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Throughput Optimization in Wireless Networks with Multi-packet Reception and Directional Antennas

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1 Introduction

Recent advances in multiuser detection techniques open up new opportunities for resolving collisions at the physical layer. These techniques permit the simultaneous reception of multiple packets by a node, which in turn increases the capacity of wireless networks [1]. However, to fully exploit multi-packet reception (MPR) capability, new architectures and protocols should be devised. These new schemes need to reformulate a historically underlying assumption in wireless networks, which states that any concurrent transmission of two or more packets results in a collision and failure of all packet receptions. For example, the IEEE 802.11 Distributed Coordination Function (DCF) adopts a backoff mechanism for which a node sensing the channel busy decreases its transmission probability.

Recently, researchers started focusing on theoretical upper and lower bounds on the throughput for MPR-capable wireless networks [1], [2], [3]. Garcia-Luna-Aceves et al. [1] demonstrated that architectures exploiting MPR capability increase the capacity of random wireless networks by a logarithmic factor with respect to the protocol model of Gupta et al. [4]. Subsequent work considered alternative schemes to compute asymptotic bounds on the throughput capacity under some homogeneous assumptions, such as nodes transmit to a single base station or access point [5], or nodes are equipped with a single omni-directional antenna[1] [6].

In this paper, we present a generalized scheme to compute the optimal throughput in MPR-capable wireless networks, where nodes have *one or more* transmitter interfaces. The scheme is valid for any fixed wireless network configuration, with nodes endowed with directional or omni-directional antennas. The generality of our model empowers us with considerable flexibility in contrast to previous models that explicitly assume a single omni-directional antenna per node. Numerical results enable us to draw valuable conclusions, some of which we summarize here. First, the number of transmitter interfaces can strongly impact on the performance of MPR-capable networks. To fully exploit MPR, nodes may need to be equipped with multiple transmitter antennas. Second, the use narrow beamwidth antennas achieves better performance; however, MPR alleviates the inefficient spatial reuse of wider beamwidth antennas, and increases the throughput to approach to the performance of networks with narrower beamwidth antennas. Finally, in highly connected networks, by equipping nodes with the ability to capture *few* packets simultaneously achieves considerable throughput enhancement; however, subsequent increments in MPR capability do not significantly improve performance. In less connected networks, further improvement can be achieved with subsequent increments in MPR capability. To the best of our knowledge, this is the first work that simultaneously considers multiple transmitter interfaces, generalized antenna model and multi-packet reception.

The remainder of this paper is organized as follows. Section 2 discusses related work. Section 3 presents background terminology used in the paper, and defines the feasibility conditions for link scheduling in MPR-capable networks. Section 4 formulates the throughput optimization problem in MPR networks as a joint routing and link scheduling problem, and Section 5 presents a polynomial time heuristic to approximate to the optimal solution. Section 6 shows numerical results, and Section 7 concludes and discusses our future work.

[1]The terms antenna and interface are used interchangeably hereafter.

2 Related Work

Scheduling problem. The throughput optimization problem in wireless networks can be seen as an extension of the maximum flow (max-flow) problem, where at any time only a subset of links may be simultaneously scheduled or activated. Brar et al. [7] presented a greedy algorithm for the scheduling problem under the physical model [4]. Moscibroda et al. [8] proposed a centralized scheduling algorithm for scenarios where the traffic demands are the same on every network link. Djukic et al. [9] presented a distributed scheduler based on the Bellman-Ford algorithm running on the conflict graph[2].

Joint routing and scheduling. Jain et al. [10] presented a max-flow linear programming scheme for computing upper and lower bounds on the optimal throughput under the protocol model. The scheme, however, requires to find all independent sets in the conflict graph, which is intractable. Kodialam et al. [11] proposed a polynomial time approximation algorithm for the routing problem, and a *graph-coloring* approach for the scheduling problem. Zhang et al. [12] presented a *column generation* approach to iteratively solve the joint routing and scheduling problem.

Scheduling with directional antennas. Spyropot et al. [13] formulated the scheduling problem as a series of maximal-weight matching in a graph. Cain et al. [14] described a distributed TDMA scheduling protocol, while Capone et al. [15] presented a max-flow formulation which results in an integer linear program, and proposed a heuristic to solve it.

