OPTICAL AND OPTOMECHANICAL RESONATORS
AND THEIR APPLICATIONS
IN
COMMUNICATION AND SENSING

by

FENFEI LIU

B.S., Physics, University of Science and Technology of China, 2005
M.S., Physics, University of New Mexico, 2009

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ABSTRACT

The radiation pressure of the large circulating optical power inside micro-scale high quality factor Whispering-Gallery mode microresonators couples the mechanical deformation of the resonator structure to the optical resonance. This coupling results in damping or amplification of the corresponding mechanical modes. Self-sustained mechanical oscillation takes place when the optomechanical gain becomes larger than mechanical loss. In this dissertation, several applications of optomechanical oscillator (OMO) in communication and sensing are proposed and explored using silica microtoroid resonator. First we investigate the spectrum of the OMO and define “weak” and “strong” harmonic generation regimes based on two distinct spectral behaviors. In weak harmonic regime, an analytical method is proposed to optimize the spectral behavior of an OMO for RF-photonic communication systems. In the strong harmonic regime, we show that OMO spectrum can be used in a read-out system for resonant optical sensing applications. Next, we explore optomechanical RF mixing and its application in RF-photonics. We study optomechanical RF mixing using coupled
differential equations as well as a semi-analytical model that simplifies the calculation of mixed frequency components. Furthermore, optomechanical down-conversion of various waveforms and audio signal from an RF carrier are demonstrated. Here for the first time we show that an OMO can function as a high-resolution mass sensor based on optomechanical oscillation frequency shift. In an OMO based mass sensor, optical power simultaneously serves as an efficient actuator and a sensitive probe for monitoring optomechanical oscillation frequency variations. The narrow linewidth of optomechanical oscillation and the small effective mass of the corresponding mechanical mode result in sub-pg mass sensitivity. We analyze the performance of microtoroid OMO mass sensor and evaluate its ultimate detection limit. The outcomes of our study enable combination of resonant optical sensing with optomechanical sensing in a single device. This so-called “dual-mode” sensing can be a powerful technique for measuring the properties (mass, density and refractive index) of micro/nano-particles and molecules. To boost the optical sensitivity of the dual-mode sensor, we also demonstrate a dynamic sensing method where the resonant photonic sensitivity is improved by over 50 times through thermally induced line narrowing.
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GLOSSARY OF ACRONYMS

CG Conversion gain
EM Electromagnetic
FEM Finite element modeling
FIB Focused Ion Beam
F-SH Fundamental and second-harmonic
FWHM Full width at half maximum
IoF IF-over-Fiber
MEMS Microelectromechanical systems
NEMS Nanoelectromechanical system
OMO Optomechanical oscillator
OMR Optomechanical resonator
PZT Piezoelectricity transducer
QED Quantum electrodynamics
RBW Resolution bandwidth
RF Radio frequency
RoF RF-over-Fiber
SEM Scanning electron microscope
SHG Second harmonic generation
WGM Whispering-gallery mode
WHG Weak Harmonic generation
WSG Whispering-gallery
Chapter 1

Introduction

This dissertation presents the study of optomechanical oscillation and its applications in RF communication and sensing. It is well known that whispering gallery micro cavities can confine and store light in small volume and for relatively long period. In a micro scale whispering gallery mode (WGM) optical resonator, such as a silica microsphere, microtoroid or microring, high optical quality factor (high-$Q$) together with small mode volume ($\sim 100 \lambda^3$, $\lambda$ is wavelength) [1] results in a large circulating optical power, and enable the study of a large variety of nonlinear and quantum optical phenomena at relatively low input power. These include quantum-nondemolition measurements [2, 3], quantum information research [4], Raman scattering [5], narrow linewidth lasing [6-8], harmonic generation [9], parametric oscillation [10], cavity quantum electrodynamics [11, 12], optomechanical interaction through optical force [18, 50] and many other optical effects [15]. This dissertation is focused on self-sustained optomechanical oscillation and its applications. Although we use a specific type of WGM optical microresonator (i.e. microtoroid) as our experimental platform, most outcomes are valid and easily extendable to other types of optomechanical oscillators (OMOs).

It is well known that change in photon momentum generates force due to the momentum conservation, where $F=dp/dt$ ($\Delta p$ is photon momentum change). One of the most famous examples is that the variance of intensity distribution of optical field inserts force on the small dielectric particles. This so called “gradient force” has been used for particle trapping in “optical tweezers”, which was first observed back in 1970 [16]. For any surface explored in the optical field, the change of photon momentum induces
“radiation pressure” to the surface through absorption or reflection. Considering a perfect reflective surface is initially at rest, and then gains a momentum $mV$ through the radiation pressure of a photon ($m$ is mass and $V$ is velocity). Consequently, the surface also gains kinetic energy $\frac{1}{2}mV^2$. Due to energy conservation, the photon now should have energy $E' = hv' = hv - \frac{1}{2}mV^2$, where $v$ and $v'$ are frequencies of the initial and reflected photon. The momentum conservation results in $p = hv/c = hv'/c + mV$. This frequency shift represents the energy flow from the photon to the reflective surface. This energy-momentum transfer process is similar to Compton scattering where the electrons are replaced by the movable surface.

The frequency shift of the scattering photons can be red (Stokes sideband) or blue (anti-Stokes sideband) depending on the direction of the motion of the reflecting surface. In non-resonant case on average the number of red and blue shifted photons are equal (the motion can be simply due to the thermal vibration of the surface). In an optical cavity the spectral dependence of the resonant photons breaks the balance between red and blue shifted photons generated by vibration of the cavity boundary. When there are more red shifted photons than blue shifted ones, the optical mode losses its energy to the mechanical mode. In contrast, when there are more blue shifted photons, the optical mode extracts energy from the mechanical vibrations. The ratio of the numbers of the red and blue shifts photons is controlled by the optical detuning [50]. Optical detuning is defined as the difference between the frequency of input optical power and the resonant frequency of the cavity, or $\Delta\omega_0 = \omega_{\text{laser}} - \omega_{\text{res}}$. $\Delta\omega_0$ represents the optical detuning, and $\omega_{\text{laser}}$ and $\omega_{\text{res}}$ are the laser and resonant frequencies, respectively. When $\Delta\omega_0 < 0$ (red detuning), there are more blue shifted photons than the red shifted ones in the resonant cavity (due
to the filtering effect of the optical transfer function of the resonator). So energy flows from mechanical mode to the optical mode, and consequently reduces the mechanical energy (damps the thermal vibration) of the resonator. This is called optomechanical cooling. In contrast, when Δω > 0 (blue detuning) the filtering effect allows more red shifted photons to be generated inside the cavity (compared to blue shifted photons). It means that the optical force does net work on the cavity and amplifies the mechanical motion [50]. The amount of power flowed from the optical mode to the mechanical mode is called optomechanical gain. When optomechanical gain completely compensates the intrinsic loss of a mechanical mode of the cavity, optomechanical oscillation at the mechanical eigen frequency starts.

Although the radiation pressure based phenomena have been subjects of research since 1960’s [17], the first optomechanical oscillation was reported in 2005 using a high-Q microtoroid WGM resonator [18]. The large circulating optical power (due to high optical-Q) combined with the small effective mass of mechanical modes of the structure make the microtoroid an ideal platform for studying the interaction between optical force and mechanical degrees of freedom. Since the first observation of optomechanical oscillation, various micro/nano scale optomechanical resonator (OMR) cavities have been reported. Double-disk/double-ring [19, 20], zipper photonic crystals [21], distributed Bragg reflectors (DBR) [22] micro disk [23], crystalline disk [24] and Si/Si3N4 microring [25,26] are examples of on-chip OMRs with large optomechanical coupling. These OMRs have been used for fundamental physics studies such as back-action cooling to quantum ground state [28, 29], coherent mixing [27], single atom QED [30], and quantum information processing [31]. In 2006, the first optomechanical cooling was
reported using a silica microtoroid OMR [113]. The OMR was cooled to 11 K from room with a red detuned laser. In 2011, cooling of the mechanical motion to quantum ground state was demonstrated using a nano-scale photonic crystal OMR [29], which enables further experimental studies of quantum mechanics. In addition, coherent excitation mixing due to mechanical modes was reported in 2009 using both double disks and zipper photonic crystals OMRs. This phenomenon is similar to the electromagnetically induced transparency [27] but using the mechanical modes of the OMR instead of the quantum states of the matter. OMR also has potential application in the study of single atom cavity quantum electrodynamics (QED). Motion coupling between a micro-scale silicon nitride film and a single trapped atom has been discussed in a high-Q and high fineness resonant cavity [30]. Recently, schemes have been proposed for generating robust photon entanglement [31] and quantum information processing [114] in optomechanical interfaces via quantum interference. In general, the ability to manipulate atoms by optical force allows realization of new quantum states, and makes OMR a useful tool in quantum physics. Unlike the fundamental physics studies, the practical applications of OMRs has received less attention. In an OMO, a continuous of laser source generates self-sustained mechanical oscillations of the cavity structure without any external feedback or delay. Through optical path modulation, the mechanical oscillations imprint their frequencies and amplitudes onto the transmitted optical power. Preliminary characterization of oscillation frequency and phase noise [32] has confirmed that the microtoroid OMO can function as an optical frequency reference (photonic clock). Preliminary studies have shown that optomechanical cavity is a potential candidate in certain RF photonic systems [33, 34].
In this dissertation, we explore the applications of OMOs in RF photonic communication and sensing. We study the spectrum of optomechanical oscillation and characterize the presence of higher harmonics of the mechanical mode in the spectrum of the detected output power. Two different regimes controlled by input optical power are defined. In weak harmonic generation regime, the optomechanical oscillation energy is mainly in the fundamental mechanical frequency. This regime is suitable when OMO is employed in RF photonic communication systems as a local oscillator. A semi-analytical model is proposed to predict the spectral behavior of the OMO. In strong harmonic generation, we show that the amplitude ratio of the fundamental and harmonic terms are approximately linear to optical detuning change in a specific range. Therefore the spectrum can serve as a sensitive read-out for optical detuning that is useful for resonant optical sensing. Next, we show the feasibility of using optomechanical oscillation in microwave frequency mixing for RF photonic communication. When the optical pump is modulated at RF frequencies, the transmitted output optical power will be modulated by the mixed frequency components. Accordingly, optomechanical down-conversion is demonstrated as a new method of extracting the baseband data from and RF carrier. If the optical pump is modulated by a RF carrier with signal sidebands, and the frequency of RF carrier equals to the optomechanical oscillation frequency, the sidebands will be down-converted to baseband. So the OMO serves as a mixer and a local oscillator in the receiver, which reduces the complexity of sub-carrier RF-photonic links or replace the electronic mixer and local oscillator in a homodyne RF receiver. The down-conversion behavior is experimentally studied and two different models (i.e. the differential equation based and a semi-analytical model) are proposed for theoretical analysis. The
first optomechanical audio signal down-conversion from a carrier is also demonstrated for the first time.

Here for the first time we use the optomechanical oscillator as a mass sensor. Any external mass deposited on an OMO changes the mechanical energy of the eigen mode and consequently the optomechanical oscillation frequency. The optical pump serves both as an actuator of the optomechanical oscillation and a read-out mechanism, so the frequency shift can be easily extracted from the measured RF spectrum. We also show that by monitoring the harmonic components of the fundamental optomechanical oscillation, amplified frequency shift and better detection limit can be achieved. Moreover, we propose a dual-mode sensing method based on OMO. Since an OMO is basically a high-$Q$ microresonator, it is naturally a high resolution optical sensor. Particles in the vicinity of the optical mode will change the effective refractive index and consequently the optical resonance of the OMO. Combining the optomechanical oscillation frequency shift with the optical resonance shift, an OMO is able to detect and characterize particles/molecules through mechanical and optical modes simultaneously. We also study the dynamic line narrowing effect in optical microresonators as a tool to improve the detection limit and resolution of resonant optical sensing. When the laser wavelength is scanned through the resonant wavelength, the thermo-optic effect inside a high-$Q$ optical microresonator can broaden or narrow down the resonance (depending on the scanning direction of the laser wavelength). In the dynamic mode the narrowed resonant linewidth can improve the sensing resolution. We show that by selecting proper scanning speed and the magnitude of input optical power, the sensitivity of a microtoroid sensor can be improved by over 50 times. Finally, we report the preliminary work on
designing and fabricating hybrid microresonators based on two different low-loss materials combined to enable high optical and mechanical $Q$ in a single device. Basically the optical mode is confined in material with low optical absorption (fused silica) and mechanical energy is mainly stored in material with low mechanical loss.

1.1 Dissertation outline and collaborative work

The studies and ideas presented in this thesis were supervised and guided by Prof. Mani Hossein-Zadeh, and few graduate research assistants have also helped the author with different aspects of their efforts. Shoufeng Lan and Yang Deng from Prof. Hossein-Zadeh’s lab helped with fabrication of microtoroids and the SEM imaging. Seyedhamidreza Alaie from Prof. Leseman’s group (Mechanical engineering department) helped with theoretical modeling (Appendix B) of the mass sensor and part of the simulations done by ANSYS. Prof. Zayd C. Leseman contributed to the fabrication procedure of the hybrid optomechanical resonator and generously shared his XeF$_2$ machine for microtoroid fabrication. Edward Gonzales from Center for Integrated Nanotechnologies (CINT) performed the electron beam lithography process for fabrication of hybrid devices.

Chapter 2 serves as a detail introduction of microtoroidal WGM resonators and the optomechanical interaction in these devices. The resonant characteristics of a WGM microtoroid, such as modal properties, coupling and loss, are discussed. Next the principle of optomechanical interaction in WGM optical cavities is introduced followed by the theory of radiation pressure driven optomechanical oscillation. Finally the experimental configurations used in this study are briefly described.
In Chapter 3, the spectrum of the radiation pressure driven OMO is investigated both experimentally and theoretically. The mechanism and behavior of harmonic generation in the spectrum of the detected output power is discussed.

Chapter 4 describes RF frequency mixing and signal down-conversion inside microtoroid OMOs. The mixing mechanism is explained by the similar analytical model presented in Chapter 3, and the mixing process is characterized comprehensively. We show that an OMO can simultaneously operate as a local oscillator and a mixer for RF photonic communication.

Chapter 5 introduces mass sensing using optomechanical oscillation. Mass detection using a microtoroid OMO is demonstrated, and the sensor performance is comprehensively analyzed using finite element modeling and theoretical calculation (Appendix B).

Chapter 6 The dual-mode sensing with OMO is introduced. In this approach the presence and properties of particles/molecules are monitored using both optomechanical oscillation and optical resonance shift. Dynamic thermal line-narrowing is proposed as a technique for enhancing the resolution of resonant optical sensing.

Chapter 7 concludes all works that appear in this thesis and discusses few incomplete efforts along with future directions. New findings about chaotic behavior and period doubling in the RF spectrum of optomechanical oscillation as well as future directions in the study of optomechanical down-conversion, mass sensing and dual-mode sensing are presented. The concept of hybrid OMO, preliminary design and fabrication results and potential applications of the hybrid optomechanical resonators are discussed. Finally, bi-frequency thermo-optomechanical oscillation in a polymer coated microtoroid
and its application in resonant optical sensing are described (Note: the experiments and theoretical calculations [Appendix C] were mainly contributed by Yang Deng. The author contributed to the fabrication, testing and measurement, and helped with the theoretical modeling).
Chapter 2

Whispering gallery modes and optomechanical interaction in high-\(Q\) toroidal microcavities

2.1 WGM in microtoroid resonator

Optical whispering gallery modes (WGM) of light were first observed back in 19\(^{th}\) century by Mie while he studied light scattering by spherical shape dielectrics. In a WGM optical resonator, light is confined by the concave surface of the dielectric-air interface due to the total internal reflection, which is similar to the reflection of sound wave in a circular cathedral hall. Unlike the Fabry–Pérot optical resonators, optical WGM resonators do not require high quality mirrors. The refractive index differences between the materials of resonant cavity and the surrounding medium combined with the geometry curvature of the boundary provide the confinement and guide of light in a circular path around the resonator. The quality \(Q\) factor of a WGM resonator is proportional to the decay time of the circulating optical waves, which in turn is inversely proportional to the surface scattering rate, the optical absorption in the medium making up the WGM resonator, and the bending loss due to the curvature of the cavity. For the most common fused silica microsphere WGM resonators, \(Q\) factor about \(9 \times 10^9\) has been reported [115]. Fused silica has extremely low absorption for the light in visible and near infrared regime, and the melting process provides surface-tension induced reflow that smoothes the surface and minimizes the scattering loss. After the development of silica microspheres, silica microtoroids [37] were developed and serve as one of the on-chip high-\(Q\) (>\(10^8\)) WGM resonators. We use silica microtoroid WGM resonators instead of
silica microspheres for our study, because silica microspheres usually have large effective mass comparing to silica microtoroids resulting in large threshold powers for optomechanical oscillation. Moreover, the modal spectrum of microsphere is very dense and complicated compared to microtoroids (because of the weak confinement in azimuthal direction [38, 39]).

Fig. 2.1. (a)-(d) Diagrams show the fabrication process of silica microtoroid. (e) and (f) side and top view SEM images of a microtoroid.

The fabrication of silica microtoroids only involves simple photolithography, wet etching and XeF$_2$ dry etching [37]. Fig. 2.1 shows the fabrication process for silica microtoroids on SiO$_2$/Si wafers. Photolithography and HF wet etching is applied to fabricate SiO$_2$ disks on Si substrate. XeF$_2$ etching removes the Si under the silica disks and creates silicon pillars as shown in Fig. 2.1 (c). In the final step the SiO$_2$ disks is melted using CO$_2$ laser absorption in glass. The melting, reflow and consequent solidification of SiO$_2$ generates a silica microtoroid with extremely smooth sidewalls (Fig. 2.1 (d)). Fig. 2.1 (e) and (f) show the SEM pictures of a typical silica microtoroid on a silicon pillar. This device can support very high-$Q$ ($\sim 10^8$) WGMs due to the low
absorption loss of silica glass (at 1550 nm) as well as the low scattering loss due to the smooth surface.

2.2 Optical WGMs in microtoroid and their excitation

2.2.1 Optical WGMs of a microtoroidal optical cavity

Whispering-gallery modes (WGMs) in dielectric circular resonators have been subject of study for many years and are well understood [37, 38, 116].

Cross sectional views of WGMs in a dielectric microsphere are given in Fig. 2.2: part (a) shows the top view of a WGM in a spherical microcavity using numerical modeling, and part (b) shows the side view pictures of four different WGMs. Mode numbers \( n \), \( l \), and \( m \) are associated with the radial modes, the polar modes and the azimuthal modes, respectively. The azimuthal modes are degenerate in frequency as shown in Fig. 2.2 (b).

Besides the numerical modeling, the optical WGMs in a spherical dielectric resonator can be calculated by solving Helmholtz equation in spherical coordinates [38, 39] such that \( E_\phi = \Psi(\phi, \theta, r) \) or \( H_\phi = \Psi(\phi, \theta, r) \) for TM or TE waves. Based on the assumption that the direction of polarization associated with the electromagnetic field of a WGM resonator
can be approximated as constant along the same spherical coordinate axes at all points in space in a homogeneous dielectric [39], the field can be separated as \( \Psi(\phi, \theta, r) = \psi_r(r)\psi_\theta(\theta)\psi_\phi(\phi) \) for either TM or TE modes. TE modes possess electric field parallel to the surface of the resonator (i.e. \( E_\phi = E_r = 0, \vec{E} \parallel \hat{\theta} \)), and TM modes possess magnetic field parallel to the surface of the resonator (i.e. \( H_\phi = H_r = 0, \vec{H} \parallel \hat{\theta} \)). Mode numbers can be used to introduce the eigen-functions of the fields, such that

\[
\psi_\phi = \frac{1}{\sqrt{2\pi}} \exp(\pm i m \phi) \tag{2.2.1}
\]

And the polar dependence (\( \psi_\theta \)) satisfies the equation

\[
\frac{1}{\cos \theta} \frac{d}{d\theta} \left( \cos \theta \frac{d}{d\theta} \psi_\theta \right) - \frac{m^2}{\cos^2 \theta} \psi_\theta + l(l + 1) \psi_\theta = 0 \tag{2.2.2}
\]

Finally the radial dependence (\( \psi_r \)) satisfies the equation

\[
\frac{d^2}{dr^2} \psi_r + \frac{2}{r} \frac{d}{dr} \psi_r + \left( k^2 n_{\text{eff}}^2 - \frac{l(l + 1)}{r^2} \right) \psi_r = 0 \tag{2.2.3}
\]

Here \( n_{\text{eff}} \) is the refractive index of the resonator material, and \( k = \frac{2\pi}{\lambda} \) is the wave number.

Similar to the solution of spherical boundary [39], these two equations have solution in the terms of the generalized Legendre Polynomials \( P_m^l (\cos \theta) \) and the Bessel functions \( j_l(kr) \). The non-zero value of \( \psi_r \) when \( r \) larger than the resonator radial indicates the evanescent field leaked out to the surrounding.

Solving the close form solutions of optical WGMs in toroidal micro cavities is far more challenging. The toroidal cavity breaks the spherical symmetric and results in inseparable scalar wave equation in local toroidal coordinates. Although a perturbative analytic method utilizing a perturbation expansion combined with an iterative procedure has been proposed and used to provide predictions of optical WGMs in a microtoroid cavity [116], numerical modeling is usually applied to calculate the WGMs of microtoroidal cavities. By solving the wave equations using a commercial Finite Element
Method (FEM) software (such as COMSOL Multiphysics), we show the intensity profile of the TM WGM ($|E_\phi|^2$) in a typical silica microtoroid resonator for $n=1, m=l=111$ (Fig. 2.3(a)). Fig. 2.3(b) shows the corresponding normalized intensity in the radial direction. The optical field on the right hand side of black line represents the evanescent field. The toroidal geometry of the cavity confines azimuthal ($m$) modes significantly and therefore prevents degeneration. The wavelength of fundamental ($l = m$) and the first order azimuthal ($l = m-1$) modes are separated by few nanometers [118] in a typical microtoroid cavity. Therefore, the optical spectrum of a microtoroidal cavity is usually much cleaner than the spectrum of a microspherical cavity.

![Figure 2.3(a)](image)

![Figure 2.3(b)](image)

**Fig. 2.3.** (a) FEM modeling of the intensity profile $|E_\phi|^2$ of a silica microtoroid. Major diameter $D=41$ µm, minor diameter $d= 6$ µm. Mode numbers $n=1, m=l=111$. Brighter color means stronger intensity. (b) Normalized intensity plotted against radial $r$. The right part of the black line is the evanescent field.

### 2.2.2 Coupling to WGM resonator (microtoroid)

Generally, coupling optical wave to a resonator through evanescent field requires two conditions: mode overlap and phase matching. Mode overlap condition means the incoming optical wave has enough spatial overlap with the evanescent fielded of the WGM in space, and phase matching means matching of their wave vectors ($k$) are almost equal.
There are a few common techniques for evanescent wave coupling into optical WGM microresonators [40-42]. Note that free space coupling based on random scattering light to the WGM is very inefficient because of the difference value between wave vectors in air and the resonator. Fig. 2.4 (a), (b) and (c) show prism coupling, angle-polished fiber coupling and fiber-taper/waveguide coupling, respectively. They are all based on the energy exchange between the evanescent field of WGM and that of the guided or totally reflected wave in a high-index medium. Angled-polished fiber and prism coupling techniques are relatively complicated and hard to optimize, therefore the fiber taper coupling method [43, 44] was developed. This method uses a micro-size diameter tapered fiber to couple light to high-\( Q \) microtoroidal or spherical WGM resonators. Beyond the low transmission and coupling loss, another important benefit of fiber-taper is that the coupling strength can be easily adjusted by tuning the gap between the fiber and the resonator. Recently, waveguides coupled microspheres were also studied and reported [45]. For integrated microdisk WGM resonators, optical waveguides and the microdisk are monolithically fabricated together and on the same chip [47]. In this section, only the fiber-taper coupling will be discussed, because it is the main approach used in all the experiments presented in this thesis.

![Diagram](image-url)

Fig 2.4. Diagrams showing the evanescent optical field coupling to a WGM resonator through (a) Prism; (b) Angle polished fiber; (c) Fiber-taper or optical waveguide.
2.2.3 Pulling of a silica fiber taper

A silica fiber taper is fabricated from a single-mode fiber by heating and simultaneously pulling the fiber in opposite directions.

Fig. 2.5. (a) Pictures show the fiber pulling setup. Fiber is held by the two V-grooves with magnets, which prevent sliding during the pulling process. The holders of the fiber are on the rail, and driven by the motors symmetrically. For optimal heating, the Hydrogen flame should be placed in the appropriate position by the 3-axis stage. Microscope can monitor the fiber dimension during pulling. (b) An optical fiber taper coupling to a silica microtoroid. Red arrow is the optical power, and dash arrows show the coupling between modes in fiber taper and microtoroid. (c) Finite Elements Modeling (FEM) of the propagation of the fundamental WGM inside a field in a silica microtoroid. The microtoroid has resonance at $\lambda \sim 1551.2$ nm with major diameter $D = 45\mu$m and minor diameter $d = 6\mu$m.
A piece of SEM28+ optical fiber is well cleaned after removing the plastic cladding, and held by two movable stages. The positions of the stages are controlled by two motors and can be moved in a constant velocity slowly. The fiber is heated by Hydrogen flame while the stages pull simultaneously until the fiber is narrowed down to about 1 µm as shown in Fig.2.5 (a). A microscope monitors the diameter in real-time during the pulling process. Meanwhile the optical power transmitted through the fiber is monitored by a detector and an oscilloscope. An air clad fiber with 1 µm diameter will support a single mode with relatively large evanescent tails for coupling to the resonator.

2.2.4 Theory of optical wave coupling to WGM resonator (microtoroid)

As the fiber taper is brought close to a microtoroid by accurate Piezo controlled stages, optical wave is able to couple in and out to the microtoroid through evanescent field as shown in Fig.2.5 (b). The coupling strength (qualified by coupling coefficient κ) depends on the gap between the fiber taper and microtoroid. A time domain approach was proposed [47] and discussed comprehensively. Assume the input optical field \( U_{in}(t) = U \exp(-i \omega t) \) is coupled into the resonator, the circulating and output fields can be written as:

\[
A_0(t) = i \kappa U_{in}(t) + R_\kappa A_0(t - \tau_{RT}) \exp(i2\pi n_{eff}L/\lambda_l - \alpha L/2)
\]

\[
U_{out}(t) = R_\kappa U_{in}(t) + i\kappa A_0(t)
\]

Here \( \tau_{RT} = n_{eff}L/c \) is the round trip time of the cavity, \( n_s \) is the refractive index of the resonator, \( c \) is the speed of light, \( L = 2\pi R_0 \) is the optical path, \( R_0 \) is the radius, \( \lambda_l \) is the laser wavelength, and \( R_\kappa^2 = 1-\kappa^2 \). \( \alpha \) is the linear attenuation for the light traveling in the cavity due to intrinsic loss, and \( \lambda_0 \) is the resonant wavelength. \( n_{eff}L = m\lambda_0 \), where \( m \) is an
integer. If the propagation loss is small (which is true for high-$Q$ cavities), by expanding $A_0$ as:

$$A_0(t - \tau_{RT}) = A_0(t) - \tau_{RT} \frac{dA_0}{dt}$$  \hspace{1cm} (2.2.6)

and inserting Eq. (2.2.6) in Eq. (2.2.4) and (2.2.5), we get:

$$\frac{dA_0}{dt} + \left(\frac{\kappa^2}{2\tau_{RT}} + \frac{\alpha c}{2n_{eff}} + i\Delta_0\right)A_0 = i\frac{\kappa\eta}{\tau_{RT}}U_{in}$$  \hspace{1cm} (2.2.7)

$\Delta_0 = \omega_{laser} - \omega_{res}$ is the optical detuning, where $\omega_{laser}$ is the laser frequency and $\omega_{res}$ is the resonant frequency; $\eta \leq 1$ is a coefficient presenting the mode matching and shows how closely the field in the waveguide matches the near field of resonator mode, where $\eta = \kappa U_{in}/\kappa U_{in}$ (bold symbols represent vectors in the WGM propagating direction). $\eta = 1$ when ideal matching happens. Note that $\tau_{ex}$ is the external decay time of the cavity due to coupling, and is related to $\tau_{RT}$ by $1/\tau_{ex} = \kappa^2/\tau_{RT}$. The external quality factor (external $Q$) due to loading is $Q_{ex} = \omega_{res}\tau_{ex}$ and the intrinsic quality factor is $Q_0 = \omega_{res}\tau_0$, where $\tau_0$ is the intrinsic decay time. Therefore, Eq. (2.2.7) becomes

$$\frac{dA_0}{dt} + \left(\frac{1}{2\tau_{ex}} + \frac{1}{2\tau_0} + i\Delta_0\right)A_0 = i\frac{\kappa\eta}{\tau_{RT}}U_{in}$$  \hspace{1cm} (2.2.8)

The circulating as well as the transmission fields can be achieved by solving Eq. (2.2.4), (2.2.5) and (2.2.8) in steady state. Eq. (2.2.8) can also be solved analytically and the stationary solution of (2.2.8) is

$$A_0 = \frac{i}{1 + \frac{1}{2\tau_0} + \frac{1}{2\tau_{ex}} + i\Delta_0} \frac{\eta U_{in}}{\kappa} = \frac{i\eta}{1 + \frac{1}{2\tau_{tot}} + i\Delta_0} \left(\frac{1}{\tau_{ex}\tau_{RT}}\right)^{1/2} U_{in}$$  \hspace{1cm} (2.2.9)
here $\tau_{\text{tot}}$ is the total decay time of the cavity, where $1/\tau_{\text{tot}} = 1/\tau_{\text{ex}} + 1/\tau_0$, and $Q_{\text{tot}} = \omega_{\text{res}} \tau_{\text{tot}}$, $Q_{\text{tot}}$ is total quality factor (called total $Q$ in this thesis). Consequently the transmitted output field in steady state can be written as:

$$U_{out} = U_{in} - U_{in} \frac{\eta/\tau_{ex}}{1/2\tau_0 + 1/2\tau_{ex}} + i\Delta_0$$  \hspace{1cm} (2.2.10)

and the total output intensity has a Lorentzian shape:

$$|U_{out}|^2 = |U_{in}|^2 \left[ 1 - \frac{1}{\tau_{ex}/\tau_0} \eta^2 \right] \left[ \left( \frac{1}{2\tau_0} + \frac{1}{2\tau_{ex}} \right)^2 + \Delta_0^2 \right]$$  \hspace{1cm} (2.2.11)

The field amplitude in the resonator has a maximum at $\tau_{ex} = \tau_0$ ($Q_{ex} = Q_0$). This is known as critical coupling condition. When there is ideal mode matching ($\eta = 1$) and the input optical power is at resonance, the optical power can be coupled in the resonator completely and the transmitted optical power is zero. If the gap between the waveguide and resonator is decreased below its value for critical coupling, the cavity becomes over coupled and $\tau_{ex} < \tau_0$. On the other hand, if the gap between the waveguide and resonator is increased above its value of the critical coupling, the cavity becomes under coupled and $\tau_{ex} > \tau_0$ [48]. The resonant linewidth of the coupled cavity (or the FWHM of the optical resonance) can be written as $\delta \omega = \omega_{\text{tot}} / Q_{\text{tot}} = 1 / \tau_{\text{tot}}$. $Q_{\text{tot}}$ can be explained as:

$$\frac{1}{Q_{\text{tot}}} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ex}}} = \frac{1}{Q_{\text{abs}}} + \frac{1}{Q_{\text{scatt}}} + \frac{1}{Q_{\text{rad}}}$$  \hspace{1cm} (2.3.3.8)

where $Q_{\text{abs}} = 2\pi/(1 - e^{-\alpha/\lambda})$ is the absorption limited $Q$-factor due to the absorption loss for the optical field inside and outside the cavity; $\alpha$ is absorption coefficient; $\lambda$ is wavelength.
$Q_{\text{scatt}}$ is the scattering limited $Q$-factor due to the surface roughness, and $Q_{\text{rad}}$ is the radiation limited $Q$ (controlled by index contrast and curvature of the cavity). For a silica microtoroid, the $Q_0$ can easily exceed $10^9$ [37].

