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Formalism and the Notion of Truth

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FORMALISM AND THE NOTION OF TRUTH

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ABSTRACT

The most widely acknowledged conceptions of truth take some kind of relation to be at truth's core. This dissertation attempts to establish that an adequate conception of this relation begins with an investigation of the entanglement of the formal and the material as set forth in the model theoretical development of set theoretical mathematics. Truth concerns first and most crucially a certain commerce across the border between the formal and the material, between the ideal and the real. The entanglement of the formal and the material must be thought in itself, apart from or prior to any assimilation into philosophical schemas committed to larger metaphysical claims. This is accomplished in model theory.

The twentieth century witnessed two attempts at bringing model theoretical mathematics to bear on accounting philosophically for the concept of truth: that of Alfred

Tarski, and that of Alain Badiou. In order to investigate the relevance of model theory to the task of working out a philosophical conception of truth, this dissertation investigates, through comparative work, these two thinkers. It is necessary to see where their projects converge in important ways, as well as where their projects diverge in equally important ways. What brings their work into close proximity is their shared conviction that truth must be thought in light of model theory. Nonetheless, the two do not agree about exactly how model theory sheds light on truth. Comparative study thus reveals both a shared site for thinking and a struggle over the significance of that site.

Agreement between Tarski and Badiou concerns the excess of the purely formal over itself, marked by the generation of an undecidable statement within formal systems of a certain level of complexity. Both thinkers determine that this formal excess touches on the material, and both further determine that the consequent entanglement of the formal and the material provides the basic frame for any philosophical consideration of truth. The point of disagreement is ultimately rooted in a difference of opinion about the adequacy of thinking the concept of truth independently of an account of what philosophers of science call theory change.

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INTRODUCTION

For as long as truth has been investigated philosophically, it has been recognized that there must be some kind of *relation* at work in truth, whether that relation has been taken to be a matter of correspondence between minds and objects, of coherence among terms or claims made regardless of inaccessible states of affairs, or of fulfillment of intentions and aims in practical encounters with the world. In the course of this dissertation, I attempt to establish that an adequate conception of the relation that lies at the heart of truth begins with an investigation of the entanglement of the formal and the material *as this is set forth in the model theoretical development of set theoretical mathematics*. Truth, I argue, concerns first and most crucially a certain commerce across the border between the formal and the material, between the ideal and the real. But this means that essential to posing the question of truth is establishing the basic nature of this commerce. The entanglement of the formal and the material must be thought in itself, apart from or prior to any assimilation into philosophical schemas committed to larger metaphysical claims. This is accomplished, on my argument, in model theory. Strange as it might sound to say it, model theory presents a purely formal theory of the entanglement of the formal and the material. For this reason, it establishes what might be called the site of the problem of truth.

Significantly, the twentieth century witnessed two attempts at bringing model theoretical mathematics to bear on accounting philosophically for the concept of truth. The first was the well-known work of Alfred Tarski, undertaken over the course of the

1930s and 1940s. The second was the less-familiar work of Alain Badiou, launched in the 1960s but brought to its first real flowering only in the 1980s. To investigate the relevance of model theory to the task of working out a philosophical conception of truth, it is necessary to investigate quite closely the work of these two thinkers. As a result, much of the work I undertake in the course of this dissertation is dedicated to close comparative readings of their most important writings on truth. It is necessary to see where their projects converge in important ways, as well as where their projects diverge in equally important ways. What seems rather clearly to bring their work into close proximity is what I take to be their shared conviction that truth must be thought beginning from a chiefly formal investigation of the entanglement of the formal and the material, and that in light of model theory. On my interpretation, in other words, Tarski and Badiou agree on the site of the problem of truth. Nonetheless and unsurprisingly, the two do not agree about exactly how model theory sheds light on truth. As a result, by looking comparatively at these two thinkers' work, it is possible to present a kind of struggle over the significance of model theoretical considerations for any thinking of truth.

The essence of the agreement between Tarski and Badiou is not difficult to summarize. Both thinkers focus on a certain excess of the purely formal over itself, marked by the generation of an undecidable statement within formal systems of a certain level of complexity. Both thinkers then determine that this excess concerns, precisely, the material, and both thinkers further determine that the consequent entanglement of the formal and the material provides the basic frame for any philosophical consideration of truth. These points of agreement establish the model theoretical site of the problem of

truth. As for the debate of sorts between Tarski and Badiou, it too is not difficult to summarize. Tarski ultimately argues that a particular truth predicate associated with a particular formal language is to be defined in terms of what holds across all material models of the formal language in question. Badiou, on the other hand, ultimately argues that the truth proper to a particular language is to be defined through the model theoretical strategy of forcing, and that a very specific material model (that constructed by Paul Cohen) of a very specific formal language (classical set theory) provides a kind of general figure for truth. These points of disagreement—ultimately rooted in a difference of opinion about the adequacy of thinking the concept of truth independently of an account of what philosophers of science call theory change—make up the model theoretical struggle surrounding the problem of truth.

The site of and the struggle over the problem of truth, as I will expisit these over the course of this dissertation, are thus not difficult to summarize. But of course the details require close attention. The following chapters are, naturally, given to these details.

Chapter 1 is dedicated to a number of preliminary considerations. It opens with a more detailed presentation of the basic thesis of the dissertation than that just given, as well as with an outline of the argument to be pursued over the course of Chapters 2 through 4. A second consideration concerns the method to be used in this dissertation, which, as I have already indicated, is that of comparative study. A brief clarification and defense of this method follow the preliminary presentation of my findings. The next part of Chapter 1 responds to a specific objection, an objection to the comparative study of Tarski and Badiou in particular. This is that Badiou has apparently explicitly denied the

relevance of at least part of Tarski's work on truth to his own. This objection can, however, be met, and in a way that underscores the suggestive possibilities of comparing these two thinkers—rather than the strict impossibility of doing so. The final part of Chapter 1 considers the chief and most obvious point of divergence between Tarski and Badiou, presented by Etienne Balibar in the form of an objection to their commensurability. In addition to arguing that Balibar drastically overstates his case, I conclude the chapter by clarifying the basic point of difference that comparative study helps to highlight.