Scheduling, routing and MPR. Garcia-Luna-Aceves et al. [1] demonstrated that MPR increases the order of capacity of random wireless networks by a logarithmic factor with respect to the protocol model [4]. The same authors [2] demonstrated that throughput is also improved with respect to the physical model [4], and that MPR provides greater improvement than network coding. Moraes et al. [3] presented an architecture that exploits the advantages of multi-user detection, successive interference cancellation, array antennas, CDMA and mobility to increase the per-source throughput. However, considerable complexity is required at the nodes. Celik et al. [5] studied the negative implications of reusing legacy MAC protocols in MPR-capable networks, and how alternative backoff mechanisms can improve throughput and fairness. A max-flow model is presented by Wang et al. [6], who proposed a centralized heuristic algorithm to jointly perform routing and scheduling.

By surveying previous work, we note that issues including the following items were not studied yet: i) limitations of MPR-capability and ii) resources needed to overcome those eventual limitations; iii) behavior of MPR-capable networks under directional transmissions and iv) impact of the beamwidth of directional antennas; v) behavior of MPR-capable networks when nodes have multiple transmitter antennas. To study these open research issues, we propose a generalized model for networks with multi-packet reception.

3 Background

We represent a wireless network as a graph $G = (V, E)$, where V is the set of nodes and E the set of links. The existence of a link $(u, v) \in E$ from node u to node $v \in V$ is determined by the channel model. Link (u, v) has one *tail* (transmitter node u) and one *head* (receiver node v).

[2] In the conflict graph, there is a vertex for each link of the network, and an edge between two vertices if the corresponding links conflict with each other. Conflicting or interfering links are those links which cannot be simultaneously scheduled under the protocol model of Gupta et al. [4].

3.1 MPR and Directional Antennas

For nodes with an omni-directional MPR antenna, Ghez et al. [16] proposed the following model. A node can correctly receive a part of all transmissions from nodes located inside the radius R from itself. R represents the receiver range of the node. There exists a link $(u, v) \in E$ from node u to node v if u is in the receiver range of v. The MPR protocol model [1, 2, 6], which is a particular case of the model by Ghez et al. [16], states that the reception of all transmissions is achievable if the the number of simultaneous transmissions in the receiver range R is less than or equal to K, and other transmitters are outside of the radius $(1+\Delta)$R. Δ is a parameter that depends on the physical layer. As previous work [6], we will assume that $\Delta = 0$.

Consider a wireless network where nodes are equipped with M $\geq$ 1 antennas. For M = 1, a node cannot transmit and receive simultaneously, and the MPR antenna operates in either transmitter or receiver mode at a given time. For M $>$ 1, one antenna, which has MPR capability, can operate as in the case of M = 1 (i.e., the antenna can be scheduled to operate in transmitter or receiver mode, as specified by the scheduling algorithm), and the other antennas operate in transmitter mode exclusively. The use of at most one interface in receiver mode obeys the fact that MPR permits the reception of multiple transmissions with a single antenna. We consider directional transmission and omni-directional reception. Directional transmission improves the spatial reuse, while omni-directional reception maximizes the benefits of MPR.

The antenna model considered in this paper is the one used in previous work including [17], [18], [19]. Sidelobes and backlobes are ignored. Although this model simplifies the radiation pattern, the sidelobes are generally small enough. Moreover, the gain of the main lobe of typical directional antennas is more than 100 times the gain of sidelobes. Additionally, smart antennas often have null capability that mitigates the sidelobes and backlobes. The interference region of an antenna is principally determined by its main lobe and simplifying the radiation pattern will not lead to a fundamental change on the result of the throughput analysis [17]. To define the radiation pattern, assume that i) all nodes in the network lie in a two-dimension plane, so that the gain of the antenna is a function of the azimuth angle only; ii) the gain of the main lobe is constant (greater than zero), and zero outside it. The main lobe is characterized by the beamwidth β of the antenna; iii) the axis of the main lobe, namely the boresight, can be directed to only one direction at a time. Fig. 1 (a) shows the radiation pattern model; α represents the angle between the boresight of the transmitter antenna and the direction of a potential receiver node. We will use the notation (M, K, β)-network[3] to refer to a network with M interfaces per node, where the receiver antenna can decode up to K packets simultaneously, and the transmitter antennas have a beamwidth β.