2.3 Optomechanics and optomechanical oscillation

Here optomechanics refers to the interaction between optical field and mechanical motion through optical forces (we only discuss radiation pressure in this dissertation). Optomechanical effects are easier to observe in optical micro resonators because they provide strong optical field build-up and confinement. Optomechanical phenomena was observed and studied in early 1967 by Braginsky and Manukin [17] for the electromagnetic radiation confined in an interferometer or a resonator. The mechanical effects of the electromagnetic field was first observed in a Fabry-Perot resonator in 1983 [49]. Depending on the direction of net power flow (which will be discussed later), the optical field can either damp or amplify the mechanical motion. Here we explain the basic physics of optomechanical interaction using amplification in a radiation pressure driven optomechanical resonator (OMR), as shown in Fig.2.6 (a). The two mirrors in a Fabry-Perot cavity recirculate optical power inside the cavity when the input wavelength matches one of the resonant wavelengths supported by the cavity. The circulating power inside the cavity is much larger than the CW input power due to the high reflection rate of the mirrors (high-$Q$). The radiation pressure force of the optical field moves the right mirror attached to a spring. As the mirror moves, the resonant wavelength of the cavity shifts so the input power drops due to the frequency mismatch between the resonance and the pump laser, as shown in Fig.2.6 (b).
Fig. 2.6. (a) Schematic diagram of the optomechanical interaction between optical field and a moveable mirror inside a cavity. With a CW pump laser power, the mirror displacement $dr$ due to radiation pressure generates oscillating optical output that is shown in time domain. (b) Schematic diagram showing the relation between optomechanical modulation depth and resonant frequency shift. The Lorentzian dip represents the optical transfer function of the resonance. (c) Mechanism of optomechanical interaction.

Consequently the circulating power as well as the radiation pressure decrease, and the mirror moves back by the restoring force of the spring. This periodic oscillation caused
by purely CW laser input modulates the circulating power in the cavity as well as the optical output power. Fig.2.6 (c) shows that in an optomechanical oscillator the energy is exchanged between two energy reservoirs: mechanical resonator and optical resonator. The parameters that play important roles in optomechanical interaction in an OMR are:

1-Optical quality factor $Q$

Both intrinsic ($Q_0$) and loaded (total, $Q_L$ or $Q_{\text{tot}}$) quality factors of the resonator play important roles in cavity optomechanics. Together they define the ability of the cavity to store optical energy.

$$Q_0 = \frac{\lambda_{\text{res}}}{\delta \lambda_0} = \frac{\omega_{\text{res}}}{\delta \omega_0} = \frac{2\pi c}{\lambda_{\text{res}}} \tau_0$$

and

$$Q_L = Q_{\text{tot}} = \frac{\lambda_{\text{res}}}{\delta \lambda} = \frac{\omega_{\text{res}}}{\delta \omega} = \frac{2\pi c}{\lambda_{\text{res}}} \tau_{\text{tot}}$$

Here $\lambda_{\text{res}}$ and $\omega_{\text{res}}$ are the optical resonant wavelength and frequency ($\omega_{\text{res}} = 2\pi c / \lambda_{\text{res}}$, $c$ is the speed of light) of the cavity, respectively. $\delta \lambda_0$ and $\delta \omega_0$ are the intrinsic linewidth (bandwidth, or FWHM, full width at half maximum) of the cavity resonance in wavelength and frequency, respectively. $\delta \lambda$ and $\delta \omega$ are the loaded linewidth. $\tau_0$ and $\tau_{\text{tot}}$ are the intrinsic and loaded cavity photon decay times as defined in section 2.2.4.

2-Mechanical quality factor $Q_{\text{mech}}$

Similarly, the mechanical quality factor of the mechanical resonator represents the mechanical energy loss or the damping factor of the resonator. Larger $Q_{\text{mech}}$ means the resonator has less mechanical loss. The intrinsic mechanical-$Q$ can be written as:

$$Q_{\text{mech}} = \frac{Q_0}{\gamma_0} = \frac{2f_{\text{mech}}}{\gamma_0}$$

Here $Q_0$ is the intrinsic mechanical resonant frequency, and $\gamma_0$ is the intrinsic mechanical damping coefficient (mechanical power dissipation rate).
3-Optomechanical coupling factor

Optomechanical coupling factor \( g_{OM} \) describes the coupling strength between optical and mechanical modes of an OMR/OMO. It is defined as the sensitivity of the optical resonant frequency \( \omega_{res} \) to the mechanical deformation.

\[
g_{OM} = \frac{d\omega_{res}}{d\xi}
\]

Here \( \xi \) quantifies the mechanical deformation of a normal mechanical mode of the OMR structure that induces the change in the optical path length. For the Fabry-Perot cavity shown in Fig.2.6, \( d\xi = dr \). For a typical silica microtoroid (diameter~60 µm), \( d\xi \) is the change in radius, and \( g_{OM} \) is about 40 GHz/nm at the wavelength of 1550 nm. The optomechanically modified cavity resonant frequency is expressed as \( \omega_{res}(\xi) = \omega_{res} - g_{OM}\xi \). Generally, strong optomechanical coupling means that a small mechanical deformation can induce significant change in the resonant frequency and consequently the circulating optical power.

4-Optical detuning

Optical detuning (or detuning) refers to the pump laser frequency detuned from the optical resonant. It is defined as \( \Delta\omega = \omega_{laser} - \omega_{res} \), where \( \omega_{laser} \) is the frequency of the laser, and \( \omega_{res} \) is the resonant wavelength in the absent of optical power. Similarly it can be defined using wavelength as \( \Delta\lambda = \lambda_{res} - \lambda_{laser} \). In the presence of optomechanical interaction, \( \Delta\omega = \omega_{laser} - \omega_{res}(\xi) = \omega_{laser} + g_{OM}\xi \). A normalized detuning is defined as the detuning normalized to the optical linewidth, where \( \Delta\omega_{N} = \Delta\omega / \delta\omega \). When \( \Delta\omega < 0 \), the laser is red detuned and the radiation pressure damps the mechanical motion; when \( \Delta\omega > 0 \), the laser is blue detuned and the radiation pressure amplifies the mechanical motion. The
sign of the detuning controls the direction of energy transfer (from mechanical to optical modes or from optical to mechanical modes), and results in cooling or oscillation. Fig. 2.7 shows the spectral diagrams that help to understand this important concept when the mechanical vibration period is comparable or longer than the cavity lifetime. For a microcavity that has a mechanical eigen frequency of $\omega_0$, the mechanical vibration modulates the pump laser frequency and subsequently generates frequency sidebands at $\omega_{laser} + \Omega_0$ and $\omega_{laser} - \Omega_0$ because of the scattering by the moving microcavity.

![Diagram](image)

Fig. 2.7. Diagrams showing the blue detuned (a) and red detuned (b) pump optical fields in the OMO/OMR cavity as well as the Stokes and anti-Stokes fields of the circulating optical field (in unresolved sideband regime).

The amplitudes of these sidebands (Stokes and anti-Stokes fields [50]) are asymmetric because the cavity enhancement is different for each frequency. Since the high frequency photons have more energy than the low frequency photons ($E=h\nu$), the asymmetric sidebands results in power loss or gain for the optical mode. When the amplitude of anti-Stokes ($\omega_{laser} + \Omega_0$) field is smaller than the amplitude of Stokes ($\omega_{laser} - \Omega_0$) field, the optical mode loses power and amplifies the mechanical motion of the corresponding
mode. As shown in Fig. 2.7, the blue detuning leads to amplification of the mechanical vibrations (above the natural thermal vibrations). By the same argument, for red detuning, the optical pump damps the thermal vibration of the OMR, which is not the focus of this thesis. Both optomechanical amplification and cooling have been observed and verified experimentally [18, 50], and different theoretical approaches were proposed to analyze the behavior of an OMO/OMR.

5-Optomechanical Effective mass \( m_{\text{eff}} \)

Optomechanical effective mass here is defined as the mass involved in the motion at the direction that can change the optical path length of the resonator. This direction is the direction of laser beam for the Fabry-Perot OMR demonstrated in Fig.2.6 (a). For a microtoroid OMR, effective mass is more complicated because various types of deformation can modulate the optical path length. For the harmonic motion, \( m_{\text{eff}} \) can be written as

\[
m_{\text{eff}} = \frac{2E^2}{r_{\text{max}}^2\Omega^2}
\]

Here \( E \) is the total energy stored in the mechanical mode, and \( r_{\text{max}} \) is the maximum displacement in the direction that changes the optical.

2.4 Optomechanical oscillation on silica microtoroid

As mentioned in previous section, the focus of this thesis is the optomechanical oscillation and its applications. Silica microtoroid is the platform chosen for our study, because its high optical-\( Q \), small dimensions, unique geometry, relatively large mechanical quality factor and reasonable large effective mass that make it an ideal platform for low phase-noise optomechanical oscillation. For a silica microtoroid with
$Q_0 \sim 10^8$, 1 mW input power can result in circulating power of 100W. This level of power is more than enough to induce self-sustained optomechanical oscillation in the microscale toroidal structures. The first optomechanical oscillation in air and room temperature was observed and reported in the silica microtoroid in 2005 [18], and the characteristics of microtoroid OMO was comprehensively studied in 2006 [32].

The optomechanical interaction between mechanical motion and the variation of circulating optical field in a cavity can be explained using coupled differential equations in different equivalent forms, such as classical Newtonian mechanics [51], Lagrangian mechanics [52], decomposition to Stokes/anti-Stokes frequencies based [53] and quantum Hamiltonian mechanics [54]. In this thesis, only the simple Newtonian model of a silica microtoroid OMO is investigated. The two coupled time domain differential equations governing the dynamics of mechanical motion and resonant optical field can be driven from one dimensional harmonic oscillator equation and equation (2.2.7):

$$r(t)'' + \gamma_0 r'(t) + \Omega_0^2 r(t) = \frac{F_{rv}}{m_{eff}} = \frac{2\pi m_{eff}}{cm_{eff}} \frac{c n_{eff} \epsilon_0}{m_{eff}} s(t) |A(t)|^2$$  \hspace{1cm} (2.4.1)

$$A(t)' + A(t) \left[ \frac{\omega_l}{2Q_{tot}} + i \Delta \omega_b + i \frac{\omega_0}{R_0} r(t) \right] = i \sqrt{\frac{\frac{2}{c n_{eff} \epsilon_0 s'(t)}}{\tau_{RT} Q_{ex}}} P_{in} \omega_l$$  \hspace{1cm} (2.4.2)

The first equation describes the motion of the harmonic oscillator in the presence of radiation pressure, and the second equation describes the variation of circulating optical field ($A$) in the resonant cavity. Here the optical resonant frequency is a function of radial displacement of the microtoroid [$r(t) = r_{max} \cos(\Omega t)$], so the detuning becomes $\Delta \omega = \omega_{laser} - \omega_{res}(r) = \omega_{laser} - \omega_{res} - \omega_0 r(t)/R_0$. $\gamma_0$ is the intrinsic mechanical damping coefficient discussed in Section 2.3, $F_{rv} = 2\pi n_{eff} P_{cir}/c$ [Appendix A] is the radiation pressure in a circular cavity, where $P_{cir} = s \epsilon_0 n_{eff} |A(t)|^2/2$ is the circulating optical power. $n_{eff}$ is the
effective refractive index of the resonant cavity, \( \varepsilon_0 \) is the vacuum permittivity, \( s \) and \( s' \) are the cross-sectional areas of the optical modes in the cavity and the waveguide, respectively. \( P_{in} \) is the power of the input optical field, and \( A(t) \) is the circulating optical field as discussed in section 2.2. The presence of the radiation pressure modifies the intrinsic mechanical damping \( \gamma_0 \), the spring constant (and consequently the intrinsic eigenfrequency \( \Omega_0 \)) [32]. Eq. (2.4.1) can be written as

\[
\dot{r}(t)'' + \gamma r'(t) + \Omega^2 r(t) = 0
\]  

At blue detuning, the radiation pressure decreases the value of \( \gamma \), according to [32]:

\[
\gamma = \gamma_0 \left( 1 - \frac{P_{in}}{P_{th}(\Delta \omega_0)} \right)
\]  

where \( P_{th}(\Delta \omega_0) \) is the threshold optical pump power required for self-sustained mechanical oscillation. It refers to the minimum input optical power to excite the mechanical oscillation at blue detuning. When the power gain from the optical field overcomes the intrinsic mechanical loss, the OMR starts oscillating. When \( P_{in} < P_{th} \), the resonator is thermally driven and the optical pump only reduces the mechanical loss factor [18], which manifests as the mechanical resonant linewidth narrowing. When \( P_{in} > P_{th} \), \( \gamma < 0 \) and the mechanical loss is canceled by the optomechanical gain. The self-sustained oscillation begins and the oscillation linewidth is no longer described by Eq. (2.4.4), but is limited by the presence of different noise mechanisms in the oscillator system.

Besides, the oscillation frequency also changes in the presence of the optical pump:

\[
\Omega = \Omega_0 \left[ 1 + \zeta P(\Delta_0) P_{in} \right]
\]  

(2.4.5)
where $\zeta_P$ is a coefficient combining the thermal drift due to optical absorption in the structure and the optical spring effect [55]. When the input power is below $P_{th}$, the mechanical mode is thermally driven and the mechanical displacement (modulation depth) is very small. As demonstrated in previous study [18], the mechanical loss is reduced when the optical pump is increased, which appears as the narrowing of the resonant bandwidth. Meanwhile, the resonant frequency also shifts as the optical pump power changes. Once the optical pump goes above the $P_{th}$ the self-sustained oscillation starts and the displacement grows rapidly as a square root of the optical input power. The square root dependence is due to the relation $r \propto \sqrt{E}$ for a mechanical harmonic oscillator. Here $r$ represents the displacement and $E$ represents the energy stored in the mechanical mode.

The steady state solution of the coupled time differential equations in the form of Stoke and anti-Stokes sidebands (proposed in Ref. 50) shows that mechanical frequency shift due to optical spring and the optomechanical gain (or damping rate, depending on the detuning) in unresolved sideband regime ($\Omega << \delta \omega$) can be explained as [50]:

\[
\Delta \Omega_{os} = F^2 \frac{8n^2 \omega_0}{\Omega_0 m_{eff} c^2} C \cdot \left[ \frac{2\Delta_0 \tau_{tot}}{(4\tau_{tot}^2 \Delta \omega_0^2 + 1)} \right] P_{in} \quad C = \frac{\tau_{tot}}{4\tau_{tot}^2 \Delta \omega_0^2 + 1}
\]

\[
\Gamma_{OM} = -F^2 \frac{8n^2 \omega_0 R_0}{m_{eff} c^3} C \cdot \left[ \frac{16\Delta_0 \tau_{tot}}{(4\tau_{tot}^2 \Delta \omega_0^2 + 1)^2} \right] P_{in}
\]

here $F=2\pi \tau_{tot}/\tau_{RT}$ is the Finesse of the optical cavity. Therefore the total (effective) damping rate of the OMO is $\gamma = \gamma_0 + \Gamma_{OM}$. $\Gamma_{OM}$ is the optomechanical gain (damping). The threshold power $P_{th}$ can be calculated as [50]:

\[
\text{Stoke and anti-Stokes sidebands (proposed in Ref. 50) shows that mechanical frequency shift due to optical spring and the optomechanical gain (or damping rate, depending on the detuning) in unresolved sideband regime ($\Omega << \delta \omega$) can be explained as [50]:}

\[
\Delta \Omega_{os} = F^2 \frac{8n^2 \omega_0}{\Omega_0 m_{eff} c^2} C \cdot \left[ \frac{2\Delta_0 \tau_{tot}}{(4\tau_{tot}^2 \Delta \omega_0^2 + 1)} \right] P_{in} \quad C = \frac{\tau_{tot}}{4\tau_{tot}^2 \Delta \omega_0^2 + 1}
\]

\[
\Gamma_{OM} = -F^2 \frac{8n^2 \omega_0 R_0}{m_{eff} c^3} C \cdot \left[ \frac{16\Delta_0 \tau_{tot}}{(4\tau_{tot}^2 \Delta \omega_0^2 + 1)^2} \right] P_{in}
\]

Here $r$ represents the displacement and $E$ represents the energy stored in the mechanical mode.

The steady state solution of the coupled time differential equations in the form of Stoke and anti-Stokes sidebands (proposed in Ref. 50) shows that mechanical frequency shift due to optical spring and the optomechanical gain (or damping rate, depending on the detuning) in unresolved sideband regime ($\Omega << \delta \omega$) can be explained as [50]:

\[
\Delta \Omega_{os} = F^2 \frac{8n^2 \omega_0}{\Omega_0 m_{eff} c^2} C \cdot \left[ \frac{2\Delta_0 \tau_{tot}}{(4\tau_{tot}^2 \Delta \omega_0^2 + 1)} \right] P_{in} \quad C = \frac{\tau_{tot}}{4\tau_{tot}^2 \Delta \omega_0^2 + 1}
\]

\[
\Gamma_{OM} = -F^2 \frac{8n^2 \omega_0 R_0}{m_{eff} c^3} C \cdot \left[ \frac{16\Delta_0 \tau_{tot}}{(4\tau_{tot}^2 \Delta \omega_0^2 + 1)^2} \right] P_{in}
\]

here $F=2\pi \tau_{tot}/\tau_{RT}$ is the Finesse of the optical cavity. Therefore the total (effective) damping rate of the OMO is $\gamma = \gamma_0 + \Gamma_{OM}$. $\Gamma_{OM}$ is the optomechanical gain (damping). The threshold power $P_{th}$ can be calculated as [50]:
In the resolved sideband regime ($\Omega >> \delta \omega$, the mechanical frequency is comparable to or even exceeds the cavity decay rate), the shift of mechanical frequency and the optomechanical gain (damping) rate can be calculated as:

\[
P_{th} = \frac{\Omega_0^2}{Q_{mech}} \frac{m_{eff} c^2}{F^2 8 n_{eff}^3 \omega_0 R_0 C} \left[ \frac{16 \Delta \omega_0 \tau_{tot}}{(4 \tau_{in}^2 (\Delta \omega_0^2 + 1))^2} \right]^{-1} \tag{2.4.7}
\]

\[
\Delta \Omega_{os} = F^2 \frac{8 n_0^2 \omega_0}{\Omega_0 m_{eff} c^2} C \tau_{tot} \left[ \frac{\Delta \omega_0 - \Omega_0}{4(\Delta \omega_0 - \Omega_0)^2 \tau_{tot}^2 + 1} + \frac{\Delta \omega_0 + \Omega_0}{4(\Delta \omega_0 + \Omega_0)^2 \tau_{tot}^2 + 1} \right] P_{in} \tag{2.4.8.a}
\]

\[
\Gamma_{OM} = -F^2 \frac{8 n_0^2 \omega_0}{\Omega_0 m_{eff} c^2} C \cdot \left[ \frac{1}{4(\Delta \omega_0 - \Omega_0)^2 \tau_{tot}^2 + 1} - \frac{1}{4(\Delta \omega_0 + \Omega_0)^2 \tau_{tot}^2 + 1} \right] P_{in} \tag{2.4.8.b}
\]

The threshold power $P_{th}$ can be calculated as [50]:

\[
P_{th} = \frac{\Omega_0^2}{Q_{mech}} \frac{m_{eff} c^2}{F^2 8 n_{eff}^3 \omega_0 R_0 C} \left[ \frac{1}{4(\Delta \omega_0 - \Omega_0)^2 \tau_{tot}^2 + 1} - \frac{1}{4(\Delta \omega_0 + \Omega_0)^2 \tau_{tot}^2 + 1} \right]^{-1} \tag{2.4.9}
\]

The threshold power is proportional to $m_{eff}$ and $\Omega_0$, and inversely proportional to $Q_{mech}$ and $1/Q_{tot}^3$[18], so high-$Q$ (both optical and mechanical) resonators are needed for low threshold optomechanical oscillation.

The oscillation linewidth of a microtoroid OMO has also been studied theoretically and experimentally [32]. In the absence of optical pump, the resonant linewidth $\delta \Omega$ is dominated by the mechanical loss, and can be extracted from $Q_{mech}$ as $\delta \Omega_0 = \Omega_0/Q_{mech}$. When the optical pump power is below threshold ($P_{in} < P_{th}$), $\delta \Omega$ is narrowed by the radiation pressure and Eq. (2.4.4) can be used to estimate the linewidth ($\delta \Omega \sim \gamma$). Above threshold ($P_{in} > P_{th}$), oscillation linewidth decreases dramatically and
enters the sub-Hz regime. It is limited by different noise mechanisms in the oscillator system and required a modified system of equations including the impacts of optical pump noise and thermal noise. To avoid going off the main objective, which is the applications of optomechanical oscillation, a simple explanation [32] based on the general theory of line narrowing in self-sustained oscillators is introduced here. This model is obtained from the similar theories for optical oscillators (lasers) and electronic oscillators [56-58]. The same way the spontaneous emission is the dominant noise mechanism in laser cavity above threshold, the governing noise source in an OMO is thermal noise. The corresponding oscillation linewidth can be written as [32]:

\[ \delta \Omega = \frac{k_B T}{2P_d} (\delta \Omega_0)^2 \]  

(2.4.10)

\( P_d \) is the oscillator output power (dissipated power from the oscillator \( P_d = \Omega_0 E_{\text{stored}}/Q_{\text{mech}} \)), \( k_B \) is the Boltzmann constant, \( T \) is temperature, and \( \delta \Omega_0 \) is the linewidth in the absent of optical power. Experimental measurements have verified that the oscillation linewidth varies inversely with the optomechanical oscillation amplitude and therefore the mechanical energy stored in the oscillator according to Eq. (2.4.10). So the short-term stability of the OMO at room temperature is limited by thermo-mechanical noise (also referred to as Brownian motion). The Equation (2.4.10) can be written in terms of measurable parameters [32]:

\[ \delta \Omega = \left( \frac{4k_B T Q_{\text{tot}}^2}{m_{\text{eff}} \Omega_0^2 R_0^2} \right) \frac{\Gamma \delta \Omega_0}{M^2} \]  

(2.4.11)

where \( M = P_{\text{mod}}/P_{\text{max}} \) is the optical modulation depth induced by resonator motion, \( P_{\text{mod}} \) and \( P_{\text{max}} \) are the modulated and maximum power of the output (see Fig. 2.6 (b)), \( \Gamma \) is the
optical modulation transfer function that can be estimated by solving Eq. (2.4.1) and (2.4.2).

Finally, we briefly discuss the oscillatory mechanical modes of the microtoroid OMO. A large number of mechanical eigen modes inhibit in a microtoroidal resonator structure, but most of them are weakly coupled to the circulating optical power through radiation pressure. In other words, a large number of modes (deformations) have very small \( g_{OM} \). Optomechanical interaction requires that the deformations are able to generate enough optical path length change (large \( g_{OM} \)). Fig. 2.8 (a) shows the fundamental WGM in a microtoroid cavity and the direction of the radiation pressure \( F_{rp} \). The torque generated by \( F_{rp} \) results in various deformations of the toroidal structure. Coupling to different WGMs (i.e. \( l = m-1 \) mode) can slightly change the torque. Clearly, radiation pressure cannot transfer energy to the mode that only has rotation deformation (in \( \phi \) direction). Note that eigen modes with axial symmetric deformation may have more chance to be excited because of the radiation pressure distribution. Moreover, for efficient energy transfer, the periods of the eigen modes should be longer than the photon lifetime of the loaded cavity, which limits the eigen frequencies to those smaller than the optical bandwidth. Combining the two factors discussed above, the threshold power \( P_{th} \) is used as a figure of merit in the oscillation mode selection. Eigen modes that satisfy the above conditions can essentially be excited if the input optical power is above \( P_{th} \). However, the eigen mode with lowest \( P_{th} \) will be excited first and dominates the mechanical motion. The power is mainly coupled into this eigen mode, and increasing optical power increases the oscillation amplitude. Because \( P_{th} \) is a complex function of several parameters, (i.e. optical coupling, detuning, microtoroid dimensions, \( \Omega_0 \), and \( m_{eff} \)),
which vary among eigen modes and microtoroids, the dominant oscillatory mode depends on the OMR properties and the input parameters and cannot be predicted easily. In a single microtoroid, oscillating mode selection by adjusting optical coupling and/or detuning has been reported [59]. In thesis, we use the eigen modes naturally excited in our experiments, previously reported results [18, 32, 51] and the study in Ref. 60.

Fig. 2.8. (a) A diagram shows the cross section of the mode and motion of a microtoroid. The red regime shows the WGM, and \( F_{RP}(t) \) is the radiation pressure. (b) and (c): FEM simulated mechanical displacement of a microtoroid for the 1st and 3rd axial symmetric modes. Color indicates the amount of displacement. Color indicates the displacement. \( \text{D}=50 \ \text{µm}, \text{D}_P=10 \ \text{µm} \). (d), (e) and (f): FEM simulated mechanical displacement of a microtoroid for 3rd axial symmetric mode and two asymmetric modes with different microtoroid dimensions. \( \text{D}=133 \ \text{µm}, \text{D}_P=10 \ \text{µm} \). The color represents the relative displacement normalized to the microtoroid itself. \( f_{\text{mech}}=\Omega_0/2\pi \).

Fig. 2.8 (b) and (c) show the deformation of the 1st (radially symmetric flexural mode) and 3rd (radial breathing mode) axial symmetric modes of a microtoroid with pillar
diameter $D_p=10$ µm and disk diameter $D=50$ µm using FEM simulation. Fig. 2.8 (d), (e) and (f) shows the 3rd axial symmetric mode and two other asymmetric modes of a microtoroid with different disk dimension ($D_p=10$ µm, $D=133$ µm). The color represents the relative displacement normalized to the microtoroid itself. As the dimension of the disk increasing, the deformation of the 3rd axial symmetric mode is changed significantly and has more ripples. Note that asymmetric modes can appear on either small or large microtoroids.