With preliminary considerations out of the way, I turn in Chapter 2 to a close investigation of the role played by the set theoretical apparatus in Tarski's work on truth. Although many summaries of Tarski's project have appeared in print, little to none of the literature begins by asking exactly how set theory guides and structures his project. Starting from that question allows emphasis to be laid on Tarski's criterion of material adequacy—an element of his thought that will prove essential to what I mean to accomplish in this dissertation. My exposition in Chapter 2 distinguishes between two major ways set theory organizes Tarski's project. To understand the first, material adequacy is key. Understanding the full implications of this criterion—going far beyond the way that it prescribes the structure of Tarski's famous T-schema—requires recognizing the way that it, combined with Tarski's discovery of an ontologically consequential theorem that still bears his name, forced him to pursue what he called a semantic approach to truth. The same business of material adequacy guides Tarski's strategy in providing strict definitions of truth (for particular languages), but in concert with a second guiding criterion—that of formal correctness. Most of what is usually

associated with Tarski's name in connection with truth concerns this second criterion: the distinction between object-language and metalanguage, the ramification of the concept of truth into so many concepts of truth severally attached to particular formalized languages, and even the strategy of defining truth in terms of the broader semantic concept of satisfaction. However familiar these features of Tarski's work are, they deserve close exposition as well so that the precise role played in them by the set theoretical apparatus is fully clear.

Chapter 3 continues in a general expository vein but shifts focus from Tarski to Badiou. Given how recently Badiou's work on truth has appeared—especially in English translation!—it is unsurprising that it is less familiar and has been only lightly and occasionally expounded, particularly its more formidable formal presentation. Much more than providing basic exposition, however, Chapter 3 is meant to show preliminarily that, as with Tarski, set theory organizes Badiou's work on truth in two distinct ways. To appreciate the first of these, it is necessary to clarify the relationship between Badiou's thought and that of Martin Heidegger. Badiou understands set theory to constitute the science of being, and specifically of being as Heidegger understood the word. (Ontology, for Heidegger, means investigating the meaning of being, rather than investigating what actually is.) Some exposition of Badiou's reliance on and debate with Heidegger makes clear the basic motivations of Badiou's project and its ontological bearings. When Badiou turns his attention from general ontological concerns to the specific task of exposing the nature of truth, Heidegger retreats to allow Georg Cantor to step into the spotlight. The undecidability of the latter's continuum hypothesis, a central focus in work on set theory through much of the twentieth century, provides the resources for Badiou's fixing of the

notion of truth. This point is ultimately what distinguishes Badiou from Tarski, but the point of Chapter 3 is just to place Badiou's project in its entirety on display.

Chapter 4, finally, exchanges exposition and interpretation for direct comparison and philosophical reflection. With Tarski's and Badiou's projects clarified in terms of their respective forms of dependence on the set theoretical apparatus, I give much of Chapter 4 to a discussion of merely apparent points of divergence and much realer points of convergence. What most crucially marks the close relationship between Tarski's and Badiou's projects is, as I have already noted, what I argue is their shared (minimal) ontology. This ontological parity especially requires detailed attention. As I make clear in Chapter 4, it concerns the philosophical status of a specific—and a specifically Tarskian—development within set theory (namely, model theory) and the ramifications of a specific—and, again, specifically Tarskian—criterion (namely, material adequacy). After giving this point its due, I give my attention at last to the rival proposals of Tarski and Badiou regarding the appropriate resources, within set theory and model theory, that should be used to provide a full elucidation of the concept of truth. This allows for the final isolation of a philosophical problematic made clear by comparison between Tarski and Badiou.

I follow Chapter 4 with a brief conclusion, stating my findings and pointing out directions for further study.

PRELIMINARY CONSIDERATIONS

As I have already indicated in the Introduction, I mean in this first chapter only to accomplish a few preliminary tasks before turning my attention in subsequent chapters to the work of expositing, interpreting, and drawing from the writings of Alfred Tarski and Alain Badiou on the notion of truth. First and before all else, I present in this chapter a summary of my conclusions and an outline of the argument presented in the course of this dissertation, already introduced in a preliminary way in the Introduction.

Beyond that, however, at least a few words concerning methodology seem necessary, at the very least because I will occupy much of this dissertation with comparative study. If there is any truth in conventional disciplinary wisdom, analytic thinkers tend to work directly on already-established problems while continental thinkers tend to give their efforts to defending the philosophical importance of certain thinkers through exegetical work. Here I propose to do something that either is a kind of weave of these two usual approaches to the philosophical task or, perhaps, amounts to something else altogether. Unlike the proverbial analytic philosopher, I am here uninterested in already-established problems; I do not propose to engage with the *current* debate about truth, to intervene in the conversation that has developed in the decades since Tarski produced his work. And unlike the proverbial continental philosopher, I am here uninterested in defending the philosophical importance of any particular thinker, preferring just to identify a problematic. My intention is thus to work at the blurry boundary between the analytic and continental traditions, closely reading a major thinker

hailing from each tradition in the hopes of isolating a problem that seems largely to have gone unnoticed in philosophy.¹

After defending the methodology to be employed, I turn in a third section to an important objection—an objection not to the comparative method in general, but to the particular proposal of this dissertation: comparative study of Tarski and Badiou. The objection concerns the fact that Badiou has explicitly denied the relevance of at least part of Tarski’s work on truth to his own. I show, however, that Badiou’s statement concerning the irrelevance of part of Tarski’s work to his own project, if read in the context of Badiou’s philosophical development between the 1960s and the 1980s, implies much less than appears to be the case.