3.2 Scheduling in (M, K, β)-networks

A *schedulable set* $S \subseteq E$ is a set of links which can be scheduled simultaneously. The set S can be characterized by a *schedulable vector* p_S of size $|E|$, where $|\cdot|$ denotes cardinality. The j^{th} element of this vector is set to one if the link $e_j \in E$ is a member of S, and to zero otherwise. The definition of the vector p_S assumes that the set of links in the network are ordered in a determined way, such that $E = \{e_1, e_2, ..., e_{|E|}\}$. Any schedulable vector p_S can be regarded as a point in a $|E|$-dimensional space, which also becomes a vertex of the the convex hull of the set of schedulable vectors. To illustrated these concepts, consider the network topology of Fig. 1 (b), where $E = \{(a,b),(c,d),(e,f)\}$ and the links

[3] We use this notation to simplify the explanation. It is straightforward to generalize to networks where M, K and β are not the same at every node.

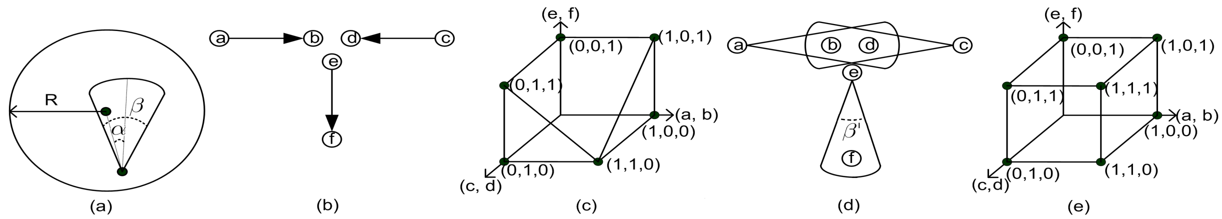

Figure 1: (a) Radiation pattern model. (b) A wireless network where links (a,b), (c,d) and (e,f) interfere with each other. (c) Schedulable vectors and corresponding convex hull for the network in (b), assuming a (1, 2, 2π)-network. (d) A feasible schedule for the network in (b), assuming a (1, 2, β′)-network, and (e) corresponding schedulable vectors and convex hull.

conflict with each other. For a (1, 2, 2π)-network, links can be scheduled individually, or they can be combined in groups of two (because K = 2). Fig. 1 (c) shows the corresponding schedulable vectors and convex hull. Now, if nodes are endowed with directional antennas so that the network becomes a (1, 2, β′)-network, a feasible schedulable set is shown in Fig. 1 (d), which *enlarges* the convex hull as depicted in Fig. 1 (e).

To define the feasibility conditions for scheduling, we need to define the following terminology. Let $\delta^+(u) \subseteq S$ denote the set of links in S having node u as tail (transmitter). Similarly, let $\delta^-(v) \subseteq S$ be a set of links such that, $\forall e \in \delta^-(v)$, the corresponding tail of e, say node u, is in the receiver range of v and schedules its antenna in a direction for which $\frac{-\beta}{2} \leq \alpha_{uv} \leq \frac{\beta}{2}$; α_{uv} is the angle between the boresight of the transmitter antenna at u and the direction of v from u. $\delta^-(v)$ represents the set of links for which node v is within a distance R to the corresponding transmitter antennas and lies in the main lobe of them. Finally, let $I(v)$ be a binary variable equal to one if $\exists\ e = (u,v) \in S$, and zero otherwise. For a (M, K, β)-network, a set $S \subseteq E$ is a feasible schedulable set if $\forall\ e = (u,v) \in S$:

$$
\begin{aligned}
|\delta^+(u)| + I(u) &\leq M, && (1)\\
I(v) \cdot |\delta^-(v)| &\leq K. && (2)
\end{aligned}
$$

The first term on the left hand side of Eq. (1) is the number of links having node u as transmitter (i.e., $|\delta^+(u)|$ interfaces are used in transmitter mode), while the second term is equal to one if S includes at least one link having u as receiver (i.e., the antenna that can operate as transmitter or receiver must be scheduled to operate in receiver mode). Eq. (2) states that the receiver node v can decode at most K packets.

4 Problem Formulation

We formulate the joint routing and scheduling problem in (M, K, β)-networks as a max-flow problem. Let N be the set of end-to-end flows. Each flow is characterized by a 3-tuple (s_n, d_n, f_n), which denotes the source node, the destination node and the flow[4] in bits per second (bps) transmitted from s_n to d_n respectively. The problem can be divided into: i) routing, which ignores the impact of wireless interference, and attempts to maximize throughput by routing through (potentially) multiple paths connecting each source-destination pair, and ii) scheduling, which deals with finding schedulable sets and the fraction of time allocated to each set.