2.5 A brief overview of existing optomechanical cavities

Optomechanical interaction has been observed in variety of optical microresonators. As discussed in the previous sections, silica microtoroid OMO has simple and low cost fabrication process. With relatively low threshold power (less than 1mW) [32], we can excite the optomechanical oscillation of a silica microtoroid (in atmosphere and room temperature environment). The optomechanical oscillation phase noise of a silica microtoroid is low compared to other micro-size OMOs reported so far (mainly due to the larger effective mass of its mechanical modes). Low phase noise results in sub Hz linewidth [32] that is very important for the communication and sensing applications. However, $g_{OM}$ (optomechanical coupling factor) that is limited by the device geometry and dimensions, is smaller for a silica microtoroid. Another disadvantage of microtoroid is the fact that it is not fully integrated and requires an off-chip fiber-taper for optical coupling. In spite of these disadvantages, for exploring the applications of OMO (the main objective of this thesis), simplicity, reproducibility, low phase noise and controllable coupling were the dominant factors that justified using silica microtoroids.
Note that most of the outcomes of our study are general and can be applied on all radiation pressure driven OMOs.

The closest OMO to a regular silica microtoroid OMR is the spoke-supported microtoroid OMR that has been fabricated by removing the membrane material of a silica microtoroid [96]. Fig. 2.9 (a) shows the SEM image of the device [96]. The optomechanical interaction in this type of cavity is the same as a regular microtoroid, but the mechanical-$Q$ is significantly increased due to reduction of the clamping loss. $Q_{\text{mech}}$ above $10^5$ has been reported on this cavity (compared to $\sim 3000$ for a regular silica microtoroid.

Double micro-disk resonator [19,127,129], which is shown in Fig. 2.9 (b), is another type of OMOs that oscillate in room pressure and temperature. The cavity geometry, consisting of a pair of silica disks separated by a nanoscale gap, enables very strong optomechanical coupling through the gradient force from the WGMs. Although the effective mass of this OMO is small, its low mechanical-$Q$ in air ($Q_{\text{mech}}\sim 4$, due to damping effect of gas molecules trapped in the gap between two microdisks) results in large threshold power in air. Similar to a microtoroid, a fiber tapers is used to coupled light to the WGMs of the double micro-disk OMO.

Recently, Si and SiN micro-ring OMRs were reported in Ref 25 and 26. Fig. 2.9 (c) and (d) show the SEM pictures of the devices. The major advantage of these OMRs is the full integration: both the resonators and the coupling waveguides are fabricated on the same chip and the coupling gap is steady. The CMOS compatible fabrication process allows mass production and makes this type of OMOs a good candidate for applications such as local oscillator. However, due to the material absorption of light (at 1550 nm) for
SiN and the surface quality dependent on etching process, the optical-$Q$ of Si/SiN micro-ring OMO ($Q_0 \sim 10^5$) is significantly less than silica microtoroid. Moreover lower effective mass of the device results in larger phase-noise.

Fig. 2.9 (a) Spoke-supported OMR [96]. (b) Double disk OMR [19]. (c) and (d) Si and SiN micro-ring OMRs [25, 26]. (e) Si microdisk OMR. (f) CaF$_2$ crystalline OMR [24]. (g) Zipper photonic crystal OMR [27]. (h) Nanobeam photonic crystal OMR [29]. (i) Waveguide-DBR OMR [22]. (j) SiN membrane placed in a Fabry-Perot cavity [138].

Besides the micro-ring geometry, Si micro-disk OMR has also been reported [23] as shown in Fig. 2.9 (e). By shrinking the WGM cavity to 4 µm, its $g_{OM}$ is significantly increased ($\sim 722$ GHz/nm) and optomechanical oscillation above GHz is observed in air. The effective mass is small and threshold power of Si micro-disk OMO is relatively low due to the small cavity and the optical-$Q$ of the resonator is limited by the fabrication process, which is similar to Si/SiN micro-ring OMO.

Large dimension OMRs up to millimeter has also been reported in the literature [24] (picture shown in Fig 2.9 (f)). By polishing CaF$_2$ or MgF$_2$ crystals, one can achieve
extremely large $Q$-factors ($Q_0 \sim 10^{10}$, $Q_{\text{mech}} \sim 10^5$) disk WGM resonators, and the optomechanical interaction through radiation pressure coupling has been reported in these cavities. In addition, the crystalline material (i.e. CaF$_2$) has very low absorption in mid-infrared regime comparing to fused silica, and may provide interesting research contents. But the large size and effective mass of the crystalline OMR make it very difficult to excite optomechanical oscillation despite the high-$Q$s.

Optomechanical interaction in photonic crystals resonator with small dimensions down to nanometer has been reported. Zipper [21,27,127] and nanobeam [29,127] photonic crystals are the two most well known cavities. For zipper cavities, the optical modes couple to the mechanical motion of the two beams (as shown in Fig. 2.9 (g)) through gradient force from the uneven optical field. For the nanobeam cavity (as shown in Fig. 2.9 (h)), the optomechanical interaction is caused by the radiation pressure. The extremely small optical mode volumes and device dimensions, results in very strong optomechanical coupling (large $g_{\text{OM}} \sim 800$ GHz/nm) and high oscillation frequencies ($f_{\text{OMO}} \sim \text{GHz}$) in these devices. However, the optical-$Q$ of these photonic crystal OMOs are about 1000 times less than the WGM based OMRs (i.e. microtoroid).

Optomechanical interaction has also been observed in Fabry-Perot cavity with a moving mirror. One of the recently reported Fabry-Perot OMR [126] has mechanical modes with sub-MHz frequencies. Due to the large cavity length ($\sim 25$ mm) and the large effective mass ($\sim 400$ ng) of this device the optomechanical phenomena has been only observed in vacuum.

Optomechanical oscillation has also been demonstrated in the waveguide coupled on-chip DBR (distributed bragg reflector) resonator [22]. As shown in Fig. 2.9 (i), the
optical power is coupled by the waveguide to the cavity and the radiation pressure is able to drive the mechanical oscillation reflectors. However, the optomechanical oscillation of the device has only been observed in vacuum environment.

Finally, strong dispersive coupling has been reported between a Fabry-Perot cavity and a movable SiN thin membrane inside it [128]. The SiN membrane (~50 nm in thickness) is shown in Fig. 2.9 (j). The main advantage of this OMR configuration is the fact the mechanical resonator is not part of the optical boundary of the high-Q optical cavity allowing independent optimization of optical and mechanical quality factors (a challenging task in other configurations) The unique mechanical properties of SiN membrane have $Q_{\text{mech}} \sim 10^7$, and the high reflection of the mirrors offers extremely large optical quality factor $Q_0 \sim 10^9$. However, $g_{\text{OM}}$ for this type of cavities are very small (~0.1 MHz). Note that the Fabry-Perot cavity should be large enough to allow mounting the SiN membrane inside it.

The table in Fig. 2.10 summarizes the parameters of the major optomechanical cavities that have been reported in the literatures. Generally, self-sustained optomechanical oscillation in air and low input pump power requires that the OMR has large optical and mechanical $Q$s, large $g_{\text{OM}}$ and small $m_{\text{eff}}$. To the best of the author’s knowledge, self-sustained optomechanical oscillation has been reported on the optomechanical cavities of silica microtoroid, double micro-disk, Si/SiN micro-ring, Si micro-disk, photonic crystal and waveguide DBR. Considering the low phase noise requirement that we need for our communication and sensing applications, silica microtoroid is a good platform comparing to other OMOs. (Low $m_{\text{eff}}$ results in higher oscillation frequencies but also higher phase noise.)
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Fig. 2.10 Table summarizing the characteristics of the major optomechanical cavities.
Chapter 3

Spectrum of optomechanical oscillator

3.1 Introduction

Optomechanical oscillation involves both mechanical vibration and optical field variation in the cavity. The nonlinearity of the optical transfer function of the optical resonator (Lorentzian shape) translates the single tone mechanical oscillation to an optical output power spectrum that includes harmonic frequencies of the fundamental optomechanical oscillation [18, 32, 51, 61]. The second and higher order nonlinearities are present even at low input optical power (for instance, 2 times of the oscillation threshold [32]). In extreme case, the radiation pressure nonlinearity can results in chaotic optomechanical motion [62]. So far minimal effort has been dedicated to the study and analysis of spectral behavior of OMO. The RF spectrum of OMO with weak harmonic frequency components has been reported in two papers [32, 51, 55], but the behavior of the spectrum (specially in the strong nonlinear regime) and the impact of the optical input power and laser wavelength detuning on different frequency components have not been studied. Considering recent progress toward fabrication of monolithic OMOs [25, 26] that enable their usage as practical devices through integration with on-chip optoelectronic devices and systems, this chapter is dedicated to a more careful analysis of the RF spectrum of detected optical output power of OMO.

Here we explore the spectral characteristics of an OMO both numerically and experimentally. The results show in contrast to previous observations, at certain power and detuning range the amplitude of the second harmonic frequency component exceeds the amplitude of the fundamental oscillation frequency ($f_{OMO}$, that is one of the
mechanical eigen frequencies of the structure [32, 50, 63]). Accordingly we define the strong and weak harmonic generation regimes and show that in weak harmonic generation regime the oscillating transfer-function model is in good agreement with the experimental results. Moreover we show that the power crossing between the fundamental and second harmonic components can be used to extract the resonant optical wavelength at a fixed laser wavelength. This is an important feature that may simplify the readout in resonant optical sensing applications. Although we use silica microtoroid as our experimental platform, given the similarity of the interaction mechanism in all radiation pressure based optomechanical oscillators, our analysis and corresponding results are valid for all kinds of radiation pressure driven OMOs.

3.2 Harmonic generation of optomechanical oscillation

The nonlinearity of optomechanical oscillation can been predicted using the time coupled differential equations shown in section 2.3. Fig.3.1 (a) shows the temporal variation of the mechanical displacement (in radial direction) $r(t)$, circulating optical power $P_{\text{ci}}(t)$ and output power $P_{\text{out}}(t)$ time in a microtoroid OMO calculated using Eq. (2.4.1) and (2.4.2). Because of the nonlinearity of the cavity transfer function (Lorentzian), the linear harmonic motion of the cavity geometry (top figure) induces nonlinear fluctuation of the optical field (middle and bottom figures), and consequently the harmonic components in the frequency spectrum as shown in Fig.3.1 (b). $P_1$, $P_2$, $P_3$ and $P_4$ indicate the amplitude of fundamental, 2$^{nd}$, 3$^{rd}$ and 4$^{th}$ order harmonic components, respectively. The transduction process between the mechanical vibration and the optical field in optomechanical oscillation is similar to the light passing through a modulator.
with a nonlinear transfer function. If optical input is extremely large, the optomechanical oscillation may show chaotic behavior as reported in Ref.62 and section 3.5.

Fig. 3.1. (a) From top to bottom: displacement $r(t)$, circulating power $P_{cir}(t)$ and output power $P_{out}(t)$ plotted against time using coupled time-domain differential equation calculation (Equations 2.4.1 and 2.4.2), respectively. (b) RF spectrum of the output. $P_1$, $P_2$, $P_3$ and $P_4$ indicate the fundamental, 2nd, 3rd and 4th order components, respectively.

Fig.3.2 (a) shows the diagram of the experimental setup used for studying the optomechanical oscillation spectrum. The OMO used here is a silica microtoroid driven by a tunable laser source (center wavelength $\sim$ 1550 nm) through a silica fiber-taper. The transmitted optical power is detected by a photodetector and the frequency spectrum of the detected power is analyzed using an RF spectrum analyzer. Above threshold effective radius of the circulating optical mode and therefore its resonant frequency ($\nu_{res}=\omega_{res}/2\pi$) behave like a harmonic oscillator with a frequency $f_{OMO}$ that is almost equal to the mechanical eigen frequency ($f_0$) of the corresponding mechanical mode [18, 32, 50]. At a fixed laser wavelength the oscillation of this nonlinear transfer function modulates the
optical output power at $f_{OMO}$ and its higher harmonics (i.e. $2f_{OMO}$, $3f_{OMO}$, $4f_{OMO}$ ...). The magnitude of the harmonic components depend on the strength of the nonlinearly in the vicinity of the laser frequency ($\nu_{\text{laser}}$) and the oscillation amplitude. All the \textit{optomechanically} generated frequency components should to be within the transfer function in order to resonate inside the cavity. For the $m$th harmonic this condition can be approximately quantified as $m \times f_N < 2$ (here $f_N = f_{OMO}/\delta \nu$ where $\delta \nu$ is the linewidth of the loaded optical resonance). The schematic diagram in Fig. 3.2 (b) shows the oscillatory transfer function (the Lorentzian function centered at $\nu_0$) and the alignment of the pump laser frequency and optomechanically generated frequency components in the cavity. The oscillation amplitude is proportional to the optomechanical gain $\Gamma_{OM}$. $\Gamma_{OM}$ is proportional to rate of energy transferred from optical mode to mechanical mode, and is controlled by optical detuning ($\Delta \lambda = \lambda_0 - \lambda_{\text{laser}}$, where $\lambda_0$ is the resonant optical wavelength of the microcavity) and the input optical power $P_{in}$ as discussed in section 2.3. For the sake of simplicity here we use normalized input optical power ($P_N = P_{in}/P_{th}$, where $P_{th}$ is the minimum threshold power for self-sustained optomechanical oscillation $P_{th}$) and normalized detuning ($\Delta \lambda_N = \Delta \lambda/\delta \lambda$, where $\delta \lambda$ is the linewidth of the loaded optical resonance, $\delta \lambda = (\lambda_{\text{res}}/\nu_{\text{res}}) \delta \nu$) to characterize the optomechanical output spectrum. Fig. 3.2 (c)-(e) show the RF spectrum of the detected optical power at three different $\Delta \lambda_N$’s and $P_N$’s.

In contrast to the previously reported OMO spectrums [32, 51] where $P_1 > P_2 > P_3$ ($P_m$ is the detected RF power at $m \times f_{OMO}$, $m=1,2,3$), for certain values of $P_N$ and $\Delta \lambda_N$ (Fig.
3.2 (c)) we observe that second harmonic component \( P_2 \) is larger than the fundamental oscillation \( P_1 \).

![Diagram](image)

Fig. 3.2. (a) Schematic diagram of the experimental setup. (b) Diagram showing the frequency components of the optical power circulating inside an OMO. (c)- (e) Measured RF spectrum of a microtoroidal optomechanical oscillator at three different powers and detunings. Here loaded optical quality factor \( Q_L \) is equal to \( 2.67 \times 10^6 \) and \( f_{OMO} = 24.45 \text{ MHz} \). We call this regime the strong harmonic regeneration (SHG) regime. Although at the first glance it seems natural that if \( \Delta \lambda_N \) approaches zero the quadratic behavior of the Lorentzian transfer should modulate the pump laser at \( 2f_0 \) (large \( P_2 \)), a small \( \Delta \lambda_N \) also translates to a small \( g_{OM} \) and oscillation amplitude [50] resulting in a small \( P_2 \) (quadratically dependent on the oscillation amplitude). Moreover the \( 2f_0 \) component is also filtered by the optical transfer function. For example a comparison between Fig. 3.2 (c) and (e) shows that at a fixed detuning depending on \( P_N, P_1 \) can be larger or smaller than \( P_2 \). So in general harmonic generation is a complex interplay among the shape of the transfer function, \( g_{OM}, \Delta \lambda_N \) and \( f_N \).
Fig. 3.3. (a) The detected optical power at fundamental ($f_{\text{OMO}}$), 2nd ($2f_{\text{OMO}}$) and 3rd ($3f_{\text{OMO}}$) harmonic frequencies plotted against normalized input power ($P_N$). Here $Q_L = 1.54 \times 10^6$, $f_{\text{OMO}}=37.29$ MHz, $f_N=0.3$, $P_{th} = 96$ μW, mechanical quality factor ($Q_{\text{mech}}$)=1883 and $\Delta \lambda_N$ is fixed at 0.68. The solid lines are guides for the eye. (b) $P_1$ (solid lines and triangles), $P_2$ (dashed line and squares) plotted against $\Delta \lambda_N$ at four different $P_N$s. Here $P_{th} = 41$ μW, $Q_L = 1.11 \times 10^7$ and $f_{\text{OMO}}=11.6$ MHz ($f_N=0.67$). The solid and the dashed lines are polynomial fits to the measured data. The large circles indicate the F-SH crossing points. (c) Measured (markers) and simulated (solid lines) values of $\Delta \lambda_N^{(1,2)}$ plotted against $P_N$ for 3 different OMOs. T1: $Q_L = 3.06 \times 10^5$, $f_{\text{OMO}}=24.5$ MHz ($f_N=0.39$); T2: $Q_L = 1.3 \times 10^7$, $f_{\text{OMO}}=9.88$ MHz; T3: $Q_L=1.11 \times 10^7$ ($f_N=0.66$), $f_{\text{OMO}}=11.6$ MHz ($f_N=0.67$).

Fig.3.3 (a) shows detected power of an OMO at different frequency components as a function of $P_N$ (here $\Delta \lambda_N$ is fixed at 0.68). Note that when $P_N$ is larger than 1.8 the strength of the second harmonic component ($P_2$) exceed the fundamental oscillation ($P_1$).

Fig.3.3 (b) shows $P_1$ and $P_2$ plotted against $\Delta \lambda_N$ at four different $P_N$s. At low powers although $\Delta \lambda_N$ controls the relative amplitude of different frequency components, $P_1>P_2>P_3$ for all values of $\Delta \lambda_N$. Above certain $P_N$ (called $P_N^{(1,2)}$) we can always find a detuning (called $\Delta \lambda_N^{(1,2)}$) at which $P_1 = P_2$. Below $\Delta \lambda_N^{(1,2)}$, $P_2$ is larger than $P_1$ and above $\Delta \lambda_N^{(1,2)}$, $P_1$ is larger than $P_2$. We call this phenomenon fundamental and second-harmonic
(F-SH) crossing. The F-SH crossing is an important behavior that has not been reported and studied before.

These measurements show that for a given OMO the level of harmonic suppression is controlled by $P_N$ and $\Delta \lambda_N$, and low distortion oscillation at the fundamental frequency ($f_{\text{OMO}}$) can be achieved only if $P_N < P_N^{(1,2)}$ or $\Delta \lambda_N > \Delta \lambda_N^{(1,2)}$ (if $P_N > P_N^{(1,2)}$). Fig. 3.3 (c) shows the measured (markers) and simulated (solid lines) values of $\Delta \lambda_N^{(1,2)}$ plotted against $P_N$ for the OMO used in part (b) (T3) as well as two other OMOs (T1 and T2). As we increase $P_N$ the second harmonic oscillation dominates through a larger detuning range.

Two main theoretical approaches are used to study the dynamics of radiation pressure optomechanical oscillation: time domain and frequency domain analysis [32, 50, 51, 53]. In the time domain approach radiation pressure force couples the differential equations governing the mechanical displacement of the harmonic oscillator and circulating optical power inside the microcavity (Eq. (2.4.1) and (2.4.2)). In the frequency domain approach radiation pressure couples differential equations governing the mechanical displacement and three photon fluxes (i.e. pump laser and the stokes and anti-stokes generated through optomechanical interaction). Using the Fourier transform the time domain approach can predict the RF spectrum of the optical power output while extending the second approach to include the sidebands of the second and third harmonics is a difficult task. As shown in Fig.3.3(c) the experimental results are in good agreement with the theoretical estimations (solid lines) that are calculated using the time domain approach.
The dependence of RF spectrum on $\Delta \lambda_N$ suggests that at a fixed laser wavelength the resonant wavelength shift ($\Delta \lambda_0$) can be directly extracted from the RF spectrum. Given the importance of resonance shift measurement in optical sensing applications [64], this feature may create new applications for OMO. The general behavior of $P_1$ and $P_2$ versus $\Delta \lambda_N$ and at different $P_N$’s is relatively complicated and non-monotonic (see Fig.3.3 (b)). However a careful study shows that near F-SH crossing point $\Delta \lambda_N$ can be extracted from $P_1$ and $P_2$ measured with a high resolution. These techniques have been discussed in detail in section 3.4. Note that although the overall spectral behavior of all OMOs are similar, the effect of the optical and mechanical properties of the optomechanical resonator on the magnitude of $P_N^{(1,2)}$ and $\Delta \lambda_N^{(1,2)}$ is complicated and requires more investigation.

3.3 Weak harmonic generation (WHG) regime and oscillatory transfer function method

In most applications where OMO is used as an RF local oscillator (i.e. RF-photonic communication), a pure oscillation at the fundamental frequency is required and the harmonic components should be suppressed as much as possible. For example, in RF frequency mixing [59] based on an OMO, high purity of the fundamental frequency is required because the presence of harmonic components will generate unwanted signal [65] in the output spectrum. In these applications the fundamental frequency ($f_{OMO}$) should be the dominant component in the RF spectrum of OMO. Here we define the weak harmonic generation (WHG) regime as the power range where $P_1 > P_2 > P_3$ independent of $\Delta \lambda_N$. 

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Clearly to operate in WGH regime $P_N$ should be smaller than $P_N^{(1,2)}$. Assuming $f_N < 1$, in WHG regime the oscillation amplitude of different frequency components can be estimated using an oscillatory transfer function (OTF) instead of the time domain coupled differential equation method used in the previous section. Beyond its simplicity this approximation provides a direct explanation for harmonic generation mechanism in an OMO. In this approach we consider that the pump laser is modulated by the oscillating optical transfer function of the OMO (a Lorentzian with a center frequency oscillating at $f_{OMO}$). From Eq. (2.2.11), we have

$$
|U_{out}(t)|^2 = |U_{in}(t)|^2 \left[ 1 - \frac{1}{\tau_0 \xi} \eta^2 \left( \frac{1}{2\tau_0} + \frac{1}{2\tau_{ex}} \right)^2 + \Delta_0^2 \right] = |U_{in}|^2 \left[ 1 - \frac{1}{\tau_0 \xi} \eta^2 \left( \frac{1}{2\tau_0} + \frac{1}{2\tau_{ex}} \right)^2 + \Delta_0^2 + 4\pi^2 (\nu_{laser} - \nu_{res})^2 \right]
$$

(3.3.1)

where $\nu_{res} = \nu_{res} [1 + H_A \cos(2\pi f_{OMO} t)]$

$U_{in}$ is the input optical field, $U_{out}$ is the transmitted optical field, $H_A = r_{max}/R_0$ is the normalized oscillation amplitude that can be estimated using the optomechanical gain equation (Eq. 2.4.6 .b), $r_{max}$ is the maximum displacement in radial direction, and $R_0$ is the radius of the microtoroid. As remind, $\Delta_0$ is the detuning in frequency $\omega$, $\nu_{laser}$ is the pump laser frequency, and $\nu_{res} = c/\lambda_{res}$. So spectrum of the detected optical output power can be written as:

$$
P_{out}(f) \approx \mathcal{F}\left\{P_{out}(t)\right\} = \frac{n_{eff} E_0 c}{2} \mathcal{F}\left[ |U_{out}(t)|^2 \right]
$$

(3.3.2)

Where $\mathcal{F}$ represents the Fourier transform between time and RF frequency domain. Based on this model the nonlinearity of the optical transfer function at $\nu_{laser}$ results in nonlinear amplitude modulation and generation of harmonic components ($2f_{OMO}, \nu_{res}$).
$3f_{\text{OMO}}, \ldots$) in the RF spectrum. This process is similar to the harmonic generation in electro-optic modulators (Mach-Zehnder and microdisk modulators) that are driven by external source (as opposed to the self-sustained oscillation of an OMO) [66, 67]. Note that in strong harmonic generation regime the optical power distribution among the fundamental and harmonic sidebands inside the cavity makes the estimation of optomechanical gain $\Gamma_{\text{OM}}$ very complicated, and this approximation is invalid.

Fig. 3.4. (a) Measured $P_1$, $P_2$ and $P_3$ (markers) and OTF (solid lines) plotted against $\Delta \lambda_N$. (b) Measured (circles) and simulated (using OTF, solid line) values of $P_1^2/P_2$ plotted against $\Delta \lambda_N$. Here $Q_1= 3.04 \times 10^6$, $Q_0= 3.03 \times 10^7$, $P_{\text{th}}=282 \mu\text{W}$, $P_N= 1.42$, $Q_{\text{mech}}=1600$ and $f_{\text{OMO}}=37.3$ MHz ($f_N=0.59$).

Fig. 3.4 (a) shows the measured $P_1$, $P_2$ and $P_3$ at WHG regime plotted against $\Delta \lambda_N$. The solid lines are the calculated results using the OTF method. For OMO (and most oscillators), lower input power results in less non-linearity but in the same time reduces fundamental oscillation. For an OMO we have a second degree of freedom because even at small powers ($P_N < P_N^{(1,2)}$) the wavelength detuning can affect the harmonic generation. So an optimized detuning can be obtained by maximizing $P_1 \times (P_1/P_2)$ as a function of $\Delta \lambda_N$ to optimize the intensity of fundamental component as well as the oscillation linearity.

Fig. 3.4(b) shows the measured values of $P_1^2/P_2$ plotted against $\Delta \lambda_N$ that is in good
agreement with the calculated results using OTF (showing an optimized value of 0.6 for $\Delta \lambda_N$). Note that in WHG regime the 3rd harmonic component is always smaller than the second harmonic and the above optimization will also guarantee small 3rd order nonlinearity.

3.4 Using the RF spectrum of OMO for sensing application

Resonant sensing is one of the most important applications of optical microresonators [64, 68]. It has been shown that optical microresonators can function as very sensitive molecules detectors both in gas and aqueous environment [69, 70]. The detection mechanism is based on resonant wavelength shift caused by effective refractive index ($n_{\text{eff}}$) change through interaction of the optical evanescent field with surrounding medium [64]. The magnitude of $\Delta n_{\text{eff}}$ ($\Delta n_{\text{eff}} \propto \Delta \lambda_{\text{res}}$, $\Delta \lambda_{\text{res}}$ is the resonant wavelength shift) is proportional to the concentration of molecules in the vicinity of the optical cavity. The narrow resonant linewidth of high quality factor optical microresonators enables high resolution measurement of $\Delta \lambda_{\text{res}}$. Fig. 3.5 is the schematic diagram showing the idea of resonant optical sensing. The resonance of the microresonator is monitored by scanning the laser wavelength constantly in a small range. The surface of the resonator is usually coated by specific molecules to have selectivity to the target molecules. The binding between the coated and target molecules, for example, covalent bonds, can change the effective refractive index ($n_{\text{eff}}$) through the evanescent field. In contrast to the fluorescent-based or radiation-based detection, this protocol does not involve any labeling tags of the target molecules, so the molecules are measured in their natural forms.
This type of detection is relatively easy fast and lower cost, and attracts tremendous attention in the past twenty years [64, 68].

![Fig. 3.5. A schematic diagram showing the principle of resonant optical sensing. Tunable laser is coupled to a functionalized microresonator, and the Lorentzian resonance is monitored. The shift of the cavity resonance shows that the binding happens on the surface of the microresonator. The value of resonance shift is proportional to the density of target molecules.](image)

Usually $\Delta \lambda_{\text{res}}$ is detected by either continuous monitoring of $\lambda_{\text{res}}$ using a tunable laser, or measuring variations of transmitted optical power at a fixed laser wavelength. Here we propose a new method to detect the resonance shift based on the strength of harmonic components in the OMO spectrum. As evident from Fig. 3.4 (b), when $P_N$ is sufficiently larger than $P_N^{(1,2)}$, near the F-SH crossing point $\partial(P_1)/\partial(\Delta \lambda_N)$ is positive constant and $\partial(P_2)/\partial(\Delta \lambda_N)$ is a negative constant. As a result near $\Delta \lambda_N^{(1,2)}$, the difference between $P_1$ and $P_2$ is sensitive linear function of $\Delta \lambda_N$. Since $\partial(P_1-P_2)/\partial(\Delta \lambda_N)>\partial(P_1)/\partial(\Delta \lambda_N)$ and $\partial(P_1-P_2)/\partial(\Delta \lambda_N)<|\partial(P_2)/\partial(\Delta \lambda_N)|$, the value of $P_1-P_2$ is more sensitive to the change of $\Delta \lambda_N$ than $P_1$ or $P_2$ alone. Typically $\partial(P_1-P_2)/\partial(\Delta \lambda_N)$ is constant for $|\Delta \lambda_N|$ variations smaller than 0.4 near $\Delta \lambda_N^{(1,2)}$, which is large enough given that in most resonant optical systems $\Delta \lambda_N<0.1$. 


Fig. 3.6. (a) Schematic diagram of an OMO sensor based on $P_1-P_2$. Measured (b) and (c) calculated (time domain differential equations) values of $P_1-P_2$ plotted against $\Delta \lambda_N$ for three different input powers. (d) Red squares: $P_1-P_2$ plotted against $\Delta \lambda_N$ for $P_N=2.65$. Green diamonds: $P_1-P_2$ plotted against sample temperature change ($\Delta T$) for $P_N=2.65$. The properties of the microtoroid OMO used for these measurements: $Q_0=1.91 \times 10^7$, $Q_L=2.67 \times 10^6$, $f_{OMO}=24.54$ MHz ($f_0=0.34$), $P_{th} \sim 300$ µW, and $Q_{mech} \sim 500$.

