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Common, Multiple and Parametric Lyapunov Functions for a Class of Hybrid Dynamical Systems

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Abstract

This paper considers Lyapunov stability of discontinuous dynamical systems. It is assumed that discontinuities in the system dynamics are caused by some internal (*e.g.*, component failures), and/or external (*e.g.*, controller commands) discrete events. This kind of systems are often called Hybrid Dynamical Systems. Three cases are discussed in this work. First, we consider that a set of continuous-state systems is given and each system from the set shares a common Lyapunov function. Second, stability of sequentially switched systems is investigated by means of multiple Lyapunov functions. Third, some preliminary results are provided for the case when a hybrid system switches between systems with parametric uncertainty.

1. Introduction

Real-time hybrid complex systems (*e.g.*, aircraft, robot) are subject to discontinuities in their dynamics caused by discrete events. Discrete events may result from component failures, changing operating conditions, switching control algorithms, choosing different sensor outputs, etc. In this paper we assume that if an event occurs, the evolution of the continuous-state is governed by a new set of differential/difference equations. Related topics include variable structure controllers, gain scheduling, sample-data systems, motion control systems, systems with parametric uncertainty, and many other dynamical systems, see [1] for references.

Stability (*e.g.*, Lyapunov stability), and cycling behavior of hybrid control systems are difficult to analyze because classical stability theory [9] needs to impose various continuity constraints on the dynamical system. Even very simple hybrid systems violate such continuity requirements. Although some rigorous results have been reported in the literature [3], [13], [20], the field is not yet mature and there still are many issues that require deeper investigation.

We consider three situations that may arise in practical applications of hybrid control systems. First, it is assumed that a set of continuous-state systems is given and each system from the set shares a common Lyapunov function. This approach has been used to prove stability, for instance in robust stability of uncertain systems [10], and asymptotic stability of a class of fuzzy systems [18]. In this paper we present a more practical stability criterion for fuzzy systems than the result in [18]. Furthermore, we use a common Lyapunov function to decouple the design of a multimodal aircraft controller from the design of the discrete-event supervisor. Second, stability of sequentially switched systems is investigated by means of multiple Lyapunov functions [3], [4], [13], and more general and less conservative stability criteria than the results in those references are obtained. Finally some preliminary results are provided for the case when the hybrid system switches between systems with parametric uncertainty [2].

The remainder of the paper is organized as follows. In Section 2 we present some mathematical preliminaries and the problem formulation. Sections 3, 4, and 5 discuss stability of hybrid dynamical systems by means of *common*, *multiple*, and *parametric* Lyapunov functions, respectively. Finally, Section 6 gives some concluding remarks.

2. Preliminaries

2.1 Hybrid Dynamical System

Since we are interested in the qualitative analysis of hybrid systems, a suitable qualitative model of such systems is needed. We adopt the definition of hybrid dynamical system presented in [20] which is an extension of the notion of general dynamical system introduced in [11]. A *hybrid dynamical system* is represented by a five-tuple $\mathcal{H} = \{(T,\rho), (X,d), S, A, T_0\}$ consisting of a time space with metric ρ, a state space with metric d, a family of *motions* S, $A \subset S$, and $T_0 \subset T$. Details are discovered in the original references.

Two results in [20] are important in our work: 1) a hybrid system is defined on an abstract fully ordered time space T which can be *e.g.*, $T \equiv \{(t,k) \in \Re \times N : t \geq 0, k = \lfloor t \rfloor\}$, and 2) any hybrid system $\mathcal{H}$ can be embedded into another dynamical system $\tilde{H}$ defined on real time space $\Re^{+}$. The second result is important because stability properties of $\mathcal{H}$ can be deduced from stability properties of $\tilde{H}$.