Finally, the last section of this first chapter is dedicated to what might at first seem to be a further objection—a point that has, at any rate, been presented as a kind of objection by one interpreter: Etienne Balibar. In effect, Balibar claims that what Tarski and Badiou respectively understand by the word “truth” is ultimately so distinct that the two projects are fundamentally incommensurable. Balibar, however, overstates this point, resting his case on a deeply problematic account of the relationship between philosophy in France and philosophy as practiced elsewhere. Nonetheless, as I show, Balibar’s objection harbors an important truth: that Tarski and Badiou do in fact begin their work with distinct pre-theoretic notions of truth. But far from constituting an objection to comparative study, it is precisely this difference that motivates comparative study. What Tarski and Badiou share helps in crucial ways to reveal the genuine stakes of what they

¹ I thus entirely agree with Paul Livingston when he says that “the continued assumption of an analytic/continental distinction today shelters methodological prejudices on both sides which tend to debar access to the unified formal problematic[s]” that most deserve attention. Livingston, *Politics of Logic*, xii–xiii.

clearly do not share.

With all of these preliminaries out of the way, I turn in subsequent chapters to comparative work and the task of isolating the philosophical problematic revealed by such work.

Thesis and Argument

As is already clear, my general aim in this dissertation is to bring Alfred Tarski's and Alain Badiou's respective investigations of truth, both of which draw heavily on set theoretical mathematics, into philosophically productive conversation. In a certain way, it is enough for me just to make fully clear *that* and *how* the two projects can speak to one another in interesting ways. By no means is it possible to argue convincingly that the two projects amount to one, or that comparison reveals that every attempt to bring set theory to bear on truth will produce the same results. Nonetheless, comparison does reveal that *certain* quite similar convictions draw these thinkers to the same formal resources in their attempts to become clear about truth. A major task of this dissertation, therefore, is to become clear about what Tarski and Badiou in fact share, despite crucial differences in their respective conceptions of truth. On my argument, these two thinkers share what I will call an ontology, a specifiable conception of the basic entanglement of the formal and the material that both take to lie at the foundation of every consideration of truth.

What especially unites Tarski and Badiou as regards these minimal ontological commitments is their shared investment in a certain development internal to historical set theory: the field, arguably invented by Tarski himself, of model theory. Both thinkers argue in their own ways that the problem of truth must be approached with an eye to the

formal problems bound up with model theory. Because model theory basically amounts to the investigation of undecidable formal languages—that is, languages in which it is possible to construct sentences neither the affirmation nor the negation of which can be derived from the resources of the language—the point of crucial convergence between Tarski and Badiou amounts to the idea that truth in its most basic or foundational form concerns questions of undecidability. The scientifically rigorous notion of undecidability regulates in the most formal fashion the boundary between the strictly formal and the minimally material. If truth is to be thought in its concept, clearly and distinctly, it is at that border that thinking must take place.

Of course, it should be no surprise that, once this point of genuine convergence is established, Tarski and Badiou nonetheless diverge quite drastically on the question of exactly *how* undecidability relates to the basic concept of truth. Hence a second chief task in this dissertation: to clarify the basic nature and stakes of a kind of debate between Tarski and Badiou on what model theory might have to say about truth. Both Tarski and Badiou draw on the resources of set theory and model theory in their attempts to fix, in a formal way, the concept of truth—Tarski through the construction of rigorous definitions of truth for particular formal languages, and Badiou through the identification of rigorous operations modeling the production of truth for particular historical situations. But, of course, each draws on set theory and model theory quite distinctly in his work, determining the importance of undecidability in quite different ways. Their debate might be summarized as follows.

Tarski's work captures the general features of undecidability and outlines a strategy in light of those general features for defining truth in particular cases. To this

end, he puts satisfaction, the most basic operation lying at the foundations of model theory, to work: truth is to be defined in terms of satisfaction—more specifically, as whatever holds across all models of a specific formal language, each of which satisfies the formal language. Badiou, however, insists that truth can be thought in its concept only if one considers a quite specific instance of undecidability: the undecidability of Georg Cantor's continuum hypothesis with respect to the axioms of classical set theory. For this reason, Badiou privileges the model theoretical operation of forcing, a much more recent development, taking this operation to figure the material production of truth through the transformation of knowledge.

This debate, grounded on a shared ontology, reveals the problematic I mean to isolate in the course of this dissertation. To think in the most formal way about the basic relation that lies at the core of the concept of truth, one must investigate the question of undecidability, particularly as this is worked out most rigorously in model theory. But bringing the question of truth into relation with the right set of resources in no way decides what must be said in order to clarify truth as a concept. Rather, pre-theoretic notions of truth will inevitably guide the distinct ways in which one will work with the resources on offer in model theory. The problem of truth finds in a certain minimal ontology its proper site, but a variety of possible solutions to that problem remains, and these solutions will follow the lines of the pre-theoretic considerations one brings to the question.

This is what I will argue over the course of this dissertation. Obviously, all of this depends on being able to tease out of the writings of Tarski and Badiou both a genuinely shared (minimal) ontology—which secures the commensurability of their projects—and a

real debate regarding the proper use of the resources of that ontology—which marks their divergence nonetheless. This requires a comparative methodology, about which it is proper at this point to say a few words.

Comparative Study

Comparative work in philosophy all too easily looks like a kind of unsophisticated exercise in compare and contrast. And, unfortunately, comparative work indeed *can* easily and *does* often devolve into an attempt either to downplay important differences in the name of highlighting overinflated and generally uninteresting points of contact or to underscore obvious but trivial dissimilarities in order to stage dismissive arguments against the strawman opponent of a favored philosophical worldview. As I conceive of the comparative task here, however, something rather different is at stake.² In the best comparative work, the task is first to develop an entire network of similarities and differences between apparently divergent phenomena. Similarities are brought to the fore principally in order to make all the starker points of dissimilarity. And dissimilarities are brought into focus chiefly so that the stakes of any genuine points of similarity can be measured.³ What ideally emerges in the course of rigorous comparative work is a set of *forms*, each form distinct in each of its several instantiations but indicative of what remains largely invariant across those same instantiations. To the extent that such forms—repeated configurations of networks of similarity and difference—become discernible through careful comparative work, it becomes possible to ask about their

² I have learned much, methodologically, from philosophically-driven comparative study in religion—especially the work of Jad Hatem. See, for instance, Hatem, *Postponing Heaven*.