[4] Although a flow is characterized by (s_n, d_n, f_n), we will also use the term flow to informally refer to f_n.

4.1 Routing

Let x_{ij}^n be the variable representing the amount of the n^{th} flow routed on link (i,j). The routing problem is defined by:

$$\max F = \sum_{n\in N} f_n \tag{3}$$

$$\sum_{j:(i,j)\in E} x_{ij}^n - \sum_{j:(j,i)\in E} x_{ji}^n = \begin{cases} f_n; \text{if } i = s_n \\ -f_n; \text{if } i = d_n \\ 0; \text{otherwise} \end{cases} ;n \in N \tag{4}$$

$$x_{ij}^n \geq 0; \qquad n \in N, (i,j) \in E \tag{5}$$

$$\sum_{n\in N} x_{ij}^n \leq c_{ij}; \qquad (i,j) \in E \tag{6}$$

Eq. (3) is the aggregated throughput, or simply throughput, which must be optimized. Eq. (4) represents the flow conservation constraint. Eq. (5) restricts the amount of flow on each link to be non-negative, and Eq. (6) states that the total amount of flow on a link (i,j) cannot exceed its capacity c_{ij}. We will refer to Eqs. (3)-(6) as linear program 1 (LP1).

4.2 Scheduling

A schedule specifies the schedulable sets and the fraction of time allocated to each of them. Let $\Gamma = \{S_1, S_2, ..., S_{|\Gamma|}\}$ be a set of schedulable sets which satisfy Eqs. (1) and (2), and $\lambda_k, 0 \leq \lambda_k \leq 1$, be the fraction of time allocated to S_k. We may write the time interval $[0,1]$ as $\cup_k [t_k, t_{k+1}]$, where links in S_k are active for the activity period $t_{k+1} - t_k = \lambda_k$. We will call the variables λ's as *activity period variables*. The schedule restriction can be written as:

$$\sum_{S_k\in\Gamma} \lambda_k = 1 \tag{7}$$

Let $U = (u_1, u_2, ..., u_{|E|})$ be a $|E|$-dimensional *usage vector*, where u_i indicates the fraction of time link $e_i \in E$ is active. Jain et al. [10] proved that for a scheduling problem under the protocol model, a vector U is feasible if U lies within the *independent set polytope.* For (M, K, β)-networks, we can extend this proposition as follows.

Proposition 1 *A usage vector U is feasible if and only if it lies within the convex hull of the schedulable vectors.*

The proof is given in Appendix A. The scheduling problem also imposes link capacity constraints. For wireless networks, the link capacity constraint given by Eq. (6) should be redefined as follows. The total capacity of a link (i,j) is given by sum of the product of the fraction of time the link is active on a scheduled period and the capacity of the link on that period:

$$\sum_{n\in N} x_{ij}^n \leq \sum_{S_k|(i,j)\in S_k} \lambda_k c_{ij}^{\lambda_k}; \qquad (i,j) \in E \tag{8}$$

$c_{ij}^{\lambda_k}$ represents the capacity of the link (i,j) during the activity period λ_k. The MPR protocol model [1] sets this value as a constant ($c_{ij}^{\lambda_k} = c_{ij}$). We use this model to compute c_{ij} as:

$$SNIR_{ij} = \begin{cases} \zeta \cdot r_{ij}^{-\gamma}; & \text{if } \frac{-\beta}{2} \leq \alpha_{ij} \leq \frac{\beta}{2} \\ 0; & \text{otherwise} \end{cases} \tag{9}$$

$$c_{ij} = \log_2(1 + SNIR_{ij}) \tag{10}$$

Eq. (9) states that the $SNIR_{ij}$ decays exponentially according to the distance r_{ij} between nodes i and j, and it is zero if the receiver node j is outside of the main lobe of the transmitter antenna. ζ and γ depend on the path loss model. Eq. (10) is the normalized Shannon capacity. A similar capacity model is frequently used for omni-directional antennas [20]. Eqs. (9) and (10) model the common case where shorther transmission distance implies higher link capacity [20]. We will refer to Eqs. (3)-(5), (7) and (8) as LP2. If Γ includes *all* schedulable sets, then LP2 yields the global optimal solution. However, since there can be an exponential number of schedulable sets, we present a polynomial time heuristic in the next section.

5 Polynomial Time Heuristic

The heuristic consists of three steps: i) solve LP1; ii) create the set Γ by using a greedy approach; iii) solve LP2.