2.2 The Problem Definition

Consider the continuous-time system given by

$$\dot{x}(t) = f_i(x(t)), \quad i \in \{1,2,\ldots,N\} \equiv \underline{N} \tag{1}$$

and the discrete-time system

$$x(k+1) = f_i(x(k)), \quad i \in \underline{N} \tag{2}$$

where $x \in X \subseteq \Re^n$ is the continuous state, and $i \in \underline{N}$ is the discrete state. The evolution function $f(\cdot)$ is assumed to be globally Lipschitz, *i.e.*, $f(\cdot) \in C^{\infty}(\Re^n)$. Let Ω_{ij} denote a region of $\Re^n$ such that for any $i \in \underline{N}$ we have: (1) Ω_{ij} with $j \in \underline{N}$ cover the state space $\Re^n$, and (2) Ω_{ij} and Ω_{ik} with $j \neq k$ do not overlap. It is assumed that if the continuous state hits a certain boundary $\partial\Omega_{ij}$ (*i.e.*, a switching event) the discrete state is given as

$$i(t) = g(i(t^-), x(t)) = j, \text{if } i(t^-) = i \text{ and } x(t) \in \Omega_{ij}, \tag{3}$$

where g: $\underline{N} \times X \rightarrow \underline{N}$ is the discrete dynamics.

In this work the discrete dynamics is abstracted away in order to study stability of the continuous-state part of the hybrid system. The hybrid system $\mathcal{H}$ given by (1),((2)) and (3) is *embedded* into a switched system $\tilde{H}$ whose motions are discontinuous.

Some definitions are required in order to proceed.

Definition 2.1 A *valid switching sequence* $\mathcal{S}$, for a given initial continuous state x_0, is defined by the pair (i_k, t_k) as follows

$$\mathcal{S} = \{(i_0, t_0), (i_1, t_1), \ldots, (i_{k-1}, t_{k-1}), (i_k, t_k), \ldots\}, \tag{4}$$

where $t_0 < t_1 < \ldots < t_{k-1} < t_k$, and $i_k \in \underline{N}$. As it can be seen, the evolution of the system is governed by the vector field $f_{i_k}(\cdot)$ on the interval $t_k \leq t < t_{k+1}$. We shall call t_k a *switching time*.

Definition 2.2 We define an increasing time sequence T_k

$$T_k = \{t_0, t_1, t_2, \ldots, t_{k-1}, t_k, t_{k+1}, t_{k+2}, \ldots\} \tag{5}$$

Definition 2.3 If in a ball $B(r_0)$, the function $V(x)$ is positive definite and has continuous partial derivatives, and if its time derivative along any state trajectory $x(\cdot)$ of system $\dot{x} = f(x)$, is negative semi-definite, *i.e.*, $\dot{V}(x) \leq 0$, then $V(x)$ is said to be a *Lyapunov function* for the system $\dot{x} = f(x)$ [16].

Definition 2.4 x_e is an equilibrium point of the hybrid system (1),(2) if $f_i(x_e) = 0$ for all $i \in \underline{N}$.

Definition 2.5 The equilibrium point of the hybrid system (1),(2) is *stable* if for every $\varepsilon > 0$, and $t_0 \in T_0$ there exists a $\delta = \delta(\varepsilon, t_0) > 0$ such that for any $\|x(t_0) - x_e\| < \delta$ and for any $i(t_0) \in \underline{N}$, we have $\|x(t) - x_e\| < \varepsilon$ for all $t \geq t_0$. The equilibrium point is said to be *uniformly stable* if $\delta = \delta(\varepsilon)$. Moreover, the equilibrium point is said to be *asymptotically stable* if it is stable and $\|x(t) - x_e\| \rightarrow 0$ as $t \rightarrow \infty$.

The variable structure nature of hybrid systems provides interesting stability behaviors.

Example 2.6 Consider the following systems:

$$A_{[1,2,3,4]} = \begin{bmatrix} [-0.5,-0.5,0.5,0.5] & [1,7,1,7] \\ [-7,-1,-7,-1] & [-0.5,-0.5,0.5,0.5] \end{bmatrix}.$$

Note that $\lambda(A_1) = \lambda(A_2) = -0.5 \pm j\,2.6458$ and $\lambda(A_3) = \lambda(A_4) = 0.5 \pm j\,2.6458$, then the system is stable for $i = 1, 2$ and unstable for $i = 3, 4$.

Case 1: If A_1 is selected in the second and fourth quadrants and A_2 is selected in the first and third quadrants, the hybrid system is unstable.

Case 2: If A_3 is selected in the first and third quadrants and A_4 is selected in the second and fourth quadrants, the hybrid system is stable.

Case 3: Suppose that a periodic sequence $\mathcal{S}_1$ = {[(1,0), (2,4), (3,6), (4,7.5)], [(1,8),....} is used. The eigenvalues of the transition matrix $\Phi(t_0+\Delta, t_0)$ with period Δ, are $\lambda(\Phi)$ = {-3.8669, -0.0047}. From Willems' theorem [7] we conclude the hybrid system is unstable.

