood since they say their empirical tests are only preliminary evidence and not conclusive tests. Barges notes a number of factors that might exist within an industry which could cause differences in the yield required by the stockholders. The major portion of one chapter in Barges' book is devoted to discussing the concept of a risk class, yet he does not test to see if there may be separate risk classes within the industries used as his samples. He shows that the coefficient of variation of earnings could provide an adequate measure of the degree of certainty of the expected earnings, but does not propose a criterion for separation of the risk classes, nor the number of risk classes into which the industry should be divided.

It is thought, by this writer at least, than an investigator should make an examination to see if he has satisfied the underlying assumptions before being able to say that the tests he makes are actually complete tests of the hypotheses he is studying. A sample consisting of firms of a given industry does not necessarily imply the firms will have the same degree of uncertainty attached to their expected earnings. Investor advisory publications, in which the firms of an industry are categorized as different grades

or classes, provides an example that the investor does not necessarily perceive all firms of an industry to be a single risk class. Therefore, it is proposed that an investigator should first make a test on the sample to insure that the industry is a homogeneous risk class before testing the Modigliani-Miller propositions.

C. The Objectives

The first objective of this study is to examine the procedure for determination of the degree of uncertainty of the average expected earnings as proposed by Barges, then to test if a sample of firms within a given industry could be classified into two risk classes having different rates of return. After the appropriate statistical test is made, the conclusion may then be drawn whether the use of this industry would be the same as taking a homogeneous risk class as proposed by Modigliani-Miller.

The next objective is to test the Modigliani-Miller and Traditional propositions of the relationship between the rate of return on the common stock and leverage, for each risk class.

D. Methodology

The method used for determining the uncertainty of the average expected earnings for a firm will be to calculate

the coefficient of variation about a least squares trend line for past earnings. This method would be suitable for an industry if the firms can be considered subject to the same economic conditions during the period for which data was gathered. The differences in the stability of earnings would reflect operational dissimilarities among firms and could be measured by the coefficient of variation of the earnings.

The industry will be separated into two risk classes, using the average coefficient of variation of earnings for all firms as the dividing point. The use of two risk classes for this study follows the classification of firms by investor guides for the sample selected.

To test if the risk classes have significantly different rates of return to the stockholder, a one-way analysis of variance test will be used. From the result of this test, it may be concluded that the industry is a homogeneous risk class or the industry contains two homogeneous risk classes.

The effect of leverage on the rate of return to the common stockholder will be examined through a regression analysis for each of the risk classes found in the above step. The opposing viewpoints of the Modigliani-Miller and Traditional propositions of the relation of rate of return

and leverage would then be tested for the risk classes.

E. <u>The Hypotheses</u>

The first hypothesis which will be tested in this study may be stated as follows: When firms within an industry are classified into two risk classes based upon the degree of uncertainty of the average expected earnings in the manner proposed by Barges, there will be a significant difference in the required rate of return on the common stock for firms belonging to the different risk classes. It is also hypothesized that this relationship will be such that the firms belonging to the risk class having the greater degree of uncertainty of the average expected earnings will have the higher rate of return.

For each risk class found in the above step, the relationship of the required rate of return and leverage will be tested to find if the stockholders require an increased rate of return for firms with higher leverage. This could constitute a test of the Traditional proposition, which claims the relationship is zero. Should the results of this test indicate there is a positive relationship - the slope of the regression line is significantly different than zero - the Modigliani-Miller proposition will then be tested. The latter proposition holds that the rate of return will

increase with increases in leverage at a rate equal to the spread between the rate of return on a pure equity firm and the interest rate. For this test the hypothesis will be that the slope of the regression line does not differ significantly from the difference between the intercept on the rate of return axis and the interest rate.

III. THEORETICAL FRAMEWORK

A. Definition of a Homogeneous Risk Class

The environment in which firms operate is such that the performance of the firms cannot be predicted with certainty. Thus the valuation of firms in the marketplace by investors places a premium on the return which will accrue to them over receiving a sure income stream. The "riskiness," or degree of uncertainty in which the business enterprise operates varies from industry to industry. For those industries which have exhibited a stableness of earnings, it is expected that the degree of uncertainty will tend to be less among firms of those industries, as compared to an industry which has shown to have very large fluctuations in their firms earnings. Within an industry, though, there may exist heterogeneous factors that result in wider fluctuations of earnings for a firm than common among the other firms of that industry. A few such factors could be unusually high labor costs, a continuing battle with a local labor union, inefficient management or production methods, unwise investment or product promotion decisions, a declining market, or perhaps the failure to operate competitively in the marketplace. For any number of reasons, there can be firms within an otherwise stable industry which can be

considered as having a greater degree of uncertainty than other firms belonging to the industry.

Taking an external view of the firm, there are two types of risk which are normally distinguished from each other. These are business risk and financial risk.[1] By business risk it is meant the risk inherent in the operation of an enterprise and the inability to insure stable sales, costs, and profits. Financial risk is independent of business risk, being attributable only to the incurrence of debt funds and the requirement of meeting fixed interest charges and principal payments. Since the use of debt funds will give rise to greater fluctuations of the net income available for distribution to the common stockholder, Modigliani-Miller have equated leverage with financial risk. Due to the tax deduction of interest under the present tax laws, this concept is not common in any of the financial articles other than that of Modigliani-Miller's, at least for moderate amounts of debt.

Before further examining the Modigliani-Miller propositions it would be instructive to define just what is meant by risk in this paper.

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[1] Robichek and Myers, op. cit.

The usual distinction between uncertainty, per se, and risk is noted as being only a matter of degree.[2,3] Under conditions of uncertainty, the outcomes of an event cannot be predicted with complete accuracy. Risk, on the other hand, is the situation under which the possible outcomes can be assigned a definite probability distribution even though the outcome cannot be predicted with complete accuracy. Hence, risk is said to be a degree of uncertainty.

As an illustration of the concept of risk, consider the following hypothetical situation. An investor is contemplating purchasing the stock of Company "K." Assume this investor knows the maximum production possible and estimates the earnings for the next year will be A dollars. Also assume the investor estimates the earnings with no production to be zero dollars. If the investor considered earnings to be equally likely between these two values, his subjective probability distribution would be as illustrated in Figure 1., which is a uniform probability distribution. For each value

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[2] R. Byrne, et. al., Some New Approaches to Risk, Carnegie Institute of Technology, Pittsburgh, Pa., 1967.

[3] Irving N. Fisher, et. al., Risk and Corporate Rate of Return, Rand Corporation, Santa Monica, California, November, 1967.

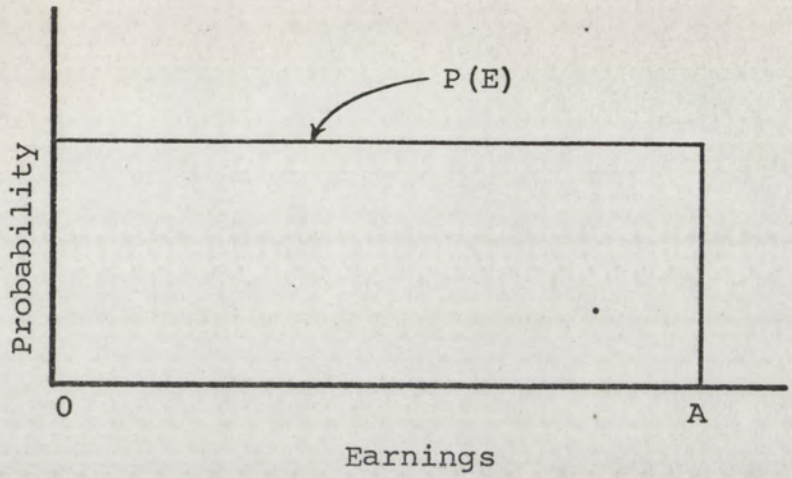

Figure 1.

A Uniform Probability Distribution

of earnings, shown on the horizontal axis, between zero and A, the probability of occurrence, P(E), is the same. The probability of occurrence is shown by the vertical distance from the earnings axis.

It would not be expected that the investor would perceive every earnings between zero and A as having an equal chance of occurring. If the investor has examined the past performance of Company "K" and had found that the earnings had been fairly stable for the past 50 years, having only random fluctuations from year to year, the distribution of this past earnings record might be that of the distribution shown in Figure 2., which is a normal probability distribution. If this were the situation, the investor would likely expect

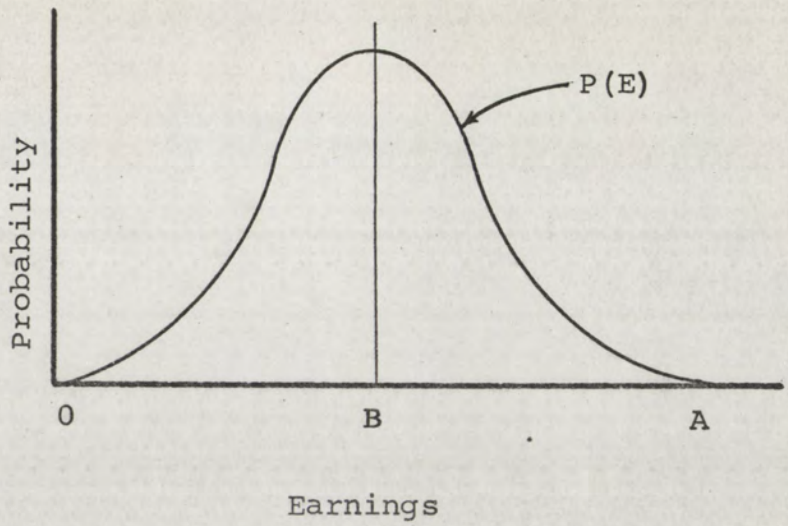

Figure 2.

A Normal Probability Distribution

that the earnings in the next year would be someplace close to the value B, the mean of the distribution. In fact he would have his "best estimate" of the expected earnings for the next year as being B dollars.

This illustration does not by any means exhaust all of the possible subjective distributions that the investor might have of the earnings for the Company "K." In fact, two different investors might have completely different estimates of the earnings because of information differences.

It is assumed that investors are normally averse to risk and will prefer an alternative which is less risky to one which is riskier if the expected yields are the same for

each alternative. By a less risky alternative, it is meant that the probability assigned to the expected yield for the alternative is greater than the probability assigned to the other alternative. This relationship can be viewed as being similar to the consumer utility function of economics and this relationship has often been used in the development of discussions in risk.[4]

Returning to the discussion of risk classes of firms, the method in which Modigliani-Miller introduce the concept of a homogeneous risk class is for firms having only equity funds in their capital structures. Their description is as follows:

> The next assumption plays a strategic role in the rest of the analysis. We shall assume that firms can be divided into "equivalent return" classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on the shares issued by any other firm in the same class . . . It follows that all relevant properties of a share are uniquely characterized by specifying (1) the class to which it belongs and (2) its expected return . . . The significance of this assumption is that it permits us to classify firms into groups within which the shares of different firms are "homogeneous," that is, perfect substitutes for one another . . .[5]

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[4] Nils Hemming Hakansson, Optimal Investment and Consumption Strategies for a Class of Utility Functions, Western Science Inst., University of California, Los Angeles, Calif., 1967.

[5] Modigliani and Miller, *op. cit.*, pp. 268-269.

In their development of the valuation of the uncertain streams of income, it is the degree of uncertainty that is attached to the expected return which determines the rate of return in the case of the pure equity firm. Once debt has been substituted for equity funds they hold that the same risk class exists, since there is the same degree of uncertainty of the earnings before interest charges, though the shares are no longer perfect substitutes.