³ I intentionally use language here suggestive of structuralist comparative work in the human sciences. The methodology of such work has perhaps been most illuminatingly analyzed in Deleuze, “How Do We Recognize Structuralism?”

philosophical importance: What tasks does the existence of such and such a form give to philosophical reflection?

The description I have just offered likely seems rather abstract. It might, though, be illustrated with a brief example. Because of its relevance to the topic of this dissertation (although I will not draw on it in later chapters), I will illustrate the method by comparing a well-known text from Plato with a piece by Badiou. Quite recently, Badiou has tried his hand at translating the most important work of Western philosophy's founder: *The Republic*. In what he calls a "hyper-translation," Badiou updates Plato's text, altering it in significant and telling ways while nonetheless following the original Greek text.⁴ Because what Badiou offers in this text is a translation, the network of relevant similarities and differences between the two thinkers is readily discernible. But because it is a *hyper*-translation, in which Badiou feels quite free to alter Plato's original to suit his own philosophical proclivities, real divergences are just as readily discernible. All this ready discernibility makes the productive results of comparative work yield themselves in basic outline without extensive work. I will, of course, focus only on a short—but telling—passage from *The Republic*, one that conveniently concerns the subject of truth.

Late in Book VI of the *Republic*, Plato presents the well-known allegory of the sun. The stated purpose of the allegory is to clarify, albeit indirectly through an allegory, the nature of the good. In Plato's text, sunlight serves as a *tertium quid* that mediates between human sight and visible things, making the relation between the two possible. As Socrates explains, "unless a third kind of thing is present, which is naturally adapted for this very purpose, you know that sight will see nothing, and the colors will remain

⁴ For an explanation of what constitutes a "hyper-translation," see Badiou, *Plato's "Republic"*, xxxi–xxxv.

unseen.”⁵ This “third kind of thing” is “caused and controlled” in turn by the sun, one of the gods in heaven.⁶ The sun serves as the cause of sight (and is therefore in no way equivalent to sight), and it can, significantly, be seen by sight, but continued exposure to the sun will end up ruining sight. Now, all this Socrates compares to knowledge. Just as sunlight serves to mediate between visible things and human sight, allowing the two to relate to one another, the form of the good mediates between the forms that give shape to visible things and the individual intellect or soul, allowing these two to relate to one another. The Platonic good thus organizes a kind of primal fittedness necessary to the possibility of knowledge. Because the intellect or the soul would be ruined by any (sustained) direct encounter with the good (like eyes staring into the sun), the good is only (or at least better) known indirectly, through concrete encounters with *goods*. Just as visible things of various sorts are realizations of the various forms, goods are formal realizations of the good. Goods might be conceived of as localized realizations of a kind of fittedness between the intellect or the soul and the forms that give shape to the visible realm. All such local goods are ruled over and caused by *the* good.⁷

In his hyper-translation of *The Republic*, Badiou makes two crucial changes to the terminology of this allegory—changes crucial enough that they uniquely receive mention in the volume’s preface: “the form of the good” becomes “the idea of the true” or sometimes just “Truth,” while “the soul” or “the intellect” becomes “the subject.”⁸ Thus, in Badiou’s version of Plato’s allegory, what governs the realm of the forms and thereby fits the forms to the intellect—or, in Badiou’s terminology, exposes the subject to the

⁵ Plato, *Republic*, 507d–e.

⁶ *Ibid.*, 508a.

⁷ I am indebted for some details of this interpretation to Hancock, *The Responsibility of Reason*, 60–63.

⁸ Badiou, *Plato’s “Republic”*, 204–207.

invariance of the ideas—is the idea of the true, Truth with a capital “T.”⁹ Consequently, in Badiou’s picture, what one discerns among the visible things, themselves shaped by the forms, is not *goods* but *truths*, truths that indirectly give the idea of the true to be known because of the derivative relationship between them and that governing idea. Each truth, moreover, bears witness not to the moral goodness of the ordered world, but to the subject’s exposure to the invariant forms that repeatedly (if nonetheless rarely) reemerge as configurations of being that underlie the realm—or really, the realms—of appearance.

Now, what is to be learned by considering Plato’s and Badiou’s iterations of *The Republic* comparatively? As I have already noted, because Badiou’s *Republic* is a translation of Plato’s, the network of relevant similarities and differences between the two thinkers’ projects is ready to hand, easily discernible. The schema of the allegory of the sun remains the same across the two texts, with the sun and its light serving the same role in each case, namely that of fitting visible things to human sight. And although Plato’s “form of the good” and his “intellect” or “soul” are respectively replaced with Badiou’s “idea of the true” and “subject,” the network of relations among the several terms remains constant across the two texts: the form of the good or the idea of the true brings the intellect or the subject into a direct relation of exposure to the invariant forms that give order to visible things; and goods or truths in the plural are in each case indirect witnesses for the intellect or the subject to the form of the good or the idea of the true. But these similarities—similarities of structure, one might say—serve to bring out the essential differences between Plato and Badiou. Indeed, it is perhaps *only* in light of essential similarities that the relevant force of the following sentence from Plato’s

⁹ For Badiou, the subject is in no way the Cartesian knowing subject. Instead, the subject is a product of a truth procedure. These points will be discussed at greater length in Chapter 3.

Republic can be felt: “Here it is right to think of knowledge and truth as godlike but wrong to think that either of them is the good—for the good is yet more prized.”¹⁰ Plato explicitly distinguishes truth from the good, coupling it with knowledge and locating both of these together at, so to speak, a different level (they are not to be prized quite so much as is the good).¹¹ For his part, Badiou reworks this line from Plato to locate knowledge (“scientific knowledge and exactness,” the latter replacing Plato’s reference to truth) at one level and truth (taking the place of the good) at another: “We’ll say that it’s correct to regard science and veridical knowledge as partaking in Truth, but it’s wrong to equate them with Truth itself, because a higher rank should be accorded to the specific nature of Truth.”¹² Badiou’s basic notion of truth, quite distinct from Plato’s, emerges with real clarity when his hyper-translation and Plato’s original are set side by side. And these differences in turn serve to clarify what Plato and Badiou unmistakably share: a certain notion of participation or (in reverse) realization, understood to be causal in some (largely unexplained) fashion, and a schema of indirect witnessing bound up with that notion of participation.¹³

There emerges in the foregoing analysis—overly brief though it is—the barely discernible outlines of a handful of *forms*, philosophical concepts that position themselves at the border between the two thinkers whose work is compared. Some of these forms distinctly fall on one or the other side of that border, constituting the specific

¹⁰ Plato, *Republic*, 508e–509a.