Case 4: Suppose the previous sequence is rescheduled, then $\mathcal{S}_2$ = {[(4,0), (2,0.5), (3,2.5), (1,4)], [(4,8),....}, *i.e.*, the first and the last systems were interchanged. It is easy to verify that the system is asymptotically stable and the eigenvalues of Φ are $\lambda(\Phi)$ = {-0.0545, -0.3363}.

□

Example 2.6 shows some of the stability problems one may encounter in switched hybrid systems. In the remainder of the paper we present some Lyapunov tools that may help in the stability analysis of a large class of systems.

3. Common Lyapunov Functions in Linear Time-Invariant Systems

For the LTI case, *i.e.*, $\dot{x} = A_i x$ with $i \in \underline{N}$ and $A_i \in \Re^{n \times n}$, if a common Lyapunov function exists, then there exist symmetric positive definite matrices P and Q_i such that

$$A_i^T P + P A_i = -Q_i, \quad \forall i \in \underline{N}. \tag{6}$$

We can conclude that the set of systems is robustly stable [10]. Furthermore, any arbitrary *fast* switching sequence between elements in the set is stable [5], and any linear positive combination of the elements of the set is also stable.

A fundamental problem in this section is how to find such a common quadratic Lyapunov function. An explicit construction, assuming that the stability matrices A_i commute pairwise is given in [12]. We present a lemma for the linear continuous time-invariant case which will be useful in the stability analysis presented in this section.

Lemma 3.1 Let Γ_P be the set of asymptotically stable matrices $A \in \Re^{n \times n}$ that share a common quadratic Lyapunov function $V(x) = x^T P x$. Then any linear positive combination of A_i belongs to Γ_P. In a more compact form, we have

$$\forall A_i \in \Gamma_P, \alpha_i \geq 0, \text{If } M = \sum_{i=1}^{N} \alpha_i A_i \Rightarrow M \in \Gamma_P. \tag{7}$$

3.1 Application to Fuzzy Logic Control Systems

A Fuzzy Control System is a special class of hybrid systems where a finite rule base interacts with a continuous-state system. The communication between the fuzzy controller and the controlled plant is by using the so-called *fuzzifier* and *defuzzifier* interfaces. Note that a closed-loop fuzzy system can be seen as a system that *switches* between matrices in a convex set.

Example 3.2 Takagi-Sugeno's Fuzzy Model

The *i*th rule of the this fuzzy system is given by

Rule i: IF x_1 is $\mathbf{X}_{1i}$ and ... and x_n is $\mathbf{X}_{ni}$

THEN $\dot{x} = A_i x + B_i u$, $i = 1,2,\ldots,r$

The inferred final output of the fuzzy system is

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i [A_i x(t) + B_i u(t)]}{\sum_{i=1}^{r} w_i}, \tag{8}$$

The fuzzy controller is given by

Rule i: IF x_1 is $\mathbf{X}_{1i}$ and ... and x_n is $\mathbf{X}_{ni}$

THEN $u_i = -K_i x$, $i = 1,2,\ldots,r$

it yields the following closed-loop fuzzy control system.

$$\dot{x}(t) = \frac{\sum_{i=1}^{r(r+1)/2} v_i H_i x(t)}{\sum_{i=1}^{r(r+1)/2} v_i}, \tag{9}$$

where

$$k = \sum_{m=1}^{j} (m-1) + i, \quad v_k = \begin{cases} w_i w_j & \text{if } i = j \\ 2 w_i w_j & \text{if } i < j, \end{cases}$$
$$H_k = \begin{cases} A_i - B_i K_j & \text{if } i = j \\ \dfrac{(A_i - B_i K_j) + (A_j - B_j K_i)}{2} & \text{if } i < j. \end{cases} \tag{10}$$

The next theorem [18] gives a sufficient condition for stability of system (9).