It is important to note that the uncertainty of the average expected earnings is not the expected earnings for any single year, but it is the uncertainty of the mean value of the revenue stream accruing to the stockholder over time.

B. <u>The Model Used to Define the Risk Measure</u>

From the above discussion it might be inferred that the homogeneous risk class, as defined by Modigliani-Miller, is actually a group or set of firms which have the same amount of business risk attached to their operations, over time. Reviewing the definition of business risk presented earlier, it was said to be the risk inherent in the operation of an enterprise and its inability to insure stable sales, costs, and profits. If the inability to insure stable sales and costs should be reflected in the

inability to insure profits, then, by the same line of reasoning, the ability to provide stable profits might be used to differentiate the business risk among firms. The following assumptions will be made about an industry: (1) The technological sophistication is the same for all firms; (2) There is homogeneity of markets within the industry; (3) The products of the firm are homogeneous; (4) The general economic forecast is the same for all firms; and (5) The accounting procedures are relatively consistent within the industry. If it is also assumed that the uncertainty of future earnings can be estimated by the record of earnings in the past, then the variability of the past earnings would be a measure of the degree of uncertainty of the average expected earnings. Using the method suggested by Barges, the coefficient of variation would give a measure of the degree of uncertainty of earnings that could be used for comparative purposes among firms.[6] To allow for growth, the coefficient of variation for this study will be defined as the standard error of estimate about the least squares approximations of earnings, divided by the average of these

[6] Barges, op. cit., p. 17.

earnings. The method of least squares for the approximation of expected earnings would give the best unbiased estimate if the yearly earnings are assumed to be random variables, as they were by Modigliani-Miller.[7,8] By the use of the coefficient of determination, firms which have experienced a larger fluctuation of earnings, with the same average earnings, would have larger coefficients of determination and will be considered as having a higher degree of uncertainty of the average expected earnings.

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[7] Michael J. Brennan, Preface to Econometrics, South-Western Publishing Company, Cincinnati, Ohio, 1965, p. 354.

[8] Modigliani-Miller, *op. cit.*, p. 267.

IV. PROBLEM FORMULATION

A. The Method Used to Separate the Risk Classes

If it is assumed that an industry is not a homogeneous risk class, the first step is to devise a method to categorize the firms within an industry into the appropriate risk classes. The manner in which this is achieved by investors could vary, as the investor's interpretation of the firm's risk is subjective. Different investors could have completely different ideas of how many risk classes are appropriate, and where to end one class and begin another. This would depend on the sophistication of the tools of analysis, the breadth of the portfolio, what information is available, and the investor's subjective estimations of the business environment in the future.

For this introductory study, the rather broad classification used by an investor's guide was chosen to estimate the number of risk classes in the industry.[1] Using the coefficient of variation as described in the preceding section, the industry will be divided into two risk classes. Those firms which have a coefficient of variation above the average for the industry will be taken as being members of

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[1] Standard and Poor's Stock Guide, Standard and Poor's Corporation, Publishers, New York, March, 1968.

one risk class, which will be called Risk Class I. Firms having a coefficient of variation equal to or below the average for the industry will be taken as the remaining risk class, which will be called Risk Class II.

The method of least squares which can provide the best fit for the data of the period covered will be used, whether it be a simple linear, curvilinear, power, or logarithmic function.

The coefficient of variation will be calculated in the following manner:

$$C_v = \frac{S_x}{\bar{X}} \qquad (1)$$

where C_v = the coefficient of variation of the earnings

S_x = the standard error of estimate of the least squares approximation of earnings

$\bar{X}$ = the average of the past earnings.

Following the definitions of Modigliani-Miller for defining the variables used in their empirical study, earnings will be taken as the sum of interest, preferred dividends, and stockholders income net of corporate taxes.[2] The rate

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[2] Modigliani and Miller, op. cit., p. 276.

of return to the common stockholder is taken as the expected net income divided by the market value of the shares outstanding. The average expected net income was approximated in the same manner as the average expected earnings, using the method of least squares of past new incomes.

For the values of debt and equity, it was shown by Barges that the use of market values will introduce bias into the statistical tests, which Modigliani-Miller had mentioned as being a possibility.[3] By the use of book values, assuming accounting procedures to be relatively consistent, this bias was shown to be eliminated. Thus debt is taken to be the book value of all securities senior to the common stock. Similarly, equity is taken to be the book value of the common stock and premium on common stock plus the earnings retained by the company.

Throughout the remainder of this paper, when the Modigliani-Miller proposition is noted, it will be meant their proposition that investors will require a higher rate of return on the common stock as debt is added to the capital structure. Analogously, the Traditional proposition will be taken to mean investors will not require a higher rate of

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[3] Barges, loc. cit., pp. 22-33.

return on the common stock for moderate amounts of debt in the capital structure.

The method of testing the first hypothesis of this paper is by finding if the rate of return for the risk classes, as defined above, are significantly different. If R_1 and R_2 are defined as the rate of return on the common stock of Risk Class I and Risk Class II, respectively, the test will be to see if the rate of return on the common stock of one risk class is significantly different than the rate of return of the other. This is asking whether the difference in the rate of return is really different or can the difference be attributed to chance.

The second hypothesis will be tested by finding if the regression coefficient of the least squares linear regression line relating yield and leverage is significantly different from zero, which is a test of the traditional position. If the difference is greater than zero, the third hypothesis will be tested to see if the regression coefficient is significantly different from the spread between the interest rate and the rate of return on an unlevered firm, which could constitute a test on the Modigliani-Miller proposition.

$$\text{If } R = a + bL \qquad (2)$$

where R = the required rate of return on the common stock

a = the least squares constant coefficient which is the intercept on the R axis, and which would be the required rate of return on an unlevered firm in the Modigliani-Miller proposition

b = the least squares regression coefficient which is the slope of the regression line and indicates the relation between the rate of return and leverage

L = the debt/equity ratio or leverage

then $b \neq 0$ would be the test for the traditional position and if this is found to be true then

$b \neq (a-i)$ would be the test for the Modigliani-Miller proposition, where

i = the interest rate on debt securities.

Further tests will be made on assumptions of Modigliani-Miller and one noted by Barges for which the data were readily available.[4] These assumptions are that the rate of return is not substantially influenced by size, growth, or dividend policies.

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[4] Barges, op. cit., p. 25-26

B. Definition of Variables

The variables used in this study may be classified into three separate groups as follows:

1. Variables used in the testing of the difference in the rate of return between risk classes:

   K = number of years for which reported earnings and net income are collected.

   X_t = reported annual earnings, which is the sum of interest, preferred dividends, and stockholders income net of corporate taxes for years t_1 to t_k.

   $\bar{X}_t$ = average of the reported annual earnings.

   X_c = estimated annual earnings by use of the method of least squares.

   $\bar{X}$ = average expected earnings which is taken to be the estimated earnings for the year t_{k+1}.

   S_x = the standard error of estimate for the observed and estimated earnings.

   C_v = the coefficient of variation of the earnings. (See discussion in text of section III-B)

P = the arithmetic mean of the high and low quoted common stock price for year t_{k+1}.

N = the number of common shares outstanding for year t_{k+1}.

R = required rate of return to the common stockholder.

2. Variables used in testing the rate of return and leverage relationship:

R = as above

D = the book value of debt as taken from the annual report for year t_{k+1}, debt being defined as the book value of all securities senior to the common stock.

E = the book value of equity as taken from the annual report for year t_{k+1}, equity being defined as the sum of the book values of the common stock, premium on common stock, and the retained earnings.

L = the leverage of a firm defined as the ratio of the book value of debt to the book value of equity.

a = the least squares constant coefficient of the regression line with rate of return as

the dependent variable and leverage as the independent variable, taken to be the rate of return of the unlevered, or pure equity, firm in the test of Modigliani-Miller's proposition.

b = the least squares regression coefficient of the regression line immediately above, taken to be the rate of increase in required rate of return in the test of Modigliani-Miller's proposition.

3. Variables used in the testing of the assumptions of Modigliani-Miller.

P = as above

M = the dividends declared for year t_{k+1}.

Y = the dividend yield to the common stockholder.

D = as above

E = as above

H = a measure of the size of the firm taken to be the sum of the book values of debt and equity.

B = a measure of growth of the firm taken to be the slope of the least squares line of

the reported earnings.

c & d = the constant and regression coefficients, respectively, of the least squares linear regression for the variable size, growth and dividend yield.

The proposed relationships between the variables are as below:

1. The coefficient of variation to be used as a measure of uncertainty of the expected average earnings:

$$C_v = \frac{S_x}{\bar{X}} \tag{3}$$

2. The required rate of return to the common stockholder:

$$R = \frac{\bar{X}}{P \cdot N} \tag{4}$$

3. The relationship between the required rate of return and leverage:

$$L = D/E \tag{5}$$

$$R = a + bL \tag{6}$$

b = 0 for the tradition position

b = (a-i) for the Modigliani-Miller proposition

4. The relationship of the variables not assumed to have any bearing on the rate of return on the common stock:

   a. Size: $R = c + dH$ (7)

   b. Growth: $R = c + dB$ (8)

   c. Dividend Yield: $R = c + dY$ (9)

C. Assumptions

The assumptions made in the development of the risk class model are outlined below:

1. Stocks are assumed to be traded in perfect markets.
2. The accounting procedures are relatively consistent within the industry and provide a consistent measure of the state of the firm.
3. The markets for the products of the firms are homogeneous.
4. The size of the firm has no effect on the performance, costs, or valuation of the firm.
5. All firms have the same expected rate of growth.
6. All firms have essentially the same technology and economic forecast.
7. All investors have estimated the same average

expected earnings for any given firm.

8. The degree of uncertainty of the average expected earnings may be quantified by the use of the coefficient of variation about the least squares approximations of past earnings.
9. Investors are averse to risk and must receive a greater rate of return for an investment which is subject to a greater degree of uncertainty than that of other investments.
10. The number of risk classes used represents the number of risk classes perceived by investors.
11. The use of the average expected earnings - price ratio measures the required rate of return by the investor.
12. The use of the arithemetic mean of the high and low stock prices is a measure of the price the investor is willing to pay for the average expected earnings.
13. The investor is indifferent whether the net income available to him is reinvested by the firm or paid out to him as dividends.
14. The effect of laws and regulatory bodies is the same for all firms of the industry.

In addition to the assumptions noted, a further assumption is made for the tests of the traditional and Modigliani-Miller propositions. This is the assumption that the effect of tax deduction for debt funds is negligible.

Statistical assumptions are that the variables employed, with the exception of the time series data on earnings and net income, are independent and normally distributed. The effect of autocorrelation on the time series data is assumed to be negligible and, through the Chebychev Inequality Rule, the variance of the earnings about the least squares approximation of earnings is assumed to be a valid measure for probabilistic connotations of the coefficient of variation. With these assumptions the use of the appropriate parametric statistical tests would be permitted.

D. Limitations of the Study

There are numerous limitations of this study as there are in any dynamic situation where controlled experimentation is impossible. It is attempted to measure and quantify expectations of the future with relationship to static and historical data. In the real world, where the model must be verified, future expectations change with every introduction of legislation, newspaper headline, presidential speech, and many other occurrences, even

including weather forecasts.

The limitations of this paper are twofold. First there are limitations brought about thru the use of historical data and the sources of the data. There are also limitations by the neglection of variables in the proposed relations set forth in the previous section.