Beyond the sensitivity, measuring $P_1-P_2$ instead of a single component suppresses the optical power fluctuations and reduces the overall noise in the electronic detection.
(through differential measurement). Fig. 3.6 (a) shows the schematic diagram of a system for measuring $P_1 - P_2$. Here the photodetector output passes through two bandpass filters with center frequencies at $f_{OMO}$ and $2f_{OMO}$. Next the two output voltages are subtracted resulting in a low-noise signal proportional to $P_1 - P_2$. Fig. 3.6 (b) shows $P_1 - P_2$ plotted against $\Delta \lambda_N$ for three values of $P_N$ for a single microtoroid. When $\Delta \lambda_N$ is close to $\Delta \lambda_N^{(1,2)}$, $(P_1 - P_2)/\Delta \lambda_N$ is almost constant, and its value is proportional to $P_N$ as the input optical power increases. Fig. 3.6 (c) shows the calculated values of $P_1 - P_2$ based on time domain approach that is in good agreement with the experimental results. For a fixed laser wavelength, sensitivity to $\Delta \lambda_N$ is equivalent to the sensitivity to effective refractive index change ($\Delta n_{eff}$) used for resonant optical sensing. So measuring $P_1 - P_2$ provides a simple and high-resolution technique for monitoring $\Delta n_{eff}$ using the OMO spectrum. If the laser wavelength is fixed such that in the absence of any molecule $\Delta \lambda_N \approx \Delta \lambda_N^{(1,2)}$, then $P_1 - P_2$ is proportional $\Delta \lambda_{res}$ (for $\Delta \lambda_{res} < 0.4 \delta \lambda$). So the corresponding detection limit for $\Delta n_{eff}$ can be written as:

$$\Delta n_{eff, min} = n_{eff} \frac{\Delta \lambda_{res, min}}{\lambda_{res}} = n_{eff} \frac{\delta \lambda \times \Delta P}{\lambda_{res} \times \eta} \quad (3.4.1)$$

Here $\eta$ is the value of $\partial (P_1 - P_2)/\partial (\Delta \lambda_N)$ at $\Delta \lambda_N^{(1,2)}$. $\Delta P$ is the minimum RF power difference that can be measured (limited by the measurement instrument and optical noise), and $\Delta \lambda_{res, min}$ is the corresponding minimum detectable wavelength shift. Clearly the minimum detectable index change depends on the sensitivity of RF power detection as well as $\eta$. As shown in Fig. 3.6, $\eta$ can be increased by increasing the input optical power. As a proof of concept we have also measured $P_1 - P_2$ as $n_{eff}$ was changed by increasing the ambient temperature (through thermo-optic effect). Fig. 3.6(d) shows $P_1$-
P₂ is plotted against temperature change ΔT (green curve). The red curve shows P₁-P₂ measured versus Δλₜ (at the same Δλₜ⁽¹,2⁾ and Pₜ) to show the correlation between Δλₜ and ΔT. A resonant wavelength shift sensitivity of better than 0.01×δλ (∼5.8×10⁻⁶ nm) is clearly observed in this experiment. Both Pₜ and Δλₜ are adjustable and consequently the dynamic range and sensitivity can be tuned to certain extent.

In Chapter 5, we also demonstrate that OMO can also function as a high-resolution mass sensor where mass deposition on a microtoroid OMO (down to picogram level) is detected by monitoring the RF spectrum (measuring ΔfOMO) [71]. Using the above method the same measurement can also reveal Δλₚₑₛ, enabling simultaneous monitoring of optomechanical oscillation frequency and optical resonance shift using the RF spectrum.

### 3.5 Observation of chaotic spectrum

Chaotic behavior in optomechanical oscillation has been reported in previous studies of the microtoroid OMO [51, 62]. It has been shown that the temporal behavior of regular sinusoidal oscillation can turn into chaotic oscillation with very large Pₘ (Pₘ~ 20 mW) [62]. However, in our experiment aperiodic behavior has been observed at relatively low input powers (Pₘ<1 mW) in microtoroids with high optical-Qs but very low mechanical-Qs. These microtoroids have large toroidal part (>10 µm) compared to regular microtoroids. Fig. 3.7 (a) shows the optical microscope image of one of the microtoroid in our experiment. Moreover we have been able to switch the behavior of an OMO from normal oscillation to chaotic quivering only by changing the wavelength detuning at a fixed input power as shown in Fig. 3.7 (b). As Δλₜ is reduced, first we
observed period doubling and then non-periodic fluctuation. This behavior may be explained by larger circulating optical power and strong nonlinearity at the bottom of the Lorentzian transfer function. These results show that more investigation is needed to fully understand the parameters and conditions that control the transition to chaotic regime.

![Image](image.png)

Fig. 3.7. (a) Image showing the microtoroid with chaotic oscillation observed in our experiment. (b) The spectrum of the detected RF power at different wavelength detuning ($\Delta \lambda_N$) for constant $P_N$. Here $Q_L=2.45 \times 10^7$, $Q_0=2.64 \times 10^7$, $P_{th}=177 \mu W$, $P_N \sim 4.55$, $Q_{mech} \sim 136$ and $f_{OMO}=5$ MHz.

Usually the mechanical-$Q$ of is governed by two kinds of loss mechanism: clamping loss and material loss. The low mechanical-$Q$ of microtoroids is mainly due to large clamping loss that becomes very large when the silicon pillar is off center or non-circular. It seems that when the silicon pillar is not exactly at the center of the disk or not circular, perturbation appears in the optomechanical oscillation of the microtoroid and results in chaotic behavior. By changing the detuning, the distribution of the circulating optical power in the cavity at fundamental and harmonic frequencies is changed, and varies the threshold of chaotic generation. It also implies that optomechanical oscillator
can intentionally turn into a platform for study of chaotic dynamic by controlling the XeF₂ etching process. If the concentration of XeF₂ gas is unevenly distributed in the etching chamber (i.e. constantly flow from one to another ports), the difference in etching rate of silicon will easily produce controllable off center pillars.

Other than harmonic generation and chaotic motion, the period doubling state provides oscillatory output at frequencies other than the fundamental and its harmonics. It may have interesting research content in the aspects of engineering application such as RF comb generation (note that the inherent low phase noise in near chaotic regime is major limitation).

3.6 Summary

We have investigated the spectrum of a typical radiation-pressure driven OMO at different input power levels and wavelength detunings. Through this study we have also identified two different regimes of weak and strong harmonic generation. An analytical method based on oscillating transfer function (OTF) is developed to estimate the harmonic components in weak harmonic generation regime without solving coupled time-domain differential equations, and to provide an intuitive understanding of the harmonic generation mechanism. Using the behavior of the fundamental and second-harmonic components in strong harmonic generation regime, a method has been proposed to directly extract resonant wavelength shift from the RF spectrum of OMO. This approach can be used to design a simple and high-resolution readout system for resonant optical sensing applications. Finally, we report the first observation of detuning
dependent aperiodic oscillation at relatively low input pump power, which may be interesting for studying the nonlinear dynamic of optomechanical systems.
Chapter 4

Characterization of optomechanical RF frequency mixing/down-conversion and its application in photonic RF receivers

4.1 Introduction to RF frequency mixing and signal down-conversion

Radio frequency (RF) photonics or so called microwave photonics is the study of photonic devices and systems (e.g. lasers, detectors, modulators, links, signal processors, etc.) operating at microwave frequencies and their applications in microwave systems [75-80]. In RF photonic communication, microwave signals are transported over the optical link (optical fiber) with electrical-to-optical and optical-to-electrical conversions at the transmitting and receiving ends [76]. The photonic technology is either used to reduce the power consumption, weight, cost, complexity and susceptibility to electromagnetic interference or to enable functions that are difficult and sometimes impossible to achieve in microwave domain and solely using microwave devices. Microwave photonic systems offer low and constant loss over the entire microwave modulation frequency range, low dispersion, and high data rate. For example, a typical optical fiber link has its weight of 1.7kg/km and loss about 0.5dB/km, comparing to 567 kg/km and 360dB/km at 2GHz for a coaxial microwave cable [75]. Fig. 4.1 shows the typical configuration of RoF down conversion links as receivers [81-85]. The information sidebands over RF carrier is received by an antenna. It modulates the amplitude of the source laser after passing through an amplifier, resulting in an microwave signal up-converted to optical domain. After transmission over fiber, the optical signal can be detected by a photodetector (photodiode). Once the RF output of the detector mixes with
the signal supplied by a local oscillator (LO) at frequency $f_{RF}$, the baseband information will be extracted from the RF carrier. It is also possible to down-converted the signal (to IF or baseband) before the optical transportation, in which case only the IF (or baseband) signals are carried over fiber (IF-over-Fiber or IoF links).

Fig. 4.1. A diagram showing the configuration of RoF at receiver side. $\nu_{\text{laser}}$ is the laser frequency. EO modulator: Electro-optic modulator.

Frequency mixing plays an important role in signal baseband conversion in RF photonic communication. It is the key function to process information signal using RF carrier. A frequency mixer is a nonlinear electrical device that multiplies two input signals and generates mixed frequencies components. The output from an RF mixer can be written as [76]:

$$V_{\text{out}} = A \times (V_1 \cos \omega_1 t + V_2 \cos \omega_2 t) + B \times (V_1 \cos \omega_1 t + V_2 \cos \omega_2 t)^2 + ...$$

$$\approx A \times (V_1 \cos \omega_1 t + V_2 \cos \omega_2 t) + B \times (V_1^2 \cos^2 \omega_1 t + V_2^2 \cos^2 \omega_2 t + 2V_1V_2 \cos \omega_1 t \cos \omega_2 t)$$

$$= A \times (V_1 \cos \omega_1 t + V_2 \cos \omega_2 t) + B \times (V_1^2 \cos^2 \omega_1 t + V_2^2 \cos^2 \omega_2 t) + BV_1V_2 \cos(\omega_1 - \omega_2) t + BV_1V_2 \cos(\omega_1 + \omega_2) t$$

(4.1.1)
where $V_1$ and $V_2$ are the two input signals at two different frequencies $\omega_1$ and $\omega_2$, respectively. $A$ and $B$ represent the amplitudes of the linear and square terms generated by the mixer, respectively. The crossing term $[\cos\omega_1 t \times \cos\omega_2 t]$ results in the mixed frequency components at frequency $\omega_1 \pm \omega_2$. Through the frequency mixing process, the IF or baseband signals can be up or down-converted to a higher frequency carrier for microwave transportation.

### 4.2 RF frequency mixing and signal down-conversion in OMO

The basic idea of RF frequency mixing and signal down-conversion based on optomechanical oscillation was proposed and demonstrated in Ref. 33. Preliminary experiments have shown that if the optical pump power is modulated with a suppressed-carrier RF signal with a carrier frequency equal to optomechanical oscillation frequency, the output spectrum of the optomechanical oscillator (OMO) will contain the baseband signal [33]. In other words OMO may simultaneously function as local oscillator and RF mixer in optomechanical domain.

Here for the first time we characterize the performance of the OMO as an RF frequency mixer and down converter. Our experiments and theoretical calculation show that an OMO has linear response to the RF/optical input. Moreover, the power efficiency of an OMO mixer is higher comparing to the traditional electronic mixer.

#### 4.2.1 RF frequency mixing using optomechanical oscillation

In this section, we consider the situation that the RF modulation is single tone frequency signal. Fig. 4.2 (a) is the experimental configuration used for characterizing the
RF frequency mixing using an microtoroid OMO. The optical pump power is amplitude modulated by the RF signal through an EO modulator, then coupled to an OMO. When the optical power is above threshold, the OMO functions as a local oscillator (LO) and a mixer, and mixes the modulation frequency ($f_{RF}$) with its own oscillation frequency $f_{OMO}$. Two mixed sidebands at frequencies $f_{OMO} \pm f_{RF}$ can be detected by a RF spectrum analyzer. Fig. 4.2 (b) shows the detected RF spectrum from an OMO mixing setup in part (a).

The response of radiation pressure driven OMO to modulated optical pump can be evaluated using the time-domain differential equations (Eq. (2.4.1) and (2.4.2)). When the optical input power is modulated by an RF signal (in general an RF carrier modulated by a baseband signal), $P_{in}$ is a time varying function. If we use a MZ modulator $P_{in}(t)$ can be written as:

$$P_{in}(t) = \frac{P_0}{2} \left\{ 1 + \cos \left[ \phi_0 + \frac{\pi V_{RF}}{V_\pi} \cos(2\pi f_{RF} t) \right] \right\}$$  \hspace{1cm} (4.2.1.1)

Where $f_{RF}$ is the RF modulation frequency, $P_0$ is the input optical power to the modulator, $V_\pi$ is the half-wave voltage of the modulator, $\phi_0$ is a phase factor of the MZ modulator, and $V_{RF}$ is the amplitude of the RF voltage. To maximize the linearity of the amplitude modulation the modulator should be biased at quadrature ($\phi_0 = \pi/2$) and $V_{RF}/V_0 << 1$. So Eq. (4.2.1.1) can be simplified as:

$$P_{in}(t) = \frac{P_0}{2} \left[ 1 + M \cos(2\pi f_{RF} t) \right] = \frac{P_0}{2} Q(f_{RF})$$  \hspace{1cm} (4.2.1.2)

$M$ is the optical modulation index that is determined by $V_{RF}$ and for MZ modulator is equal to $\pi V_{RF}/V_\pi$. 

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In order to simulate the output spectrum of the OMO in the presence of a modulated input power, we can derive \( P_{\text{out}} (\propto |E_{\text{out}}(t)|^2) \) using Eq. (2.4.1), (2.4.2), and (4.2.1.2), and then calculate its Fourier transform. We refer to this method as time domain differential equation method or TDE. A typical calculated spectrum is shown in Fig. 4.2 (c).

![Diagram showing an OMO based frequency mixing configuration.](image)

Fig. 4.2. (a) Diagram showing an OMO based frequency mixing configuration. (b) Detected RF spectrum of the output of an OMO mixer at (a). \( f_{\text{mod}}=5 \) MHz, \( f_{\text{OMO}}=25.1 \) MHz, \( Q_{\text{tot}}=3.09 \times 10^6 \), \( Q_0=1.1 \times 10^7 \), \( Q_{\text{mech}}\approx 1300 \), \( P_{\text{th}}=87\mu W \) and \( P_{\text{in}}=3.9 P_{\text{th}} \). (c) Calculated spectrum by time coupled differential equations.

Although the TDE approach can predict the output of a the OMO driven by a modulated optical pump, it does not clearly show the role of various parameters in the optomechanical RF mixing process. Here we use an analytical approach based on oscillatory transfer function (OTF) approximation to estimate the output spectrum of the OMO with modulated pump. This approach provides a direct relation between...
optomechanical oscillation and mixing/down-conversion and can be used as a semi-analytical technique for practical applications.

When \( f_{\text{OMO}} \) is smaller than optical resonant bandwidth (unresolved sideband regime), and the pump power is small enough such that \( P_{\text{OMO}}^{(1)} > P_{\text{OMO}}^{(n)} \) (\( P_{\text{OMO}}^{(n)} \) is the \( n^{\text{th}} \) harmonic in the spectrum of the OMO [72]) the optomechanical oscillation can be explained as the modulation of the pump laser by an oscillating Lorentzian transfer function [72]. The oscillatory transfer function (OTF) of the optomechanical cavity above threshold (\( P_{\text{in}} > P_{\text{th}} \)) can be written as:

\[
T_{\text{OMO}} = \frac{U_{\text{out}}}{U_{\text{in}}} = 1 - \frac{1}{\frac{1}{\tau_{\text{ex}}} + \frac{1}{2\tau_{\text{ex}}} + i[\omega_{\text{laser}} - \omega'_{\text{res}}(t)]} \tag{4.2.1.3}
\]

\[
\omega'_{\text{res}}(t) = \omega_{\text{res}}[1 + H_{A}(P_{\text{in}}/P_{\text{th}}, \Delta \omega_{0})\cos(2\pi f_{\text{OMO}} t)] \tag{4.2.1.4}
\]

Here \( \omega_{\text{laser}} \) is the laser frequency, \( \omega_{\text{res}} \) is the resonant frequency of the cavity in the absence of optical power, \( H_{A} = r_{\text{max}}/R_{0} \) is the normalized oscillation amplitude that can be estimated using the optomechanical gain equation (Eq. 2.4.6.b), \( \Delta \omega_{0} \) is the optical detuning, \( r_{\text{max}} \) is the maximum displacement in radial direction, and \( R_{0} \) is the radii of the microtoroid. \( \tau_{\text{ex}} \) and \( \tau \) are the external and total decay times of the optical cavity. Clearly the oscillation of the transfer function is caused by radiation pressure but in this approach the \( H_{A} \) is considered a known function of \( P_{\text{in}}/P_{\text{th}} \), \( \Delta \omega_{0} \) and characteristics of the optomechanical resonator. Therefore, the spectrum of the optical output power can be written as:
The second term in Eq. (4.2.1.5) is simply the output of OMO in the absence of pump modulation. So it can be written as:

\[ P_{OMO}^{(0)} + P_{OMO}^{(1)} \cos(2\omega_{OMO} t) + P_{OMO}^{(2)} \cos(2 \times 2\omega_{OMO} t) + \text{h.o.t} \]  

(4.2.1.6)

Where \( P_{OMO}^{(n)} \) (\( n = 1, 2, \ldots \)) is the amplitude of the optical power oscillating at \( n\omega_{OMO} \) when input power \( P_{in} \) is constant and \( P_{in} \) is equal to \( P_0 \). \( P_{OMO}^{(0)} \) is the portion of the transmitted optical power that is not modulated by the optomechanical oscillation (at a given optical power). In other words \( P_{OMO}^{(n)}/P_{OMO}^{(0)} \) is the \( n \)th order optomechanical modulation index. Since the OTF approach is only valid when \( P_{OMO}^{(1)}> P_{OMO}^{(2)}> P_{OMO}^{(3)} \), and we are only interested in the first order mixing effects, we can ignore the higher order terms (\( n > 1 \)).

\[
P_{out}(t) = \frac{1}{2} Q(f_{RF}) \times \left[ P_{OMO}^{(0)} + P_{OMO}^{(1)} \cos(2\omega_{OMO} t) \right] \]

\[
\left[ \frac{1}{2} + \frac{M}{2} \cos(2\omega_{RF}) \right] \times \left[ P_{OMO}^{(0)} + P_{OMO}^{(1)} \cos(2\omega_{OMO} t) \right] = \frac{1}{2} \left[ P_{OMO}^{(0)} + P_{OMO}^{(1)} \cos(2\omega_{OMO} t) \right] + \frac{M}{2} \cos(\omega_{RF}) \left[ P_{OMO}^{(0)} + P_{OMO}^{(1)} \cos(2\omega_{OMO} t) \right] \]

(4.2.1.7)

When input optical power is constant (\( M=0 \)), Eq. (4.2.1.7) results in the typical spectrum of an OMO with an input power of \( P_0/2 \) that consists of \( f_{OMO} \) and its harmonics. If the optical input power is modulated by a single-tone RF frequency (\( M\neq0 \)), the second term in Eq. (4.2.1.7) represents the optomechanical oscillation at \( f_{OMO} \) and the harmonic
frequencies on the DC background. If the optical input power is modulated by a single frequency ($M \neq 0$), Eq. (4.2.1.7) becomes

$$P_{out}(t) = \frac{P^{(0)}_{OMO}}{2} + \frac{P^{(0)}_{OMO}}{2} \cos(2\pi f_{RF} t) + \frac{P^{(1)}_{OMO}}{2} \cos[2\pi f_{OMO} t] +$$

$$\frac{MP^{(1)}_{OMO}}{4} \{ \cos[2\pi (f_{OMO} + f_{RF}) t] + \cos[2\pi (f_{OMO} - f_{RF}) t] \} \tag{4.2.1.8}$$

The last term in equation (4.2.1.8) is the modulated optical power at mixed frequencies ($f_\pm = f_{OMO} \pm f_{RF}$) as shown in Fig. 4.2 (b), so the average optical power at $f_\pm$ can be written as:

$$P_\pm = \frac{1}{4} M \times P^{(1)}_{OMO} \tag{4.2.1.9}$$

and the average detected RF power at $f_\pm$ can be written as:

$$P_{RF,\pm} = \left[ \frac{G(0.25 \times M \times P^{(1)}_{OMO})}{2R_L} \right]^2 \tag{4.2.1.10}$$

Where $G$ (unit: V/W) is the photodetector conversion gain ($G = \text{responsivity (A/W)} \times \text{transimpedance gain (V/A)}$) and $R_L$ is the load resistor driven by the photodetector.

Using the configuration shown in Fig. 4.2 (a), we characterize the RF frequency mixing in an OMO. The optical input power ($P_{in}$) to OMO is provided by a tunable near–Infrared laser ($\lambda_{laser}=1550$ nm, $\nu_{laser}=194$ THz). The amplitude of the laser output is modulated in a Mach-Zehnder electro-optic (EO) modulator with $V_\pi = 4.3$ V and Ins. Loss = 2.1 dB. The modulated optical input power is coupled to the OMO using a standard silica fiber-taper and the optical output power ($P_{out}$) is detected by a photodetector (bandwidth = 125 MHz). The detected signal is monitored by an oscilloscope and analyzed in an RF spectrum analyzer.
According to Eq. (4.2.1.9), the amplitude of the two sidebands around $f_{OMO}$, defined as $f_+ = f_{OMO} + f_{RF}$ and $f_- = f_{OMO} - f_{RF}$, should be equal to $(M/4)P_{OMO}^{(1)}$, where $P_{OMO}^{(1)}/2$ is the modulated optical output power at $f_{OMO}$ in the absence of pump modulation. The detected RF power at each frequency component (measured by feeding the photodetected current to the RF spectrum analyzer) is given by Eq. (4.2.1.10). The RF power detected at $f_{RF}$ corresponds to the second term in Eq. (4.2.1.8). Fig. 4.3 (a) shows detected RF power at $f_+$ and $f_-$ plotted against normalized wavelength detuning ($\Delta \lambda_N = \Delta \lambda_0/\Delta \lambda_L$, $\Delta \lambda_0 = \lambda_{res} - \lambda_{laser}$), at constant $P_{RF-in}$ and $P_{in}$. Fig. 4.3 (b) shows the detected RF power at $f_+$ and $f_-$ plotted against modulating RF power ($P_{RF-in}$) at constant $P_{in}$ and wavelength detuning. Fig. 4.3 (c) shows $f_+$ and $f_-$ plotted against normalized optical input power ($P_{in}/P_{th}$), at constant $P_{RF-in}$ and $\Delta \lambda_0$. The solid lines in all these plots are the calculated behavior using the OTF method (Eq. (4.2.1.9) and (4.2.1.10)). Fig. 4.3 (d) shows $f_+$ and $f_-$ plotted against modulating frequency ($f_{RF}$), at constant $P_{RF-in}$, $P_{in}$ and $\Delta \lambda_0$. The solid lines in part (d) are calculated using TDE since in the OTF method the filtering effect of the optical transfer function is not taken into account and therefore it cannot predict the effect of $f_{RF}$ variations on the amplitude of the mixed components (i.e. $f_+$ and $f_-$) specially when $f_{RF} \geq \Delta \nu_L$. Fig. 4.3 (e) shows the simulated 3dB bandwidth for the up (down)-converted detected RF powers as a function of bandwidth of the loaded optical resonance. Qualitatively the reduction of $P_{RF,+}$ and $P_{RF,-}$ as $f_{RF}$ gets closer to the optical bandwidth of the loaded optical mode ($\delta \nu_L = \nu_{res} / Q_L$) can be explained by the spectral dependence of the circulating optical power inside the cavity. In Eq. (4.2.1.5) it is assumed that $P_0$ enters the cavity and drives the OMO (that is true for constant $P_0$).
However, in the presence of the modulation, the optical input power is distributed between the CW component and sidebands (at $\nu_L \pm f_{RF}$).

Fig. 4.3. (a) Detected RF power of the mixing components plotted as a function of normalized detuning $\Delta \lambda_N$. $P_{th}=3.6 P_{th}$. (b) Measured RF power of $P_{RF, \pm}$ vs. modulated RF power $P_{RF}$ with $\Delta \lambda_N = 0.55$ and $P_{in}=3.6 P_{th}$. For all cases there are $f_{RF} = 5$ MHz, $Q_{tot}=3.1 \times 10^6$. (c) Detected optical power of the mixing components plotted against normalized power $P_{in}/P_{th}$. $\Delta \lambda_N = 0.55$ and $M=0.15$. (d) Measured $P_{RF, \pm}$ plotted against modulating frequency ($f_{RF}$), for $P_{RF} \sim 0$ dBm. TDE: Calculated by time differential equations. (e) The simulated 3dB bandwidth for the up(down)-converted detected RF powers is plotted against the loaded optical bandwidth of the optical resonance, for a constant $\Delta \lambda_N$. 
As long as $\Delta v_0 (=v_{\text{laser}}-v_{\text{res}})<\delta v_L$, the CW components enters the cavity while depending on magnitude of $f_{RF}$ the sidebands can be partially or fully rejected by the cavity. Consequently $P_0$ depends on $f_{RF}$ and when $f_{RF} > \delta v_L$ effectively only the CW component drives the oscillation and $P_0$ is reduced by two times the optical power in the sidebands. Clearly when $f_{RF} < \delta v_L$ the variation of $P_0$ is a relatively complicated function of $f_{RF}$ and depends on $\delta v_L$ and $\Delta v_0$. When $f_{RF} > \delta v_L$, still the interference between the components that miss the cavity (i.e. $v_{\text{laser}} \pm f_{RF}$) and the optomechanically modulated wave coupled out of cavity generate mixed components. Fig. 4.4 shows the RF spectrum of OMO detected output spectrum when $f_{RF} > \delta v_L > f_{OMO}$. The optomechanical sidebands up to second order harmonic are visible as sidebands around $f_{RF}$.

![Figure 4.4](image.png)

**Fig. 4.4.** RF spectrum of the OMO detected output spectrum when $f_{RF} > \delta v_L > f_{OMO}$. $f_{RF} = 12$ MHz, $\delta v_L = 50$ MHz, $f_{OMO} = 500$ MHz.

### 4.2.2 Optomechanical down-conversion from an RF carrier

In this case we consider that the optical power is modulated by an RF carrier with two signal sidebands. Similar to the signal down conversion using an electronic mixer [78, 79], here the signal can be converted to baseband from the RF carrier through the mixing...
process of an OMO. Fig. 4.5 shows the experimental configuration. The devices in the black box function as a source, which provides the signal sidebands over RF carrier. A signal generator at frequency \( f_{RF} \) provides the carrier, and a function generator provides baseband signal (IF) at frequency \( f_b \). An electronic mixer is used to up-converted the IF to RF carrier as two sidebands.