Theorem 3.3 The equilibrium of system (9) is globally asymptotically stable if there exists a positive definite matrix P such that

$$H_i^T P + P H_i < 0, \qquad i = 1,2,\ldots,\tfrac{r(r+1)}{2}. \tag{11}$$

□

Suppose we have a rule base with r rules, we would need to verify condition (11) $r(r+1)/2$ times. If the rules are chosen carefully, the applicability of Theorem 3.3 improves drastically. For instance, in many applications a fuzzy model is obtained from a linearized model of the plant, and all the consequents of the fuzzy rules share a common matrix B. In this situation we need to check condition (11) only r times. The advantage of this simplification is evident when r becomes larger. We formalize this result with the following proposition.

Proposition 3.4 The equilibrium of system (9) is globally asymptotically stable if there exists a positive definite matrix P such that

$$H_i = A_i - B_i K_i, H_i^T P + P H_i < 0, i = 1,\ldots,r \tag{12}$$

provided that $B_i = B$ for all $i = 1,2,\ldots,r$.

Proof: For a given r, equation (9) can be rewritten as

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} \hat{v}_i H_i x(t)}{\sum_{i=1}^{r(r+1)/2} v_i}, \text{ where } \hat{v}_i = w_i \sum_{j=1}^{r} w_j, \text{ or}$$

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i H_i x(t), \ \sum_{i=1}^{r} \alpha_i = 1, \ \alpha_i \geq 0, \forall i = 1,\ldots r.$$

Asymptotic stability follows directly from the fact that each individual matrix H_i shares a common Lyapunov function. However in this case only r Lyapunov equations have to be solved.

3.2 Application to a Stable Supervisory Behavior Control System

In intelligent control the decisions of the supervisor are associated with changes in the closed-loop system behavior. From the continuous-state point of view a *behavior* is determined by selecting a control algorithm, an output function, and a reference trajectory from the corresponding libraries, see Fig. 1 below.

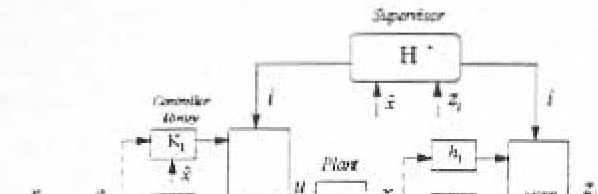

Fig. 1. Closed-loop behavior selection.

Unfortunately, stability of the continuous-state part of the system depends on the decisions of the supervisor. We present an approach to *decouple* the stability of the continuous-part of the hybrid system from the decisions of the higher-level controller. The main idea is to design stable *behaviors* that share a common Lyapunov function. Therefore, any arbitrary sequence between them can be proven to be stable. Note that a *separation principle* holds for the class of hybrid systems which this design approach can be applied. In other words, the design of the logic-based controller and the servo controller can be carried out independently.

Example 3.6 Aircraft Control Design

As an example of a practical hybrid system, we consider the longitudinal dynamics of the F-16 aircraft shown in Fig. 2 [17]. We are interested in three control modes: (1) normal acceleration control N_z, (2) pitch-rate control q, and (3) angle of attack control α. The discrete-state (*i.e.*, finite state machine) of this hybrid system is depicted in Fig. 3. Note that stability of this class of systems is normally carried out by extensive simulation an effective technique which may not reveal the complete behavior of the system.

The differential equations for each control mode are given by

$$\begin{aligned} \dot{x} &= A_i x + B_i u + G_i r, \\ y &= x, \quad u = -K_i y, \\ z &= H_i x + F_i u \end{aligned} \qquad (13)$$

where the state vector $x = [\alpha \;\; q \;\; w]^T$ consists of the angle of attack, the pitch-rate, and the output of an integral controller respectively. The input command $u \equiv \delta_e$ is the angle of the elevator.

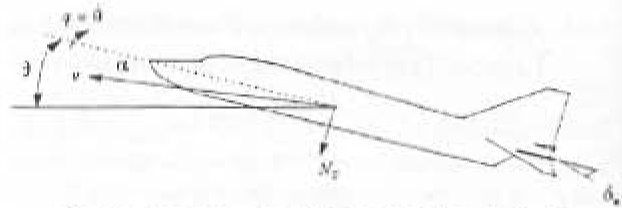

Fig. 2. Notations for the aircraft longitudinal dynamics.

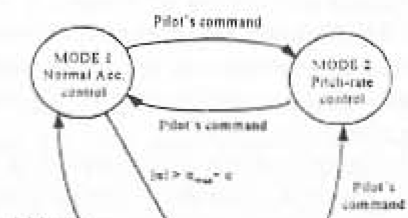

Fig. 3. Discrete-state dynamics.