By the very nature of the study, we are using data which are the results of historical cost reporting procedures of the firms. The extent to which the accounting procedures are consistent is not known, but this is the only data available and we have to assume the accounting procedures are relatively consistent to be able to conduct the study. Differences in reporting of earnings must be assumed to be negligible if we are trying to find out how the investor capitalizes earnings and then make comparisons among these capitalization rates. The use of the dividends paid (ignoring the possibility of stock dividends) has been suggested as a measure that could bypass this accounting dilemma, but there are also problems that can occur with this method. Even if it was possible to use the stream of dividends as a measure for the capitalization rates, we would not be able to work in the framework set by Modigliani-Miller and purport to make tests of their propositions. The same argument is true for

risk classes if we have defined risk classes as Modigliani-Miller have defined them. The use of the average of the high-low stock prices for a given year has been found to not be statistically different than the average price paid for a share during the year, and this is assumed to be true for this study also.[5] The use of book values for debt and equity, while it may not introduce bias into the statistical aspect of the analysis, does draw questions of the accuracy of the measurements that are being made. Perhaps not so much for debt, which is a fixed liability, but for equity, as it reflects again the results of the reporting of earnings, and their retention in the firm over a number of years. It is then the extent to which the measurements are reliable that the mathematic and statistical results will be meaningful.

Up to this point the limitations that have been noted are concerned with the reliability of the data. For the Modigliani-Miller proposition of the relationship of leverage and the rate of return, we can only work within the framework of their assumptions to make a test of their hypothesis.

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[5] Charles E. Edwards and James G. Hilton, "A Note on the High-Low Price as an Estimator on Annual Average Stock Prices," Journal of Finance, Vol. 21, March, 1966.

For the tests of the risk classes, there are a number of variables which were not included in the study that were expressly assumed not to be relevant. Only three of the variables have been included in testing of the assumptions, which are size, growth, and dividend yield. Aspects which are noted by Standard and Poor's that are used in the categorizing of the industry used as the sample include the following: (1) The composition of the market served by the firm in terms of industrial growth and population growth; (2) The age and condition of the assets; (3) The management; and (4) The regulatory bodies and laws effecting the operation of the firm.[6] In addition to these items, variables that may have an effect on the rate of return could be the eligibility of the firms common stock for purchase by institutions, and the coveriance of the earnings of this industry with others. It is the extent to which these variables, other than the three noted above, were not considered and the part they have in the determination of the rate of return that the study has its limitations.

The conclusions that we are able to reach will thus be valid, reliable, and relevant only to the extent to which

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[6] Standard and Poor's Stock Guide, *op. cit.*, p. 4.

the assumptions that have been made are valid, reliable, and relevant.

V. RESEARCH TECHNIQUES TO BE USED

A. The Selection of the Sample

An industry which would satisfy the requirements set forth earlier was obtained by using the Gas Utility Industry. This is an industry which is marked by its stability, has sufficiently consistent accounting methods, a homogeneous product, and a state-of-the-art in technology which is similar among firms. It is also an industry for which there is sufficient information available for investors to be well informed. To insure that there was an active trading of the common stock of the firm, the sample was chosen from a list of gas utilities provided by an investor's guide publication.[1] Firms which had experienced deficit earnings or net losses were eliminated since the model utilized cannot adequately handle negative values for these two items. Also firms whose operating income had substantial amounts of revenue from other than gas sales were removed from the sample. A complete list of the firms in the sample can be found in Appendix I.

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[1] Standard and Poor's Stock Guide, *op. cit.*

B. Sources of the Original Data

After having obtained the list of firms as noted, the data necessary was compiled by the use of the latest information available in the Moody's Public Utility Manual for the reported earnings and net income for the years 1958 thru 1966, and the 1967 book values of debt and equity were found in the same source.[2] The high and low stock prices, number of shares outstanding and dividends declared for 1967 were all taken from the latest issue of the Standard and Poor's Stock Guide available.[3]

C. The Statistical Tests

The IBM Model 360 Computer, available for student use at the University of New Mexico Computer Center, was used to make all numerical calculations and statistical tests. The program, written in FORTRAN IV, was checked thoroughly to eliminate all errors by the use of handwritten

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[2] Moody's Public Utility Manual, Moody's Investor Service, New York, 1959-1967.

[3] Standard and Poor's Stock Guide, op. cit.

calculations and thru the use of data from statistical texts.[4,5,6,7]

The test of the hypothesis that there is a difference in the two risk classes is to be made as follows:

1. Null Hypothesis:

   The required rate of return for Risk Class I, R_1, is not significantly different than the required rate of return for Risk Class II, R_2.

$$H_{\theta 1}: R_1 = R_2 \qquad (10)$$

2. Hypothesis:

   The required rate of return for Risk Class I, R_1, having greater than average coefficients of variation, will be significantly different than

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[4] Fred N. Kerlinger, Foundations of Behavioral Research, Holt, Rinehart & Weston, Inc., N. Y. 1967.

[5] Robert Ferber and P. J. Verdorn, Research Methods in Economics and Business, the Mac Millan Company, N. Y., 1962.

[6] Brennan, loc. cit.

[7] System/360 Scientific Subroutine Package (360A-CM-03X) Version II - Programmer's Manual, International Business Machines Corporation, Technical Publication Department, White Plains, N. Y., 1967.

the required rate of return for Risk Class II, R_2, having less than or equal to the average coefficients of variation.

$$H_1: R_1 \neq R_2 \qquad (11)$$

3. The level of significance at which the null hypothesis will be rejected is alpha = .05.

4. The One-way Analysis of variance is used to test if the required rate of return is significantly different between risk classes:

$$F = V_b / V_w \qquad (12)$$

where V_b = the between classes variation = SV/N

V_w = the within risk classes variation = $\sum (R - \bar{R})^2/(N-1)$

with degrees of freedom = (N-1)/N(n-1)

SV = the variance of R

N = the number of risk classes

n = the number of firms in the sample

$\bar{R}$ = the mean of rate of return of all firms in the class.

5. If the calculated value of F is less than or equal to the value in the F table for alpha = .05 and

(N-1)/N(n-1) degrees of freedom the Null Hypothesis, $H_{\theta 1}$, that there is no difference in the rate of return for the two risk classes will be accepted.

6. If the calculated value of F exceeds the value in the F table for alpha = .05 and (N-1)/N(n-1) degrees of freedom the Hypothesis, H_1, that there is a significant difference in the rate of return between the two risk classes will be accepted, and the Null Hypothesis rejected.

The following test will be used for each risk class if there is a significant difference in the rate of return between the risk classes (H_1 is accepted) or for the entire sample if there is no significance in the rate of return between the risk classes ($H_{\theta 1}$ is accepted), to test the traditional position.

1. Null Hypothesis:

   The slope of the regression line, b, with the required rate of return as the dependent variable and leverage as the independent variable is not significantly different from zero as proposed by the traditional position.

$$H_{\theta 2}: b = 0 \qquad (13)$$

2. Hypothesis:

The slope of the regression line, b, with the required rate of return as the dependent variable and leverage as the independent variable is significantly different from zero:

$$H_{2}: \; b \neq 0 \qquad (14)$$

3. The level of significance at which the null hypothesis will be rejected is alpha = .05.

4. The Z- test will used to test if the slope of the regression line is significantly greater than zero:

$$Z = \frac{b}{S_{\hat{b}}} \qquad (15)$$

where b = the value of the regression coefficient

$S_{\hat{b}}$ = the standard error of the regression coefficient

5. If the calculated value of Z is less than or equal to the value of Z obtained from the normal probability tables for alpha = .05, the null hypothesis, $H_{\theta 2}$, that the slope of the regression line does not differ significantly from zero will be accepted.

6. If the calculated value of Z is greater than the value of Z obtained from the normal probability tables for alpha = .05, the hypothesis, H_2, that the slope of the regression line does differ significantly from zero will be accepted, and the null hypothesis rejected.

The following test will be performed only if the regression line does differ significantly from zero (H_2 is accepted) to test the Modigliani-Miller proposition.

1. Null Hypothesis:

   The slope of the regression line with rate of return as the dependent variable and leverage as the independent variable is not significantly different from the spread between the rate of return of an unlevered firm and the interest rate as proposed by Modigliani-Miller.

$$H_{\theta 3}: \quad b = a-i \qquad (16)$$

2. Hypothesis:

   The slope of the regression line with rate of return as the dependent variable and leverage as the independent variable is significantly different from the spread between the rate of

return of an unlevered firm and the interest rate.

$$H_{3:}\ b \neq a-i \qquad (17)$$

3. The level of significance at which the null hypothesis will be rejected is alpha = .05.
4. The use of a Z- test is used to test if the slope of the regression line differs significantly from the spread between the rate of return of an unlevered firm and the interest rate:

$$Z = \frac{b-(a-i)}{S_{\hat{b}}} \qquad (18)$$

where b = as above

a = the value of the constant coefficient

i = the interest rate

$S_{\hat{b}}$ = as above

5. If the calculated value of Z is less than or equal to the value of Z obtained from the normal probability tables for alpha = .05, the null hypothesis, $H_{\theta 3}$, that the slope of the regression line does not differ significantly from the spread between the rate of return or an unlevered firm and the interest rate will be accepted.

6. If the calculated value of Z is greater than the value of Z obtained from the normal probability tables for alpha = .05, the hypothesis, H_3, that the slope of the regression line does differ significantly from the spread between the rate of return on an unlevered firm and the interest rate will be accepted and the null hypothesis rejected.

In testing the assumptions of Modigliani-Miller the same test will be used a number of times and the procedures will be similar in each instance. To avoid further repetition the hypothesis and null hypothesis will be defined only once for the three variables, as will be the calculation of the Z value and acceptance or rejection of the hypothesis.

1. Null Hypothesis:

   $H_{\theta 4}$: there is no significant relation between growth (as an independent variable) and rate of return.

   $H_{\theta 5}$: there is no significant relation between size (as an independent variable) and rate of return.

   $H_{\theta 6}$: there is no significant relation between dividend policy (as an independent variable and rate of return.

2. Hypothesis:

   There is a significant relation between the variables and rate of return, $H_{\theta 4}$, $H_{\theta 5}$, $H_{\theta 6}$, as

proposed by the Modigliani-Miller proposition.

3. The level of significance at which the null hypothesis will be rejected is alpha = .05.
4. If the calculated value of Z is less than or equal to the value of Z obtained from the normal probability tables, accept the null hypothesis.
5. If the calculated value of Z is greater than the value of Z obtained from the normal probability tables, accept the hypothesis and reject the null hypothesis.