Step 1. This step is intended to identify *good* paths for each flow, such that the throughput is maximized. The output of the step 1 is the set of links which are assigned a positive flow value by LP1, namely $E_{LP1} = \{(i,j) \in E \,|\, \sum_{n \in N} x_{ij}^n > 0\}$.

Step 2. Given that step 1 may (likely) produce an unfeasible solution (LP1 ignores the conditions for the feasibility of scheduling imposed by Eqs. (1) and (2)), step 2 finds feasible schedulable sets for E_{LP1}. The main idea is to find a *small* number of schedulable sets, so that they can be found in polynomial time. The schedulable sets, however, should be as *good* as possible. To this end, we consider maximal[5] sets. The detailed process is shown in Algorithm 1. The *Greedy Scheduler* creates sets $S_1, S_2, ..., S_{|\Gamma|}$, where every link of each set satisfies Eqs. (1) and (2). Line 3 sets the mode the antennas operate at. For M $>$ 1, M - 1 antennas operate in transmitter mode. The remainder antenna, which has MPR capability and can operate in either transmitter or receiver mode according to Section 3.1, is set to exclusively work in receiver mode. For M = 1, the operation mode of the MPR antenna is decided later by the algorithm (e.g., if a link (u,v) is scheduled for a certain period of time, the interfaces at nodes u and v are set to operate in transmitter and receiver mode for that period). In line 10, a link is greedily chosen to belong to the set S_k according to $\mu_{ij} = \frac{\sum_{n \in N} x_{ij}^n}{c_{ij}}$. The value μ_{ij} represents the utilization (in fraction of time) of link (i,j). The greedy selection attempts to group links with similar utilizations in the same set.

Step 3. The last step solves LP2, which produces an optimal solution with respect to Γ (i.e., the best convex combination of the schedulable vectors considering the schedulable sets in Γ is obtained). The solution of LP2 gives the amount of flow x_{ij}^n routed through each link (i,j), and establishes the fraction of time λ_k allocated to each schedulable set S_k.

Define $\tau_k = \max\{\mu_{ij} | (i,j) \in S_k\}$, $S_k \in \Gamma$, and $\tau = \sum_{S_k \in \Gamma} \tau_k$. The flow value F_{LP2} (Eq. (3)) is bounded by:

$$\frac{F_{LP1}}{\tau} \leq F_{LP2} \leq F_{LP1} \tag{11}$$

This proof is given in Appendix B.

To demonstrate that the heuristic has a polynomial running time, we will show that the running time and the size (number of variables and constraints) of steps 1 and 3 are bounded (step 2 is a greedy algorithm, which obviously runs in polynomial time). Step 1 is a linear program (LP1), with $O(|N| \cdot |E|)$ total positive flow variables (Eq. (5)). The number of flow conservation (Eq. (4)) and link capacity (Eq.

[5] A schedulable set that is maximal under inclusion is called a maximal schedulable set.

Algorithm 1 Greedy Scheduler

INPUT: E_{LP1}, $G(V,E)$
OUTPUT: Set Γ of schedulable sets.

Initialize the operation mode of antennas
$\Gamma = \{\}$
$k = 0$
while ($E_{LP1} \neq \{\}$) **do**
    $k = k+1$
    $S_k = \{\}$
    while ($\exists\ (i,j) \in E_{LP1} | S_k \cup \{(i,j)\}$ is a schedulable set) **do**
        $(i,j) = \arg\max\{\mu_{ij} | S_k \cup \{(i,j)\}$ is a schedulable set; $(i,j) \in E_{LP1}\}$
        $S_k = S_k \cup \{(i,j)\}$
        $E_{LP1} = E_{LP1} - \{(i,j)\}$
    end while
    $\Gamma = \Gamma \cup S_k$
end while
return Γ

(6)) constraints are $O(|N| \cdot |E|)$ and $O(|E|)$ respectively. Thus, the number of variables and constraints are polynomially bounded in $|N| \cdot |E|$ and can be solved by a polynomial time linear programming algorithm. LP2 is similar to LP1, except that one additional constraint is added, namely Eq. (7), and $O(|E|)$ additional variables (i.e., the activity period variables).

