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Soft Neutrosophic Ring and Soft Neutrosophic Field

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Abstract. In this paper we extend the theory of neutrosophic rings and neutrosophic fields to soft sets and construct soft neutrosophic rings and soft neutrosophic fields. We also extend neutrosophic ideal theory to form soft neutrosophic ideal over a neutrosophic ring and soft neutrosophic ideal of a soft neutrosophic ring . We have given many examples to illustrate the theory of soft neutrosophic rings and soft neutrosophic fields and display many properties of of these. At the end of this paper we gave soft neutrosophic ring homomorphism.

Keywords: Neutrosophic ring, neutrosophic field,neutrosophic ring homomorphism, soft neutrosophic

1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set, intuitionistic fuzzy set and interval valued fuzzy set. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

Molodtsov in $[8]$ laid down the stone foundation of a richer structure called soft set theory which is free from t**he** parameterization inadequacy, syndrome of fuzzy se theory, rough set theory, probability theory and so on. In many areas it has been successfully applied such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. Recently soft set theory has attained much attention since its appearance and the work based on several operations of soft sets introduced in $[2,9,10]$. Some more exciting properties and algebra may be found in $[1]$. Feng et al. introduced the soft semirings $[5]$. By means of level soft sets an adjustable approach to fuzzy soft sets based decision making can be seen in $[6]$. Some other new concept combined with fuzzy sets and rough sets was presented in $[7,8]$. AygÄunoglu et al. introduced the Fuzzy soft groups $[4]$.

Firstly, fundamental and basic concepts are given for neutrosophic rings neutrosohic fields and soft rings. In the next section we presents the newly defined notions and results in soft neutrosophic rings and neutrosophic fields. Various types of soft neutrosophic ideals of rings are defined and elaborated with the help of examples. Furthermore, the homomorphisms of soft neutrosophic rings are discussed at the end.

2 Fundamental Concepts

Neutrosophic Rings and Neutrosophic Fields

Definition 1. Let R be any ring. The neutrosophic ring $\langle R \cup I \rangle$ is also a ring generated by R and I under the operations of R. I is called the neutrosophic element with the property $I^2 = I$. For an integer n, $n + I$ and nI are neutrosophic elements and $0.I = 0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Definition 2. Let $\langle R \cup I \rangle$ be a neutrosophic ring. A proper subset P of $\langle R \cup I \rangle$ is said to be a neutrosophic subring if P itself is a neutrosophic ring under the operations of $\langle R \cup I \rangle$.

Definition 2. Let $\langle R \cup I \rangle$ be any neutrosophic ring, a non empty subset P of $\langle R \cup I \rangle$ is defined to be a neutrosophic ideal of $\langle R \cup I \rangle$ if the following conditions are satisfied;

1. P is a neutrosophic subring of $\langle R \cup I \rangle$.
2. For every $p \in P$ and $r \in \langle R \cup I \rangle$, rp and $pr \in P$.

Definition 4. Let K be a field . We call the field generated by $K \cup I$ to be the neutrosophic field for it involves the indeterminacy factor in it. We define $I^2 = I$, $I + I = 2I$ i.e., $I + I +, ..., + I = nI$, and if $k \in K$ then $k.I = kI, 0I = 0$. We denote the neutrosophic field by $K(I)$ which is generated by $K \cup I$ that is $K(I) = \langle K \cup I \rangle$. $\langle K \cup I \rangle$ denotes the field generated by K and I.

Definition 5. Let $K(I)$ be a neutrosophic field, $P \subset K(I)$ is a neutrosophic subfield of P if P itself is a neutrosophic field.

Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov [8] defined the soft set in the following manner:

Definition 6. *A pair* (F, A) *is called a soft set over* U *where* F *is a mapping given by* $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Definition 7. For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if

1) $A \subseteq B$ and
2) $F(a) \subseteq H(a)$, for all $a \in A$.

This relationship is denoted by $(F, A) \subset (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \supset (H, B)$.

Definition 8. Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).

Definition 9. Let (F, A) and (K, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F, A) \cap_R (K, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap K(c)$ for all $c \in C = A \cap B$.

Definition 10. The extended intersection of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H \quad c$ is defined as

$$H(\mathrm{c}) = \begin{cases} F(\mathrm{c}) & \text{if c} \in A - B \\ \mathrm{K(c)} & \text{if c} \in B - A \\ F(\mathrm{c}) \cap \mathrm{K(c)} & \text{if c} \in A \cap B. \end{cases}$$

We write $(F, A) \cap_{\varepsilon} (\mathrm{K}, B) = (H, C)$.

Definition 11. The restricted union of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and for all $c \in C$, $H \quad c$ is defined as the soft set $(H, C) = (F, A) \cup_{R} (K, B)$ where $C = A \cap B$ and $H(\mathrm{c}) = F(c) \cup \mathrm{K}(c)$ for all $c \in C$.