VI. RESULTS

The separation of the firms into two risk classes, with the average coefficient of variation for the industry of 0.02075 being the dividing point, placed sixteen firms into Risk Class I and thirty-one firms into Risk Class II. The firms in each risk class are listed in order of increasing

TABLE 1

Listing of Firms in Risk Class I

	Firm	Rate of Return	Coefficient of Variation
1.	Pioneer Natural Gas	5.643	2.084
2.	Alabama Gas	8.216	2.196
3.	Consolidated Natural Gas	8.727	2.217
4.	Mississippi Valley Gas	9.111	2.332
5.	Michigan Gas Utilities	7.752	2.365
6.	Kansas-Nebraska Natural Gas	7.486	2.480
7.	North Penn Gas	8.029	2.551
8.	Atlanta Gas Light	9.307	2.635
9.	Pacific Lighting	9.025	2.938
10.	Houston Natural Gas	4.118	3.194
11.	Gas Service	8.189	3.248
12.	Western Kentucky Gas	8.260	3.481
13.	National Gas & Oil	8.878	3.722
14.	Minneapolis Gas	8.079	4.348
15.	Chattanooga Gas	9.357	6.617
16.	Commonwealth Gas	9.925	10.461

coefficients of variation in Tables 1 and 2. The rate of return for each firm is also listed in the tables and one can see there is no clear pattern or indication that a

TABLE 2

	Listing of Firms in Risk Class II		
	Firm	Rate of Return	Coefficient of Variation
1.	South Jersey Gas	6.719	0.525
2.	Springfield Gas Light	9.128	0.559
3.	United Gas Improvement	8.262	0.753
4.	Northern Illinois Gas	7.725	0.776
5.	Columbia Gas System	8.528	0.840
6.	Northwest Natural Gas	7.800	0.878
7.	Washington Natural Gas	7.994	0.880
8.	Brooklyn Union Gas	8.838	0.931
9.	Washington Gas Light	10.401	0.963
10.	Florida Public Utilities	10.560	1.031
11.	Providence Gas	8.728	1.121
12.	American Natural Gas	7.855	1.141
13.	National Fuel Gas	9.002	1.183
14.	Oklahoma Natural Gas	8.390	1.246
15.	Peoples Gas Light & Coke	8.424	1.353
16.	Arkansas Louisiana Gas	7.492	1.370
17.	Laclede Gas	9.023	1.423
18.	Equitable Gas	8.812	1.473
19.	New Jersey Natural Gas	8.184	1.487
20.	Lone Star Gas	6.049	1.513
21.	Piedmont Natural Gas	9.851	1.545
22.	Union Gas of Canada	3.697	1.573
23.	Southern Union Gas	8.461	1.601
24.	Elizebethtown Gas	7.338	1.684
25.	Northern Natural Gas	7.963	1.687
26.	International Utilities	7.114	1.733
27.	Indiana Gas & Water	7.478	1.778
28.	Public Service of North Carolina	9.724	1.829
29.	Mountain Fuel Supply	8.281	1.830
30.	Arkansas Western Gas	8.778	1.938
31.	Rio Grande Valley Gas	5.340	2.033

relation would exist between the rate of return and the coefficient of variation. In fact the highest two rates

of return are found in Risk Class II, and the second lowest rate of return is found in Risk Class II, which is the reverse of what would be expected. Table 3 lists the average rate of return for each risk class, along with the average coefficient of variation for each risk class and the industry average of these two items. With the average rate of return for Risk Class I only minutely greater than the average rate of return for Risk Class II, it is not surprising to find

TABLE 3

Rates of Return for Risk Classes I and II

	Risk Class I	Risk Class II
Average rate of return in class	8.1314%	8.1270%
Average coefficient of variation in class	3.554%	1.312%
Average rate of return for the industry	8.1285%	
Average coefficient of variation for the industry	2.075%	

there is no statistical difference in the rates of return for the two risk classes. See Appendix II for the details of this test. From the result of this test, the use of the industry as a homogeneous risk class would be accepted as meeting the criterion of a homogeneous risk class as defined

in this paper. Although there were no empirical studies found similar to that of this portion of the paper, a number of articles in the financial literature have noted that industries with stable earnings, such as found in the utilities, would be considered as the most appropriate industries to be employed as homogeneous risk classes for the testing of the Modigliani-Miller propositions.[1,2] The results of the foregoing test has given evidence that this is true, at least for the gas utility industry.

The test on the relationship of the rate of return and leverage was next made, using the industry as a homogeneous risk class. Following the notation set forth earlier, with R denoting the rate of return on the common stock and L denoting the ratio of debt to equity, the results of the linear regression are as follows:

$$R = 8.077 + 0.00037\ L \quad (\pm\ 0.0028) \qquad (19)$$

$$r = 0.0197,\ z = 0.1324$$

The value in the parentheses is the standard error of the

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[1] J. Fred Weston, "A Test of Cost of Capital Propositions," Southern Economic Journal, Vol. 30, (Oct.), 1963.

[2] Fred D. Arditti, "Risk and the Required Rate of Return," Journal of Finance, Vol. 22, Dec. 1967.

regression coefficient. Below the regression equation are the values of the correlation coefficient, r, and the calculated z value. The results of the statistical test show that the increase in the rate of return does not differ significantly from zero as the ratio of debt to equity increases. The square of the correlation coefficient gives the percentage of variation of R that is explained by the variable L. In the regression above there is about 0.04% of the variation of R explained by L. Figure 3. shows the results of the regression in the form of a scatter diagram.

The results of the regression are very similar in form to the regression equation for Modigliani-Miller's empirical test using the electrical utility industry as their sample. Their results are shown below:

Modigliani-Miller results (1947-1948): (20)

$$R = 6.6 + 0.00017\ L \quad (\pm\ 0.00004)$$

$$r = 0.53$$

Their study had shown the regression to be highly significant, e.g., there is a relation between leverage and the rate of return. However, the statistical test for the gas utility industry shows there is no significant relation of leverage and the rate of return. Since the method used in this paper

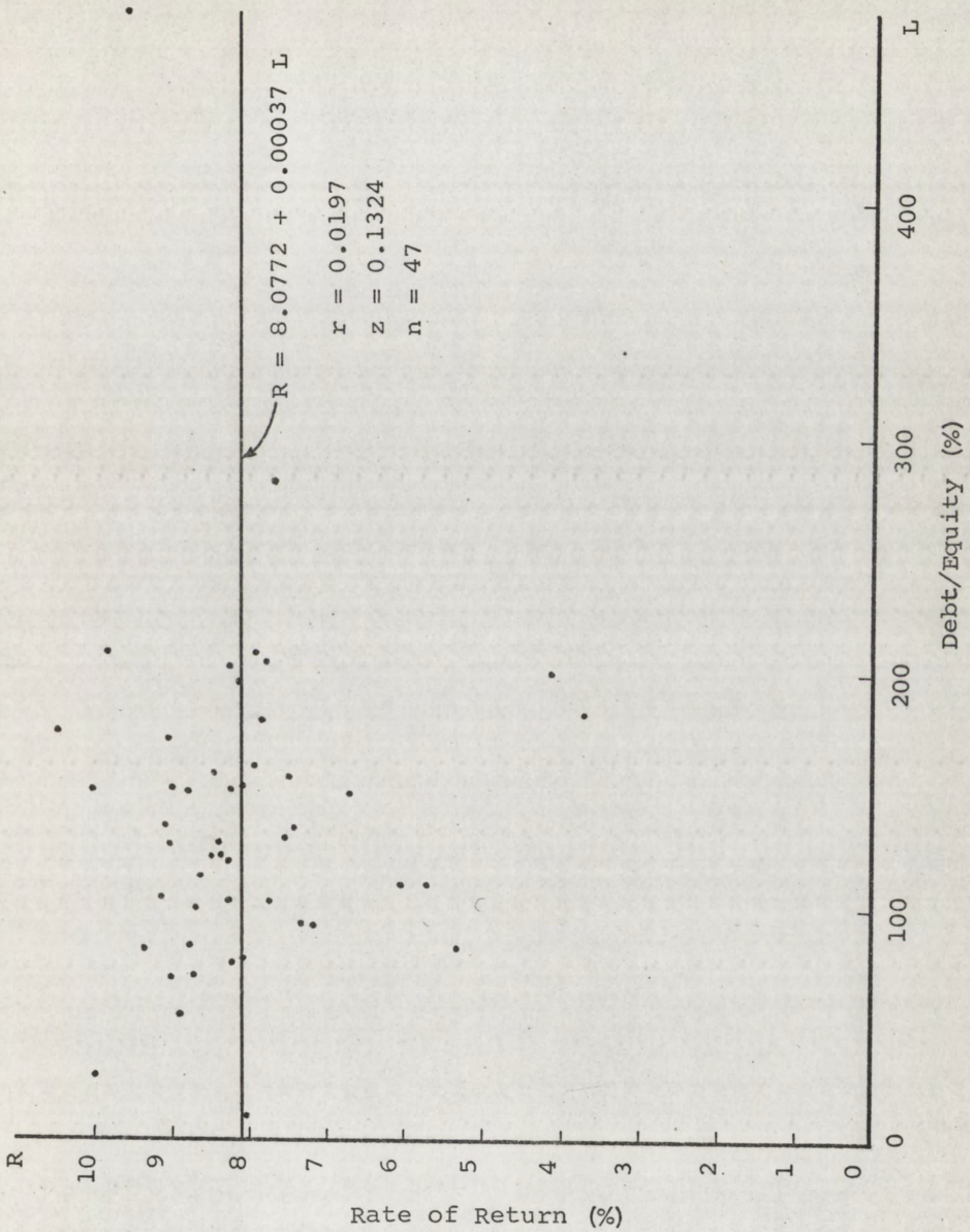

Figure 3.

Rate of Return Vs. Leverage

follows the procedure outlined by Modigliani-Miller in all but two respects, it might be thought that the difference can be explained by one, or both, of the different definitions used in this paper. These are the use of book values for the measurement of leverage, and the use of the method of least squares in estimating the average expected earnings. Barges has noted that the use of book values in place of the market values for the measure of leverage did not effect the significance of Modigliani-Miller's test. This implies that the difference would most likely be in the means of estimating the average expected earnings. To make a check on this, the average earnings for the last two years of the period covered were used as the estimation of the expected average earnings, which is the same method used by Modigliani-Miller. The results were the same as previously found with the regression showing the relation between rate of return and leverage being not significantly different from zero. While this does not prove conclusively that the diverse results cannot be due to dissimilarity in the measurements, (it could be the effect of the use of both book values and the means of estimating the average expected earnings at the same time), it does imply there may be differences in the industries as perceived by the investors.

With the rejection of the hypothesis that the relation between the required rate of return and leverage was significantly different than zero, the test of Modigliani-Miller's proposition was not conducted.

The assumptions that growth, size, and dividend policy would not effect the required rate of return were next evaluated. With R denoting the required rate of return on the common stock as before, and letting B represent a measure of growth (B is the slope of the least squares line for the earnings for the period 1958-1966), H a measure of size (H is the sum of the book values of debt and equity), and Y denoting the dividend yield, the results of the regression analysis are as in Table 4.

TABLE 4

Growth, Size, and Dividend Yield Effects on Required Return

1.	Rate of Return and Growth	$R = 8.269 - 0.00156\ B\ (\pm\ 0.0017)$ $r = -0.13261$
2.	Rate of Return and Size	$R = 8.134 - 0.00000\ H\ (\pm\ 0.0000)$ $r = -0.00527$
3.	Rate of Return and Dividend Yield	$R = 6.406 + 0.38333\ Y\ (\pm\ 0.1226)$ $r = 0.4225$

From Table 4 it is seen that the assumptions that growth and size do not have a significant relation to the required rate of return are upheld. This differs with Weston's study on the electric utility industry, where he had found a significant relation between the rate of return and growth, using a different measure of growth than used in this paper. The dividend yield was found to have a significant relation on the required rate of return with a correlation coefficient of 0.4225 and the computed z-value being 3.127. The positive regression coefficient for dividend yields with the required rate of return could be interpreted as increased dividend yields causing a increase in the rate of return. This would mean a premium is placed for higher dividend yields and the assumption of Modigliani-Miller that dividends make no difference would not be true for this industry. There is one factor that makes the results of this regression less clear. Both the dividend yield and the required rate of return have the same quantity, the average market price of the common stock, in the denominator. It may be that an expected earnings-dividend ratio has a relationship with the stock price as has been suggested by Arditti.[3]

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[3] *Ibid.*

To fully investigate the effects of dividends and the rate of return is beyond the scope of this paper, and it is sufficient to say that there is a significant relationship of dividends on the rate of return found in this study.