6 Numerical Results

We present numerical results based on the heuristic presented in Section 5, which was implemented as a *solver* in C language. For LP1 and LP2, the solver incorporates the package lpsolve [21]. Although several topologies were considered, we report here the results from one representative topology. The network topology, for different values of R, is shown in Fig. 2. We considered a connected network with one hundred nodes and fifty flows. Nodes were uniformly distributed over a 1000 x 1000 square-meter area. The source and destination of each flow were randomly selected, so that each node acts as either a source or a destination of one flow. A node can also operate as relay node for other flows. We assumed a link capacity $c_{ij} = 10$ units when the distance r_{ij} between nodes i and j is equal to R (maximum distance from which a node can decode a packet). The path loss exponent γ was set to 4, which corresponds to the two-ray model. Having set these values, any link capacity can be computed according to Eqs. (9) and (10). We considered several scenarios with different values of β, K, M and R. The results were evaluated in terms of the objective function (Eq. (3)), and normalized to the upper bound F_{LP1}.

Impact of the number of interfaces. Fig. 3 (a) shows the throughput as a function of K, for different number of antennas with $\beta = \frac{\pi}{3}$. Note that the throughput for M = 1 and M = 2 increases monotonically until K = 3, and that further increments of K have no impact. The corresponding curves are almost overlapped because they have the same bottleneck: only one interface can operate in transmitter mode. By adding transmitter antennas, the MPR capability is better exploited and the throughput is increased. Note also that the throughput increases approximately linearly until K equals the number of transmitter antennas.

Impact of the beamwidth. Fig. 3 (b) shows the curves of throughput vs K with M = 1, and different values of β. The best performance is obtained with the minimum beamwidth value ($\beta = \frac{\pi}{3}$), due to a better spatial reuse. However, the disadvantage of having wider beamwidth antennas can be compensated by increasing the MPR capability; for $M \geq 11$, even the use of omni-directional antennas produces an optimal performance. The effect of increasing the number of interfaces to three (two interfaces operate in transmitter mode) is shown in Fig. (3) (c): the maximum throughput increases from about 0.19 to 0.35. Note also that, for both, M = 1 and M = 3, and for a given value of K, say 5, a beamwidth $\beta = \frac{\pi}{2}$ produces about the same result as a beamwidth $\beta = \frac{\pi}{3}$ (i.e., a beamwidth of $\frac{\pi}{2}$ is narrow enough for optimal performance, and narrower beamwidths would not produce significant improvement). This fact is better highlighted in Fig. 3 (d).

Impact of the receiver range R. Fig. 3 (e) shows, for M = 1, the normalized throughput F_{LP2}/F_{LP1}', where F_{LP1}' is the flow value when R = 400. LP1 produces the maximum upper bound on throughput when R increases, because of a higher connectivity (the average node degrees for R = 200, 300 and 400 meters (m) were 10.26, 21.92 and 35.7 respectively). For omni-directional transmitter antennas, the best performance is obtained when R = 400; the throughput monotonically increases until 0.054 at K = 11, and higher values of K do not produce any improvement. For small values of K, and R = 200 or R = 300, increments in K produce smaller improvement than for the case of R = 400. Similarly, for $\beta = \frac{\pi}{3}$ and $K \leq 3$, the best performance is obtained when R = 400. On the other hand, for $K > 4$, higher throughput is obtained when R = 200. These results allow us to infer the following: i) for high values of R, the high connectivity permits the routing of flows through short paths in term of hops. Thus, *few links* are needed to be scheduled, which can be achieved with small values of K. For higher values of K, the improvement in throughput is not significant because of two bottlenecks: the large number of conflicting links due to the high connectivity, and the limited number of transmitter antennas; ii) for small values of R, the increments of K have more impact than for the case of high values of R, because more links can be simultaneously scheduled due to the lower connectivity. Fig. (3) (f) shows this fact, where the average number of links scheduled per second was computed as $\sum_{S_k \in \Gamma} \lambda_k |S_k|$. Similar conclusions can be obtained by increasing M. However, those results are not included in this paper because of space limitation.

7 Conclusion

In this paper, we have presented a generalized model for the throughput optimization problem in wireless networks. To the best of our knowledge, this is the first work that simultaneously considers multiple transmitter interfaces, generalized antenna model and multi-packet reception. We have divided the problem into two subproblems: routing and scheduling. Because of the hardness of the scheduling subproblem, we have also proposed a polynomial time heuristic based on a combination of greedy and linear programming paradigms.