Definition 12. The extended union of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(\mathrm{c}) = \begin{cases} F(\mathrm{c}) & \text{if c} \in A - B \\ \mathrm{K(c)} & \text{if c} \in B - A \\ F(\mathrm{c}) \cup K(\mathrm{c}) & \text{if c} \in A \cap B. \end{cases}$$

We write $(F, A) \cup_{\varepsilon} (\mathrm{K}, B) = (H, C)$.

Soft Rings

Definition 13. Let R be a ring and let (F, A) be a non-null soft set over R. Then (F, A) is called a soft ring over R if $F(a)$ is a subring of R , for all $a \in A$.

Definition 14. Let (F, A) and (K, B) be soft rings over R. Then (K, B) is called a soft sub ring of (F, A), If it satisfies the following;

1. $B \subset A$
2. $K(a)$ is a sub ring of $F(a)$, for all $a \in A$.

Definition 15. Let (F, A) and (K, B) be soft rings over R. Then (K, B) is called a soft ideal of F, A , If it satisfies the following;

1. $B \subset A$
2. $K(a)$ is an idela of $F(a)$, for all $a \in A$.

3 Soft Nuetrosophic Ring

Definition. Let $\langle R \cup I \rangle$ be a neutrosophic ring and (F, A) be a soft set over $\langle R \cup I \rangle$. Then (F, A) is called soft neutrosophic ring if and only if $F(a)$ is a neutrosophic subring of $\langle R \cup I \rangle$ for all $a \in A$.

Example. Let $\langle Z \cup I \rangle$ be a neutrosophic ring of integers and let (F, A) be a soft set over $\langle Z \cup I \rangle$. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Then clearly (F, A) is a soft neutrosophic ring over $\langle Z \cup I \rangle$, where

$$F(a_1) = \langle 2Z \cup I \rangle, F(a_2) = \langle 3Z \cup I \rangle$$

$$F(a_3) = \langle 5Z \cup I \rangle, F(a_4) = \langle 6Z \cup I \rangle.$$

Theorem . Let (F, A) and (H, A) be two soft neutrosophic rings over $\langle R \cup I \rangle$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. The proof is straightforward.

Theorem. Let (F, A) and (H, B) be two soft neutrosophic rings over $\langle R \cup I \rangle$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. This is straightforward.

Remark. The extended union of two soft neutrosophic rings (F, A) and (K, B) over $\langle R \cup I \rangle$ is not a soft neutrosophic ring over $\langle R \cup I \rangle$.

We check this by the help of following Example.

Example. Let $\langle Z \cup I \rangle$ be a neutrosophic ring of integers. Let (F, A) and (K, B) be two soft neutrosophic rings over $\langle Z \cup I \rangle$, where

$$F(a_1) = \langle 2Z \cup I \rangle, F(a_2) = \langle 3Z \cup I \rangle, F(a_3) = \langle 4Z \cup I \rangle,$$

And

$$K(a_1) = \langle 5Z \cup I \rangle, K(a_3) = \langle 7Z \cup I \rangle.$$

Their extended union
$(F, A) \cup_E (K, B) = (H, C)$, where

$$H(a_1) = \langle 2Z \cup I \rangle \cup \langle 5Z \cup I \rangle,$$

$$H(a_2) = \langle 3Z \cup I \rangle,$$

$$H(a_3) = \langle 5Z \cup I \rangle \cup \langle 7Z \cup I \rangle.$$

Thus clearly $H(a_1) = \langle 2Z \cup I \rangle \cup \langle 5Z \cup I \rangle$, $H(a_3) = \langle 5Z \cup I \rangle \cup \langle 7Z \cup I \rangle$ is not a neutrosophic rings.

Remark. The restricted union of two soft neutrosophic rings (F, A) and (K, B) over $\langle R \cup I \rangle$ is not a soft neutrosophic ring over $\langle R \cup I \rangle$.

Theorem. The OR operation of two soft neutrosophic rings over $\langle R \cup I \rangle$ may not be a soft neutrosophic ring over $\langle R \cup I \rangle$.

One can easily check these remarks with the help of Examples.

Theorem. The extended intersection of two soft neutrosophic rings over $\langle R \cup I \rangle$ is soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. The proof is straightforward.

Theorem. The restricted intersection of two soft neutrosophic rings over $\langle R \cup I \rangle$ is soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. It is obvious.

Theorem. The AND operation of two soft neutrosophic rings over $\langle R \cup I \rangle$ is soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. Easy.

Definition. Let (F, A) be a soft set over a neutrosophic ring $\langle R \cup I \rangle$. Then (F, A) is called an absolute soft neutrosophic ring if $F(a) = \langle R \cup I \rangle$ for all $a \in A$.