VII. SUMMARY AND CONCLUSIONS

Within the limitations noted, the implications of this study are:

1. <u>The selection of the gas utility industry would satisfy the criterion that the degree of uncertainty of the average expected earnings is not significantly different among firms of the industry and the industry may be considered as a homogeneous risk class.</u>

This conclusion would be a verification of the comments found in a number of articles which state that industries which exhibit a high stability of earnings, such as found in the utilities, would be homogeneous risk classes.[1,2] The generalization of this conclusion would be possible only for the industries which are very similar to the gas utility industry.

2. <u>The Modigliani-Miller proposition that increased amounts of leverage in the capital structure will cause higher required rates of return on the common</u>

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[1] J. Fred Weston, "The Management of Corporate Capital: A Review Article," <u>Journal of Business</u>, April, 1961.

[2] Arditti, <u>op</u>. <u>cit</u>.

stock does not hold for the gas utility industry.

This conclusion may be taken as support of the traditional proposition, increases in the debt to equity ratio for moderate amounts of debt will not cause an increased rate of return to the common stock, for the firms of the gas utility industry. This conclusion cannot be generalized to other industries, even within the utilities since there was shown to be a relation between leverage and the rate of return for the electric utilities by Modigliani-Miller and Weston.[3,4] The use of a coverage measure as suggested by Wippern and Barges would perhaps be a better measure of the financial risk than the ratio of debt to equity, but whether this can be placed into a theoretical framework remains to be seen.[5,6] The conclusion that can be made from this portion of the study is that the Modigliani-Miller proposition does not hold for the gas utility industry.

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[3] Weston, op. cit.

[4] Modigliani and Miller, loc. cit.

[5] Barges, loc. cit.

[6] Ronald F. Wippern, "Financial Structure and the Value of the Firm," The Journal of Finance, Vol. 21, No. 4, Dec. 1966.

3. <u>The assumption that dividends do not make a difference in the required rate of return is not true for this industry</u>.

This conclusion indicates that the Modigliani-Miller assumption does not hold true for this industry. As this assumption has a critical role in the development of the proposition that is being studied, the application of this Modigliani-Miller proposition for this industry, would have to be rejected on a theoretical basis. If dividends were not capitalized at the same rate as retained earnings, the relationship of the Modigliani-Miller proposition would not be applicable.

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APPENDIX I

APPENDIX I

Listing of Firms in the Sample

1. Alabama Gas
2. American Natural Gas
3. Arkansas Louisiana Gas
4. Arkansas Western Gas
5. Atlanta Gas Light
6. Brroklyn Union Gas
7. Chattanooga Gas
8. Columbia Gas System
9. Commonwealth Gas
10. Consolidated Natural Gas
11. Elizebethtown Gas
12. Equitable Gas
13. Florida Public Utilities
14. Gas Service
15. Houston Natural Gas
16. Indiana Gas and Water
17. International Utilities
18. Kansas-Nebraska Natural Gas
19. Laclede Gas
20. Lone Star Gas
21. Michigan Gas Utilities
22. Minneapolis Gas
23. Mississippi Valley Gas
24. Mountain Fuel Supply
25. National Fuel Gas
26. National Gas & Oil
27. New Jersey Natural Gas
28. North Penn Gas
29. Northern Illinois Gas
30. Northern Natural Gas
31. Northwest Natural Gas
32. Oklahoma Natural Gas
33. Pacific Lighting
34. Peoples Gas Light & Coke
35. Piedmont Natural Gas
36. Pioneer Natural Gas
37. Providence Gas
38. Public Service of North Carolina
39. Rio Grande Valley Gas
40. South Jersey Gas
41. Southern Union Gas
42. Springfield Gas Light
43. Union Gas of Canada
44. United Gas Improvement
45. Washington Gas Light
46. Washington Natural Gas
47. Western Kentucky Gas

APPENDIX II

FIRM	RATE OF RETURN	COEFFICIENT OF VARIATION
ALABAMA GAS	8.216	2.196
AMERICAN NATURAL GAS	7.855	1.141
ARKANSAS LOUISIANA GAS	7.492	1.370
ARKANSAS WESTERN GAS	8.778	1.938
ATLANTA GAS LIGHT	9.307	2.635
BROOKLYN UNION GAS	8.838	0.931
CHATTANOOGA GAS	9.357	6.617
COLUMBIA GAS SYSTEM	8.528	0.840
COMMONWEALTH GAS	9.925	10.461
CONSOLIDATED NATURAL GAS	8.727	2.217
ELIZEBETHTOWN GAS	7.338	1.684
EQUITABLE GAS	8.312	1.473
FLORIDA PUBLIC UTILITIES	10.560	1.031
GAS SERVICE	8.189	3.248
HOUSTON NATURAL GAS	4.118	3.194
INDIANA GAS & WATER	7.478	1.778
INTERNATIONAL UTILITIES	7.114	1.733
KANSAS-NEBRASKA NATURAL GAS	7.485	2.480
LACLEDE GAS	9.023	1.423
LONE STAR GAS	6.049	1.513
MICHIGAN GAS UTILITIES	7.752	2.365
MINNEAPOLIS GAS	8.079	4.348
MISSISSIPPI VALLEY GAS	9.111	2.332
MOUNTAIN FUEL SUPPLY	8.281	1.830
NATIONAL FUEL GAS	9.002	1.183
NATIONAL GAS & OIL	8.878	3.722
NEW JERSEY NATURAL GAS	8.184	1.487
NORTH PENN GAS	8.029	2.551
NORTHERN ILLINOIS GAS	7.725	0.776
NORTHERN NATURAL GAS	7.963	1.687
NORTHWEST NATURAL GAS	7.800	0.878
OKLAHOMA NATURAL GAS	8.390	1.246
PACIFIC LIGHTING	9.025	2.938
PEOPLES GAS LIGHT & COKE	8.424	1.353
PIEDMONT NATURAL GAS	9.851	1.545
PIONEER NATURAL GAS	5.643	2.084
PROVIDENCE GAS	8.728	1.121
PUBLIC SERVICE OF NORTH CAROLINA	9.724	1.329
RIO GRANDE VALLEY GAS	5.340	2.033
SOUTH JERSEY GAS	6.719	0.525
SOUTHERN UNION GAS	8.461	1.601
SPRINGFIELD GAS LIGHT	9.128	0.559
UNION GAS OF CANADA	3.697	1.573
UNITED GAS IMPROVEMENT	8.252	0.753
WASHINGTON GAS LIGHT	10.401	0.963
WASHINGTON NATURAL GAS	7.994	0.880
WESTERN KENTUCKY GAS	8.260	3.481

LISTING OF FIRMS IN ORDER OF INCREASING COEFFICIENT OF VARIATION

FIRM	RATE OF RETURN	COEFFICIENT OF VARIATION
1. SOUTH JERSEY GAS	6.719	0.525
2. SPRINGFIELD GAS LIGHT	9.128	0.559
3. UNITED GAS IMPROVEMENT	8.262	0.753
4. NORTHERN ILLINOIS GAS	7.725	0.776
5. COLUMBIA GAS SYSTEM	8.528	0.840
6. NORTHWEST NATURAL GAS	7.800	0.878
7. WASHINGTON NATURAL GAS	7.994	0.880
8. BROOKLYN UNION GAS	8.838	0.931
9. WASHINGTON GAS LIGHT	10.401	0.963
10. FLORIDA PUBLIC UTILITIES	10.560	1.031
11. PROVIDENCE GAS	8.728	1.121
12. AMERICAN NATURAL GAS	7.855	1.141
13. NATIONAL FUEL GAS	9.002	1.183
14. OKLAHOMA NATURAL GAS	8.390	1.246
15. PEOPLES GAS LIGHT & COKE	8.424	1.353
16. ARKANSAS LOUISIANA GAS	7.492	1.370
17. LACLEDE GAS	9.023	1.423
18. EQUITABLE GAS	8.812	1.473
19. NEW JERSEY NATURAL GAS	8.184	1.487
20. LONE STAR GAS	6.049	1.513
21. PIEDMONT NATURAL GAS	9.851	1.545
22. UNION GAS OF CANADA	3.697	1.573
23. SOUTHERN UNION GAS	8.461	1.601
24. ELIZEBETHTOWN GAS	7.338	1.684
25. NORTHERN NATURAL GAS	7.963	1.687
26. INTERNATIONAL UTILITIES	7.114	1.733
27. INDIANA GAS & WATER	7.478	1.778
28. PUBLIC SERVICE OF NORTH CAROLINA	9.724	1.829
29. MOUNTAIN FUEL SUPPLY	8.281	1.830
30. ARKANSAS WESTERN GAS	8.778	1.938
31. RIO GRANDE VALLEY GAS	5.340	2.033
32. PIONEER NATURAL GAS	5.643	2.084
33. ALAPAMA GAS	8.216	2.196
34. CONSOLIDATED NATURAL GAS	8.727	2.217
35. MISSISSIPPI VALLEY GAS	9.111	2.332
36. MICHIGAN GAS UTILITIES	7.752	2.365
37. KANSAS-NEBRASKA NATURAL GAS	7.486	2.480
38. NORTH PENN GAS	8.029	2.551
39. ATLANTA GAS LIGHT	9.307	2.635
40. PACIFIC LIGHTING	9.025	2.938
41. HOUSTON NATURAL GAS	4.118	3.194
42. GAS SERVICE	8.189	3.248
43. WESTERN KENTUCKY GAS	8.260	3.481
44. NATIONAL GAS & OIL	8.378	3.722
45. MINNEAPOLIS GAS	8.079	4.348
46. CHATTANOOGA GAS	9.357	6.617
47. COMMONWEALTH GAS	9.925	10.461

LISTING OF FIRMS IN RISK CLASS I

FIRM	RATE OF RETURN	COEFFICIENT OF VARIATION
1. PIONEER NATURAL GAS	5.643	2.084
2. ALABAMA GAS	8.216	2.196
3. CONSOLIDATED NATURAL GAS	8.727	2.217
4. MISSISSIPPI VALLEY GAS	9.111	2.332
5. MICHIGAN GAS UTILITIES	7.752	2.365
6. KANSAS-NEBRASKA NATURAL GAS	7.486	2.480
7. NORTH PENN GAS	8.029	2.551
8. ATLANTA GAS LIGHT	9.307	2.635
9. PACIFIC LIGHTING	9.025	2.938
10. HOUSTON NATURAL GAS	4.118	3.194
11. GAS SERVICE	8.189	3.248
12. WESTERN KENTUCKY GAS	8.260	3.481
13. NATIONAL GAS & OIL	8.878	3.722
14. MINNEAPOLIS GAS	8.079	4.348
15. CHATTANOOGA GAS	9.357	6.617
16. COMMONWEALTH GAS	9.925	10.461