Therefore, Eq. (4.2.1.1) becomes:

\[
P_{in}(t) = \frac{P_0}{2} \left[ 1 + \cos(\phi_0) + M[1 + m\cos(2\pi f_b t)\cos(2\pi f_{RF} t)] \right]
\]

(4.2.2.1)

\( m \) is the RF modulation index. It is determined by the baseband power and the properties of the electronic mixer, and can be calculated as the ratio between modulation sideband and the RF carrier, such that \( m = \frac{2V_{SB}}{V_{RF}} \). Here \( V_{SB} \) is the peak to peak voltage of the modulation, and \( V_{RF} \) is the voltage of RF carrier. Consequently Eq. (4.2.1.2) becomes:

\[
P_{in}(t) = |E_{in}(t)|^2 = \frac{P_0}{2} \left[ 1 + M \left( \cos(\omega_{RF}) + \frac{m}{2}\cos(2\pi(f_{RF} + f_b)t) + \frac{m}{2}\cos(2\pi(f_{RF} - f_b)t) \right) \right]
\]

(4.2.2.2)
Following the similar process, Eq. (4.2.1.7) becomes:

\[
P_{\text{out}}(t) = \frac{1}{2} \left[ 1 + M \left[ \cos(\omega_{RF}) + \frac{m}{2} \cos[2\pi(f_{RF} + f_b)t] + \frac{m}{2} \cos[2\pi(f_{RF} - f_b)t] \right] \right] \times \left[ P_{\text{OMO}}^{(0)} + P_{\text{OMO}}^{(1)} \cos(\omega_{OMO}t) \right]
\]

\[= \frac{1}{2} \left[ P_{\text{OMO}}^{(0)} + P_{\text{OMO}}^{(1)} \cos(2\pi f_{OMO}t) \right] + \frac{M}{2} \cos(2\pi f_{RF}) \left[ P_{\text{OMO}}^{(0)} + P_{\text{OMO}}^{(1)} \cos(2\pi f_{OMO}t) \right] + \left[ \frac{Mm}{4} \cos[2\pi(f_{RF} + f_b)t] + \frac{Mm}{4} \cos[2\pi(f_{RF} - f_b)t] \right] \times \left[ P_{\text{OMO}}^{(0)} + P_{\text{OMO}}^{(1)} \cos(2\pi f_{OMO}t) \right]
\]

(4.2.2.3)

When \(m = 0, M \neq 0\), the last term in Eq. (4.2.2.3) vanishes and Eq. (4.2.2.3) is the same as Eq. (4.2.1.7). When \(m \neq 0, M \neq 0\) and \(f_{RF} = f_{OMO}\), the optomechanical mixing inside the oscillating cavity results in down-conversion and generation of the baseband modulated optical power. For the sake of simplicity we consider a single frequency baseband \(f_b\). The third term of Eq. (4.2.2.3) results in the down-conversion and generation of baseband modulation optical power:

\[
P_{\text{out}}(t) = \left[ \frac{Mm}{4} \cos[2\pi(f_{OMO} + f_b)t] + \frac{Mm}{4} \cos[2\pi(f_{OMO} - f_b)t] \right] \times \left[ P_{\text{OMO}}^{(1)} \cos(2\pi f_{OMO}t) \right]
\]

(4.2.2.4)

\[= \frac{MmP_{\text{OMO}}^{(1)}}{4} \cos(2\pi f_b t) + \frac{MmP_{\text{OMO}}^{(1)}}{8} \left[ \cos(2\pi f_{OMO} + f_b)t \right] + \left[ \cos(2\pi f_{OMO} - f_b)t \right] \]

The first term is the down-converted optical signal so the average RF power detected at \(f_b\) is equal to:

\[
P_{RF-b} = \left[ \frac{G(0.25 \times M \times m \times P_{\text{OMO}}^{(1)})}{2R} \right]
\]

(4.2.2.5)

In our experiments we have used a suppressed carrier RF signal \((m \gg 2)\) to modulate optical input power to the OMO. The RF carrier \((f_{RF})\) is modulated by a single-tone baseband \((f_b)\) signal with an modulation index of \(m \sim 20\) (RF carrier power is 20 dB smaller than the sidebands). As predicted in Eq. (4.2.2.4) when \(f_{RF} = f_{OMO}\), the mixing of
$f_{\text{OMO}}$ with the sidebands ($f_{\text{RF}} \pm f_b$) in optomechanical domain, generates a down-converted signal at the baseband frequency ($f_b$). Upon detection in a photodetector and low-pass filtering an electric current proportional to the baseband signal will be generated. So as shown previously [33], OMO simultaneously serves as mixer and local oscillator in optical domain and is effectively a photonic homodyne RF-receiver [33]. Fig. 4.6 (a) and (b) show the RF spectrum of the optical power entering and exiting the OMO respectively.

Fig. 4.6. (a) Measured RF spectrum of the $P_{\text{in}}$ when RF carrier ($f_{\text{RF}}$) is modulated by a single frequency baseband ($f_b$). (b) Measured spectrum of $P_{\text{RF-out}}$ when $f_{\text{RF}} = f_{\text{OMO}}$. $P_{\text{RF-b}}$ is the detected RF power at $f_b$ (down converted baseband signal). $P_{\text{RF-1}}$ and $P_{\text{RF-2}}$ are the detected RF power at fundamental optomechanical oscillation frequency ($f_{\text{OMO}}$) and its 2nd harmonic ($2f_{\text{OMO}}$). (c) Schematic diagram showing the spectrum of the RF and optical signals flowing through the system. (d) Calculated RF spectrum of the optical output power using the time-domain coupled differential equation (TDE).

After up-conversion to optical frequencies (in the EO-modulator) and passing through the OMO, the optomechanical oscillation boosts the modulated optical power at $f_{\text{RF}} = f_{\text{OMO}}$
and generates optical amplitude modulation at $f_b$ once mixed with the sidebands ($f_{RF} \pm f_b$). Fig. 4.6 (c) is a schematic diagram showing the spectrum of the RF and optical signals flowing through the system. Fig. 4.6 (d) shows the simulated spectrum of $P_{RF-out}$ (using Eq. (2.4.1) and (2.4.2)) that is in good agreement with the measure results.

We have characterized the behavior of the down-converted optical power ($P_b$). According to Eq. (4.2.2.4), $P_b$ is proportional to $M \times m \times P_{OMO}^{(1)}/4$ and $P_{RF-b}$ is given by Eq. (4.2.2.5). $M$ and $m$ are determined by the characteristics of the RF mixer, the LO power and the baseband signal power fed to the mixer. In the weak harmonic generation regime (where OTF approximation is valid), $P_{OMO}^{(1)}$ is proportional to $P_{in}/P_{th}$ [72]. To verify the predicted linear relation between $P_{RF-b}$ and $P_b$ with $P_{RF-in}$ and $P_{in}/P_{th}$, we have measured three different microtoroidal OMOs. Fig. 4.7 (a) shows $P_{RF-b}$ plotted against $P_{RF-in}$. Fig. 4.7 (b) shows $P_b$ plotted against $P_{in}/P_{th}$. Wavelength detuning and optical coupling strength are kept constant in these measurements.

![Diagram](image_url)
Based on OTF, it is obvious that $P_{RF-b}$ is proportional to $P_{RF-in} \propto m^2 M^2$ and $P_{RF-1}$. The slopes in Fig. 4.7 (a) are decided by the values of $P_{RF-1}$ for each microtoroid, which are tunable through the experimental setting. The measured data show that the ratio of $P_{RF-1}$ ratio for the 3 cases in the experiments is about 1 : 2.9 : 0.022 (red : black : green) and approximately matches the ratio of the slopes. In addition, with approximately constant $M$ and $m$, the slopes in Fig. 4.7 (b) are decided by the values of $P_{OMO(1)}/P_{in} \propto \Gamma_{OM}/P_{in}$, which have the ratio about 2 : 1 : 1 (red: blue: green) according to Eq. (2.4.6.b). Accordingly, OTF is proved to be a reliable approach to explain the power behavior of baseband conversion. To maximize $P_b$, one should optimize the optomechanical oscillation within the linear region, and increase the modulation power.

Wavelength detuning ($\Delta \lambda_0$) affects the down-conversion process through its impact on $P_{OMO(1)}$. Fig. 4.8 shows $P_{RF-b}$ and $P_{RF-1}$, plotted against $\Delta \lambda_N$. According to Eq. (4.2.2.4), $P_b$ is proportional to $P_{OMO(1)}$ and $P_{OMO(1)} \propto P_{in}/P_{th}$. $P_{th}$ is a function of $Q_{tot}$, $Q_0$, $Q_{mech}$ and $m_{eff}$ of the corresponding optical and mechanical modes. As expected $P_b$ follows the behavior of $P_{OMO(1)}$ when $\Delta \lambda_N$ is varied.

Fig. 4.8. Measured RF power of at $f_b$ (black circles) and $f_{OMO}$ (gray squares) plotted against $\Delta \lambda_N$. Here $Q_{tot}=3.8 \times 10^6$, $Q_0=4.5 \times 10^6$, $Q_{mech}=2700$, $f_{OMO}=22.9$ MHz, $P_{th}=62$ µW, $P_{in}=1.67 P_{th}$, and $f_b=1.5$ MHz. Solid line: OTF calculation. Dashed line: time-domain differential equations (TDE) calculation.
4.2.3 Conversion gain and noise

Similar to an electronic system, the power conversion gain for OMO based down-converter can be defined as \( G_{d-OMO} = P_{RF-b}/P_{RF-in} \). Using Eq. (4.2.2.5) and assuming an input impedance of 50 Ω for the EO-modulator:

\[
G_{d-OMO} = \frac{\pi^2 m^2 G_D^2 (P_{OMO}^{(1)})^2}{16V_\pi^2 (1 + m^2 / 2)} = \frac{\pi^2 m^2 R_I P_{RF-OMO}^{(1)}}{2V_\pi^2 (1 + m^2 / 2)} \quad (4.2.3.1)
\]

So similar to electrical down-converters, the conversion gain inside OMO is proportional to the optomechanical oscillation power. We can also define a noise figure (\( NF \)) for the optomechanical frequency down-converter:

\[
NF = \frac{SNR_{in}}{SNR_{out}} = 1 + \frac{N_{add}}{G_{d-OMO} N_{in}} \quad (4.2.3.2)
\]

Here \( N_{add} \) is the noise added by the optical system. \( N_{add} \) in the optomechanical down-conversion process is generated by the laser noise, photodetection noise, and OMO noise. The contribution of OMO is mainly through fluctuation of laser power (relative intensity noise, RIN) and the wavelength detuning (that affect the optomechanical oscillation amplitude through optomechanical gain). The tunable laser in our experiment had a RIN less than -90dB and frequency stability of about \( 4 \times 10^5 \) Hz. For \( f_{OMO}=75 \) MHz, \( P_{in}\sim180 \) µW, \( \Delta \lambda N\sim0.5 \) and \( Q_0/Q_{tot}\sim0.1 \), the calculated relative amplitude fluctuation, \( \delta P_{OMO}^{(1)}/P_{OMO}^{(1)} \), is less than -26 dB. Using Eq. (4.2.3.1) and for \( P_{RF-in} = -20 \) dBm this translates to an amplitude noise in the down-converted signal (\( \delta P_{RF-b} \)) less than -60 dBm.

In addition to amplitude fluctuations, the noise in OMO frequency (\( \delta f_{OMO} \)) can also contribute into distortion of the down-converted signal. Basically if \( f_{OMO}-f_{RF}=\varepsilon \neq 0 \) then instead of one component at \( f_b \) two components near \( f_b \) will be generated (i.e. \( f_b \)

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Optomechanical frequency fluctuation is also linked to laser power and wavelength detuning fluctuations through optical spring and thermal effect [32, 50]. For our system $\delta f_{\text{OMO}}$ was less than 50 Hz. Note that for smaller values of $m < 5$, the optomechanical injection locking mechanism [125] naturally suppresses $\varepsilon$ when $f_{\text{OMO}} - f_{\text{RF}}$ is originally small enough. Moreover as shown in Fig. 4.9, input noise modes that have their frequencies close to $f_{\text{OMO}}$ can generate distortion by generating unwanted signals near $f_b$.

![Fig. 4.9 (a) Detected RF spectrum of an OMO fed by modulated pump power ($f_{\text{RF}} \pm f_b$) near $f_{\text{OMO}}$. The frequencies labeled as $f_n$ are unwanted noisy components mainly due to weak excitations of some mechanical modes. (b) Shows the detected RF spectrum near $f_b$ showing the down-converted noisy components that appear as distortion.](image)

4.2.4 Optomechanical waveform down-conversion

To examine the fidelity of the optomechanical down-conversion process we have also modulated the input power ($P_{\text{RF-in}}$) with anharmonic waveforms. Fig. 4.10 (a) shows temporal variation of the baseband signals before mixing with the RF carrier (in this case $f_{\text{RF}}=75.5$ MHz). After mixing (up-conversion) the resulting RF signal drives the MZ modulator (Fig. 4.5). Next the modulated optical power ($P_{\text{in}}$) is fed to the OMO and the detected power ($P_{\text{RF-out}}$) is passed through a low-pass filter (with a pass-band < 10 MHz). Fig. 4.10 (b) shows the temporal variation of the down-converted signal. This
experiments demonstrates the performance of OMO as a signal down-converter and its potential to serve as an all-optical RF receiver. The low distortion for square wave also enables digital signal processing using OMO.

![Waveforms](image)

**Fig. 4.10.** (a) Waveform of the input baseband signal to the OMO mixer. From top to bottom: sin wave, 50% square wave, and 50% triangle wave. (b) Output signal from the OMO mixer. 

In this experiment the frequency of the baseband signal was limited by the waveform generator. In order to explore the RF bandwidth limitation imposed by the optical bandwidth of the microresonator we have also studied the power variations of the down-converted harmonic signal as a function of it frequency \( f_b \). Fig. 4.11 shows \( P_{RF-b} \), \( P_{\text{fmod}} \) and \( P_{RF-1} \) plotted against \( f_b \) (\( \Delta \lambda \), \( P_{\text{in}} \) and optical modulation depth and RF modulation index are fixed). Here \( P_{\text{fmod}} \) is the detected RF power at frequency \( f_{RF} \). Clearly the variation \( P_{RF-b} \) limits the application of this method for down-converting broadband signals (in this case larger than 10 MHz).
Fig. 4.11. Measured \( P_{RF-b} \), \( P_{mod} \) and \( P_{RF-1} \) plotted against \( f_b \). The microtoroid has \( Q_{tot}=1.05 \times 10^6 \), \( Q_0=6.11 \times 10^6 \), \( Q_{mech}\sim 3200 \), \( f_{OMO}=74.53 \) MHz, \( P_{th}=285 \) µW, \( P_{in}=1.4 \) \( P_{th} \) and \( \Delta \lambda_N\sim 0.6 \). Dashed lines: TDE calculation.

4.3 Optomechanical radio receiver

To demonstrate the potential of OMO based receiver on real radio communication, we have tested our system using an audio baseband signal (\( f_b \) within 20 Hz-20 kHz range). The experimental setup is shown in Fig. 4.12. The sound wave is generated by a computer (MP3 file) and mixed with the RF carrier (\( f_{RF} = f_{OMO} = 74.5 \) MHz) in an RF mixer. The resulting signal is amplified and drives the MZ electro-optic modulator. The modulated laser power is fed to the OMO and finally the output power is fed to a photodetector that drives an amplified speaker. When the input optical power to OMO is larger than \( P_{th} \), the audio signal can be heard from the speaker. As the video linked to Fig. 4.12 shows, when the microtoroid is decoupled from the fiber taper, the sound vanishes because the speaker do not respond to the up-converted audio signal (\( f_{OMO} \pm f_b \)). The sound can be recovered by coupling back the microtoroid to the fiber taper.
Fig. 4.12. Experimental configuration used for demonstration of optomechanical audio down-conversion from 74.5 MHz RF carrier. Video: (click to view the video): the video shows the correlation between down-converted sound and optomechanical oscillation. When the microtoroid (OMO) is decoupled from the fiber-taper, the local oscillator and the mixing function (provided by the microtoroid) are eliminated and the detected signal is still the up-converted audio signal that cannot be heard from the speaker.

4.4 Application of OMO in RF-over-fiber and IF-over-fiber links

In a typical RoF communication system, the OMO based signal down conversion may provide a simpler and lower power consumption solution comparing to the conventional RF-over-Fiber/IF-over-Fiber configurations. Similar to RoF, IF-over-Fiber (IoF) [121-124] is a common configurations in microwave photonics linkss used in various applications such as Fiber-Wireless or Satellite-over-Fiber systems [121-124]. In an IoF after down-conversion the IF signal is distributed over the optical link and detected by a photodetector for further processing. Fig. 4.13 (a) shows a conventional IF-over-Fiber link architecture (based on electronic local oscillator and mixers) and Fig. 4.13 (b) is the proposed OMO based link. $P_{RF-in}$ is the RF power received from the antenna, and $P_{LO}$ is the local oscillator power. When an OMO is employed, electronic local
oscillator and mixer is no longer necessary in the system resulting in reduced complexity and power consumption.

![Fig. 4.13. (a) Conventional IF-over-fiber link. (b) OMO base IF-over-fiber link. (c) Conventional RF-over-fiber link.](image)

Fig. 4.13. (a) Conventional IF-over-fiber link. (b) OMO base IF-over-fiber link. (c) Conventional RF-over-fiber link.

Fig. 4.14 shows $P_{RF-b}$ plotted against $P_{RF-in}$ for the configuration shown in Fig. 4.13 (a) and (b). The laser power, optical modulation index and amplification rate of the power amplifier are kept constants. Clearly the OMO mixer based link not only provides a better power response, but a simpler configuration because the $P_{LO}$ is from the mechanical oscillation of the OMO.

![Fig. 4.14. Measured down-converted RF power $P_{b-RF}$ ($P_b$) plotted against $P_{RF-in}$ using OMO mixer (Fig. 4.13 (a)) and electronic mixer (Fig. 4.13 (b)).](image)
Clearly the power consumption of OMO based link is also less than a conventional RF-over-fiber system (Fig. 4.13 (c)) where not only a fast photodetector (with a bandwidth matched to the RF carrier) should be used but also post detection amplification is needed to drive the mixer.

4.5 Summary

In summary, RF frequency mixing using optomechanical oscillation is demonstrated and characterized experimentally using silica microtoroid OMO. An analytical method (OTF method) is proposed to explain the frequency mixing mechanism and predict the mixing power other than solving time coupled differential equations. Accordingly, baseband signal conversion using OMO is also demonstrated and characterized for RF photonic signal processing. The outcomes of OTF method calculation have good agreement to the observed behavior and power amplitude of the baseband converted signal. In addition, power efficiency, waveform distortion, conversion bandwidth, conversion gain and noise figure are briefly discussed for an OMO based link. A demonstration of audio signal conversion using OMO is also shown for the first time. Our study shows that OMO based mixer has reasonable performance in RoF/IoF links as an “OMO radio” (where all electronic elements are replaced by a laser, EO modulator, OMO and photodetector). This is an important step towards the practical application of OMO in RF communication domain. Given the rapid development of monolithic OMO and OMOs with high oscillation frequencies [25], this study may lead into RF photonic links/systems that exploit the unique properties of OMOs to reduce power consumption and provide immunity to electromagnetic interference.
Chapter 5

Mass sensing using optomechanical oscillation

5.1 Introduction to mass sensing using mechanical resonators/oscillators

During the past ten years a plethora of various micro and nano-electromechanical (MEMS and NEMS) mass sensors have been demonstrated that function based on monitoring the mechanical resonant frequency shift, or the bending induced by the mass adding [86-88]. In the dynamic operational mode, almost universally, resonant/oscillation frequency shift is monitored by electric or optical transducers. The magnitude of the frequency shift is proportional to the amount of external mass deposited. Typically, the motion of the resonator/oscillator is excited thermally (for resonators) [89] or by a Piezoelectricity transducer (PZT) actuator (for oscillators) [90]. And various methods such as optical deflection [89, 91], integrated piezoresistive [92], or piezoelectric [87,93] are used to monitor the vibrational frequencies. Fig. 5.1 (a) is a diagram showing the basic principle of a micro-scale cantilever sensor. Part (b) is a SEM picture of the actual micro-cantilever sensor from Ref. 92. The motion of this cantilever is excited by the Piezoelectricity transducer (PZT) actuator and monitored by the laser reflection detected by a detector. When external mass is deposited on the sensor, the resonant/oscillation frequencies shift is proportional to the amount of mass [87]. Therefore a sensitivity slope $\zeta$ can be defined as $|\Delta f_0/\Delta m|$ where $\Delta f_0$ is the frequency shift for corresponding mechanical mode due to the added mass $\Delta m$. Accordingly, the sensing resolution is limited by $\zeta$ and the ability to measure $\Delta f_0$. When $\zeta$ is decided by the dimensions of the sensor, position of the mass and the selected mechanical mode, the best resolution of the
frequency shift is fundamentally limited by the linewidth of the mechanical resonance and the noise level.

Fig. 5.1. (a) A diagram showing the principle of cantilever based micro/nano mass sensor. The motion of the cantilever is excited by the PZT actuator and monitor by the laser reflection. (b) The SEM picture of an actual micro cantilever sensor from Ref. 92.

5.2 Mass sensing using optomechanical oscillation

I-Method

Here we use optomechanical oscillation [18, 49] (as opposed to electromechanical oscillation) to monitor the mass deposited on a micro-mechanical resonator. As discussed in previous chapters, in a high-$Q$ optomechanical resonator, the large circulating optical power results in a strong coupling between the optical and mechanical modes through radiation pressure. The interplay between resonant optical wavelength and the mechanical deformation may result in optomechanically induced gain or loss for the motion of the corresponding mechanical mode (depending of pump laser detuning). In the blue detuned regime ($\Delta\lambda=\lambda_{\text{res}}-\lambda_{\text{laser}}>0$) beyond certain threshold power ($P_{\text{th}}$) the optomechanical gain cancels the mechanical loss and self-sustained optomechanical oscillation ensues. Consequently the transmitted optical power (couple out of microresonator) will be modulated at the mechanical oscillation frequency with an amplitude proportional to optical input power (pump power). So in an optomechanical
oscillator (OMO) the optical power serves both as an efficient actuator and sensitive read-out to monitor resonant frequency of mechanical modes. Since the optomechanical oscillation has an extremely narrow linewidth, $\Delta f_0$ can be measured with very high resolution making the OMO a high-resolution mass sensor. Typically optomechanical oscillation frequency ($f_{\text{OMO}}$) is very close to the natural frequency of the corresponding mechanical mode ($f_0$) and therefore depends on the dimension, geometry and material properties. Therefore an OMO can function as a mass sensor with an integrated excitation and readout components only using a pump laser and a photodetector. Compared to the MEMS/NEMS sensors with expensive piezo based excitation and optical/piezoresistive readout, OMO sensors only require a waveguide to carry both pump power and read-out signal. In addition, the optomechanical oscillation frequency and the optical read-out have no limitation in frequency bandwidth as opposed to piezo based excitation and piezo-resistive read-out sensors, which have their operating frequency limited by the piezoelectric materials [94]. Piezoelectric excitation and transducers of such sensors also require complicated co-fabrication of the transducers and the mechanical resonators, and result in high cost. Furthermore, piezoelectric based sensors may have limitation in working in harsh environment with strong electromagnetic field, which does not affect the all-optical OMO sensors.

Here we use silica microtoroid OMO to perform our mass sensing study because of its well studied optomechanical oscillatory behavior [71]. Note that recently various other micro/nano scale optomechanical oscillators have been proposed and demonstrated [23-27], however all of them have more complicated fabrication process than the device used here and have relatively small capture areas for mass sensing application. Even in
atmospheric pressure and room temperature silica microtoroid OMO has relatively low threshold power [32] and very narrow oscillation linewidth (sub-Hz linewidth has been reported with only 300 µW of optical input power). Compared to other OMO cavities, the silica microtoroid has a large sensitive area for mass sensing [23-27], (because of the large membrane involved in mechanical vibration). Moreover, the relatively simple fabrication process [37] of silica microtoroids also allows for their massive production.

Fig. 5.2 (a) shows the experimental arrangement used for optomechanical oscillation mass sensing. Optical input power is provided by a near-IR tunable laser (λ~1550 nm) that is coupled into and out of the microtoroid cavity using silica fiber-taper. When the laser wavelength is blue detuned (Δλ>0) and the input power exceeds the optomechanical oscillation threshold power (i.e. \( P_{\text{in}}>P_{\text{th}} \)) [32], the amplitude of the transmitted optical power will be modulated at \( f_{\text{OMO}} \) with a modulation depth proportional to \( P_{\text{in}}/P_{\text{th}} \). Upon detection in a photodetector the optical signal generates an electric signal that is measured and analyzed using an oscilloscope and RF spectrum analyzer. Fig. 5.2 (b) shows a schematic diagram showing the cross-section of a microtoroid, mechanical deformation of the silica membrane (associated with the 3rd axial symmetric mechanical mode), and the circulating WG optical mode. The radiation pressure is applied on the structure in radial direction (blue arrows). Fig. 5.2 (c) shows the SEM image of the silica microtoroid used in this study. The silica microtoroid has a major diameter of \( D = 131 \) µm minor diameter of \( d =7.4 \) µm. The silica membrane is 2 µm thick and is attached on top of a silicon pillar with a diameter of \( D_p=5.2 \) µm.
Fig. 5.2. (a) Experimental arrangement used for optomechanical oscillation mass sensing. PD: Photodetector. (b) A schematic diagram showing the cross-section of a microtoroid, mechanical deformation of the silica membrane (associated with 3rd mechanical mode), and the circulating WG optical mode. (c) SEM picture of a silica microtoroid optomechanical resonator/oscillator. Inset: a polymer microbead (mass) on the toroidal region of the resonator. (d) Photograph of a microprobe (loaded with microbeads) used for randomly placing microbeads on the OMO.

Fig. 5.2 (d) shows the photograph of the microprobe loaded with microparticles. The micro-beads are placed using a silica probe with microspherical tip attached to a micromanipulator. The silica microsphere was covered by micro-beads and then put in contact with the microtoroid. So micro-beads were randomly distributed on the silica disk and its toroidal section. The inset in Fig. 5.2 (c) shows a micro-bead with a diameter of 1.3 µm landed on the toroidal section of the microresonator.
2-Experiments and results

Fig. 5.3 shows the spectrum of a microtoroid OMO as external mass (fused silica spheres) is loaded. The microtoroid has pillar diameter $D_P=21.2$ µm, major diameter $D=42$ µm, and minor diameter $d=5.4$ µm. To study the effect of the added mass on $f_{OMO}$, we have added silica microspheres with diameter between 1-5 µm on the silica microtoroid.

![Graph showing frequency vs. detected RF power](image)

![FEM image showing deformation of oscillation mode](image)

![Images showing microtoroid and mass added](image)

Fig. 5.3 (a-1) RF spectrum of the detected optical power for the OMO under test. (b) FEM image showing the deformation of the oscillation mode. (c-1) to (c-3) Images showing the microtoroid in the experiment and the mass added (silica spheres within the circles). (c-4) An SEM image showing the microtoroid and mass added.

When mass (~191 pg) is loaded on the edge of the microtoroid (Fig. 5.3 (c-2)), the oscillation frequency shift about 16 kHz from 13725 kHz (Blue peak) to 13709 kHz (Red peak). Next additional mass (~290 pg) is loaded on top of the microtoroid pillar (white area) as shown in Fig. 5.3 (c-3), and the oscillation frequency stays without shift (Green peak).
peak). Regarding the mechanical deformation in the microtoroid cross-section shown at Fig. 5.3 (b) by FEM simulation, an intuitive understanding is that the frequency shift requires displacement of the loaded mass. When the mass is deposited at location with no displacement (on the pillar region), optomechanical oscillation frequency shift is negligible. Therefore, microtoroids with smaller pillars are considered better candidates for mass sensing, because apparently they have larger free moving areas with the same values of $D$.

A microtoroid OMO with extremely small pillar ($D_p\approx11.3 \ \mu\text{m}$) and large disk ($D\approx133 \ \mu\text{m}$, $d\approx7.2 \ \mu\text{m}$) is measured in the experiment. We have placed Polyethylene microspheres (density\~1.3 $\text{g/cm}^3$, FM Series Microspheres, Cospheric) with diameters between 0.5-2 $\mu\text{m}$ and masses between 0.1 – 4 pg on the silica microtoroid. The microtoroid has optomechanical oscillation with $f_{\text{OMO}}=24.88 \ \text{MHz}$, and the changes of $f_{\text{OMO}}$ are measured accurately and shown in Fig. 5.4 (a). Fig. 5.4 (b-1) to (b-4) show the corresponding mass distribution. (b-5) are the close-up SEM images of the particles. As we add mass on the microtoroid, the optomechanical oscillation frequency $f_{\text{OMO}}$ is reduced gradually. Fig. 5.4 (c) shows $\Delta f_{\text{OMO}}$ plotted against the values of loaded mass. The value of the added mass is calculated using the density and the volume of the particles. The volume of particles were estimated by Scanning Electron Microscope (SEM) imaging after each step. The sensitivity of the fundamental oscillation frequency to mass variation is about 72 Hz/pg that is in good agreement with the results using FEM simulation (black triangles connected with a dashed line). The inset of Fig. 5.4 (c) shows the deformation of the oscillation mode using FEM simulation.
Fig. 5.4. (a) Measured RF spectrum of the optomechanical oscillation for mass loaded cases from (b-1) to (b-4). RBW: resolution bandwidth of the RF spectrum analyzer. (b-1) to (b-4) Microscope images of the corresponding mass distribution. (b-5) SEM close-up pictures of selected areas of (b-4) showing the distributions of microspheres. (c) Measured and simulated frequency shift $\Delta f_{\text{OMO}}$ and 5th harmonic frequency shift plotted against the amount of mass loaded. Inset: FEM image showing the deformation of the mechanical eigen mode.
By decreasing the pillar diameter and increasing the disk diameter, we achieve larger capture area (sensitive to mass loaded). As trade-off, $f_{OMO}$ drops and consequently reduces the sensing sensitivity. To compensate the decrease of $f_{OMO}$, we monitor the harmonic frequencies instead of the fundamental frequency of optomechanical oscillation. Fig. 5.4 (c) shows the measured frequency shift of the 5th order harmonic component (blue squares). As expected the sensitivity of the 5th harmonic is about five times larger (344 Hz/pg). Note that the higher harmonics are generated by the nonlinear optical transfer function of the optical microresonator as described in Ref. 32, 51, 72 and Chapter 3. Basically the oscillation of the Lorentzian transfer at $f_{OMO}$ (caused by mechanical oscillation) modulates the transmitted optical power at $f_{OMO}$ and the nonlinearity generates the harmonic frequencies. So a mass induced mechanical frequency shift of $\Delta f_{OMO}$ is translated to optical amplitude modulation frequency shift of $n \times \Delta f_{OMO}$ (where $n$ is the harmonic component whose shift is measured). Since the oscillation linewidth is almost equal for all harmonics, the intrinsic optical harmonic generation mechanism functions as an amplifying mechanism that enhances the frequency shift at higher harmonics [72]. Although here we use the 5th order harmonic, higher order harmonic generation can be also excited [72] using larger optical input power and optimized detuning. In principle this approach can be applied to all kinds of OMO based mass sensors to increase the slope response and signal-to-noise ratio (SNR) for frequency shift measurement and therefore high resolution mass detection. Note that as shown later the sensitivity $\varsigma = |\Delta f_0/\Delta m|$ is different for each mechanical mode and depends on the location of the added mass. Independent of what mode is excited; the induced shift of the
corresponding $n^{th}$ harmonics frequency will always shift $n$ times more than $\Delta f_0$ for a given mass and mass location.