¹¹ Presumably what is at issue here is a conviction, on Plato’s part, that truth can be equated with the knowable, while the good cannot. In this regard, a fundamental distinction between Plato and Badiou might be said to be replicated in the debate between Badiou and Tarski. This point will be developed in Chapter 4.

¹² Badiou, *Plato’s “Republic”*, 207.

¹³ It is worth noting that Badiou has often presented his philosophical project as a reinvigoration—with revisions—of Plato’s work. He first made this “gesture,” as he has called it, in 1989 in Badiou, *Manifesto for Philosophy*, 96–109. In a more recent interview, however, Badiou has suggested that even his earliest philosophical work, dating to the 1960s, can be understood as an attempt to determine the nature of Platonic participation. See Badiou, *The Concept of Model*, 92.

points of their disagreements: Badiou's concept of truth as working at a level distinct from that of knowledge over against Plato's placement of knowledge and truth on a single level; Plato's totalizing concept of the good over against Badiou's implicit emphasis on multiplicity and his relative uninterest in moral categories.¹⁴ Others of these forms are unmistakably shared between the two thinkers, constituting the site of their disagreements: the causally construed notion of participation or (in reverse) realization; and the schema of indirect witness bound up with participation. Each of these forms, once discernible thanks to the differential network in which it takes on a more definite shape, can then be given more direct philosophical attention. How might we think about the participation-witnessing schema, and what work does it do for each of these two thinkers, given their differences? How might we rigorously distinguish Plato's and Badiou's respective conceptions of truth given their points of convergence, and what might such distinctions tell us about what it means to think philosophically about truth?

Borrowing from Gilles Deleuze, I want to claim that these *forms*, sites for philosophical reflection, are simply what philosophers usually call *problems* or *problematics*. The point in doing comparative work is to allow networks of similarities and differences among philosophers to bring to light problems that call for thought. Problems are, according to Deleuze, configurations of sense, points of confluence or torsion in the structure of thought.¹⁵ The task of thinking is first to identify, and then to give itself to, philosophical problems. This is one reason it might productively begin with comparative work, work that aims just at allowing problems to make themselves manifest

¹⁴ Badiou *can* of course be said to be interested in certain ways in questions of morality, but his interest is always expressed in terms that drastically reconfigure the notion of ethics. See, on this score, Badiou, *Ethics*.

¹⁵ See Deleuze, *Difference and Repetition*, 153–64.

without yet feeling any particular need to answer them (at least, not so as to do away with them). Indeed, once problems come to light, they demand reflection and thought, solutions in the plural; they in no way represent mere questions that require correct and conclusive answers. As Deleuze says, rightly in my view, “teachers already know that errors or falsehoods are rarely found in homework. . . . Rather, what is more frequently found—and worse—are nonsensical sentences, remarks without interest or importance, banalities mistaken for profundities, ordinary ‘points’ confused with singular points, badly posed or distorted problems—all heavy with dangers, yet the fate of us all.”¹⁶ What good comparative work helps to facilitate is the identification of genuine problems that demand philosophical thought. The mark of bad comparative work is that it pretends to pose questions rather than answers and then pretends to answer those questions definitively.

In the course of this dissertation, as I have already indicated, I mean to undertake comparative work with the aim just of establishing a single problematic, one that concerns the relationship between model theory and the notion of truth. What allows for the clarification of the genuine debate between Tarski and Badiou is, first, the network of similarities—what I have above called their shared (minimal) ontology. But that network of similarities serves first and foremost to clarify the basic philosophical stakes of their genuine points of difference. To the extent that such comparative work helps to clarify these points of difference, it does important philosophical work.

Of course, one might concede the comparative method in rather general terms while nonetheless dismissing the particular proposal set forth here: to bring Tarski and Badiou into productive philosophical conversation. That such can be done in a useful way

¹⁶ *Ibid.*, 153.

will be demonstrated over the course of the following chapters. Nonetheless, it would be helpful to consider an important objection to this project from the outset—particularly because the objection in question levels an *immanent* critique against the project. In an important passage penned by Badiou, it seems clear that he rejects the relevance of Tarski to his work.¹⁷ This deserves close attention.

Badiou on Tarski

In the introduction to *Being and Event*, his magnum opus, Badiou provides a brief account of the years leading up to his work on truth. As he presents his history there, the problem that set him thinking was—unsurprisingly—political. As a young professor, Badiou was politically radicalized by the events of May 1968 in Paris, with the consequence that he dedicated most of his attention in the 1970s to strictly political tasks, whether practical or theoretical.¹⁸ While the slow collapse of French Maoism in the second half of that same decade led many of Badiou’s comrades to denounce publicly

¹⁷ It is worth noting that a related but distinct comparative project suggests in advance the viability of what I undertake in this dissertation. To the extent that Badiou’s work represents a certain formalizing development of notions already on offer in the writings of Martin Heidegger, and to the extent that the subtle insistence in Tarski’s work that his formalizations could be brought to bear on natural language has been developed by Donald Davidson, it is significant that there has developed in recent years a literature productively comparing Heidegger and Davidson on truth. The upshot of such comparative work has been to establish that Davidson and Heidegger share a certain interest in truth-conditions—that is, in the conditions under which it is possible to say that a sentence or a belief is true—while they nonetheless fundamentally disagree about what those conditions are. This comparative project might seem to bear directly on my work in this dissertation, but closer observation suggests otherwise. Neither Heidegger nor Davidson pays especially close attention to the strictly formal—and certainly not to the details of set theory and model theory. Moreover, neither Tarski nor Badiou gives strong attention to the particulars of truth-conditions, the question central to comparative study of Heidegger and Davidson. For representative comparative studies of Heidegger and Davidson on truth, see Wrathall, *Heidegger and Unconcealment*; Okrent, *Heidegger’s Pragmatism*; Rorty, *Objectivity, Relativism, and Truth*; and Malpas, *Donald Davidson and the Mirror of Meaning*.