LISTING OF FIRMS IN RISK CLASS II

FIRM	RATE OF RETURN	COEFFICIENT OF VARIATION
1. SOUTH JERSEY GAS	6.719	0.525
2. SPRINGFIELD GAS LIGHT	9.128	0.559
3. UNITED GAS IMPROVEMENT	8.262	0.753
4. NORTHERN ILLINOIS GAS	7.725	0.776
5. COLUMBIA GAS SYSTEM	8.528	0.840
6. NORTHWEST NATURAL GAS	7.800	0.878
7. WASHINGTON NATURAL GAS	7.994	0.880
8. BROOKLYN UNION GAS	8.838	0.931
9. WASHINGTON GAS LIGHT	10.401	0.963
10. FLORIDA PUBLIC UTILITIES	10.560	1.031
11. PROVIDENCE GAS	8.728	1.121
12. AMERICAN NATURAL GAS	7.855	1.141
13. NATIONAL FUEL GAS	9.002	1.183
14. OKLAHOMA NATURAL GAS	8.390	1.246
15. PEOPLES GAS LIGHT & COKE	8.424	1.353
16. ARKANSAS LOUISIANA GAS	7.492	1.370
17. LACLEDE GAS	9.023	1.423
18. EQUITABLE GAS	8.812	1.473
19. NEW JERSEY NATURAL GAS	8.184	1.487
20. LONE STAR GAS	6.049	1.513
21. PIEDMONT NATURAL GAS	9.851	1.545
22. UNION GAS OF CANADA	3.697	1.573
23. SOUTHERN UNION GAS	8.461	1.601
24. ELIZEBETHTOWN GAS	7.338	1.684
25. NORTHERN NATURAL GAS	7.963	1.687
26. INTERNATIONAL UTILITIES	7.114	1.733
27. INDIANA GAS & WATER	7.478	1.778
28. PUBLIC SERVICE OF NORTH CAROLINA	9.724	1.829
29. MOUNTAIN FUEL SUPPLY	8.281	1.830
30. ARKANSAS WESTERN GAS	8.778	1.938
31. RIO GRANDE VALLEY GAS	5.340	2.033

ANALYSIS OF VARIANCE CALCULATIONS

SUMMARY OF DATA AND ANALYSIS OF VARIANCE DATA

	RISK CLASS I.	RISK CLASS II.
SUM OF Z(I)'S	130.1027	251.9379
SUM SQUARED	16926.7010	63472.7054
MEAN OF Z(I)'S	8.1314	8.1270
SUM OF SQUARES	1089.5993	2105.4724

NUMBER OF FIRMS IN RISK CLASS I. 16

NUMBER OF FIRMS IN RISK CLASS II. 31

TOTAL NUMBER OF FIRMS 47

TOTAL SUM OF Z(I)'S FOR BOTH RISK CLASSES	382.0406
SQUARE OF THE TOTAL SUM FOR BOTH RISK CLASSES	145954.9874
MEAN OF Z(I)'S FOR BOTH RISK CLASSES	8.1285
SUM OF SQUARES FOR BOTH RISK CLASSES	3195.0718

SOURCE	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F-RATIO
BETWEEN GROUPS	1.	0.000203	0.000203	0.0001
WITHIN GROUPS	45.	89.646298	1.992140	
TOTAL	46.	89.646501		

APPENDIX III

VALUES USED IN REGRESSION ANALYSIS

DEBT TO EQUITY RATIO	GROWTH FACTOR	SIZE MM$S	DIVIDEND YEILD
153.691	9.943	59.143	0.584
178.301	381.733	1009.750	4.847
131.911	236.603	299.929	4.149
84.454	14.176	23.774	4.923
170.428	59.378	129.646	5.080
148.365	96.305	231.618	5.412
81.211	1.331	9.867	5.143
122.390	445.406	1324.378	5.473
2.990	8.671	20.450	2.061
70.252	363.753	852.958	5.577
90.206	26.970	45.450	1.568
113.397	37.482	137.808	5.616
172.110	4.476	11.535	5.556
195.784	34.111	120.623	5.993
197.607	93.579	154.847	1.693
129.328	26.074	61.129	4.829
92.196	70.138	317.638	0.333
158.526	22.994	70.243	4.846
151.179	63.269	148.652	5.464
109.025	95.198	393.386	4.607
281.337	22.586	37.080	5.766
78.714	40.452	70.439	5.683
134.688	7.192	30.247	5.774
101.639	41.640	123.397	5.554
77.704	76.208	244.411	5.680
55.015	-1.910	7.185	0.008
152.639	36.039	52.385	4.692
10.334	3.324	57.655	4.000
100.982	296.462	410.787	4.211
159.828	323.303	665.987	4.918
203.219	57.697	122.070	4.172
120.667	64.580	168.799	5.302
127.445	155.296	807.903	5.455
127.291	379.694	728.886	5.268
207.899	34.264	57.467	5.538
109.290	51.180	98.870	3.692
99.272	9.835	32.150	5.027
486.528	23.492	42.519	5.026
82.059	5.104	15.368	3.797
147.342	21.196	46.910	4.838
156.515	65.855	145.562	4.351
102.714	7.245	23.411	5.928
182.501	225.035	175.323	1.953
77.480	60.251	122.369	5.363
149.026	77.550	225.987	5.437
206.488	56.631	95.084	4.667
201.502	12.713	27.274	5.341

VARIABLE	EXPLAINATION
1	RATE OF RETURN
2	COEFFICIENT OF VARIATION
3	DEBT-TO-EQUITY RATIO
4	GROWTH FACTOR
5	SIZE
6	DIVIDEND YIELD

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFICIENT	STD. ERROR OF REG.COFF.	COMPUTED Z VALUE
2	2.07544	1.67052	0.13804	0.11536	0.12338	0.93498
DEPENDENT						
1	8.12848	1.39598				

INTERCEPT	7.88907
MULTIPLE CORRELATION	0.13804
STD. ERROR OF ESTIMATE	1.39789

ANALYSIS OF VARIANCE FOR THE REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	1	1.70827	1.70827	0.87419
DEVIATION FROM REGRESSION	45	87.93462	1.95410	
TOTAL	46	89.64287		

VARIABLE	EXPLAINATION
1	RATE OF RETURN
2	COEFFICIENT OF VARIATION
3	DEBT-TO-EQUITY RATIO
4	GROWTH FACTOR
5	SIZE
6	DIVIDEND YIELD

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFICIENT	STD. ERROR OF REG.COFF.	COMPUTED Z VALUE
3	138.15741	74.27939	0.01974	0.00037	0.00280	0.13241
DEPENDENT						
1	8.12848	1.39598				

INTERCEPT	8.07724
MULTIPLE CORRELATION	0.01974
STD. ERROR OF ESTIMATE	1.41113

ANALYSIS OF VARIANCE FOR THE REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	1	0.03491	0.03491	0.01753
DEVIATION FROM REGRESSION	45	89.60796	1.99129	
TOTAL	46	89.64287		

VARIABLE	EXPLAINATION
1	RATE OF RETURN
2	COEFFICIENT OF VARIATION
3	DEBT-TO-EQUITY RATIO
4	GROWTH FACTOR
5	SIZE
6	DIVIDEND YIELD

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFICIENT	STD. ERROR OF REG.COFF.	COMPUTED Z VALUE
4	90.30826	118.48157	-0.13261	-0.00156	0.00174	-0.89752
DEPENDENT						
1	8.12848	1.39598				

INTERCEPT	8.26959
MULTIPLE CORRELATION	0.13261
STD. ERROR OF ESTIMATE	1.39894

ANALYSIS OF VARIANCE FOR THE REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	1	1.57646	1.57646	0.80554
DEVIATION FROM REGRESSION	45	88.06642	1.95703	
TOTAL	46	89.64287		

VARIABLE	EXPLAINATION
1	RATE OF RETURN
2	COEFFICIENT OF VARIATION
3	DEBT-TO-EQUITY RATIO
4	GROWTH FACTOR
5	SIZE
6	DIVIDEND YIELD

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFICIENT	STD. ERROR OF REG.COFF.	COMPUTED Z VALUE
5	214.00581	292.45215	-0.00527	-0.00003	0.00071	-0.03537
DEPENDENT						
1	8.12848	1.39598				

INTERCEPT 8.13387

MULTIPLE CORRELATION 0.00527

STD. ERROR OF ESTIMATE 1.41138

ANALYSIS OF VARIANCE FOR THE REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	1	0.00249	0.00249	0.00125
DEVIATION FROM REGRESSION	45	89.64038	1.99201	
TOTAL	46	89.64287		

VARIABLE	EXPLAINATION
1	RATE OF RETURN
2	COEFFICIENT OF VARIATION
3	DEBT-TO-EQUITY RATIO
4	GROWTH FACTOR
5	SIZE
6	DIVIDEND YIELD

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFICIENT	STD. ERROR OF REG.COFF.	COMPUTED Z VALUE
6	4.49350	1.53859	0.42248	0.38332	0.12259	3.12688
DEPENDENT						
1	8.12848	1.39598				

INTERCEPT	6.40602
MULTIPLE CORRELATION	0.42248
STD. ERROR OF ESTIMATE	1.27926

ANALYSIS OF VARIANCE FOR THE REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	1	16.00058	16.00058	9.77734
DEVIATION FROM REGRESSION	45	73.64230	1.63650	
TOTAL	46	89.64288		