A third microtoroid with an alternative mechanical eigen mode is measured. The microtoroid has $d \sim 7.8 \mu m$, $D \sim 132 \mu m$, and smaller pillar $D_p \sim 5.2 \mu m$ comparing to the previous one. The optomechanical oscillation frequency $f_{OMO} = 8.5$ MHz. Fig. 5.5 shows the microtoroid and the mass distribution in the experiment. The corresponding $f_{OMO}$ are measured as shown in Fig 5.6 (a). In addition, the $5^{th}$ order harmonic component of the fundamental oscillation frequency is also measured as shown in Fig. 5.6 (b).
Fig. 5.6. (a) Measured RF spectrum of the optomechanical oscillation. RBW: resolution bandwidth of the RF spectrum analyzer. (b) Measured RF spectrum of the 5th harmonic frequency component. (c) Measured and simulated frequency shift $\Delta f_{\text{OMO}}$ plotted against the amount of mass loaded. Inset: FEM simulation showing the deformation of the mechanical mode. ($Q_0 = 6.8 \times 10^7$, $Q_{\text{tot}} = 6.72 \times 10^6$, $f_{\text{OMO}} = 8.5$ MHz, $Q_{\text{mech}} = 1300$, $P_{\text{th}} = 553.7$ $\mu$W, $P_{\text{in}} = 2.4$ $P_{\text{th}}$, $\Delta \lambda_N = 0.56$.)

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In Fig. 5.6 (c), the frequency shift $\Delta f_{OMO}$ is plotted versus loaded mass for both fundamental and 5\textsuperscript{th} order harmonic components when the polyethylene microspheres are added. The square-dashed line represents the FEM simulation and agrees the experiment measurements of fundamental frequency shift with $\varsigma \sim 266$ Hz/pg. The 5\textsuperscript{th} order harmonic shows greater frequency shifts significantly, and the corresponding sensitivity $\varsigma$ is improved about 5 times to 1380 Hz/pg. The inset of Fig. 5.6 (c) shows the deformation of the mechanical mode using FEM simulation.

5.3 Characterization of microtoroid OMO based mass sensor

We characterize the microtoroid OMO based mass sensor. The impact of mass distribution, microtoroid dimensions, and mechanical modes on frequency response are studied in detail. The different noise sources of microtoroid OMO sensors are also investigated. The outcomes of these studies define a framework for exploring optimized device design for molecules detection and mass measurement. It paves the ground for design and fabrication of optimized OMO-based mass sensors. In the first part (5.3.1) of this section, we focus on the optimization of frequency shift for different applications. The noise performance and detection limit of OMO mass sensing are analyzed in the second part (5.3.2).

5.3.1 Sensitivity of a microtoroid based mass sensor

The modal and position dependence of $\varsigma$ suggests the optimal design for an OMO mass sensor may vary for three different applications: 1) lumped mass measurement, 2) lumped mass detection and 3) distributed mass detection/measurement. For lumped mass
measurement the OMO is used to measure the mass of a particle that can be placed at a desired position on the OMO. So for best resolution the OMO dimensions, excited mechanical mode and the position of the particle should be selected such that \( \zeta(r, \theta) \) is maximized. For lumped mass detection, the microtoroid is used to detect the existence of particles on its active area. The distribution of the particles is unknown (particles randomly fall on different locations on the device) and maximum probability of detection is desired.

Fig. 5.7. (a-1) to (a-4) FEM simulation showing the deformation of the microtoroid for the 4 eigen modes. The microtoroid has \( D_r \approx 11.25 \, \mu m \), \( D \approx 133 \, \mu m \), and \( d \approx 7.4 \, \mu m \). \( f_0 \) is the eigen frequency where \( f_0 = f_{OMO} \). Color represents the displacement normalized to the microtoroid itself. (b) Calculated sensitivity \( \zeta \) plotted versus radial position of the added mass \( r \) (at \( \theta = 0 \)) for the four modes in part (a). Solid lines represent the calculation based on Eq. (5.3.1) and dashed lines represent the FEM simulation by putting nodal mass. The microtoroid has \( D_r \approx 11.3 \, \mu m \), \( D \approx 133 \, \mu m \), and \( d \approx 7.2 \, \mu m \). \( \Delta m = 1 \) pg.
Therefore the OMO dimensions and the excited mechanical mode should be chosen to provide the largest area that translates the presence of the particle into a measurable frequency shift. For detection and measurement of distributed mass that is almost uniformly distributed over the OMO, the OMO dimensions and the excited mechanical mode should result in maximum oscillation frequency shift for a given surface density.

In our study we focus on four mechanical eigenmodes of the silica microtoroid shown in Fig. 5.7 (a-1) to (a-4), with lowest threshold powers (based on our experiment and the previous studies in Chapter 2 and Ref. 60). The 1st 2nd and 4th modes have been observed in the experiments in Fig. 5.3, 5.4 and 5.5, respectively. Using first order perturbation theory and dynamic equations for an elastic medium [Appendix B], it can be shown that the frequency shift due to added mass of $\Delta m$ at the position $r$ and $\theta$ on the microdisk can be written as:

\[
\Delta f_{OMO} \approx -f_{OMO} \frac{E_{\Delta m}}{2E_{total}} = -\pi^2 f_{OMO} \frac{\Delta m(r,\theta)U^2(r,\theta)}{E_{total}}
\]  

(5.3.1)

where $E_{total}$ is the total energy stored in the OMO, $E_{\Delta m}$ is the energy change due to external mass, $\Delta m(r,\theta)$ is the external mass at position $(r,\theta)$, $U(r,\theta)$ is the corresponding maximum total displacement (in 3 dimensions) at the point $(r,\theta)$. Note that here we assume the external mass is positioned on the flat surface of the non-optical vibrating plate so that $z=z_0$. It is assumed that the added mass is small such that it only perturbs the frequency and not the mode shapes. Furthermore the particle is physically small enough that the effects of its momentum of inertial is negligible (i.e. nodal mass). In spite of these limiting conditions Eq. (5.3.1) can be used to study and optimize the mass sensitivity for different scenarios. The required simulation of the natural frequency, and
the corresponding mode of mechanical oscillation is a straightforward approach. In our study the sensitivity $\varsigma$ is calculated using two methods. The first method is based on Eq. (5.3.1) where all the required parameters of the passive mechanical resonator (i.e. $f_{OMO}$, $U$ and $E_{total}$) are calculated computationally. We refer to this approach as the Energy Method. The second method determines the eigenfrequencies of the resonator before and after adding a nodal mass and subsequently extracts the frequency shift. We use a finite element modeling software (ANSYS) for the calculation and we refer to it as FEM Method. The second method takes into account possible perturbation of the mode shape due to the added mass. The simulation results in Fig. 5.4 (c) and 5.6 (c) are calculated using the FEM Method and agree with all experimental results in this work. Fig. 5.7 (b) shows $\varsigma$ plotted against radial position of mass ($\theta = 0$) for the modes shown in part (a) using Energy and FEM Methods. Agreement between the two methods justifies the use of energy method that significantly simplifies the calculation processes for multiple masses and different positions (only one time FEM modeling in the absence of mass is needed). In addition by connecting the frequency shift to the mechanical displacement associated with the corresponding mode, the Energy Method provides an insight to the mass sensing process.

For mass measurement, i.e. where the mass can be placed at the desired position, the mode order ($i$) and optimal position ($r_m, \theta_m$) should be chosen such that maximize the slope sensitivity should be chosen (i.e. $\varsigma_{\text{max}} = \varsigma_i, \text{max} = \varsigma_i(r_m, \theta_m)$, $\varsigma_{\text{max}}$ represents absolute maximum for the modes studied here). According to Fig. 5.7 (b) for the toroid in Fig. 5.7, the best choice is the 3$\text{rd}$ mode and the best position is $r = 17 \mu\text{m}$ and arbitrary value $\theta$ (due to axisymmetry). Near this specific position $\varsigma_3 \approx 500$ Hz/pg for the fundamental
optomechanical oscillation. As discussed previously the sensitivity of optically monitored oscillation can be as large as $n \times 500$ Hz/pg where $n$ is the order of harmonic modulation measured. Fig. 5.8 shows $\varsigma_{\text{max}}$ plotted against $D/D_P$ for two different values of $D$ and the four selected eigenmodes ($i = 1, 2, 3, 4$). Generally $\varsigma_{\text{max}}$ increases as $D$ is decreased and for a given $D$, shows an asymptotic behavior as a function $D/D_P$ reaching a saturated value for $D/D_P > 10$. This behavior can be explained using Eq. (5.3.1) as follows: the mechanical energy is stored in the pillar and the disk; when a larger portion of energy is stored in the disk (larger $D/D_P$ makes the disk more compliant so more energy is stored in the disk) more displacement ($U$) is expected however a larger $D/D_P$ value lowers the resonant frequency ($f_{\text{OMO}} \approx f_0$). So since $\Delta f_{\text{OMO}} \propto f_{\text{OMO}}^3 \times U^2$ these two effects balance each other when $D/D_P$ is large enough. On the other hand decreasing $D$ increases $f_{\text{OMO}}$ but the ratio $E_{\Delta m}/E_{\text{total}}$ is almost constant for very small $\Delta m$.

![Fig. 5.8. Calculated maximum sensing sensitivity $\varsigma_{\text{max}}$ plotted against $D/D_P$ for two different values of $D$ and the four selected eigen modes.](image_url)
Note that as expected this study suggests that the oscillation frequency of smaller OMOs and those with smaller effective mass (For example, zipper photonic crystal [27] and spoke-supported silica microtoroid or silicon nitride microrings [25, 26, 96]) should be more sensitive to the added mass compared to the silica microtoroid. However, loading mass to the most sensitive location of these OMOs is a difficult task and in most cases the presence of the particle interferes with the resonant optical field that can easily degrade the optical-$Q$ to a level that quenches the optomechanical oscillation or at least broadens its linewidth. Comparing to cantilever oscillator, the 2-dimension geometry of microtoroid allows many exotic deformations that may have extremely sensitive spots for external mass, and is suitable for mass measurement.

For lumped mass detection, the goal is to detect the existence of mass particles on the OMO. Therefore the major concern is to maximize the capture ability of the OMO and keep the induced frequency shift above the minimum detectable frequency change ($\Delta f_{\text{min}}$, that is defined by different noise mechanisms and the resolution of RF spectrum analyzer). So for a given mass change $\Delta m$, the sensitive area ($A_s$) is defined by $\Delta f_{\text{OMO}}>\Delta f_{\text{min}}$. Fig. 5.9 shows $A_s$ plotted against $D$ for the four selected eigen modes a fixed major diameter of with $D_p=115$ $\mu$m, particle mass of $\Delta m=1$ pg and $\Delta f_{\text{min}}=40$ Hz (chosen based on the limitation of our measurement system). Note that in this situation we expect that $D$ has strong impact to $A_s$ because it directly defines the surface area. Except the first mode, which has a frequency shift below $\Delta f_{\text{min}}$, each eigen mode shows an optimized major diameter ($D_{\text{opt}}$) resulting maximum values of $A_s$. In addition, $D_{\text{opt}}$ plotted versus mass value ($\Delta m$) is shown in the inset of Fig. 5.9. The optimized major
diameter of the microtoroid is proportional to the mass of the added particle, which suggests that larger microtoroids provide more sensitive area to catch heavy particles.

Fig. 5.9. Sensitivity area $A_s$ plotted against $D$ for the four eigen modes. Inset: Optimized microtoroid diameter $D_{opt}$ plotted against the values of added particle mass.

The microtoroid OMO can be also used to detect uniform mass deposition on its surface. This capability is effectively translated to measuring the surface density ($\rho_s$) of gas molecules that are adsorbed on the OMO surface. Since in thermodynamic equilibrium $\rho_s$ is proportional to the concentration of the molecules in the surrounding medium one can extract the volume density from oscillation frequency shift. Note that in case of biomolecules the adsorption probability can be significantly enhanced by functionalizing the surface (e.g. an organic monolayer deposited on top of the microtoroid). Hence, for a known molecular mass, given microtoroid and mechanical mode, the frequency shift is proportional to $\rho_s$. For the distributed mass, Eq. (5.3.1) can be modified as:

$$\Delta f_{OMO} \approx -\pi^2 f_{OMO}^3 \rho_s(r, \theta) \Delta a U^2(r, \theta) E_{total}$$

(5.3.2)
where $\rho_s(r, \theta)$ is surface mass density at position $(r, \theta)$ on the microdisk and $\Delta a$ is the area in contact with the external source of mass. To evaluate the contribution of the whole capture area, the figure of merit $\Lambda$ is defined as:

$$\Lambda = \frac{\pi^2 f_{OMO}}{E_{total}} \int_A \rho_s(r, \theta)U^2(r, \theta)da$$

(5.3.3)

$\Lambda$ has units of Hz and represents the total frequency shift of a microtoroid due to the surface density of absorbed gas molecules, $\rho_s$. This term, $\Lambda$, also contains the effects of microtoroid dimensions, which directly defines the capture area, frequency of oscillation and mechanical deformation (for a specific mode) for a particular mass distribution.

![Figure 5.10](image)

Fig. 5.10. Calculated $\Lambda$ plotted against $D/D_P$ for different values of $D$ and the four eigen modes.

Fig. 5.10 shows $\Lambda$ is plotted against $D/D_P$ for two different values of $D$ and the four selected eigenmodes as shown in Fig. 5.7 (a), assuming a uniform mass distribution with a constant surface mass density of $\rho_s = 0.1 \text{ ng/mm}^2$. Except the first mode, the result shows decrease relation of $\Lambda$ versus $D$ and non-monotonic behavior of $\Lambda$ over $D/D_P$, which can be explained similarly by the discussion in mass weighting but with an
integrated $E_{\Delta m}$ over capture area. It shows that the reduction of capture area is compensated by the growth of oscillation frequency, and suggests better sensitivity for some smaller devices.

5.3.2 Noise and detection limit of a microtoroid based mass sensor

Noise in a microtoroid OMO based mass sensor comes from multiple sources and limits the resolution of frequency shift measurement by broadening the optomechanical oscillation linewidth ($\delta f_L$) and drifting the center frequency. Besides, the instrument resolution in the measurement limits the accuracy of frequency detection. In this section, we analyze the ultimate detection limit of optomechanical mass sensing.

1-Oscillation Linewidth

In the absence of oscillation frequency fluctuations, the oscillation linewidth defines a fundamental limit for mass sensing resolution through $\Delta m_{\text{min}} = (\Delta f_{\text{min}}/\zeta) = (\delta f_L/\zeta)$. According to previous studies [32], thermo-mechanical noise (or the Brownian noise) is the dominant noise mechanism that limits $\delta f_L$ and is affected by temperature, mechanical loss, $m_{\text{eff}}$ and the input optical power. Note that the equilibrium temperature of the OMO is controlled by the temperature of the surrounding medium and optical input power (due to optical absorption). The linewidth of a microtoroid OMO have been characterized [32] as:

$$
\delta f_L = \frac{k_B T}{P_d} (\delta f_0)^2 = \frac{k_B T}{4\pi^2 m_{\text{eff}} f_0^2 (R_0 H_d)} \delta f_0^2 = \left( \frac{k_B T Q_{\text{tot}}^2}{m_{\text{eff}} \pi^2 f_0^2 R_0^2} \right) \left( \frac{\Gamma^2 \delta f_0^2}{M^2} \right) \tag{5.3.4}
$$

$P_d$ is the oscillator output power. the linewidth is determined by the Boltzmann constant $k_B$, absolute temperature $T$, $Q_{\text{tot}}$ is the total optical $Q$ factor, $m_{\text{eff}}$ is optomechanical effective mass, $\delta f_0$ is the intrinsic linewidth ($\delta f_0 \approx f_0/Q_{\text{mech}}$, $Q_{\text{mech}}$ is mechanical quality
factor) of the mechanical mode, \( R_0 \) is the radius of the microtoroid, \( H_A = r_{\text{max}}/R_0 \) is the relative oscillation amplitude that can be estimated by optomechanical gain \( \Gamma_{\text{OM}} \), \( r_{\text{max}} \) is the mechanical oscillation amplitude, \( \Gamma \) is the optical modulation transfer function, and \( M \) is modulation depth. When the optical path length in an optical resonator is modulated, the finite response time of the resonator is translated to a low-pass filter function. We call this filter function “optical modulation transfer function” of the resonator or \( \Gamma = \Gamma(f) \). It can be achieved by solving the dynamic time differential equations (Eq. 2.4.1 and 2.4.2).

Modulation depth \( M \) is directly decided by the resonator displacement due to radiation pressure, and there is \( M = P_{\text{mod}}/P_{\text{max}} \). \( P_{\text{mod}} \) is the modulated optical power due to optomechanical oscillation; \( P_{\text{max}} \) is the maximum transmitted optical power (Fig. 2.6 (b) in Chapter 2). Apparently high mechanical \( Q \) (small \( \delta f_0 \)) and low temperature result in small oscillation linewidth, and by increasing the pump power, the oscillation linewidth can be effectively reduced. However, the effect of increasing power is not trivial because the overall temperature is coupled to \( P_{\text{in}} \) due to the thermal absorption of silica. Fig. 5.11 (a) plots \( \delta f_L \) against \( P_{\text{in}} \) for a typical microtoroid, the temperature is estimated by FEM simulation. The optomechanical oscillation is considered within linear regime so that \( P_d \approx P_{\text{in}} \). The behavior shows that \( \delta f_L \) monotonically decreases as input power \( P_{\text{in}} \) increases over \( \mu \text{W} \) to \( \text{mW} \) range. Besides directly increasing the input optical power, \( \delta f_L \) can also be reduced by adjusting coupling and optical detuning to maximize oscillation amplitude \( H_A \). Eq. (5.3.4) also shows that the oscillation linewidth is governed by the geometry that affects \( m_{\text{eff}}, f_0, T, \delta f_0 \) and \( R_0 \). To study the impact of geometry we plot \( \delta f_L \) as a function of \( D/D_p \) for 3 different values of \( D \) as shown in Fig. 5.11 (b).
Fig. 5.11. (a) Calculated $\delta f_L$ plotted against $P_{in}$ for a typical microtoroid by FEM. $D=133$ µm, $\Delta\omega_N=0.5$, $f_0=8$ MHz, $Q_{mech}=1300$, and $Q_0=1\times10^7$. (b) Calculated $\delta f_L$ plotted against $D/D_P$ for 3 different values of $D$. $P_{in}=400$ µW, $\Delta\omega_N=0.5$, $Q_{mech}=1300$, $P_{in}=3 P_{th}$.

Here $D=2R_0$ is the major diameter, and $D_P$ is the pillar diameter. The mechanical quality factor $Q_{mech}$, intrinsic optical quality factor $Q_0$, coupling, detuning and input power are constant, and $P_{in}$ is always above threshold. The environment temperature is assumed constant, so the variation of $T$ is only contributed by the presence of optical power and simulated by FEM as well as $m_{eff}$ and $f_0$. These results show that large $P_{in}$ and large $D/D_P$ reduce the oscillation linewidth. Note that for a regular silica microtoroid, 1 Hz oscillation linewidth is easily achievable with reasonable input power (below milliwatt) and dimension ($D \sim 60$ µm) [32].

2-Oscillation Frequency Fluctuation

In the presence of circulating optical power the oscillation frequency ($f_{OMO}$) will be shifted relative to the mechanical eigenfrequency ($f_0$) due to “optical spring effect” and “thermal effect” [32, 50, 55]. This shift ($\Delta f_P = f_{OMO} - f_0$) translates the uncertainties in input parameters, mainly $\Delta\omega_N$, $Q_{tot}$ and $P_{in}$, to fluctuations of $f_{OMO}$ ($\Delta\omega_N$ is the normalized detuning where $\Delta\omega_N=\Delta\omega_0/\delta\omega = (\omega_{laser}-\omega_{res})/\delta\omega$). Depending on its magnitude this
optomechanical oscillation frequency fluctuation, or \( \delta(\Delta f_P) \), may screen the frequency shift due to mass variations \( (\Delta f_{\text{OMO}}) \) and limit the mass sensing resolution.

Optical spring effect on a microtoroid refers to rigidity modification of the structure by radiation pressure: when radiation pressure applies on a microtoroid cavity, it changes the stiffness (spring constant) of the oscillator, and consequently the oscillation frequency. This so called “optical spring” effect has been discussed in section 2.4. The frequency shift due to optical spring effect at above threshold has been studied \([50, 55]\) and can be written as:

\[
\Delta f_{\text{os}} = F^2 \cdot \frac{2n_{\text{eff}}^2 \omega_0}{\pi^2 f_0 m_{\text{eff}} c^2} \left( \frac{2 \Delta \omega_0 \tau_{\text{tot}}}{(4 \tau_{\text{tot}}^2 \Delta \omega_0^2 + 1)} \right) P_0 \;
\]

for unresolved sideband regime \( (f_0 << \delta \omega / 2\pi) \). \( F = 2\pi \tau_{\text{tot}} / \tau_{\text{RT}} \) is the Finesse of the optical cavity; \( \tau_{\text{RT}} \) is the round trip time; \( n_{\text{eff}} \) is the effective refractive index of the microtoroid; \( \omega_0 \) and \( f_0 \) are the optical and mechanical resonant frequency, respectively. \( \tau_{\text{tot}} \) is the total decay time of the cavity, where \( \frac{1}{\tau_{\text{tot}}} = \frac{1}{\tau_{\text{ex}}} + \frac{1}{\tau_0} \), with \( Q_{\text{tot}} = \omega_0 \tau_{\text{tot}} \), \( Q_{\text{ex}} = \omega_0 \tau_{\text{ex}} \) and \( Q_0 = \omega_0 \tau_0 \).

The temperature change of the microtoroid due to circulating optical power and the environment can also shift the oscillation frequency. In the presence of circulating optical power, absorption heat of the optical power generates temperature gradient in the structure proportional to \( P_{\text{circ}} \). This effect is strongly geometry dependent and has linear behavior to the oscillator temperature \([32]\), and the frequency shift can be written as \( \Delta f_{\text{th}} = f_0 \eta_T (T - T_0) \). Here \( f_0 \) is the oscillation frequency at temperature \( T_0 \), and \( \Delta f_{\text{th}} \) represents the temperature shift from \( T_0 \) to \( T \), where \( T - T_0 << T_0 \). \( \eta_T \) is a linear coefficient summarizing all thermal effects. If we ignore the environment effect and only consider the thermal absorption due to \( P_{\text{circ}} \), there is
\[ \Delta f_{th} = N \beta \gamma \frac{\lambda Q_{tot}^2}{\pi^2 n Q_{ex}} \frac{\delta \omega^2}{\omega_0^2 + \delta \omega^2} P_{in} \]  

(5.3.6)

where \( G \) and \( N \) are the parameters determined by geometry of the microtoroid and the material properties, and \( f_0 = G + NR_0 \). \( \beta \) is the linear expansion coefficient of silica, and \( \gamma \) is the proportionality factor between the circulating optical power and the temperature increment so that \( T - T_0 = \gamma P_{circ} \).

Therefore, the total frequency shift due to the presence of \( P_{circ} \), including optical spring effect and thermal effect can be written as:

\[
\Delta f_p = \Delta f_{as} + \Delta f_{th} = \left\{N \beta \gamma \frac{\lambda Q_{tot}^2}{\pi^2 n Q_{ex}} \frac{\delta \omega^2}{\omega_0^2 + \delta \omega^2} + \frac{2 n^2 \omega_0}{\pi^2 f_0 m_{eff} c^2} \frac{2 \Delta \omega_0 \tau_{tot}}{(4 \tau_{tot}^2 \Delta \omega_0^2 + 1)} \right\} P_{in}
\]

(5.3.7)

for \( f_0 \ll \) optical resonant bandwidth that is usually the case for optomechanical oscillation. \( \Delta f_p \) is a complicate function of optical input, detuning, coupling, oscillation frequency and microtoroid dimensions. Note that optical spring and thermal effects are generated naturally in the presence of circulating optical power, and cannot be eliminated in optomechanical oscillation. Therefore any fluctuation of input parameters, such as detuning or \( P_{in} \), will transfer to fluctuation of \( f_{OMO} \). For mass sensing application, this optomechanical oscillation frequency fluctuation noise \( \delta(\Delta f_p) \) is the major concern, instead of the total frequency shift from the initial value. In our experiment, there are three major parameters \( \delta P_{in}, \delta(\Delta \omega_N) \) and \( \delta(Q_{tot}/Q_0) \) contributing to \( \delta(\Delta f_p) \) as:

\[
\delta(\Delta f_p) = \left[ \frac{\partial(\Delta f_p)}{\partial P_{in}} dP_{in} + \frac{\partial(\Delta f_p)}{\partial (Q_{tot}/Q_0)} d\left( \frac{Q_{tot}}{Q_0} \right) + \frac{\partial(\Delta f_p)}{\partial (\Delta \omega_N)} d\Delta \omega_N \right]
\]

(5.3.8)

Fig. 5.12 (a), (b) and (c) show \( \Delta f_p \) plotted against \( P_{in}, Q_{tot}/Q_0 \) and \( \Delta \omega_N \) respectively. The slopes of \( \Delta f_p \) in Fig. 5.12 represent the efficiencies of translating the fluctuation of \( P_{in}, \)
$Q_{\text{tot}}/Q_0$ and $\Delta \omega_N$ to $f_{\text{OMO}}$. Apparently, it is possible to minimize $\delta(\Delta f_P)$ (for the same amount of instabilities in $Q_{\text{tot}}/Q_0$ and $\Delta \omega_N$) by choosing the optical coupling ($Q_{\text{tot}}/Q_0$) and detuning ($\Delta \omega_N$) values that also result minimum slopes. Note that these coupling and detuning values may result in smaller optomechanical oscillation amplitude [50] than the optimized amplitude, and therefore reduce the signal-to-noise ratio in the frequency peak detection. However, increasing $P_{\text{in}}$ can retrieve the optomechanical gain, which means that we can reduce both oscillation linewidth and fluctuation by consuming more optical power.

![Diagram](image)

Fig. 5.12. (a) Calculated $\Delta f_{\text{os}}$, $\Delta f_{\text{th}}$ and $\Delta f_P$ plotted against $P_{\text{in}}$. (b) Calculated $\Delta f_{\text{os}}$, $\Delta f_{\text{th}}$ and $\Delta f_P$ plotted against $Q_{\text{tot}}/Q_0$. (c) Calculated and measured $\Delta f_{\text{os}}$, $\Delta f_{\text{th}}$ and $\Delta f_P$ plotted against $\Delta \omega_N$. All other parameters are fixed for (a)-(c). $Q_0=5.78 \times 10^7$, $Q_{\text{tot}}=6.8 \times 10^6$, $P_{\text{in}}=92 \ \mu\text{W}$, $f_0=16 \ \text{MHz}$. $\Delta f_{\text{os}}$ and $\Delta f_{\text{th}}$: frequency shift due to optical spring and thermal effects, respectively. $\Delta f_{\text{os}} + \Delta f_{\text{th}} = \Delta f_P$. 

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Besides the optimization of coupling and detuning, we have also studied the impact of the microtoroid dimensions on $\delta(\Delta f_P)$, because both optical spring and thermal effects are dependent on the microtoroid dimensions. Fig. 5.13 (a) shows calculated frequency fluctuation $\delta(\Delta f_{os})$, $\delta(\Delta f_{th})$, and $\delta(\Delta f_P)$ plotted against $D/D_P$ for three different values of $D$. Here $\delta(\Delta f_{os})$ and $\delta(\Delta f_{th})$ are the frequency fluctuation induced by optical spring and thermal effects ($\delta(\Delta f_P) = \delta(\Delta f_{os}) + \delta(\Delta f_{th})$) calculated for estimated values of fluctuations in our experiment (i.e. $\delta Q_{tot}/Q_0 = 1\%$, $\delta P_{in} = 1.3 \times 10^{-6}$ µW, $\delta(\Delta \omega_N) = 0.017$) near $Q_{tot} = 6.8 \times 10^6$, $P_{in} = 1.3$ mW, $\Delta \omega_N = 0.5$, and $Q_{mech} = 1300$ (parameters are chosen based on the laser, piezo and microtoroid used in our experiment). The frequency fluctuations due to both thermal effect and optical spring effect decrease as we increase the major diameter $D$ as anticipated. When the pillar diameter reduces, the thermal fluctuation increases because of the poor heat dissipation through Si pillar, but the optical spring fluctuation decreases because more mass is involved in the oscillation and increases $m_{eff}$. When the optical spring effect dominates, larger value of $D/D_P$ is preferred for low noise.