The feedback control gains $K_i = [k_\alpha \;\; k_q \;\; k_w]$ were selected by using the LQ design with the following cost function per mode

$$J_i = \int_0^\infty (x^T Q_i x + u^T R_i u)\,dt. \qquad (14)$$

Next step was to verify if the closed-loop matrices $A_{C_i} = (A_i - B_i K_i)$ for i = 1,2,3, share a common Lyapunov function P such that

$$A_{C_i}^T P + P A_{C_i} = -Q_i. \qquad (15)$$

The problem of finding the feedback gains K_i can be cast into a convex programming framework for which very efficient algorithms have been developed [6]. Let K denotes the set of stabilizing controller gains that satisfy a given performance index per mode, and $K_P \subset K$ denotes the set of controllers gains for which a common Lyapunov function exists. Assuming that $K_P \neq \emptyset$, we are interested in finding at least one element $\kappa \in K_P$. If such a κ exists, we can conclude that any arbitrary switching sequence between elements of the set $\{A_{C_1}, A_{C_2}, A_{C_3}\}$ is asymptotically stable.

4. Multiple Lyapunov Functions

In general finding a common Lyapunov function that represent a set of systems (linear, nonlinear or both) is not an easy task. *Multiple Lyapunov Function Theory* (MLF) may be used to study stability of switched and hybrid systems.

Lyapunov stability of sequentially switched vector fields (1), (2) is addressed in [3]. In that work it is assume that each individual system is stable. On the other hand, switching between unstable modes is addressed in [13].

We should like to study stability of a more realistic hybrid system where switching sequences may be periodic, aperiodic, finite or infinite, and include some unstable modes. In the next theorem we provide some stability criteria which are more general and less conservative than the results in [3], [13].

Theorem 4.1 Assume that there exists a finite number of scalar positive definite functions $V_i(x)$: $X \to \Re^+$, with $i = 1,2,\ldots,N$ and continuous first order partial derivatives, corresponding to the continuous-state vector fields $\dot{x} = f_i(x)$ with $f_i(0) = 0$, for all i.

(*a*) Let $\mathcal{S}$ be the set of all valid switching sequences associated with the system, and t_k be the switching times. If the following holds

(i) There exists a positive definite function $\phi(x)$ such that

$$DV(x(t_j)) \le -\phi(x), \qquad (16)$$

where

$$DV(x(t_j)) \equiv V_{i_{j+1}}(x(t_{j+1})) - V_{i_j}(x(t_j)), \forall i_j \in \underline{N}. \quad (17)$$

(ii) V_i is radially unbounded, *i.e.*, $\lim_{\|x\| \to \infty} V_i(x) = \infty$,

(iii) $t_{min} \le t_{k+1} - t_k < \infty$ with $t_{min} > 0$ $\forall t_k \in T_k$,

then the state of the system globally $\|x(t)\| \to 0$ as $t \to \infty$ over $\mathcal{S}$.

(*b*) If conditions (i)-(iii) in part (*a*) are satisfied and in addition we have

(iv) V_i is nonincreasing and $\dot{V}_i < 0$ $\forall i$,

then the origin of the continuous state space of the hybrid system is a globally uniformly asymptotically stable equilibrium point over $\mathcal{S}$.

(*c*) Let $\mathcal{S}_N = \{(i_0,t_0),(i_1,t_1),\ldots,(i_N,t_N),(i_0,t_0+\Delta),\ldots\}$ be a periodic valid switching sequence with period Δ. If assumptions (ii) and (iii) hold, and (i) is modified as follows

(i*) There exists a positive definite function $\phi(x)$ such that

$$V_{i_j}(x(t_j + \Delta)) - V_{i_j}(x(t_j)) \le -\phi(x), \ \forall i_j \in \underline{N}. \quad (18)$$

then the state of the system globally $\|x(t)\| \to 0$ as $t \to \infty$ over the periodic sequence $\mathcal{S}_N$.