APPENDIX IV

```
      DIMENSION FIRM(50,10),EARN(50),CALER(10),DIFF(10),COFV(50)
      DIMENSION XLEVER(50),AFIRM(50,10),ACOFV(50),AROFR(50),ALEVER(50)
      DIMENSION BFIRM(50,10),BCOFV(50),BROFR(50),BLEVER(50),ROFR(50)
      DIMENSION X(50),VAR(10),Y(10)
      DOUBLE PRECISION EARN,CALER,DIFF,COFV,XLEVER,AFIRM,ACOFV,AROFR
      DOUBLE PRECISION ALEVER,BFIRM,BCOFV,BROFR,BLEVER,ROFR,X
      DOUBLE PRECISION TOT10,AVE10,TOTJE,A,B,TOTVAR,TOTRE,PRICE,SHARES
      DOUBLE PRECISION EXPERN,SUMCO,AVECO
      DOUBLE PRECISION ADD1,ADD2,SUM1,SUM2,SQUR,AMEAN,SQR1,SQR2,BMEAN
      DOUBLE PRECISION ADDT,TSQR,TMEAN,SUMT,TKN,AK,C,TSS,BN,BSS,WSS,WDF
      DOUBLE PRECISION WMS
      WRITE (6,20)
   20 FORMAT(1H1,39X,'RATE OF',5X,'COEFFICIENT',4X,'DEBT-TO-EQUITY',4X,'
     1GROWTH',5X,'SIZE',5X,'*COEFFICIENT'/,5X,'FIRM',32X,'RETURN',10X,'O
     2F',12X,'RATIO',9X,'FACTOR',4X,' MM*S ',10X,'OF',/,53X,'VARIATION',
     343X,'DETERMINATION',/)
      READ IN FIRM & THE DATA FOR THE FIRM.
      READ (5,26) NO
   26 FORMAT (I3)
      XNO=NO
      DO 100 I=1,NO
      READ (5,1) (FIRM(I,J),J=1,10)
    1 FORMAT (10A4)
      READ (5,2) (EARN(J),J=1,9)
    2 FORMAT (8F10.0/F10.0)
      READ (5,3) PRIC1,PRIC2,SHARES
    3 FORMAT (F7.3,3X,F7.3,3X,F10.0)
      READ (5,4) DEBT,BKVAL,DIV
    4 FORMAT (2F10.0,1X,F6.3)
      READ (6,5) (Y(J),J=1,9)
    5 FORMAT (9F7.0)
      AVERAGE EARNINGS FOR NINE YEARS IS CALCULATED
      TOT10 = 0.
      DO 110 J=1,9
  110 TOT10 = TOT10 + EARN(J)
      AVE10 = TOT10 / 9.
      REGRESSION EQUATION IS CALCULATED
      TOTJE = 0.
      DO 120 J=1,9
      XJ = 10 - J
  120 TOTJE = TOTJE + XJ * EARN(J)
      B = ( 9. * TOTJE - 45. * TOT10) / 540.
      A = AVE10 - B * 5.
      CALCULATED VALUES OF EARNINGS WITH REGRESSION EQUATION
      TOTVAR = 0.
      TOTRE = 0.
      DO 130 J=1,9
      XJ = J
      L = 10 - J
      CALER(L) = A + B * XJ
      DIFF(L) = (CALER(L) - EARN(L)) * (CALER(L) - EARN(L))
      VAR(L) = (EARN(L) - AVE10) * (EARN(L) - AVE10)
      TOTVAR = TOTVAR & VAR(L)
  130 TOTRE = TOTRE + DIFF(L)
      COEFFICIENT OF VARIATION IS CALCULATED
      VARANC = SQRT(TOTVAR) / 9.
```

```
      COFDET = SQRT(TOTRE) / 9.
      COFV(I) =(COFDET / AVE10) * 100.
      R = (TOTVAR - TOTRE) / TOTVAR
      RATE OF RETURN IS CALCULATED
      TOTINC = 0.
      DO 140 J=1,9
  140 TOTINC = TOTINC & Y(J)
      AVEINC = TOTINC / 9.
      TOTMON = 0.
      DO 141 J=1,9
      XJ = J
  141 TOTMON = TOTMON & XJ * Y(J)
      BB = (9. * TOTMON - 45. * TOTINC) / 540.
      AA = AVEINC - BB * 5.
      EXPERN = AA & B3 * 10.
      PRICE = .5 * (PRIC1 + PRIC2) * SHARES
      ROFR(I) = (EXPERN / PRICE) * 100000.
      THE DEBT-TO-EQUITY RATIO IS CALCULATED
      XLEVER(I) =(DEBT / BKVAL) * 100.
      PRINTOUT OF FIRM, COEFFICIENT OF VARIATION, RATE OF RETURN, &
      DEBT-TO-EQUITY RATIO
      BD = B / 10000.
      ASS = (DEBT & BKVAL) / 1000000.
      R = R * 100.
      WRITE (6,21) (FIRM(I,J),J=1,8),ROFR(I),COFV(I),XLEVER(I),BD,ASS,R
   21 FORMAT(5X,8A4,3X,F7.3,7X,F7.3,9X,F7.3,6X,F8.3,3X,F8.3,6X,F7.3)
      YLD =(DIV / ((PRIC1 & PRIC2) * 0.5)) * 100.
      WRITE (3,921) ROFR(I),COFV(I),XLEVER(I),BD,ASS,YLD
  921 FORMAT (4(F8.3,3X),F8.3,F7.3)
  100 CONTINUE
      END FILE 3
      REWIND 3
      THIS PORTION OF THE PROGRAM DIVIDES UP THE FIRMS INTO THE TWO RISK
      CLASSES
      WRITE (6,665)
  665 FORMAT(1H1,9X,'LISTING OF FIRMS IN ORDER OF INCREASING COEFFICIENT
     1 OF VARIATION',//)
      WRITE (6,223)
  223 FORMAT(9X,'FIRM',33X,'RATE OF',6X,'COEFFICIENT OF',/,47X,'RETURN',
     18X,'VARIATION',/)
      DO 150 I=1,NO
      BCOFV(I)=0.
  150 X(I)=COFV(I)
      J=NO
      DO 160 I=1,NO
      DO 170 K=1,NO
      IF (BCOFV(J)-X(K)) 180,170,170
  180 BCOFV(J)=X(K)
      KT=K
  170 CONTINUE
      X(KT)=0.
      DO 190 L=1,10
  190 BFIRM(J,L)=FIRM(KT,L)
      BROFR(J)=ROFR(KT)
      BLEVER(J)=XLEVER(KT)
      J=J-1
```

```
160 CONTINUE
    SUMCO = 0.
    DO 165 I=1,NO
165 SUMCO = SUMCO + COFV(I)
    AVECO = SUMCO / XNO
    DO 666 I=1,NO
666 WRITE (6,24) I,(BFIRM(I,J),J=1,8),BROFR(I),BCOFV(I)
 24 FORMAT(5X,I2,'. ',8A4,5X,F7.3,9X,F7.3)
    K = 0
    DO 200 I=1,NO
    DO 201 J=1,10
201 FIRM(I,J) = BFIRM(I,J)
    ROFR(I) = BROFR(I)
    XLEVER(I)=BLEVER(I)
200 COFV(I) = BCOFV(I)
    DO 202 I=1,NO
    IF(AVECO - COFV(I)) 203,203,202
203 K = K & 1
    ACOFV(K) = COFV(I)
    DO 204 J=1,10
204 AFIRM(K,J) = FIRM(I,J)
    AROFR(K) = ROFR(I)
    ALEVER(K) = XLEVER(I)
202 CONTINUE
    N = NO - K
    WRITE (6,22) AVECO,K,N
 22 FORMAT (///,5X,'THE AVERAGE OF THE COEFFICIENT OF VARIATION IS',1X
   1,F8.4,/,5X,'THE NUMBER OF FIRMS IN RISK CLASS I IS',2X,I2,/,5X,'TH
   2E NUMBER OF FIRMS IN RISK CLASS II IS',1X,I2)
C     PRINTOUT OF FIRMS IN EACH RISK CLASS
    WRITE (6,23)
 23 FORMAT (1H1,5X,'LISTING OF FIRMS IN RISK CLASS I',/)
    WRITE (6,223)
    DO 210 I=1,K
210 WRITE (6,24) I,(AFIRM(I,J),J=1,8),AROFR(I),ACOFV(I)
    WRITE (6,25)
 25 FORMAT(1H1,5X,'LISTING OF FIRMS IN RISK CLASS II',/)
    WRITE (6,223)
    DO 220 I=1,N
220 WRITE (6,24) I,(BFIRM(I,J),J=1,8),BROFR(I),BCOFV(I)

                     THIS IS THE END OF THIS PORTION OF THE PROGRAM.
                     THE NEXT STEP IS TO MAKE THE STATISTICAL TESTS
                     & A NEW PORTION OF THE PROGRAM WILL BE WRITTEN
                     & TESTED SEPERATELY WITH DATA FROM DUNCAN
    ADD1 = 0.
    ADD2 = 0.
    SUM1 = 0.
    SUM2 = 0.
    WRITE (6,50)
 50 FORMAT (1H1,//5X,'ANALYSIS OF VARIANCE CALCULATIONS')
    WRITE (6,51)
 51 FORMAT (/,5X,'RISK CLASS I.',/,9X,'Z(I)',11X,'Z(I)**2')
    DO 300 I=1,K
    ADD1 = ADD1 + AROFR(I)
```

```
      SQUR = AROFR(I) * AROFR(I)
      SUM1 = SUM1 + SQUR
      WRITE (6,52) AROFR(I),SQUR
   52 FORMAT (5X,F8.3,9X,F9.3)
  300 CONTINUE
      AK = K
      AMEAN = ADD1 / AK
      SQR1 = ADD1 * ADD1
      WRITE (6,53) ADD1,SQR1,AMEAN,SUM1
   53 FORMAT (//,5X,15HSUM OF Z(I)'S    ,F10.3,/,5X,'SUM SQUARED',4X,F11.
     13,/,5X,'MEAN',13X,F9.3,/,5X,'SUM OF SQUARES ',F11.3)
      WRITE (6,50)
      WRITE (6,54)
   54 FORMAT (/,5X,'RISK CLASS II.',/,9X,'Z(I)',11X,'Z(I)**2')
      DO 310 I=1,N
      ADD2 = ADD2 + BROFR(I)
      SQUR = BROFR(I) * BROFR(I)
      SUM2 = SUM2 + SQUR
      WRITE (6,52) BROFR(I),SQUR
  310 CONTINUE
      BN = N
      BMEAN = ADD2 / BN
      SQR2 = ADD2 * ADD2
      WRITE (6,53) ADD2,SQR2,BMEAN,SUM2
      WRITE (6,50)
      WRITE (6,55)
   55 FORMAT (/,5X,'SUMMARY OF DATA AND ANALYSIS OF VARIANCE DATA',//,22
     1X,'RISK CLASS I.',5X,'RISK CLASS II.',/)
      WRITE (6,56) ADD1,ADD2,SQR1,SQR2,AMEAN,BMEAN,SUM1,SUM2
   56 FORMAT (5X,13HSUM OF Z(I)'S,6X,F10.4,8X,F10.4,/,5X,'SUM SQUARED',2
     1(7X,F11.4),/,5X,14HMEAN OF Z(I)'S,6X,F9.4,9X,F9.4,/,5X,'SUM OF SQU
     2ARES',4X,F11.4,7X,F11.4)
      WRITE (6,57)
   57 FORMAT(4X,'--------------------------------------------------------
     1----------',/)
      KN = K + N
      WRITE (6,58) K,N,KN
   58 FORMAT (5X,'NUMBER OF FIRMS IN RISK CLASS I. ',I3,//,5X,'NUMBER OF
     1 FIRMS IN RISK CLASS II.',I3,//,7X,'TOTAL NUMBER OF FIRMS',10X,I3,
     2///)
      ADDT = ADD1 + ADD2
      TSQR = ADDT * ADDT
      TKN = KN
      TMEAN = ADDT / TKN
      SUMT = SUM1 + SUM2
      WRITE (6,57)
      WRITE (6,59) ADDT,TSQR
   59 FORMAT (5X,41HTOTAL SUM OF Z(I)'S FOR BOTH RISK CLASSES,5X,F12.4,/
     1/,5X,'SQUARE OF THE TOTAL SUM FOR BOTH RISK CLASSES ',F12.4/)
      WRITE (6,60) TMEAN,SUMT
   60 FORMAT (5X,36HMEAN OF Z(I)'S FOR BOTH RISK CLASSES,13X,F9.4,//,5X,
     136HSUM OF SQUARES FOR BOTH RISK CLASSES,10X,F12.4//)
      WRITE (6,57)
      C = TSQR / TKN
      TSS = SUMT - C
      BSS = (SQR1 / AK & SQR2 / BN) - C
```

```
      WSS = TSS - BSS
      WDF = TKN - 2.
      WMS = WSS / WDF
      FRATIO = BSS / WMS
      TDF = TKN - 1.
      WRITE (6,61) BSS,BSS,FRATIO,WDF,WSS,WMS,TDF,TSS
   61 FORMAT (5X,'SOURCE',9X,'DEGREES OF',3X,'SUM OF',10X,'MEAN',8X,'F-R
     1ATIO',/,21X,'FREEDOM',5X,'SQUARES',8X,'SQUARES',/,5X,'BETWEEN GROU
     2PS',5X,'1.',3X,F11.6,4X,F11.6,6X,F7.4,/,5X,'WITHIN GROUPS',5X,F3.0
     3,3X,F11.6,4X,F11.6,/7X,'TOTAL',11X,F3.0,3X,F11.6)

                           THIS IS THE END OF THE ANALYSIS
                           OF VARIANCE PORTION OF THE PROGRAM.