Definition. Let (F, A) be a soft set over a neutrosophic ring $\langle R \cup I \rangle$. Then (F, A) is called soft neutrosophic ideal over $\langle R \cup I \rangle$ if and only if $F(a)$ is a neutrosophic ideal over $\langle R \cup I \rangle$.

Example. Let $\langle Z_{12} \cup I \rangle$ be a neutrosophic ring. Let $A = \{a_1, a_2\}$ be a set of parameters and (F, A) be a soft set over $\langle Z_{12} \cup I \rangle$. Then clearly (F, A) is a soft neutrosophic ideal over $\langle R \cup I \rangle$, where

$$F(a_1) = \{0, 6, 2I, 4I, 6I, 8I, 10I, 6 + 2I, ..., 6 + 10I\},$$

$$F(a_2) = \{0, 6, 6I, 6 + 6I\}.$$

Theorem. Every soft neutrosophic ideal (F, A) over a neutrosophic ring $\langle R \cup I \rangle$ is trivially a soft neutrosophic ring.

Proof. Let (F, A) be a soft neutrosophic ideal over a neutrosophic ring $\langle R \cup I \rangle$. Then by definition $F(a)$ is a neutrosophic ideal for all $a \in A$. Since we know that every neutrosophic ideal is a neutrosophic subring. It follows that $F(a)$ is a neutrosophic subring of $\langle R \cup I \rangle$. Thus by definition of soft neutrosophic ring, this implies that (F, A) is a soft neutrosophic ring.

Remark. The converse of the above theorem is not true.

To check the converse, we take the following Example.

Example. Let $\langle Z_{10} \cup I \rangle$ be a neutrosophic ring. Let $A = \{a_1, a_2\}$ be a set of parameters and (F, A) be a soft neutrosophic ring over $\langle Z_{10} \cup I \rangle$, where

$$F(a_1) = \{0, 2, 4, 6, 8, 2I, 4I, 6I, 8I\},$$

$$\mathrm{F}(a_2) = \{0, 2I, 4I, 6I, 8I\}.$$

Then obviously (F, A) is not a soft neutrosophic ideal over $\langle Z_{10} \cup I \rangle$.

Proposition. Let (F, A) and (K, B) be two soft neutosophic ideals over a neutrosophic ring $\langle R \cup I \rangle$. Then

1. Their extended union $(F, A) \cup_E (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
2. Their extended intersection $(F, A) \cap_E (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
3. Their restricted union $(F, A) \cup_R (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
4. Their restricted intersection $(F, A) \cap_R (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
5. Their OR operation $(F, A) \vee (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
6. Their AND operation $(F, A) \vee (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.

Proof. Supoose (F, A) and (K, B) be two soft neutrosophic ideals over $\langle R \cup I \rangle$. Let $C = A \cup B$. Then for all $c \in C$, The extended union is $(F, A) \cup_E (K, B) = (H, C)$, where

$$H(\mathrm{c}) = \begin{cases} F(c), \text{If c} \in A - B, \\ K(c), \text{If c} \in B - A, \\ F(c) \cup K(c), \text{If c} \in A \cap B. \end{cases}$$

As union of two neutrosophic ideals is again a neutrosophic ideal of $\langle R \cup I \rangle$. Hence the extended union H, C is a soft neutrosophic ideal over $\langle R \cup I \rangle$.

Similarly $(2), (3), (4), (5),$ and (6) can be proved respectively.

Definition. Let (F, A) and (K, B) be two soft neutrosophic rings over $\langle R \cup I \rangle$. Then (K, B) is called soft neutrosophic subring of (F, A), if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic subring of $F(a)$ for all $a \in A$.

Example. Let $\langle C \cup I \rangle$ be the neutrosophic ring of complex numbers. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then (F, A) be a soft neutrosophic ring over $\langle C \cup I \rangle$, where

$$F(a_1) = \langle Z \cup I \rangle, F(a_2) = \langle Q \cup I \rangle,$$

$$F(a_3) = \langle R \cup I \rangle.$$

Where $\langle Z \cup I \rangle, \langle Q \cup I \rangle$ and $\langle R \cup I \rangle$ are neutrosophic rings of integers, rational numbers, and real numbers respectively.

Let $B = \{a_2, a_3\}$ be a set of parmeters . Let (K, B) be the neutrosophic subring of (F, A) over $\langle C \cup I \rangle$, where

$$\mathrm{K}(a_2) = \langle Z \cup I \rangle, \mathrm{K}(a_3) = \langle Q \cup I \rangle.$$

Theorem. Every soft ring (H, B) over a ring R is a soft neutrosophic subring of a soft neutrosophic ring (F, A) over the corresponding neutrosophic ring $\langle R \cup I \rangle$ if $B \subseteq A$.