The schematic diagram in Fig. 5.13 (b) summarizes the parameters affecting $\delta f_L$ and $\delta(\Delta f_P)$ in the aspect of actual experiment setting. Blue arrows represent an inverse relation; so large $f_0$, $m_{eff}$ (effective mass of optomechanical oscillation) and $D$ can reduce both $\delta f_L$ and $\delta(\Delta f_P)$. In our system, the frequency fluctuation noise dominates ($\delta(\Delta f_P) \gg \delta f_L \sim 1$Hz).

The noise of the photodetector (photodiode), such as shot noise, dark noise, and thermal noise, are not considered here, because they do not affect the frequency detection for the large signal-to-noise ratio (SNR). Since the oscillation linewidth of the microtoroid OMO is below 1 Hz in our experiment, the detection limit is dominated by oscillation frequency $\delta(\Delta f_P)$. 

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Fig. 5.13. (a) Calculated $\delta f_p$ plotted against $D/D_p$ for different values of $D$. $\Delta$ os represents the optical spring frequency fluctuation. $\Delta Q_{\text{opt}}/Q_0=1\%$, $\Delta P_w=1.3\times 10^{-6}$ µW, $\delta (\Delta \omega_0)=0.017$, and all other parameters are fixed: $Q_{\text{opt}}=6.8\times 10^6$, $P_w=1.3$ mW, $\Delta \omega_0 =0.5$, and $Q_{\text{mech}}=1300$. (b) Schematic diagram summarizing the relation amount the parameters to $\delta f_L$ and $\delta f_P$.

3-Instrument resolution

Besides the noise from the OMO, the detection limit of the instrument has to be considered too. It limits the accuracy to measure the frequency shift on the optomechanical oscillation spectrum. In our experiment, a RF spectrum analyzer (Agilent
N9320B) is used to monitor the spectrum of optomechanical oscillation, and its resolution bandwidth (RBW) limits the detection resolution. Any frequency shift less than the RBW cannot be detected because it is below the resolution of the device. For an RF spectrum analyzer, its RBW is adjustable, but a smaller RBW requires longer sweep time to cover the whole frequency region. In our experiment, RBW = 30 Hz is used.

Therefore, the detection limit of optomechanical frequency shift of a microtoroid OMO mass sensor can be predicted by the following formula:

\[ \Delta f_{\text{Limit}} \geq \delta f_L + \delta (\Delta f_p) + \delta f_I \]  (5.3.9)

\( \delta f_I \) represents the instrument limitation and is constant. In our experiment, it equals to the RBW. If harmonic sensing approach is applied, Eq. (5.3.9) becomes:

\[ n \times \Delta f_{\text{OMO}} \geq \delta f_L + n \times \delta (\Delta f_p) + \delta f_I \]  (5.3.10)

Here \( n \) is the order of harmonic component. The harmonic terms have the same linewidth as the fundamental component of the optomechanical oscillation, because the harmonic and fundamental oscillation components in optical domain are all generated through the same process of optical power modulation by mechanical vibration (as shown in Chapter 3). Use \( \zeta = \Delta f_{\text{OMO}}/\Delta m \), the minimum detectable mass can be written as:

\[ \Delta m_{\text{min}} \approx \frac{\delta f_I + \delta f_L + \delta (\Delta f_p)}{n \times \zeta} \]  (5.3.11)

It shows that the first term of Eq. (5.3.11) can be reduced using harmonic sensing approach, and consequently the second term dominates the value of \( \Delta m_{\text{min}} \). Fig. 5.14 (a) plots \( \Delta m_{\text{min}} \) as a function of \( D/D_p \) for the four selected mechanical eigenmodes and two
different major diameters for $n = 5$. The detection limit is improved as we increase $D$ and $D/D_P$, because large values of $D$ and $D/D_P$ results in less $\delta(\Delta f_I)$. For the microtoroid used in our experiment ($D=133 \ \mu m$, $D/D_P \sim 11.8$), the detection limit is about 0.15 pg for the $2^{nd}$ mechanical eigen modes.

In principle $\delta(\Delta f_I)$ can be reduced by decreasing noise contributed from different noise sources. For example, on-chip waveguide can be used [25, 26] other than fiber-taper to couple light to the OMOs. In integrated OMOs typically the contribution of coupling uncertainty can be ignored. Laser frequency stabilization techniques (i.e. Pound-Drever-Hall [97, 98] method, Hänsch-Couillaud [99] method, etc.) can be used to lock $\Delta \omega_N$ resulting in very small $\delta(\Delta \omega_N)$. Laser frequency stabilization within 500 Hz for 60 hours has also been reported in Ref. 119 (consequently reduction of $\delta(\Delta f_I)$ below 0.1 Hz).

The impact of laser RIN is very small ($<10^{-5}$ Hz for -90 dBm ) comparing to other effects so it is ignored here. In this case, the detection limit will be determined by the oscillation linewidth $\delta f_L$ and instrument resolution $\delta f_I$. Fig. 5. 14 (b) plot the ultimate value of $\Delta m_{\text{min}}$ against normalized detuning $\Delta \omega_N$ for the four selected mechanical eigen modes for $\delta(\Delta f_I)<0.1$ Hz and $n=5$. Here we consider $\delta f_I = 0$ Hz for ultimate detection limit. The behavior of $\Delta m_{\text{min}}$ is mainly due to the variation of $\delta(\Delta f_I)$ and $\delta f_L$ as detuning change. Fig. 5. 14 (c) shows the value of $\Delta m_{\text{min}}$ plotted against order of harmonics $n$ for the $2^{nd}$, $3^{rd}$ and $4^{th}$ mechanical eigen modes for both $\delta f_I = 30$ Hz (solid curves) and $\delta f_I = 0$ Hz (dashed curves). By using higher order harmonics, the frequency measurement uncertainty from linewidth ($\delta f_L$) and RWB ($\delta f_I$) are gradually reduced and $\Delta m_{\text{min}}$ approaches to the limit of the OMO mass sensor ($\Delta m_{\text{min}} \sim 10$ fg) for $\delta f_I = 30$ Hz. Note that for optimizing the
detection limit, the mass has to be added at the locations that have maximum displacement (red regions of the inset figures).

Fig. 5.14. (a) \( \Delta m_{\text{min}} \) plotted against \( D/D_P \) for \( D=103 \) and 133 µm for the selected four modes. \( \Delta \omega_N=0.56 \), \( Q_0=6.8\times10^7 \), \( Q_{\text{mech}}\sim1300 \), and \( P_{\text{n}}\sim2 \ P_{\text{th}} \). Uncertainty of \( Q_{\text{tot}}/Q_0 \) and detuning are 1% and 400 KHz, respectively. \( \delta f_I = 30 \) Hz. (b) \( \Delta m_{\text{min}} \) plotted against normalized detuning \( \Delta \omega_N \) for the selected four mechanical eigen modes. The microtoroid has \( D=133 \) µm and \( D_P=11.3 \) µm, \( Q_0=6.8\times10^7 \), \( \delta(Q_{\text{tot}}/Q_0) \) and \( \delta f_I \) are neglected and \( \delta(\Delta \omega_N) \) is 0.63 Hz, respectively. \( \delta f_I = 0 \) Hz. (c) \( \Delta m_{\text{min}} \) plotted against the order of harmonics \( n \) for the 2nd, 3rd and 4th mechanical eigen modes. The microtoroid has \( D=133 \) µm and \( D_P=11.3 \) µm, \( \delta \omega_N = 0.24 \). Maximum values of \( \zeta \) for each mode are used here (the red spots of the inset images of the microtoroids). Solid lines: \( \delta f_I = 30 \) Hz, dashed lines: \( \delta f_I = 0 \) Hz.

5.4 Summary

In summary, we have demonstrated a microtoroid based OMO mass sensor with sensitivity slope above 1300 Hz/pg and low power consumption (<2 mW). We have also
developed a harmonic sensing approach that can improve sensing sensitivity for all kinds of potential OMO sensors. In addition, the mass sensitivity and noise performance are comprehensively characterized using our theoretical model and FEM. The impact of microtoroid dimensions, mechanical mode shape, position of mass landed, optical input power, coupling factor and optical detuning are analyzed in detail. A resolution better than 150 fg is predicted for a typical microtoroid OMO sensor using our current measurement equipments in room temperature and atmosphere environment. This work lays the foundation for a new class of mass sensors that can be used for characterization and detection of particles and molecules in fully integrated all-optical devices. Note that MEMS/NEMS mass sensors are fundamentally limited by the operating frequency of piezo based excitation. Piezoelectric excitation and transducers of such sensors also require complicated co-fabrication of the transducers and the mechanical resonators, and result in high cost. Furthermore, piezoelectric based sensors may have limitation in working in the presence of strong electromagnetic field, which does not affect the all-optical OMO sensors.
Chapter 6

Dual-mode sensing using OMO and thermally enhanced resonant optical sensing

6.1 Dual-mode sensing using OMO

An OMO sensor is also an resonant optical sensor because of its high-$Q$ optical resonant cavity. The principle of optical label-free biosensors using optical resonators has been briefly discussed in Chapter 3. The optical resonance of the cavity is sensitive to the particles in the vicinity of the optical mode due to the interaction of the evanescent field with molecules/particles. The presence of the molecules/particles changes the effective refractive index of the resonant optical mode and shifts the resonant wavelength/frequency. When the value of resonance shift is proportional to the effective refractive index shift, the detection resolution is limited by the $Q$-factor. The narrow resonant linewidth due to large $Q$ allows high resolution monitoring of very small frequency shifts, and is critical for low concentration bio-molecule detection. Accordingly, the extremely high-$Q$ (above $10^8$) of silica microtoroid OMO makes it an ideal platform for high accuracy optical sensing [102].

Combining the resonant optical sensing with the mechanical sensing (explained in the previous chapters), an OMO is capable to function as a “dual-mode” sensor by translating the presence of molecules both through mechanical oscillation frequency shift and optical resonant frequency shift simultaneously. The diagram in Fig. 6.1 (a) shows the principle of dual-mode sensing. Considering the microtoroid OMO has its surface functionalized to catch certain molecules through molecular binding. The target
molecules (yellow spheres) trapped on the microtoroid OMO will induce the optomechanical oscillation frequency shift by changing the effective mass (as discussed in Chapter 5). In addition to the optomechanical oscillation frequency change, the molecules trapped in the vicinity of the optical mode (red area) will change the resonance of the optical cavity.

Fig. 6.1 (a) A diagram showing the principle of OMO dual-mode sensor. The presence of all molecules will shift the mechanical frequency (top right). For molecules in the vicinity of the optical mode (red circle), the strengths of different harmonics is distorted due to optical resonance shift (that changes detuning). (b) Measured RF spectrum of the optomechanical oscillation frequency and its harmonics without and with external mass. To distinguish the peaks, the spectrums are plotted in different x-axes (bottom and top). The mass ~ 40 pg. The accuracy of the measurement is limited by the large resolution bandwidth, which is necessary to cover the frequency bandwidth in a reasonable time.

This shift can be monitored by either in optical domain by scanning the laser wavelength, or by the amplitude ratio change of the fundamental oscillation and harmonic components.
as described in Chapter 3. As shown in Fig. 3.6, the value of $P_1-P_2$ is approximately linear to optical detuning around the $P_1$-$P_2$ crossing point, and can be used to monitor the detuning change. $P_1$ and $P_2$ are the power values at optomechanical oscillation and the second order harmonic frequencies on the RF spectrum, respectively. Note that the value of $P_1$-$P_2$ is mainly controlled by detuning, and independent of the mass loaded on the micro-disk region of the microtoroid. Fig. 6.1 (b) shows the measured RF spectrum of the a microtoroid OMO without and with external mass. The amplitude ratio of the fundamental and harmonic frequencies are approximately unchanged after adding the mass on the microtoroid. The optical detuning, coupling and optical power are kept constant during the experiment. Based on our measurements, the presence of mass on the microtoroid only changes the optomechanical oscillation frequency but the relative amplitude ratios of the fundamental and harmonic frequency components remain the same. Therefore, changes in the amplitude ratios in the RF spectrum indicate the optical resonance changes. Using this feature, the RF spectrum can function as a convenient read-out for both mechanical and optical modes sensing.

This dual-mode sensing approach combines the two most well-known label-free bio-sensing technologies: mechanical sensing and resonant optical sensing. The mechanical sensing can offer accurate mass information of the molecules. Meanwhile, resonant optical sensing provides information about the dielectric properties of the molecules while the mechanical system reveals their mass. So dual-mode sensing enables the identification of molecules with similar mass but different molecular dipoles and vice versa, which may be useful in diagnostics of certain diseases. The dual-mode sensing approach using OMO extends the application scope of label-free bio-sensors, and may
provide an important tool in bio-molecule detection. Note that the same technique can be implemented using other types of OMOs.

6.2 Thermal line narrowing for resonant optical sensing

The optical-$Q$ is the key parameter for high resolution resonant optical sensing. However, the surface of the sensor has to be functionalized for molecular binding in label-free optical biosensing, and the coating layer will reduce the optical-$Q$ of the sensor due to absorption and scattering. In order to overcome this fundamental limit, great effort has been spent on pursuing high intrinsic $Q$ of the optical resonators [68]. In contrast, here we use thermally induced line-narrowing effect to improve the observed resonant linewidth. It is well known that the thermal dynamics of a high-$Q$ optical resonant cavity together with the scanning laser wavelength result in linewidth broadening or narrowing [1]. When the laser wavelength is scanned through the resonance, the interplay between thermo-optic effect and resonant power build-up generates broadened or narrowed resonant linewidth depending on the direction of laser wavelength scanning. Here we demonstrate that the linewidth narrowing effect is able to enhance sensing resolution of the optical resonant sensors operating in dynamic mode (scan laser wavelength).

The principle of resonant sensing of optical resonator has been introduced in section 3.3 of Chapter 3. A resonant sensor can translate its effective refractive index change to the resonant wavelength such that $\Delta n_s = n_{\text{eff}} \times \Delta \lambda_s / \lambda_{\text{res}}$. $n_{\text{eff}}$ is the original effective refractive index, $\lambda_{\text{res}}$ is the original resonant wavelength, and $\Delta \lambda_s$ is the resonant wavelength shift. Note that the resonance shift can be monitored by directly reading through an oscilloscope. In the case of relatively large molecules/particles, the scattering
centers will cause resonance splitting [102], where the splitting gap is proportional to the strength of the scattering caused by the particle [102]. The detection limit of a resonant optical sensor ($S_{dl}$) is proportional to $\Delta \lambda_s / \delta \lambda_0$ where $\Delta \lambda_s$ is the resonant wavelength shift or splitting caused by molecules/particles and $\delta \lambda_0$ is the linewidth of the loaded optical cavity [64]. For our analysis and proof of concept experiment we use thermal line-narrowing in silica microtoroids (due to thermo-optic effect) [1] to reduce $\delta \lambda_0$ and consequently improve $S_{dl}$, however with proper design the proposed approach can be applied to other high-$Q$ optical cavities. Theoretical analysis and experimental results show that the dynamic linewidth can be improved over 50 times for a given $Q$-factor in certain cases. Given the technological challenges associated with fabrication of monolithic optical microcavities with $Q$-factors exceeding $10^6$, our proposed approach can be an important step toward improving the sensitivity of a large variety of cavity based optical sensors using proper thermal design as opposed to actual quality factor improvement. Note that for biosensing applications the surface treatment needed for functionalizing the optical cavity reduces the intrinsic quality factor lower than the fabrication limit. So even optical microcavities with very high quality factors can benefit from the dynamic linewidth narrowing.

The basics of the thermo-optic dynamic behavior in a microtoroidal cavity has been studied in Ref. 1. Appendix C also shows the thermal dynamic equation in a high-$Q$ microtoroid. As the laser wavelength is scanned through the resonant optical wavelength ($\lambda_{res}$), the interplay between thermo-optic effect and resonant power build-up results in thermal linewidth broadening or narrowing depending on the direction of laser wavelength scanning. Although in certain applications the intrinsic locking mechanism in
the thermally broadened regime is useful, for most applications including sensing the optical input power ($P_{in}$) to the cavity is kept low enough to avoid the thermo-optical linewidth change. Here we show that as a matter of fact working in linewidth-narrowed regime and using larger laser input power improves the resolution for both resonant shift and resonant splitting detection techniques (as the main approaches to resonant optical sensing).

For cavities made of a material with positive $dn/dT$ (such as silica) when the laser wavelength ($\lambda_{laser}$) is scanned from $\lambda_1$ to $\lambda_2$ where $\lambda_1 > \lambda_{res} > \lambda_2$, the heat generated through resonant power build-up results in a dynamic linewidth ($\delta\lambda_d$) that is narrower than the actual linewidth of the loaded optical cavity ($\delta\lambda_0$). Similar effect can be created for negative $dn/dT$ by scanning the laser in the opposite direction. The line-narrowing factor can be defined as the ratio $\eta= \delta\lambda_0/\delta\lambda_d$ and serves as the main figure of merit for sensing applications. The detection limit in the dynamic mode ($S_{dl,D}$) is proportional to $(\Delta\lambda_s/\delta\lambda_d) = (\delta\lambda_s/\delta\lambda_d)$ and therefore is improved by the line narrowing factor ($S_{dl,D} = \eta S_{dl}$). The magnitude of $\eta$ is proportional to the magnitude of $dn/dT$ and the local temperature variation in the vicinity of the optical mode ($\Delta T_{mode}$) as the laser wavelength is scanned through $\lambda_{res}$. Fig. 6.2 (a) summarizes the relation between parameters that control $\eta$. The optical absorption ($\alpha$) and thermal properties of the structural material as well as the geometry of the optical cavity combined with the magnitude of the circulating optical power ($P_{circ}$) determine $\Delta T_{mode}$. $P_{circ}$ is controlled by the intrinsic and loaded optical quality factor ($Q_0, Q_L$) and the optical input power ($P_{in}$). Fig. 6.2 (b) shows $\Delta T_{mode}$ plotted against delay time relative to the time at which $\lambda_{laser}=\lambda_{res}$ for a typical silica microtoroid estimated using finite element modeling. Clearly temperature gradually saturates as the
heat generation through optical absorption is balanced by the dissipation. Here $t_s$ is defined as the time required for $\Delta T_{\text{mode}}$ to reach half of its saturated value ($\Delta T_{\text{mode, max}}/2$). That is why scanning speed plays an important role in the obtaining maximum $\Delta T_{\text{mode}}$ and therefore $\eta$.

In order to maximize $\eta$ the laser scan speed ($v_{\text{scan}}$) should be smaller than a characteristic speed that we define as $v_c = \lambda_{\text{res}}/n_{\text{eff}} \times dn/dT \times \Delta T_{\text{mode, max}}/2t_s$. Fig. 6.2 (c) shows the measured value of $\eta$ plotted against $v_{\text{scan}}$ for a typical silica microtoroid for the microtoroid under test $n_{\text{eff}} \sim 1.3$, $\lambda_{\text{res}} \sim 1550$ nm and $dn/dT \sim 8.4 \times 10^{-6}$ °C$^{-1}$. Using the simulated values of $t_s \sim 0.3$ ms and $\Delta T_{\text{mode, max}} \sim 3.4$ K (from Fig. 6.2 (a)), $v_c \sim 56$ nm/sec which is in good agreement with the experimental value ($\sim 53$ nm/sec). Fig. 6.3 shows the
measured \( \eta \) and the equivalent quality factor \( (Q_d = \lambda_{\text{res}}/\delta \lambda_d) \) plotted against \( P_{\text{in}} \). \( Q_d \) is not a real quality factor and is defined for the sake of comparison (note that at low power \( Q_d = Q_L \)). As predicted by the theoretical model (solid line) [1] larger \( P_{\text{in}} \) results in larger \( \eta \) (or \( Q_d \)). As shown in the inset using a structural material with large thermo-optic coefficient can increases \( \eta \).

![Graph showing experimental and calculated values of \( \eta \) plotted against input optical power \( P_{\text{in}} \). The inset shows \( \eta \) plotted against \( dn/dT \), \( P_{\text{in}} = 50 \mu W \). (In both cases \( Q_L = 1.4 \times 10^6 \)).](image)

In the resonant shift method the sensing parameter is the resonant wavelength shift caused by effective refractive index \( (n_{\text{eff}}) \) change. In the absence of dynamic line-narrowing, when the effective refractive index changes from \( n_{\text{eff}} \) to \( n_{\text{eff}} + \Delta n_s \) (due to presence of a particle or molecule) \( \Delta \lambda_{\text{res}} \) shifts to \( \Delta \lambda_{\text{res}} + \Delta \lambda_s \) \( (\Delta \lambda_s = \lambda_{\text{res}} \Delta n_s/n_{\text{eff}}) \). In dynamic mode, where the laser is scanned through the resonance, the resonance wavelength shift \( \delta \lambda_{s, \text{th}} \) can be written as:

\[
\Delta \lambda_{s, \text{th}} \approx \frac{\Delta n_s}{n_0} \lambda_0 + \left(1 + \frac{\Delta n_s}{n_0}\right) \left(\varepsilon + \frac{dn}{dT} \frac{1}{n_0 + \Delta n_s}\right) \Delta T \lambda_0
\]  

(6.2.1)

Where \( \varepsilon \) is the thermal expansion coefficient of the structural material, \( n_0 \) is \( n_{\text{eff}} \) at \( \Delta T = 0 \). Equation (6.2.1) shows the thermal effect not only reduces \( \delta \lambda_0 \) (to \( \delta \lambda_d \)) but also amplifies
Δλ_0 (to Δλ_{s,th}). The shift enhancement (Δλ_{s,th} - Δλ_0) is relatively small for typical microcavity; therefore the improvement of the minimum detection limit (S_{II,D} ∝ Δλ_{s,th}/δλ_d) is mainly through dynamic linewidth reduction. In Fig. 6.4 (a) Δλ_{s,th}/δλ_d (blue trace) and Δλ_0/δλ_0 (red trace) are plotted against P_{in} to show the improvement of minimum detection limit through high-power operation and dynamic line narrowing. Fig. 6.4 (b) shows the resonant wavelength shift in the presence (P_{in} = 420 µW) and absence (P_{in} = 1.82 µW) of dynamic line narrowing. Here Δn_{eff} is externally induced by changing the temperature of the surrounding medium.

![Graph](image1)

Fig. 6.4. (a) Δλ_{s,th}/δλ_d and Δλ_0/δλ_0 (no thermal effect) plotted against P_{in}. Here v_{scan} = 15.2 nm/sec. (b) Measured transmitted optical power plotted against scanning time at P_{in} = 1.82 µW with negligible thermal effect (bottom), and P_{in} = 420 µW where significant thermal narrowing is observable (top). The black trace is the original transmission response (Δn_{eff} = 0), the gray and light-gray traces are shifted response due to external perturbation (Δn_{eff} = 1.4×10^{-6} and 3.1×10^{-6} respectively). For both (a) and (b) Q_0 = 1.25×10^7 and Q_L =1.09×10^6.

In the resonant splitting method the sensing parameter is the wavelength splitting caused by the presence of small scattering centers (particles on the cavity). Obtaining a closed form formula for the magnitude of splitting in the presence of the thermal effect (Δλ_{s,th}) is a difficult task. So we have used the differential equations governing the modal coupling in whispering-gallery-type resonators [103] to estimate Δλ_{s,th}. Our calculation
shows that the magnitude of the $\Delta \lambda_{s,\text{th}}$ is slightly reduced compared to $\Delta \lambda_s$ (the actual splitting in the absence of thermal effect). However the linewidth narrowing factor still dominates and the detection limit is again improved. The magnitude of $(\Delta \lambda_{s,\text{th}} - \Delta \lambda_s)$ depends on the modal coupling factor ($\Gamma$) [103]. Modal coupling factor $\Gamma$ is defined as $\tau_0 \Delta \omega_s$, where $\tau_0$ is the intrinsic photon lifetime in the resonant cavity, and $\Delta \omega_s$ is the magnitude of splitting in frequency [104]. Note that $\Delta \omega_s$ and therefore $\Gamma$ are proportional to the size and number of scattering centers (particles/molecules).

![Normalized Transmission Power](image)

(a)

![Input Power vs. $\Delta \lambda_{s,\text{th}}/\delta \lambda_d$](image)

(b)

Fig. 6.5. (a) Measured transmitted optical power plotted against scanning time for a high-Q split mode with $(P_{\text{in}} = 177 \, \mu\text{W})$ and without thermal narrowing $(P_{\text{in}} = 34.5 \, \mu\text{W})$. (b) Measured $\Delta \lambda_{s,\text{th}}/\delta \lambda_d$ plotted against $P_{\text{in}}$.

Fig. 6.5 (a) shows the measured transmitted optical power plotted against scanning time for a high-$Q$ split mode with significant line thermal narrowing $(P_{\text{in}} = 177 \, \mu\text{W})$ and with negligible thermal narrowing $(P_{\text{in}} = 34.5 \, \mu\text{W})$. Here the splitting is induced by a small defect on microtoroid wall (resembling the presence of a particle). Fig. 6.5 (b) shows the
measured and calculated $\Delta \lambda_{s,th}/\delta \lambda_d$ plotted against optical input power for two different modal coupling values. Although $\Delta \lambda_{s,th}/\delta \lambda_d \propto S_{dl,D}$ is always enhanced compared to $\Delta \lambda_{s}/\delta \lambda_0 \propto S_{dl}$, the enhancement level is limited for small values of $\Gamma$. The value of $\Delta \lambda_{s,th}/\delta \lambda_d$ for $\Gamma = 2.3$ reaches a maximum at $P_{in} = 150$ $\mu$W and degrades at larger powers while for $\Gamma = 11.5$ it remains proportional to the input power.

In conclusion, dynamic line narrowing can improve the sensitivity limit in resonant optical sensing for both resonant shift and resonant splitting method. This approach can be directly use on our dual-channel sensing platform (silica microtoroid) and significantly improve the sensitivity at the optical side. Moreover other nonlinear optical effects (such as electro-optic effect) may be considered to generate dynamic line narrowing actively and to achieve similar resolution and detection limit enhancement by selecting proper materials for the cavity.
Chapter 7

Summary

7.1 Summary

We have investigated the RF spectrum of optomechanical oscillation driven by radiation pressure, and identified the weak and strong harmonic generation regime. An analytical method based on oscillatory transfer function has been developed to predict the spectrum of optomechanical oscillation in weak harmonic generation regime without solving coupled time domain differential equations. In strong harmonic generation regime, we have proposed a method to extract the optical resonance shift from the behavior of fundamental and second harmonic components in the output spectrum of the detected signal. This approach can be used to design a simple and high-resolution readout system for resonant optical sensing with high-$Q$ optomechanical cavities.

We have characterized the optomechanical RF frequency mixing process. OMO can function as a local RF oscillator and mixer simultaneously. In addition, OMO based RF mixers may significantly simplify the link complexity and reduce the power consumption in photonic RF receivers and RF/IF-over-fiber links. We have shown that the same analytical method that we used in the study of optomechanical oscillation spectrum can be used to explain the frequency mixing mechanism in an OMO. Accordingly, we have demonstrated and characterized the RF frequency down-conversion in an OMO and demonstrated the first “OMO radio” by down-converting an audio signal from an RF carrier. At this stage the OMO mixer/LO may not be feasible to replace its electronic competitors because of the fundamental limit in down-conversion bandwidth and tunability. The down-conversion bandwidth is a complicate function of
optical bandwidth, optical detuning and optomechanical frequency. A detail study of
relation between conversion bandwidth and all device parameters is the necessary before
moving to the next step. Moreover, the impact of optical spring and thermal effects
(while changing the input power and detuning) on frequency mixing and signal down-
conversion may be an interesting subject of study.

As another application we have proposed and demonstrated the first
optomechanical mass sensor and analyzed its performance. Our experiments have shown
that an OMO can function as a mass sensor with sub-pg sensitivity. Theoretical modeling
is used to estimate the ultimate sensitivity and the noise performance of the device. Three
different operational modes of OMO based mass sensor are proposed and analyzed.
Moreover, we have shown that tracking higher harmonic frequencies in the output
spectrum of an OMO can enhance the signal-to-noise ratio and therefore the sensitivity
specially when limited by the RF frequency monitoring system. This work is an
important step towards the development of OMO based mass sensors for various
applications such as nanoparticle characterization and bio-molecule detection in vitro.

Ideally a fully integrated OMO with large capture area but small effective mass is
desired. The device should also have large optical ($>10^7$) and mechanical quality factors
($>10^4$) to operate with low optical pump power. These challenging tasks are the next
steps toward better sensitivity. Replacing the solid pillar of a microtoroid to a hollow
pillar or a microfluidic channel may enable liquid phase mass detection that is an
attractive functionality for blood testing and diagnostics. Since most of the mechanical
energy is dissipated through the pillar of a microtoroid OMO, molecule binding on the
inner surface of the hollow pillar may change the mechanical energy dissipation (intrinsic
loss) and generate measurable optomechanical frequency shift without affecting the optical-$Q$.