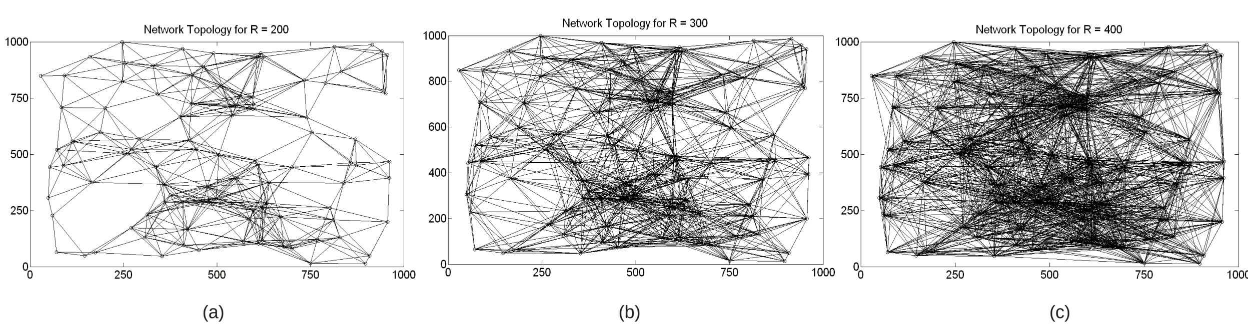

Figure 2: Network topology.

Based on the proposed heuristic, we have studied the impact of the the number of interfaces in MPR networks. We have shown that to fully exploit MPR capability, nodes may need to be endowed with multiple transmitter antennas; otherwise MPR capability may not be enough to improve performance. Additionally, we have considered the effect of the beamwidth of directional antennas on the throughput. The results include the study of increasing the MPR capability of wide beamwidth networks to achieve similar performance to narrower beamwidth networks. Finally, we have studied the impact of the receiver range of antennas on the throughput performance. As future work, we plan to evaluate our proposed scheme under the MPR physical model. We will further investigate the relations among the parameters K, M, β and R. Devising an analytical model for these relations is part of our future agenda. A distributed solution for routing and scheduling in MPR-capable networks will also be investigated.

A Appendix

Proof of Proposition 1. $\Rightarrow$ Assume that U is feasible and $\Gamma = \{S_1, S_2, ..., S_{|\Gamma|}\}$. Then, U can be computed as:

$$U = \sum_{S_i \in \Gamma} \lambda_i p_{S_i}. \tag{12}$$

By definition, the convex hull of all schedulable vectors is the set of all convex combinations of schedulable vectors:

$$\text{Convex hull} = \{\theta_1 p_{S_1} + ... + \theta_k p_{S_k} | p_{S_i} \text{ is a feasible schedulable vector}, \theta_i \geq 0, \theta_1 + ... + \theta_k = 1\}. \tag{13}$$

Note that the usage vector U given by Eq. (12) is a convex combination of the schedulable vectors in Γ, where the *weights* are given by the activity periods λ_i's. U is a particular point that satisfies Eq. (13) and therefore it belongs to the convex hull of schedulable vectors.
$\Leftarrow$ Assume that U lies within the convex hull of the schedulable vectors. Then, U can be expressed as a convex combination of a set $\Gamma = \{S_1, S_2, ..., S_{|\Gamma|}\}$ of schedulable vectors, similar to Eq. (12). By allocating λ_i seconds to $S_i \in \Gamma$ (the schedulable set that has a corresponding schedulable vector p_{S_i}), we can build a feasible schedulable, which implies that U is feasible.

B Appendix

Proof of Eq. (11). We start by proving the lower bound $\frac{F_{LP1}}{\tau} \leq F_{LP2}$. We proceed by constructing a feasible schedule, which uses the set Γ obtained in step 2 and the flow values $x_{ij}^n, (i, j) \in E, n \in N$, obtained by LP1 in step 1. The scheme is straightforward: allocate τ_k seconds to each schedulable set $S_k \in \Gamma$. During the activity period τ_k, send the aggregated flow $\sum_{n \in N} x_{ij}^n$ through link $(i, j) \in S_k$. The flow value (in bps) for this scheme is:

$$F = \frac{F_{LP1}}{\text{schedule period}} = \frac{F_{LP1}}{\sum_{S_k \in \Gamma} \tau_k} = \frac{F_{LP1}}{\tau}$$

Now, by solving the linear program LP2 with the same set of schedulable sets Γ, we obtain the optimal solution with the flow value F_{LP2}. This solution is better than any other solution (included the previous scheme) that only uses the set Γ. Therefore, $F_{LP2} \geq \frac{F_{LP1}}{\tau}$.
The bound $F_{LP1} \geq F_{LP2}$ is a trivial upper bound; if all the links can be active simultaneously (i.e., there is only one schedulable set and $\Gamma = E$), then LP2 = LP1 and $F_{LP2} = F_{LP1}$; otherwise, any link capacity c_{ij} is decreased as expressed at the right hand side of Eq. (8) and the flow value F_{LP2} may accordingly be reduced. Thus, $F_{LP2} \leq F_{LP1}$.