(*d*) Given a hybrid system $\mathcal{H}$ whose dynamics are governed by (1), (3). If assumptions (ii) and (iii) hold, and (iv) is valid for all regions Ω_{ij} and in addition we have

(v) the Lyapunov functions have the same value on the boundaries $\partial\Omega_{ij}$, that is $V_1(x) = V_2(x) = \cdots = V_N(x)$ for all $x \in \partial\Omega_{ij}$ (*c.f.*, [4]),

then the origin of the continuous state space of the hybrid system is a globally asymptotically stable equilibrium point.

Proof: See [8] for details.

Remark In part (*a*) and (*c*) a switching sequence may include some unstable systems, see Example 2.6-*Case* 2 and *Case* 4 respectively.

5. Parametric Lyapunov Functions

Parametric Lyapunov Functions are special cases of MLF. Let an uncertain system be given by

$$\dot{x} = A(q)x \qquad (19)$$

where $A(q) \in \Re^{n\times n}$, and the uncertain parameter $q \in Q$. Stability of (19) has been studied by using common Lyapunov functions [10]. However, as noted in Section 3, finding a common Lyapunov function can be a formidable problem, a better alternative seems to be the so-called *Parametric Lyapunov Function Theory* which is based on the fact: There exists a positive definite symmetric matrix function $P: Q \to \Re^{n\times n}$ such that

Continuous-time $A^T(q)P(q) + P(q)A(q) < 0$, (20)

Discrete-time $A^T(q)P(q)A(q) - P(q) < 0$, (21)

for all $q \in Q$ if and only if the polytopic family of matrices $\mathcal{A} = \{A(q) \in \Re^{n\times n} : q \in Q\}$ is robustly stable [2]. Assuming that the uncertain parameter $q \in Q$ is arbitrarily switched, then the robust stability problem becomes a hybrid stability problem.

Robust stability, *i.e.*, the system is stable for all *frozen* values of the parameter $q \in Q$, does not imply that any arbitrary switching sequence between elements of $\mathcal{A}$ is stable. To guarantee stability of the switched system the parameter time-variations need to be sufficiently slow [14], [15].

Consider a convex hull of two real $n\times n$ matrices $A(\lambda) \in \mathcal{A} = \{A_1, A_2\}$ with $\lambda \in [0,1]$, *i.e.*,

$$A(\lambda) = (1-\lambda)A_1 + \lambda A_2, \qquad (22)$$

and a discrete-time system given by

$$x(k+1) = A(\lambda_k)x(k). \qquad (23)$$

The system (23) is asymptotically stable for any arbitrary *fast* switching sequence if and only if the set $\{A_1, A_2\}$ is asymptotically stable [5]. Supposing this condition does not hold: how much can λ_k vary while still guaranteeing asymptotic stability of the hybrid system (23) ? We provide the answer to this question in the next result.

Proposition 5.1 System (23) is asymptotically stable if $\|A_1\| < \gamma_1$ and $\|A_2\| < \gamma_2$ for all k, and $|\lambda_{k+1} - \lambda_k| < \varepsilon_1$, where ε_1 is given by (26).

Proof: Consider the following Lyapunov function candidate

$$V(x_{k+1}, \lambda_{k+1}) = x_{k+1}^T P_{\lambda_k} x_{k+1}. \qquad (24)$$

By selecting $A_{\lambda_k}^T P_{\lambda_k} A_{\lambda_k} - P_{\lambda_k} = -I$, it can be shown that

$$\Delta V = x_k^T (P_{\lambda_k} - P_{\lambda_{k-1}}) x_k - x_k^T x_k. \qquad (25)$$

By using the same arguments in [14]; that is, vec(P_λ) depends continuously on A_λ. For a given $\varepsilon > 0$, say ½, there exists a $\delta(\varepsilon)$ such that if $\left\| A_{\lambda_{k+1}} - A_{\lambda_k} \right\| < \delta(\varepsilon)$, we have $\left\| P_{\lambda_{k+1}} - P_{\lambda_k} \right\| < \varepsilon$, and equation (25) is negative definite. Therefore system (23) is asymptotically stable and

$$\left\| A_{\lambda_{k+1}} - A_{\lambda_k} \right\| = \left\| (\lambda_{k+1} - \lambda_k)(A_2 - A_1) \right\| < \delta(\varepsilon),$$

then

$$|\Delta\lambda| < \frac{\delta(\varepsilon)}{\left\| A_2 - A_1 \right\|} \equiv \varepsilon_1. \qquad (26)$$

6. Conclusions

This work presents some Lyapunov stability tools for a class of hybrid systems where a discrete-state system supervises a multimodal continuous-state plant. If a *common* Lyapunov function can be associated to each discrete-state of the hybrid system, then stability of the system can be guaranteed for any arbitrarily fast switching scheme. Since this approach is quite conservative, we should expect that stability results based on common Lyapunov functions are quite conservative also. Multiple and parametric Lyapunov functions provide a more general approach to study stability of hybrid and switched systems.