Considering the optical resonant cavity of an OMO can also function as a high resolution optical sensor, evanescent field optical refractive index sensing can be combined with mass sensing. Therefore, effectively an OMO is a dual-mode sensor that can detect the presence of molecules by optomechanical oscillation frequency shift due to added mass and optical resonance shift due to refractive index change. This dual-mode approach provides more information (in mass and refractive index) than typical resonant photonic sensors. As shown in Chapter 3, the optical resonance shift (equivalent to detuning change) can be extracted from the OMO RF spectrum without directly monitoring of the optical resonance. In addition, when a molecule or nano-particle lands on the toroidal section of the microtoroid OMO, the optomechanical oscillation frequency shift ($\Delta f_{OMO}$) is controlled both by the mass change and the induced optical resonance shift. By carefully selecting the initial detuning value, the detuning induced $\Delta f_{OMO}$ can enhance the mass effect (Fig. 5.12 (c)) and increases the total frequency shift. Accordingly, an OMO dual-mode sensor may provide lower detection limit than a traditional mass sensor.

We have also shown that using thermally induced dynamic line-narrowing effect, the sensing resolution of optical mode can be enhanced by over 50 times. Dynamic line-narrowing can be an effective technique for improving the resolution of low-$Q$ resonant optical sensors as well as the dual-mode OMO sensor proposed here. Similar line-narrowing effect achieved by other nonlinear optical effects (i.e. Kerr effect) or linear
electro-optic effect with external modulation makes the narrowing effect fully controllable, and is useful in various sensing applications.

Finally, principle of hybrid OMO is shown in section 7.2. It provides the possibility of tailoring the mechanical properties without degradation of the optical-$Q$, and may have potential applications in various areas. The unusual thermo-optomechanical oscillation on a polymer coated microtoroid is also shown in section 7.3. Its application in optical sensing is explored. In section 7.4, we show the principle of line-narrowing based on EO resonator and feedback circuit, and its application in sensing.

7.2 Hybrid OMO

In dual-mode sensing, simultaneous high optical and mechanical-$Q$ are very important for high sensing resolution. When the optical-$Q$ of a silica microtoroid can easily reach $10^8$, the mechanical-$Q$ is below $10^4$ [96] due to the limitation of clamping and material loss. Although several OMO cavities with both high optical ($>10^8$) and mechanical-$Q$ ($10^4$-$10^5$) been reported [24, 96], but their geometries and dimensions are not suitable for sensing application. For example, by modifying the disk of a silica microtoroid to spokes, the clamping loss can be minimized and mechanical-$Q$ above $5\times10^4$ has been achieved. As a trade-off, the molecule capture efficiency of this device is very low due to its small surface area. Therefore, we proposed the hybrid OMO that may enable high optical and mechanical-$Q$ simultaneously without losing the capture area for dual-channel sensing. The basic idea of hybrid OMO is to confine the optical mode in a low optical loss material (i.e. fused silica for near infrared light) and the mechanical energy in a low mechanical loss material (a material with low loss coefficient) during the
optomechanical oscillation. Silicon nitride has been chosen to store the mechanical energy in the design because of its previously reported superior mechanical quality factors of silicon nitride membranes [108]. Extremely high mechanical-$Q (>10^7)$, which is usually observed only on bulk single-crystal materials with centimeter-scale, has been reported on SiN$_x$ thin films [108] in micro/nano scale. By replacing the silica disk of a regular microtoroid OMO to a SiN disk, the mechanical-$Q$ may be improved.

Besides better mechanical quality factors, the hybrid geometry may have applications in radiation detection and molecular sensing. For example, the different surface properties (i.e. hydrophobicity or surface orientation) of the hybrid OMO may allow two different functionalization (one on each material) on the same device [105]. By selecting proper chemical treatments (i.e. silanization or UV exposure), two different monolayers may be deposited on a single hybrid OMO [106, 107]. Consequently, two types of molecules can be simultaneously detected by tracking the resonant optical wavelength and mechanical oscillation frequency. Fig. 7.1 illustrates this approach: two materials of the OMO are functioned differently (blue and carmine) and are able to trap different molecules. The optical resonant shift and mechanical oscillation frequency change show the presence of the two molecules, respectively.

Fig. 7.1. A diagram showing the hybrid materials OMO sensor with two monolayers coated (blue and carmine). The change of optical resonance and mechanical oscillation frequency in RF spectrum show the presence of the two different molecules, respectively.
In addition, hybrid OMOs may have applications in radiation detection. For instance, the pillar supported geometry make the heat conduction very slow of the device. When the incident light is absorbed by the material of the disk and increases the diameter of the device by thermal expansion. The material of the disk can be replace by various materials that have strong absorption at the desired wavelength without affecting the high-$Q$ WGMs (that functions as the probe) confined in the silica section. The high optical $Q$ allows very high resolution due to the narrow optical linewidth.

Fig. 7.2 (a) shows the design and fabrication process of the hybrid materials OMO. Si$_3$N$_4$ microdisk serves as the mechanical resonator, while optical WGMs reside in the section made of fused silica. We start with a silicon substrate with a 2-$\mu$m thick Si$_3$N$_4$ layer. Si$_3$N$_4$ disks are fabricated on Si substrate by photolithography followed by RIE etching. Next the silicon substrate is oxidized using thermal oxidation (wet oxidation, at 900 °C) for about 72 hours to create a uniform SiO$_2$ layer with thickness of 1.6 $\mu$m. The oxide layer can penetrate underneath the Si$_3$N$_4$ microdisk. Longer oxidation time can increase the penetration depth as well as the oxide thickness. Next the unwanted SiO$_2$ is removed by electron beam lithography and HF$_2$ etching, and only a SiO$_2$ ring around the Si$_3$N$_4$ disk is left. Finally, the Si substrate is partially etched by XeF$_2$ gas leaving the Si$_3$N$_4$ microdisk (and SiO$_2$ ring) supported by the Si pillar. An optional CO$_2$ laser melting can be performed to achieve spherical profile (similar to a silica microtoroid) of the SiO$_2$ and improve the optical-$Q$. As shown recently [109] even without CO$_2$ laser melting, the structure can support high-$Q$ WGMs. A typical optical WGM and mechanical eigen mode by FEM are shown in Fig. 7.2 (b-1) and (b-2), respectively.
Fig. 7.2. (a) Diagram illustrates the fabrication process of Si₃N₄-SiO₂ hybrid OMO. (b) FEM simulation of the hybrid OMO; (b-1) The fundamental optical WG mode and (b-2) The deformation of a mechanical mode. (c-1) SEM image of the hybrid OMO; (c-2) An optical microscope image of the hybrid OMO with ripples on SiO₂ due to the surface stress. (c-3) A hybrid OMO without ripples by reducing the surface stress.

Optical-$Q$ above $8 \times 10^7$ is achievable of the SiO₂ wedge-resonator [109]. Fig. 7.2 (c-1) is the SEM image of a hybrid OMO that is partially removed by Focused Ion Beam (FIB) to examine the cross section profile. (c-2) shows the optical microscope image.
Obviously the oxide SiO$_2$ is deformed and has ripples naturally because of the stress residues from thermal oxidation. The oxide layer is grown in 900 °C, so the different thermal expansion coefficients of SiO$_2$ and Si$_3$N$_4$ induce different shrinking values when the device is cool down and tested in room temperature [110]. Consequently, the stress generates ripples as shown in Fig. 7.2 (c-2). After improving the thermal oxidation technique by reducing the temperature to about 600 °C and performing a post reheating process, the inner stress is reduced and ripples disappear successfully, as shown in Fig. 7.2 (c-3). The post heating process is performed before the E-beam lithography. The sample is heated to about 400 to 500 °C for 3 to 4 hours, then gradually cooled down to room temperature. By repeating this process several times, the surface stress of the oxide is reduced effectively. Besides the ripples, the junction break between the SiO$_2$ and Si$_3$N$_4$ is another issue in the fabrication. We find that the connection between SiO$_2$ and Si$_3$N$_4$ is weak and very fragile. When we tried to couple to the hybrid OMO with a fiber taper, the contact between them partially broke the connection (showing that the penetration depth of the SiO$_2$ underneath Si$_3$N$_4$ is not enough). When the SiO$_2$ ring is no longer at the horizontal level the resonator becomes extremely lossy resulting in very low-Q WGMs. This issue requires further study and experiment about improving the thermal oxidation process. For instance, very long time thermal oxidation may be necessary to achieve strong enough connection between the two materials. Meanwhile, long oxidation time may also increase the thickness of the SiO$_2$ layer and induce further impact of the device.

### 7.3 Thermo-optomechanical oscillation for sensing application

Besides the optomechanical oscillation and thermal dynamic line-narrowing on the silica microtoroid cavity, another type of temporal oscillatory behavior of transmitted
optical power has been observed in hybrid microtoroidal cavities. This so-called thermo-optomechanical oscillation is induced by both thermo-optic effect and thermal expansion on a single device, and has two different frequencies simultaneously.

In the presence of optical circulating power in the resonator, the absorption of material increases the local temperature and consequently changes the refractive index through thermo-optic effect. Recently thermally induced transmitted power variation has been reported in a PDMS coated silica microtoroid [100]. In the presence of optical circulating power in the resonator, the absorption of material increases the local temperature and consequently changes the refractive index through thermo-optic effect. It has been shown that a thin layer of polymer coating with negative thermo-optic coefficient can compensate the silica’s positive thermo-optic coefficient [101]. Moreover, the circulating optical field distributed in both of the materials heats them simultaneously, and leads to complicate thermal dynamics due to the competition of the negative and positive thermo-optic effects. In this resonator the interplay between circulating power and the resonant wavelength through thermo-optical effect in silica and PDMS results in the oscillatory behavior of the transmitted optical power as the input wavelength is scanned through the resonant wavelength.

Using a different polymer (Poly-methylmethacrylate or PMMA) and coating technique, a stable self-sustained thermo-optomechanical oscillator. In contrast to the previous report [100], in this device even at a fixed input wavelength, the optical output power is naturally modulated by a unique waveform that consists of fast and slow oscillation periods. Experimental and theoretical studies show that the slow oscillation is generated by the wavelength shift induced by thermo-mechanical deformation of the
bimorph structure, while the fast oscillation is generated by the wavelength shift due to thermal dependence of optical refractive index (thermo-optic effect) in silica and PMMA. We have studied the influence of optical input power, quality factor, and wavelength detuning on oscillation frequencies both experimentally and theoretically. Next we demonstrate the application of this bi-frequency thermo-optomechanical oscillation in monitoring ambient environment.

![Fig. 7.3. (a) SEM picture of the microtoroid before (left half) and after (right half) coating with PMMA. (D = 42µm, d = 5.5µm). The original microtoroid had an intrinsic quality factor (Q_{int}) of the 3.2×10^7. After PMMA coating, Q_{int} dropped to 2.3×10^6 due to the absorption loss in PMMA. (b). Cross sectional profile of the TE polarized WGM mode in a hybrid (silica/PMMA) toroidal microcavity. (c) Top: Measured detected transmitted optical power through the fiber-taper coupled to the hybrid microtoroid at a fixed laser wavelength detuning. The optical input power is 2.14 mW. t_1+t_2 is the period of the low-speed oscillation. Bottom: enlarged view of the fast oscillation cycles. \( f_L \) and \( f_H \) are frequencies of the slow and fast oscillations respectively.](image)

Fig. 7.3 (a) shows the SEM pictures of the bare and PMMA coated silica microtoroid. Fig. 7.3 (b) shows the calculated cross-sectional distribution of the optical power in the polymer coated microtoroid. Fig. 7.3 (c) shows the oscillation in two different frequencies simultaneously. Our theoretical analysis has confirmed these oscillations can serve as a practical readout mechanism because they translate changes in optical properties frequency change in measured in electronic domain [73, Appendix C].
7.4 Active line-narrowing with LiNbO$_3$ WGM resonators for sensing application

Similar to the thermal induced dynamic line-narrowing effect shown in section 6.2, we also study the active line-narrowing effect on optical resonator using the linear electro-optic properties of the material. In thermal dynamic line-narrowing, dynamic linewidth of the resonator is squeezed using the thermo-optic effect generated by the circulating power absorption. It requires relatively high input optical power to the micro-resonator to generate enough temperature change of the cavity, and slow scanning speed of the laser wavelength to satisfy the thermal response time. The narrowing factor ($\eta=\delta\lambda_0/\delta\lambda_d$, where $\delta\lambda_0$ is the loaded resonant linewidth and $\delta\lambda_d$ is the dynamic narrowed linewidth) is not fully controllable but dependent on the heat dissipation rate the device. Since the heat dissipation rate of the resonator is dependent on the ambiance, the thermal line narrowing is not stable in different circumstance (i.e. in atmosphere or in liquid). Therefore, we propose the active line-narrowing approach that the narrowing factor $\eta$ is independent on the ambiance and fully controllable using electro-optic effect of the resonator.

We propose LiNbO$_3$ microdisk resonator as the platform because LiNbO$_3$ has relatively high electro-optic coefficients ($g_{33} \approx 31$ p m/V). Fig. 7.4 (a), the transmitted optical power from the LiNbO$_3$ resonator is detected by the photodetector (PD) and generates a electrical signal. A feedback circuit amplifies the voltage signal and applies it on the LiNbO$_3$ disk. When the laser is scanning through the resonance, the refractive index of the resonator (a LiNbO$_3$ disk in our case) can be tuned actively by the electric field applied on it due to the electric-optic effect. The voltage may be applied on the resonator by the ring electrodes on top the microdisk. Fig.7.4 (b) shows the electric field
distribution on the disk using FEM simulation. Previously a similar system has been used to study electro-optic bistability in electro-optic microdisks [131].

The calculated dynamic spectrum (scanning the laser wavelength) of the LiNbO$_3$ is shown in Fig. 7.4 (c). The blue curve represents the simulated resonant spectrum without line-narrowing (no feedback) and the red curve is the spectrum with line-narrowing. The LiNbO$_3$ has $Q_{tot} = 5 \times 10^6$. The resonant linewidth is narrowed about 10 times in the spectrum. The mathematical model in section 6.2 is still valid in this case, which means that the narrowing factor can be enhanced by increasing the gain of the circuit. Note that
due to the very fast response time of electro-optic effect, the narrowing factor $\eta$ is no longer dependent on scan speed of the laser wavelength.
Appendix A

Radiation pressure in WGM resonators

The radiation pressure of light and Lorentz force in vacuum or dielectric materials are very fundamental concepts in electromagnetism, but they are still subjects of investigations [111]. A detailed study shows that using the Einstein-Laub formula instead of Lorentz law can explain the force exerted by electromagnetic fields [112]. However, to avoid going into too much detail of fundamental physics, we only try to provide an intuitive explanation of radiation pressure in a WGM cavity here.

As shown in Fig. A.1 (a), assuming a photon with energy \( h\nu \) and momentum \( p \) is reflected by a prefect reflective mirror, and the reflected photon has energy \( h\nu' \) and momentum \( p' \). Here bold letters represent vectors. Then the mirror has momentum \( MV \), where \( M \) is the mass of the mirror and \( V \) is the velocity. When \( \nu' \approx \nu \), the momentum \( p' \approx -p \) and \( V \approx 0 \) (\( p = h\nu/c \); the mass of the mirror \( M \) is much larger than the effective mass of the photon \( m = h\nu/c^2 \)). The magnitude of the momentum change is \( \Delta p = 2|p| \). For electromagnetic radiation (optical field) of a plane wave instead of a single photon, the same equations are valid. The radiation pressure is given by \( P = 2\langle \mathbf{S} \rangle/c \) where \( \langle \mathbf{S} \rangle \) is the time averaged of Poynting vector of the field. Note that the amplitude of the Poynting...
vector is the intensity of the optical field, the force applied on the mirror can be written in the form of optical power as \( F = s P = 2s <S>/c = 2k_0 P_{\text{opt}}/c \). \( P_{\text{opt}} \) represents the optical power, \( s \) is the beam area, and \( k_0 \) is the unit wave vector. In the case of a WGM in the dielectric material cavity as shown in Fig. A.1 (b), the velocity of light becomes \( c/n_{\text{eff}} \), and the momentum changes \( 2\pi|p| \) for the whole cycle. Hence the magnitude of the radiation pressure force is \( F = 2\pi n_{\text{eff}} P_{\text{cir}}/c \), and the direction is along the radial direction. Here \( P_{\text{cir}} \) is the circulating optical power in the WGM cavity.
Appendix B

General vibration theory of elastodynamic waves in 3-dimension isotropic medium

Consider the general vibration theory of elastodynamic waves in 3d isotropic medium:

\[ \rho(r, \theta, z) u'' - \nabla \cdot (\mu \nabla u) - \nabla[(\mu + \lambda) \nabla \cdot u] = f(t) \]  

(B.1)

Where \( u \) is the vector of three components of displacement, \( \mu \) and \( \lambda \) are Lamme coefficients and \( f(t) \) is the external forces. \( \rho(r, \theta, z) \) is the density distributed in the medium.

In a free vibration \( f(t) = 0 \) then \( u \) displacement:

\[ \rho(r, \theta, z) u'' = \nabla \cdot (\mu \nabla u) + \nabla[(\mu + \lambda) \nabla \cdot u] \]  

(B.2)

The solution of (B.2) has the form of

\[ u = \text{Re}[U(r, \theta, z)e^{i \omega t}] \]  

(B.3)

Eq. (B.2) can be written in frequency domain by Fourier transform for the \( n^{th} \) eigen mode as:

\[ \omega_n^2 \rho(r, \theta, z)U = -\nabla \cdot (\mu \nabla U) - \nabla[(\mu + \lambda) \nabla \cdot U] \]  

(B.4)

and  

\[ \omega_n^2 \rho(r, \theta, z)U_n = SU_n \]  

(B.5)

where  

\[ S = -\nabla \cdot \mu \nabla - \nabla[(\mu + \lambda) \nabla \cdot \cdot \cdot] \]  

(B.6)

Note that \( S \) is a second order tensor that operates on the vectors. Here \( \omega_n \) are the eigen values of the equation which are also called natural frequencies of mechanical vibration. \( U_n \) are eigen functions corresponding to the natural frequencies and they are also called mechanical modes of oscillation. In general these functions have both real and imaginary parts which define the amplitude and relative phase of vibration in each mode. Consider a slight perturbation of \( \rho \) is \( \delta \rho \), Eq. (B.5) becomes

\[ (\omega_n + \delta \omega_n)^2 [\rho(r, \theta, z) + \delta \rho(r - r_0, \theta - \theta_0, z)](U_n + \delta U_n) = S(U_n + \delta U_n) \]  

(B.7)
Assuming the perturbation is nodal mass (Dirac delta function distribution), so there are
\[ \delta \rho = \delta \rho(r-r_0, \theta-\theta_0), \quad \omega_n^2 = \delta \omega_n^2 + \delta \omega \psi = (\omega_n + \delta \omega_n)^2 \] and \( \hat{U}_n = U_n + \phi \delta m \delta(r-r_0, \theta-\theta_0) = U_n + \delta U_n \)
and (7) becomes
\[
(\omega_n^2 + \delta \omega \psi)[\rho(r, \theta, z) + \delta \rho \delta(r-r_0, \theta-\theta_0)]U_n + \phi \delta m \delta(r-r_0, \theta-\theta_0)] = S[U_n + \phi \delta m \delta(r-r_0, \theta-\theta_0)]
\]
(B.8)

If the perturbation is small enough so that the high order terms \((\delta m^2)\) vanish, and \(\phi\) is linear, there is
\[
\rho(r, \phi, z) \delta \rho \psi U_n + \rho(r, \phi, z) \omega_n^2 \phi \delta m \delta(r-r_0, \theta-\theta_0) + \delta \rho \delta(r-r_0, \theta-\theta_0) \omega_n^2 U_n = \phi \delta m \delta(r-r_0, \theta-\theta_0)
\]
(B.9)

To make the equation weak for approximate solution, we multiple both sides with \(U_n\) and integral over the whole domain as \(\int U_n da\), and note that \(\int U_n S \delta(r-r_0) da \rightarrow \infty\). So \(\phi\) must be zero to have right hand side convergent. The left hand side can be written as
\[
\psi = -\frac{\omega_n^2 U_n^2(r_0, \theta_0)}{\int \rho U_n^2 da}
\]
and therefore the frequency \(\omega_n\) can be written as
\[
\omega_n^2 = \omega_n^2 - \frac{\delta \rho \omega_n^2 U_n^2(r_0, \theta_0)}{\int \rho U_n^2 da} = \omega_n^2 - \omega_n^2 \frac{\delta \rho \omega_n^2 U_n^2(r_0, \theta_0)}{\frac{1}{2} \omega_n^2 \int \rho U_n^2 da} = \omega_n^2 - \omega_n^2 \frac{E_{\delta m}}{E_{\text{total}}}
\]
(B.11)

Here \(E_{\delta m}\) and \(E_{\text{total}}\) are the kinetic (strain) energy of the external mass and total oscillator, respectively. Note that \(E_{\delta m} \ll E_{\text{total}}\), using Taylor expansion there is
\[
\Delta \omega_n = \omega_n - \omega_n = \omega_n \sqrt{1 - \frac{E_{\delta m}}{E_{\text{total}}}} - \omega_n \approx -\omega_n \frac{E_{\delta m}}{2E_{\text{total}}}
\]
(B.12)
Appendix C

Thermal dynamics in a polymer spin-coated microtoroid

The resonant wavelength $\lambda_r$ of the optical mode is controlled by radius of the optical path and the effective optical refractive index of the WGM, therefore the shift of $\lambda_r$ has:

$$\frac{\Delta \lambda_r}{\lambda_r} = \frac{\Delta n_{\text{eff}}}{n_{\text{eff}}} + \frac{\Delta R_{\text{eff}}}{R_{\text{eff}}}$$  \hspace{1cm} (C.1)

where $\lambda_r$ is the resonant wavelength $n_{\text{eff}}$ is the effective index of optical mode circulating inside the PMMA-coated microtoroid and $R_{\text{eff}}$ is the radius of the circular optical path.

The effective refractive index of the WGM can be written as $n_{\text{eff}} \approx \eta_1 n_1 + \eta_2 n_2 + \eta_3 n_3$, where $n_1$, $n_2$ and $n_3$ are the indices of silica, PMMA and the surrounding medium (air) respectively. $\eta_1$, $\eta_2$ and $\eta_3$ are the fraction of optical power residing in silica, PMMA and air, respectively. Besides the thermo-optic effect, finite element method (FEM) thermal modeling shows that absorption of the circulating optical power not only changes the local temperature but also deforms the structure and changes the effective radius ($\Delta R_{\text{eff}}$) of the mode (Fig. C.1 (a)) in our spin-coated microtoroids.

![Diagram](image)

Fig. C.1. (a) FEM modeling of thermally induced deformation caused by expansion of residual PMMA underneath the toroidal structure. Here the input optical power is 1 mW (resulting in a circulating optical power of 3 W). The scale factor is 200. The color represents the temperature distribution. (b) Schematic diagram summarizing the mutual interaction between circulating optical power ($P_{\text{circ}}$) and the resonant wavelength ($\lambda_r$).
Two effects contribute to $\Delta R_{\text{eff}}$: local expansion of the PMMA layer on the toroidal part that increases $R_{\text{eff}}$, and bending of the bimorph (PMMA/silica) structure that decreases $R_{\text{eff}}$. The bending is due to the large difference in thermal expansion coefficients of silica ($\alpha_{\exp-1} = -0.55 \times 10^{-6}/K$) and PMMA ($\alpha_{\exp-2} = 2.02 \times 10^{-4}/K$). Calculations show that for the hybrid microtoroids under test the resulting $\Delta R_{\text{eff}}$ is negative (dominated by the bending effect). The abovementioned effects have different response times. The thermo-optic effect only depends on the local temperature change in the optical mode region resulting in a relatively fast response ($\tau_1 \sim \mu s$ range) while the bending of the bimorph structure (PMMA/silica) depends on the global temperature change and is relatively slow ($\tau_2 \sim ms$ range) [8, 23]. The schematic diagram in Fig. C.1 (b) shows the interplay between circulating optical power ($P_{\text{circ}}$) and resonant wavelength ($\lambda_{\text{res}}$) through optical absorption and temperature change. Considering both the thermo-optic and thermo-mechanical effects, Eq. (C.1) can be written as follows:

$$\omega_r(t) \approx \left[ \frac{1}{n_{\text{eff}}} \left( \eta_1 \frac{dn_1}{dT} \Delta T_1(t) + \eta_2 \frac{dn_2}{dT} \Delta T_2(t) \right) + \left( \alpha_{\exp} \Delta T_3(t) - \alpha_{\text{ben}} \Delta T_4(t) \right) \right]$$ \hspace{1cm} (C.2)

The first and second terms in the bracket represent the effective refractive index change and radius change, respectively. $\Delta T_1(t)$ and $\Delta T_2(t)$ denote the local temperature change of the optical mode volume in silica and PMMA respectively. $\Delta T_3(t)$ is the global temperature change in the entire bimorph structure and $\Delta T_4(t)$ is the global temperature change of the PMMA layer covering the toroidal region. $\alpha_{\exp}$ and $\alpha_{\text{ben}}$ quantify the impact of expansion in the toroidal region and the structural bending on resonant frequency. The underlying heat transfer dynamic can be described by:
Eq. (C.3) \((m=1):\) silica, and \(m=2: PMMA\) describes the heat dissipation from the mode volume; Eq. (C.4) \((n=3):\) silica, and \(n=4: PMMA\) denotes heat dissipation from the cavity structure. Here \(\gamma_{\text{th},m}\) and \(\gamma_{\text{abs},m}\) are the effective thermal relaxation and optical absorption rates in the optical mode volume; \(\tau_r (= 2\pi n_{\text{eff}} R_{\text{eff}} / c)\) is the cavity round trip time and \(E_c\) is the circulating optical field inside the cavity \((|E_c(t)| = \sqrt{P_c(t) / \tau_r})\); \(\gamma_{\text{th},n}\) and \(g_{\text{con},n}\) are the effective thermal relaxation (to the air and the substrate) and conduction (from the mode volume) rates for the entire structure. Here other high-order effects, such as Kerr nonlinearity, stress induced index variation, coupling between different modes are ignored. The dynamic of the optical field inside the cavity is governed by:

\[
\frac{d\Delta T_m(t)}{dt} = -\gamma_{\text{th},m} \Delta T_m(t) + \gamma_{\text{abs},m} \frac{E_m^2(t)}{\tau_r} \tag{C.3}
\]

\[
\frac{d\Delta T_n(t)}{dt} = -\gamma_{\text{th},n} \Delta T_n(t) + g_{\text{con},n} \frac{E_n^2(t)}{\tau_r} \tag{C.4}
\]

where \(\Delta \omega(t) = \omega_s(t) - \omega_r(t)\) represents the detuning between the laser frequency and the resonant frequency of the cavity (without optical input). \(\kappa (= \sqrt{2\delta_c \tau_r})\) and \(\delta_c (=\omega_c/2Q_{\text{ext}})\) are optical coupling factor and coupling limited linewidth respectively. As shown in Ref. 18 the thermo-optic effect \((dn/dT)\) alone can induce fast oscillatory
behavior when the laser is scanned toward the resonant wavelength. Here the slow oscillation due to expansion effects is effectively a self-generated wavelength scanning mechanism that combined with the thermo-optic effect, results in the unique waveform shown in Fig. C.2.

![Fig C.2](image)

**Fig. C.2.** Calculated temporal oscillation of the transmitted optical power. (a) The detected waveform showing eight periods of slow oscillation. The dark regions are the fast oscillation regions. The transmission is defined as $T_{\text{trans}}(t) = |E_{\text{out}}(t)|^2/|E_{\text{in}}(t)|^2$. (b) Fast oscillations resolved by a larger temporal resolution. The input power is 2 mW. $\Delta \lambda = 75$ pm. Here $Q_{\text{out}} = 1.75 \times 10^6$, $\alpha_{\text{exp}} = 2.02 \times 10^{-4}$, $\alpha_{\text{ben}} = -2.6 \times 10^{-4}$, $\gamma_{\text{th},1} = 3.56 \times 10^4$, $\gamma_{\text{abs},1} = 2.96 \times 10^4$, $\gamma_{\text{th},2} = 1.805 \times 10^7$, $\gamma_{\text{abs},2} = 3.79 \times 10^5$, $\gamma_{\text{th},3} = 26$, $g_{\text{con},3} = 18$, $\gamma_{\text{th},4} = 16$, $g_{\text{con},4} = 24$.

This power oscillation is demonstrated to be useful for gas sensing in our humidity sensing experiment [73]. The PMMA-coated microtoroid and the fiber-taper are placed in a closed chamber (atmosphere pressure). A nitrogen bubbler is employed to carry water molecules into the chamber and gradually increase the relative humidity (RH) inside the chamber. A psychrometer (humidity sensor) is used to monitor the RH inside the chamber. The measured values of $f_L$ and $f_H$ in the detected waveform plotted against RH. Here laser wavelength, coupling strength ($\kappa$) and optical input power are constant. The combination of the parabolic behavior of $f_L$ - RH and the linear behavior of $f_H$ – RH, can serve as a powerful read-out mechanism for the gas sensing. Basically $f_H$ and $f_L$ individually can be used for measuring changes in RH while $f_H - f_L$ crossing points
provide references for measuring absolute RH values. Assuming a frequency resolution of 10 Hz the slope of $f_H - \text{RH}$ (667 Hz/RHU) corresponds to a minimum detectable RH change of 0.015%.
References