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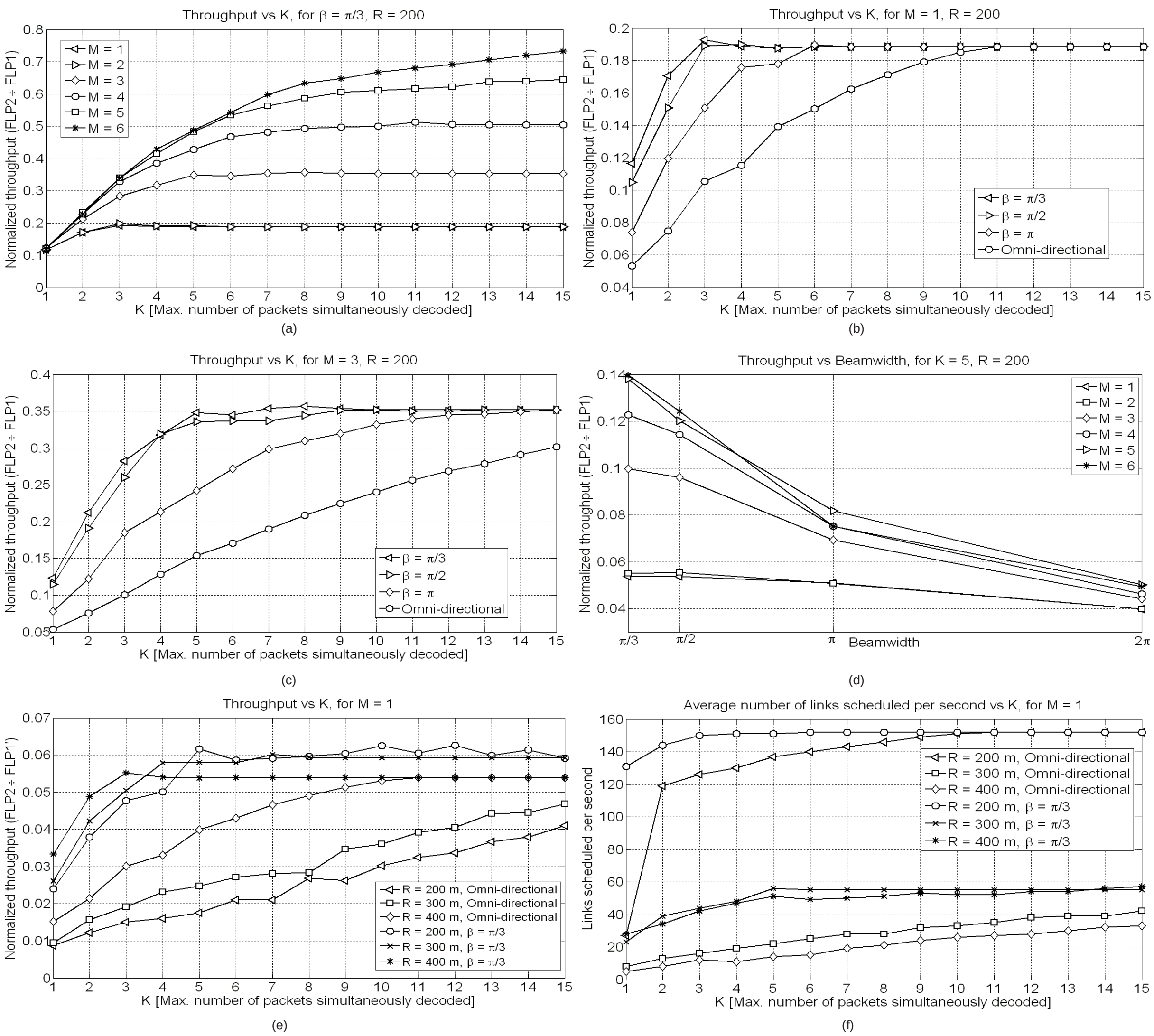

Figure 3: Numerical results.