References

[1] P. Antsaklis, W. Kohn, A. Nerode, and S. Sastry eds., *Lecture Notes in Computer Science: Hybrid Systems II*, vol. 999, Springer, 1995.

[2] B. R. Barmish, *New Tools for Robustness of Linear Systems*, Macmillan Publishing Company, New York, 1994.

[3] M. S. Branicky, "Stability of Switched and Hybrid Systems," *Proc. 33th IEEE Conference on Decision and Control*, pp. 3498-3503, Lake Buena Vista, FL. Dec. 1994.

[4] M. Dogruel, and Ü. Özgüner, "Modeling and Stability Issues in Hybrid Systems," in [1], pp. 148-165, 1995.

[5] M. Dogruel, and Ü. Özgüner, "Stability of a Set of Matrices: A Control Theoretic Approach" *Proc. 34th IEEE Conference on Decision & Control*, pp. 1324-1329, New Orleans, LA, Dec. 1995.

[6] L. El Ghaoui, R. Nikoukhah, and F. Delebecque, "LMITOOL: a Package for LMI Optimization," *Proc. 34th IEEE Conference on Decision and Control*, pp. 3096-3101, New Orleans, LA, Dec. 1995.

[7] J. Ezzine and A. H. Haddad, "Controllability and Observability of Hybrid Systems," *Int. J. Control*, vol. 49, no. 6, pp. 2045-2055, 1989.

[8] R. Fierro and F. L. Lewis, "A Framework for Hybrid Control Design," submitted to *IEEE Trans. on Systems, Man, and Cybernetics*, 1996.

[9] W. Hahn, *Theory and Application of Liapunov's Direct Method*, Prentice-Hall, Inc., Englewood Clifts, 1963.

[10] H. Horisberger, and P. Belanger, "Regulators for Linear, Time Invariant Plants with Uncertain Parameters," *IEEE Trans. on Automatic Control*, vol. AC-21, no. 10, pp. 705-708, October 1976.

[11] A. N. Michel, and K. Wang, *Qualitative Theory of Dynamical Systems*, Marcel Dekker, New York, 1995.

[12] K. S. Narendra, and J. Balakrishnan, "A Common Lyapunov Function for Stable LTI Systems with Commuting *A*-Matrices," *IEEE Trans. on Automatic Control*, vol. 39, no. 12, pp. 2469-2471, Dec. 1994.

[13] P. Peleties, and R. A. DeCarlo, "Asymptotic Stability of m-Switched Systems Based on Lyapunov-like Functions," *Proc. of the American Control Conference*, pp. 1679-1683, Boston, June 1991.

[14] W. A. Sethares, B. D. O. Anderson, and C. R. Johnson, Jr., "Adaptive Algorithms with Filtered Regressor and Filtered Error," *Math. Control Signals and Systems*, no. 2, pp. 381-403, 1989.

[15] J. Shamma, and M. Athans, "Guaranteed Properties of Gain Scheduled Control for Linear Parameter-varying Plants," *Automatica*, vol. 27, no. 3, pp. 559-564, 1991.

[16] J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, 1991.

[17] B. L. Stevens and F. L. Lewis, *Aircraft Control and Simulation*, John Wiley & Sons, Inc., New York, 1992.

[18] K. Tanaka, and M. Sugeno, "Stability Analysis and Design of Fuzzy Control Systems," *Fuzzy Sets and Systems*, vol. 45, no. 2, pp. 135-156, 1992.

[19] V. I. Utkin, "Variable Structure Systems with Sliding Modes," *IEEE Trans. on Automatic Control*, vol. AC-22, no. 2, pp. 212-222, April 1977.

[20] H. Ye, A. N. Michel, and L. Huo, "Stability Theory for Hybrid Dynamical Systems," *Proc. 34th IEEE Conference on Decision & Control*, pp. 2679-2684, New Orleans, LA, Dec. 1995.