                           ADD TWO READ STATEMENTS & THOROUGHLY
                           TEST USING DATA FROM DUNCAN.

      CALL EXIT
      END
          SIZE OF COMMON  00000    PROGRAM  21840
```

```
      DIMENSION XBAR(10),STD(10),D(10),RY(10),ISAVE(10),B(10)
      DIMENSION SB(10),T(10),W(10),RX(50),R(40),ANS(10),Z(10)
      WRITE (6,11)
   11 FORMAT(1H1,5X,'VALUES USED IN MULTIPLE REGRESSION CALCULATIONS',//
     1,5X,'RATE OF',3X,'COEFFICIENT OF',3X,'DEBT TO EQUITY',4X,'GROWTH',
     25X,'SIZE',8X,'DIVIDEND',/,6X,'RETURN',6X,'VARIATION',9X,'RATIO',9X
     3,'FACTOR'4X,' MM*S ',8X,'YEILD',/)
  100 READ (5,1) PR,PRI,N,M,NS
    1 FORMAT(A4,A2,I5,2I2)
      IO=0
      X=0.0
      CALL CORRE (N,M,IO,X,XBAR,STD,RX,R,D,B,T)
      REWIND 3
      WRITE (6,400)
  400 FORMAT (1H1,///,5X,'TABLE OF CORRELATION COEFFICIENTS',//)
      DO 420 I=1,M
      DO 410 J=1,M
      IF (I - J) 402,404,404
  402 L = I & (J *J - J) / 2
      GO TO 410
  404 L = J & (I * I - I) / 2
  410 Z(J) = R(L)
  420 WRITE (6,405) I,(Z(J),J=1,M)
  405 FORMAT (5X,'ROW',I2,/,5X,8F12.5)
      IF(NS) 108, 108, 109
  108 WRITE (6,13)
   13 FORMAT(53H1NUMBER OF SELECTIONS NOT SPECIFIED.  JOB TERMINATED.)
      GO TO 200
  109 DO 200 I=1,NS
      WRITE (6,2)
    2 FORMAT (1H1,5X,'MULTIPLE GEGRESSION',//,5X,'VARIABLE',5X,'EXPLAINA
     1TION',/,9X,'1',9X,'RATE OF RETURN'/,9X,'2',9X,'COEFFICIENT OF VARI
     2ATION'/9X,'3',9X,'DEBT-TO-EQUITY RATIO'/,9X,'4',9X,'GROWTH FACTOR'
     3,/,9X,'5',9X,'SIZE',/,9X,'6',9X,'DIVIDEND YIELD')
      WRITE (6,12)
   12 FORMAT(/,'--------------------------------------------------------
     1--------------------------------',/)
      READ (5,10) NRESI,NDEP,K,(ISAVE(J),J=1,K)
   10 FORMAT(36I2)
      CALL ORDER (M,R,NDEP,K,ISAVE,RX,RY)
      CALL MINV (RX,K,DET,B,T)
      IF(DET) 112, 110, 112
  110 WRITE (6,14)
   14 FORMAT(24H0THE MATRIX IS SINGULAR.)
      GO TO 200
  112 CALL MULTR (N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,T,ANS)
      MM=K&1
      WRITE (6,3)
    3 FORMAT(9H0VARIABLE,5X,4HMEAN,5X,8HSTANDARD,6X,11HCORRELATION,4X,
     1'REGRESSION'4X,10HSTD. ERROR,5X,8HCOMPUTED/6H    NO.,18X,9HDEVIATIO
     2N,7X,6HX VS Y,7X,11HCOEFFICIENT,3X,12HOF REG.COFF.,3X,7HZ VALUE)
```

```
      DO 115 J=1,K
      L=ISAVE(J)
  115 WRITE (6,4) L,XBAR(L),STD(L),RY(J),B(J),SB(J),T(J)
    4 FORMAT(1H ,I4,6F14.5)
      WRITE (6,5)
    5 FORMAT(/,2X,'DEPENDENT')
      L=ISAVE(MM)
      WRITE (6,4) L,XBAR(L),STD(L)
      WRITE (6,12)
      WRITE (6,6) ANS(1),ANS(2),ANS(3)
    6 FORMAT(1H /10H INTERCEPT,13X,F13.5//23H MULTIPLE CORRELATION   ,F13
     1.5//23H STD. ERROR OF ESTIMATE,F13.5)
      WRITE (6,12)
      WRITE (6,7)
    7 FORMAT(1H0,21X,39HANALYSIS OF VARIANCE FOR THE REGRESSION//5X,19HS
     1OURCE OF VARIATION,7X,7HDEGREES,7X,6HSUM OF,10X,4HMEAN,12X,7HF VAL
     2UE/30X,10HOF FREEDOM,4X,7HSQUARES,9X,7HSQUARES)
      L=ANS(8)
      WRITE (6,8) K,ANS(4),ANS(6),ANS(10),L,ANS(7),ANS(9)
    8 FORMAT(30H ATTRIBUTABLE TO REGRESSION    ,I6,3F16.5/30H DEVIATION F
     1ROM REGRESSION     ,I6,2F16.5)
      L=N-1
      SUM=ANS(4)&ANS(7)
      WRITE (6,9) L,SUM
    9 FORMAT(1H ,5X,5HTOTAL,19X,I6,F16.5)
      WRITE (6,12)
  200 CONTINUE
      CALL EXIT
      END
         SIZE OF COMMON  00000    PROGRAM  03732

      SUBROUTINE DATA (M,D)
      DIMENSION D(1)
      READ (3,1) (D(I),I=1,M)
    1 FORMAT(4(F8.3,3X),F8.3,F7.3)
      WRITE (6,2) (D(I),I=1,M)
    2 FORMAT (5X,F7.3,7X,F7.3,9X,F7.3,7X,F7.3,3X,F8.3,5X,F7.3)
      RETURN
      END
         SIZE OF COMMON  00000    PROGRAM  00352

      SUBROUTINE CORRE (N,M,IO,X,XBAR,STD,RX,R,B,D,T)
      DIMENSION X(1),XBAR(1),STD(1),RX(1),R(1),B(1),D(1),T(1)
      DO 100 J=1,M
      B(J)=0.0
  100 T(J)=0.0
      K=(M*M+M)/2
      DO 102 I=1,K
  102 R(I)=0.0
      FN=N
      L=0
      IF(IO) 105, 127, 105
  105 DO 108 J=1,M
      DO 107 I=1,N
```

```
      L=L+1
  107 T(J)=T(J)+X(L)
      XBAR(J)=T(J)
  108 T(J)=T(J)/FN
      DO 115 I=1,N
      JK=0
      L=I-N
      DO 110 J=1,M
      L=L+N
      D(J)=X(L)-T(J)
  110 B(J)=B(J)+D(J)
      DO 115 J=1,M
      DO 115 K=1,J
      JK=JK+1
  115 R(JK)=R(JK)+D(J)*D(K)
      GO TO 205
  127 IF(N-M) 130, 130, 135
  130 KK=N
      GO TO 137
  135 KK=M
  137 DO 140 I=1,KK
      CALL DATA (M,D)
      DO 140 J=1,M
      T(J)=T(J)+D(J)
      L=L+1
  140 RX(L)=D(J)
      FKK=KK
      DO 150 J=1,M
      XBAR(J)=T(J)
  150 T(J)=T(J)/FKK
      L=0
      DO 180 I=1,KK
      JK=0
      DO 170 J=1,M
      L=L+1
  170 D(J)=RX(L)-T(J)
      DO 180 J=1,M
      B(J)=B(J)+D(J)
      DO 180 K=1,J
      JK=JK+1
  180 R(JK)=R(JK)+D(J)*D(K)
      IF(N-KK) 205, 205, 185
  185 KK=N-KK
      DO 200 I=1,KK
      JK=0
      CALL DATA (M,D)
      DO 190 J=1,M
      XBAR(J)=XBAR(J)+D(J)
      D(J)=D(J)-T(J)
  190 B(J)=B(J)+D(J)
      DO 200 J=1,M
      DO 200 K=1,J
      JK=JK+1
  200 R(JK)=R(JK)+D(J)*D(K)
  205 JK=0
      DO 210 J=1,M
```

```
      XBAR(J)=XBAR(J)/FN
      DO 210 K=1,J
      JK=JK+1
  210 R(JK)=R(JK)-B(J)*B(K)/FN
      JK=0
      DO 220 J=1,M
      JK=JK+J
  220 STD(J)= SQRT( ABS(R(JK)))
      DO 230 J=1,M
      DO 230 K=J,M
      JK=J+(K*K-K)/2
      L=M*(J-1)+K
      RX(L)=R(JK)
      L=M*(K-1)+J
      RX(L)=R(JK)
  230 R(JK)=R(JK)/(STD(J)*STD(K))
      FN=SQRT(FN-1.0)
      DO 240 J=1,M
  240 STD(J)=STD(J)/FN
      L=-M
      DO 250 I=1,M
      L=L+M+1
  250 B(I)=RX(L)
      RETURN
      END
         SIZE OF COMMON  00000    PROGRAM  02852

      SUBROUTINE ORDER (M,R,NDEP,K,ISAVE,RX,RY)
      DIMENSION R(1),ISAVE(1),RX(1),RY(1)
      MM=0
      DO 130 J=1,K
      L2=ISAVE(J)
      IF(NDEP-L2) 122, 123, 123
  122 L=NDEP+(L2*L2-L2)/2
      GO TO 125
  123 L=L2+(NDEP*NDEP-NDEP)/2
  125 RY(J)=R(L)
      DO 130 I=1,K
      L1=ISAVE(I)
      IF(L1-L2) 127, 128, 128
  127 L=L1+(L2*L2-L2)/2
      GO TO 129
  128 L=L2+(L1*L1-L1)/2
  129 MM=MM+1
  130 RX(MM)=R(L)
      ISAVE(K+1)=NDEP
      RETURN
      END
         SIZE OF COMMON  00000    PROGRAM  00682

      SUBROUTINE MINV(A,N,D,L,M)
      DIMENSION A(1),L(1),M(1)
      D=1.0
      NK=-N
```

```
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
   10 IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
      J=L(K)
      IF(J-K) 35,35,25
   25 KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI) =HOLD
   35 I=M(K)
      IF(I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI) =HOLD
   45 IF(BIGA) 48,46,48
   46 D=0.0
      RETURN
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
      DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
   65 CONTINUE
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
```

```
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
      D=D*BIGA
      A(KK)=1.0/BIGA
   80 CONTINUE
      K=N
  100 K=(K-1)
      IF(K) 150,150,105
  105 I=L(K)
      IF(I-K) 120,120,108
  108 JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
  110 A(JI) =HOLD
  120 J=M(K)
      IF(J-K) 100,100,125
  125 KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
  130 A(JI) =HOLD
      GO TO 100
  150 RETURN
      END
        SIZE OF COMMON  00000     PROGRAM  01874


      SUBROUTINE MULTR (N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,T,ANS)
      DIMENSION XBAR(1),STD(1),D(1),RX(1),RY(1),ISAVE(1),B(1),SB(1),
     1           T(1),ANS(1)
      MM=K+1
      DO 100 J=1,K
  100 B(J)=0.0
      DO 110 J=1,K
      L1=K*(J-1)
      DO 110 I=1,K
      L=L1+I
  110 B(J)=B(J)+RY(I)*RX(L)
      RM=0.0
      BO=0.0
      L1=ISAVE(MM)
      DO 120 I=1,K
      RM=RM+B(I)*RY(I)
      L=ISAVE(I)
      B(I)=B(I)*(STD(L1)/STD(L))
  120 BO=BO+B(I)*XBAR(L)
      BO=XBAR(L1)-BO
      SSAR=RM*D(L1)
  122 RM= SQRT( ABS(RM))
      SSDR=D(L1)-SSAR
```

```
      FN=N-K-1
      SY=SSDR/FN
      DO 130 J=1,K
      L1=K*(J-1)+J
      L=ISAVE(J)
  125 SB(J)= SQRT( ABS((RX(L1)/D(L))*SY))
  130 T(J)=B(J)/SB(J)
  135 SY= SQRT( ABS(SY))
      FK=K
      SSARM=SSAR/FK
      SSDRM=SSDR/FN
      F=SSARM/SSDRM
      ANS(1)=BO
      ANS(2)=RM
      ANS(3)=SY
      ANS(4)=SSAR
      ANS(5)=FK
      ANS(6)=SSARM
      ANS(7)=SSDR
      ANS(8)=FN
      ANS(9)=SSDRM
      ANS(10)=F
      RETURN
      END
         SIZE OF COMMON  00000    PROGRAM  01356
```