¹⁸ Badiou has spoken of May 1968 as “a genuine road-to-Damascus experience.” Hallward, *Badiou*, 33. In a 2007 interview, he describes the two years after May 1968 as “perhaps the most activist, the most militant, the most revolutionary of the last fifty years of French history,” adding, “for my part, I was completely involved during this period.” Badiou, *The Concept of Model*, 79. For his full analysis of the significance of that political event in general terms, see Badiou, *The Communist Hypothesis*, 41–100.

their former associations, Badiou gave those years to a kind of retrenchment effort at providing Marxism-Leninism with a workable account of the militant subject. A first installment of that project was published in 1982 as *Theory of the Subject*, in the course of which Badiou determined, thanks especially to what he had come to believe was right in the thought of Jacques Lacan, that the ontology of the subject he desperately needed lay in formal considerations.¹⁹ Thereafter, as he says in *Being and Event*, he “groped around for several years among the impasses of logic—developing close exegeses of the theorems of Gödel, Tarski, and Löwenheim-Skolem.”²⁰ Badiou provides no further details about what he found useful in the early 1980s in the relevant writings of those four closely-related mathematicians, motivating his “close exegeses” of their fundamental theorems—though he *had* provided a revealing philosophical exegesis of Kurt Gödel’s incompleteness theorems in a paper published already in the late 1960s.²¹ In the introduction to *Being and Event*, however, Badiou only explains why he eventually *abandoned* his efforts at finding in the theorems of Gödel, Tarski, and Löwenheim-Skolem something he might use in his search for a theory of the subject. That Badiou explicitly states in the opening of his major work on truth that he found his philosophical way only by rejecting the relevance of Tarski’s theorem to his project—a theorem deeply enmeshed in Tarski’s work on truth—would seem rather obviously to suggest that, at least from Badiou’s perspective, there is little to gain from close comparative study of Badiou and Tarski.²² Is this right? An answer to this question will depend heavily on what

¹⁹ See, of course, Badiou, *Theory of the Subject*. For an incredibly helpful summary by Badiou of what he drew from Lacan, see Badiou and Roudinesco, *Jacques Lacan, Past and Present*, 6–9. See also Badiou, *Pocket Pantheon*, 1–4.

²⁰ Badiou, *Being and Event*, 5, translation slightly modified.

²¹ See Badiou, “Mark and Lack.”

²² Tarski’s theorem appears in the course of his most important essay on truth. See Tarski, “The Concept of Truth in Formalized Languages,” 199; as well as, for the full proof of the theorem, Tarski, “On Undecidable

exactly it was Badiou hoped to find in Tarski and related mathematicians—and what he therefore felt he did not find—when he undertook their study in the early 1980s. This must be clarified.

The exegesis of Gödel’s work published by Badiou in the late 1960s (read in close connection with his late-1960s book, *The Concept of Model*, and other related work) helps to provide some indication of what Badiou hoped to find in his investigations in the early 1980s. At the time he produced the exegesis, before his political radicalization and still heavily under the influence of Louis Althusser, Badiou argued strongly that “there is no subject of science,” using his exegesis of Gödel’s theorem to illustrate this principle.²³ His aim in the Gödel essay was actually principally to contest the claim by Lacan’s protégé Jacques-Alain Miller that set theory as specifically deployed by Gottlob Frege in *The Foundations of Arithmetic* (in providing a definition of the number zero) provides a formal model for the Lacanian subject.²⁴ Badiou mobilized Gödel’s work to demonstrate that whatever trace there might be of the Lacanian subject in Frege was merely a reflection of ideological commitments on Frege’s part—commitments that ultimately compromised Frege’s pretensions to scientificity.²⁵ After his political radicalization and consequent break with Althusser, and especially after he came to believe that Lacan had something to teach him about the notion of the subject, Badiou must have returned to Gödel’s theorem—alongside the related theorems of Tarski and Löwenheim-Skolem—in

Statements in Enlarged Systems of Logic and the Concept of Truth.”

²³ Badiou, “Mark and Lack,” 171. See also Badiou, *The Concept of Model*. For an articulation of the role Althusser’s thought played in these reflections, see Badiou, “The (Re)commencement of Dialectical Materialism.”

²⁴ See Miller, “Suture.”

²⁵ For a helpful articulation of the conception of the relationship between science and ideology operative in Badiou’s thought in the late 1960s, see Althusser, “*Philosophy and the Spontaneous Philosophy of the Scientists*” and *Other Essays*, 73–100.

the hopes of controverting his own earlier assessment.²⁶ Clearly, he sought in the work of these mathematicians some trace of the subject.

A further hint concerning what Badiou hoped to find in Gödel's and related theorems appears in *Theory of the Subject*, the last work Badiou published before he undertook his early-1980s exegeses. There, in a lecture originally given on May 4, 1979, Badiou states as his position that "the key to the theory of the subject" lies in the resolution of "the undecidable": "The undecidable is the concept of [the subject's] constitution."²⁷ In that connection, he invokes Gödel's theorems: "We know since Gödel's famous theorem that to posit 'There is some undecidable' can be the result of a demonstration. This goes to show that there is a concept of the undecidable, and thus that we firmly tie the doctrine of the subject to the possibility of a calculation. We *demonstrate* the subject."²⁸ It was clearly the possible connection between the subject and the undecidable that drew Badiou's attention back to Gödel at the end of the 1970s and, therefore, again in the early 1980s. It seems clear that the same motivation drew his attention to Tarski's theorem (along with the Löwenheim-Skolem theorem), so closely related to Gödel's incompleteness theorems.²⁹

Also essential, of course, is what Badiou has to say in *Being and Event* about his motivations for eventually rejecting the relevance of Tarski's theorem to his own work. Here, at some length, is Badiou's description of how he "had mistaken the route" in

²⁶ Bruno Bosteels has given a reading of Badiou's development that works against any strong periodization of his work. To reveal the shortcomings of such a reading, it would be necessary to undertake close comparative work on the uses of both Paul Cohen's notion of forcing and Georg Cantor's continuum hypothesis in *Theory of the Subject* and *Being and Event*. Such an investigation, however, lies beyond the scope of this dissertation. For Bosteels's, interpretation, see Bosteels, *Badiou and Politics*, 150–73.