Proof. Straightforward.

Definition. Let (F, A) and (K, B) be two soft neutrosophic rings over $\langle R \cup I \rangle$. Then (K, B) is called soft neutrosophic ideal of (F, A), if

1. $B \subseteq A$, and

2. $K(a)$ is a neutrosophic ideal of $F(a)$ for all $a \in A$.

Example. Let $\langle Z_{12} \cup I \rangle$ be a neutrosophic ring. Let $A = \{a_1, a_2\}$ be a set of parameters and (F, A) be a soft set over $\langle Z_{12} \cup I \rangle$. Then clearly (F, A) is a soft neutrosophic ring over $\langle Z_{12} \cup I \rangle$,where

$F(a_1) = \{0, 6, 2I, 4I, 6I, 8I, 10I, 6 + 2I, ..., 6 + 10I\}$,

$F(a_2) = \{0, 2, 4, 6, 8, 2\mathrm{I}, 4\mathrm{I}, 6\mathrm{I}, 8\mathrm{I}\}$.

Let $B = \{a_1, a_2\}$ be a set of parameters. Then clearly (H, B) is a soft neutrosophic ideal of (F, A) over $\langle Z_{12} \cup I \rangle$, where

$$H(a_1) = \{0, 6, 6 + 6I\},$$

$$H(a_2) = \{0, 2, 4, 6, 8\}.$$

Proposition. All soft neutrosophic ideals are trivially soft neutrosophic subrings.

Proof. Straightforward.

4 Soft Neutrosophic Field

Defintion. Let $K(I) = \langle K \cup I \rangle$ be a neutrosophic field and let (F, A) be a soft set over $K(I)$. Then (F, A) is said to be soft neutrosophic field if and only if $F(a)$ is a neutrosophic subfield of $K(I)$ for all $a \in A$.

Example. Let $\langle C \cup I \rangle$ be a neutrosophic field of complex numbers. Let $A = \{a_1, a_2\}$ be a set of parameters and let (F, A) be a soft set of $\langle C \cup I \rangle$. Then (F, A) is called soft neutrosophic field over $\langle C \cup I \rangle$, where

$$F(a_1) = \langle R \cup I \rangle, F(a_2) = \langle Q \cup I \rangle.$$

Where $\langle R \cup I \rangle$ and $\langle Q \cup I \rangle$ are the neutosophic fields of real numbers and rational numbers.

Proposition. Every soft neutrosophic field is trivially a soft neutrosophic ring.

Proof. The proof is trivial.

Remark. The converse of above proposition is not true.

To see the converse, lets take a look to the following example.

Example. Let $\langle Z \cup I \rangle$ be a neutrosophic ring of integers. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters and let (F, A) be a soft set over $\langle Z \cup I \rangle$. Then (F, A) is a soft neutrosophic ring over $\langle Z \cup I \rangle$, where

$$F(a_1) = \langle 2Z \cup I \rangle, F(a_2) = \langle 3Z \cup I \rangle$$

$$F(a_3) = \langle 5Z \cup I \rangle, F(a_4) = \langle 6Z \cup I \rangle.$$

Clearly (F, A) is not a soft neutrosophic field.

Definition. Let (F, A) be a soft neutrosophic field over a neutrosophic field $\langle K \cup I \rangle$. Then (F, A) is called an absolute soft neutrosophic field if $F(a) = \langle K \cup I \rangle$, for all $a \in A$.

5 Soft Neutrosophic Ring Homomorphism

Definition. Let (F, A) and (K, B) be the soft neutrosophic rings over $\langle R \cup I \rangle$ and $\langle R' \cup I \rangle$ respectively. Let $f : \langle R \cup I \rangle \to \langle R' \cup I \rangle$ and $g : A \to B$ be mappings. Let $(f, g) : (\mathrm{F}, \mathrm{A}) \to (\mathrm{K}, \mathrm{B})$ be another mapping. Then (f, g) is called a soft neutrosophic ring homomorphism if the following conditions are hold.

1. f is a neutrosophic ring homomorphism from $\left\langle R \cup I \right\rangle$ to $\left\langle R' \cup I \right\rangle$.
2. g is onto mapping from A to B, and
3. $f(F(a)) = K(g(a))$ for all $a \in A$.

If f is an isomorphicm and g is a bijective mapping. Then (f, g) is called soft neutrosophic ring isomorphism.

Conclusions

In this paper we extend the neutrosophic ring, neutrosophic field and neutrosophic subring to soft neutrosophic ring, soft neutrosophic field and soft neutrosophic subring respectively. The neutrosophic ideal of a ring is extended to soft neutrosophic ideal. We showed all these by giving various examples in order to illustrate the soft part of the neutrosophic notions used.

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