²⁷ Badiou, *Theory of the Subject*, 286.

²⁸ *Ibid.*, emphasis added.

²⁹ The close relationship between Gödel's and Tarski's theorem is universally recognized. Both of these are clarified in the course of Chapter 2.

pursuing the implications of Tarski's theorem (in connection with the theorems of Gödel and Löwenheim-Skolem):

Without noticing it, I had been caught in the grip of a logicist thesis which holds that the necessity of logico-mathematical statements is formal due to their complete eradication of any effect of sense, and that in any case there is no cause to investigate what these statements account for, outside their own consistency. I was entangled in the consideration that if one supposes that there is a referent of logico-mathematical discourse, then one cannot escape the alternative of thinking of it either as an "object" obtained by abstraction (empiricism), or as a super-sensible Idea (Platonism). This is the same dilemma in which one is trapped by the universally recognized Anglo-Saxon distinction between "formal" and "empirical" sciences. None of this was consistent with the clear Lacanian doctrine according to which the real is the impasse of formalization.³⁰

Badiou goes on to explain that it was only when—due to "the chance of bibliographic and technical research on the discrete/discontinuous couple"—he turned his attention specifically to Georg Cantor's continuum hypothesis that he extricated himself from the grip of unwitting logicism.³¹ But what, from Badiou's words above, can be concluded about his supposed mistaking of the way forward for his project, and about its implications for his relation to Tarski?

What Badiou seems first and foremost to have rejected in what might be called the Gödelian-Tarskian route toward the subject is its failure to touch on what Badiou,

³⁰ Badiou, *Being and Event*, 5.

³¹ *Ibid.*

following Lacan, calls “the real.” In the course of giving the lectures that became *Theory of the Subject*, Badiou came to believe that the undecidable *as such* was the key to the subject. Later, however, Badiou came to believe that the key to the subject lay not in the undecidable *as such*, but rather in *a very specific instance* of undecidability. What initially drove his early 1980s interest in Gödel and Tarski (and Löwenheim and Skolem) was the possibility of regarding undecidability in general with what Lacan called “the impasse of formalization.”³² What eventually depleted his interest in Gödel and Tarski was the idea, which he came to by “chance,” that undecidability does not *in itself* constitute a key impasse of formalization. Among all the halting steps taken in set theory’s historical development, it is one of the earliest and longest-standing obstacles that, in Badiou’s view, constitutes a genuine impasse: Cantor’s hypothesis that the powerset of a given infinite set of a given cardinality has as its cardinality the immediate successor of the cardinality of the given set.³³ Badiou’s conviction, worked out at great length in *Being and Event*, is that the problem of truth must be situated rigorously in relation to that particular impasse.³⁴

Does that conviction suggest that Badiou’s rejection of Tarski’s theorem as the key to the theory of the subject, which he hoped to construct, entails a wholesale rejection of Tarski’s work on truth as essentially irrelevant to his own? The answer to this question would seem to depend in large measure on what exactly constitutes the relationship

³² See Lacan, *On Feminine Sexuality, the Limits of Love and Knowledge*, 93: “The real can only be inscribed on the basis of an impasse of formalization.” For a helpful exposition of this point in both Lacan and Badiou, see Livingston, *Politics of Logic*, 188–92.

³³ Importantly, Badiou already noted the importance of the continuum hypothesis in his 1970s lectures that became *Theory of the Subject*. His interest in the problem at that time, however, was limited to the way in which the decision against the continuum hypothesis, validated by the work of Paul Cohen, mirrors a certain politically militant decision against the strict stability of historical situations. See Badiou, *Theory of the Subject*, 265–74.

³⁴ The technical details of the continuum hypothesis will be addressed more fully in Chapter 3.

between Badiou's theory of the subject and his conception of truth. If Badiouian truth depends in an essential way on the concept of the subject, then it might well prove to be the case that the irrelevance of Tarski's theorem—which concerns truth precisely—to Badiou's theory of the subject entails the general irrelevance of Tarski's work on truth to Badiou's. As it turns out, Badiou's conception of truth *does* in fact depend rather heavily on his concept of the subject; the Badiouian subject is constituted by its relationship, precisely, to truth (as was already suggested above in Badiou's hyper-translation of Plato's allegory of the sun).³⁵ Does this close relationship therefore directly imply that Badiou's and Tarski's respective attempts at formalizing truth are irrelevant to each other?

In one register, the answer to this question is straightforwardly negative. Badiou did not reject Tarski's theorem (and the similar theorems of Gödel and Löwenheim-Skolem) as *entirely* irrelevant to what he hoped to say about the subject, even if he failed to find there a full-blooded theory of the subject. Indeed, that theorem (along with the other, similar theorems) directly concerns the nature of undecidability, and Badiou in no way rejected the relevance of undecidability to his conception of truth. Rather, he determined just that the notions of the subject and of truth could be fully clarified—as noted above—only with reference to a very particular *instance* of undecidability. It could thus be said that, for Badiou, Tarski's theorem bears within it an important *trace* of the subject, a formal schema of subjective decision or subjective intervention, even if it does

³⁵ In *Being and Event*, Badiou argues that the subject solely exists as the producer of truth; where truth is not in production, there is no subject. In his somewhat revisionary *Logics of Worlds*, published twenty years after *Being and Event*, Badiou has altered his position on this point. It nonetheless remains the case even in Badiou's more recent work that the subject solely exists in virtue of its relationship to some process of truth production, even if the latter is only a latent possibility and the subject exists only in the form of a reactionary refusal to assume the responsibility of pursuing that possibility. See Badiou, *Logics of Worlds*, 45–78.

not directly capture the whole concept of truth in which Badiou is interested. In this connection, it is in fact worth noting that, in *Being and Event*, Badiou describes the controversial set theoretical axiom of choice as “the interventional form,” a first mathematical trace of the subject.³⁶ The importance of this particular axiom for set theory was demonstrated by Löwenheim in his first formulation of the theorem that would be developed by Skolem and become the Löwenheim-Skolem theorem.³⁷ The close relationship between this theorem and Tarski’s theorem suggests something of the relevance of Tarski’s work on truth to Badiou’s conception of truth, even if Badiou himself has never directly acknowledged that relevance.³⁸ If, as Badiou says, “it is on the basis of the couple of the undecidable event and the interventional decision that time and historical novelty result” (and because the notion of historical novelty lies at the heart of what Badiou understands by “truth”), then it would seem that, despite certain appearances, there is *some* kind of relation between Tarski’s and Badiou’s respective projects that motivates comparative study.³⁹ Indeed, much of what I will argue in Chapter 4 concerning the debate between Tarski and Badiou will amount to a restatement of what I have had to say already in this paragraph.

These considerations suggest that Tarski’s theorem, while failing to be *the* key to Badiou’s theory of the subject and therefore to Badiou’s ultimate conclusions regarding truth, nonetheless relates in a fundamental way to many of Badiou’s central concerns in his attempt to bring set theoretical mathematics to bear on the concept of truth.

³⁶ Badiou, *Being and Event*, 223.

³⁷ On this point, see Moore, *Zermelo’s Axiom of Choice*, 249–59. For the original papers in which the theorem was proven, see Löwenheim, “On Possibilities in the Calculus of Relatives”; and Skolem, “Logico-Combinatorial Investigations in the Satisfiability or Provability of Mathematical Propositions.”

³⁸ The relation between Tarski’s theorem—and his work on truth more generally—and the Löwenheim-Skolem theorem will be considered, albeit only briefly, in Chapter 4.

³⁹ Badiou, *Being and Event*, 231.

Nevertheless, it might well be asked whether the defense just given does not in a rather different way problematize the supposed relation between Badiou and Tarski. Does the fact that Tarski's work on truth seems relevant only to a *part* of Badiou's work on truth suggest that these thinkers are indeed working on fundamentally distinct notions of truth—that, to put it another way, there is something equivocal about the word “truth” that only misleadingly suggests a kind of parallel or an element of relevance between the two thinkers? More directly, one might ask whether the fact that Badiou situates the notion of truth in relation specifically to the continuum hypothesis, while Tarski undertakes his own work on the notion of truth without so much as mentioning the continuum hypothesis, suggests that there is no real relevance between these two projects that would justify undertaking comparative study.

This possibility has been presented in the form of an objection to the commensurability between Tarski and Badiou, specifically by Etienne Balibar. It deserves independent attention.

Balibar on Badiou

So far as I have been able to find, there has been only one published attempt to make sense of the relationship between Badiou and Tarski: an essay, first published in 2002, written by Etienne Balibar.⁴⁰ Only two pages of the essay in question focus on specific points of contrast between Badiou's and Tarski's projects, but those two pages come in the course of a larger and more sustained argument aimed at situating Badiou's work on truth within the context of the French philosophical tradition. One of the purposes of that larger argument is to establish that Badiou's conception of truth, before

⁴⁰ It appeared in English in 2004. See Balibar, “The History of Truth.”

and apart from any direct investment in the relevance of set theoretical mathematics to that conception, is one he inherits from a very specific French school of epistemology—a school that Balibar takes to be so drastically distinct from that in which Tarski was situated as to make their projects largely irrelevant to each other. Because Badiou, according to Balibar, inherited his basic conception of truth from Georges Canguilhem—an important but often neglected mid-twentieth-century French thinker—the only relation that could hold between his project and that of someone like Tarski is one of overt contestation. Indeed, Balibar goes so far as to put his conclusion in starkly triumphalist terms: Badiou’s conception of truth “has a potentially devastating power that could destroy the defenses of so-called analytic philosophy, to the extent that it can still recognize itself in Tarski’s semantics: it is difficult to see how it could put up any resistance.”⁴¹ In light of such a claim, it is unsurprising that Balibar ultimately concludes that Badiou should be called “the anti-Tarski.”⁴²

One way of marking the profound difference Balibar finds between Badiou’s and Tarski’s respective conceptions of truth, independently of their attempts to capture those conceptions through the employment of set theory, is to note that where Tarski claims that his formalization of truth is effectively neutral with respect to various epistemological theories (“we may accept the semantic conception of truth without giving up any epistemological attitude we may have had; we may remain naïve realists, critical realists or idealists, empiricists or metaphysicians—whatever we were before”),⁴³ Badiou insists that truth cannot be entirely disentangled from epistemological considerations even as it is irreducible to the knowable (“everything is at stake in the thought of the

⁴¹ *Ibid.*, 33.

⁴² *Ibid.*

⁴³ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 362.

truth/knowledge couple”; “the key to the problem is the mode in which the procedure [by which truth is produced] traverses existent knowledge”).⁴⁴ On Balibar’s account, this difference between Tarski’s insistence on the epistemological neutrality of the semantic conception of truth and Badiou’s insistence that truth cannot be thought apart from its complex relationship to knowledge indicates their fundamentally distinct—and ultimately irreconcilable—beginning points.

More broadly, Balibar takes the French epistemological tradition from which Badiou hails to be fundamentally distinct from the epistemological tradition inherited by philosophers at work outside of France. Thus Balibar speaks of the “relative autonomy with respect to its international environment” that characterized the French philosophical scene “from the end of the 1950s to the beginning of the 1980s.”⁴⁵ Whatever its relationship with the outside world of philosophy, the French “logico-phenomenological, and logico-epistemological, debate” he presents as having taken its orientation (however consciously or unconsciously does not matter) from a phrase originally coined by Blaise Pascal: “the history of truth.”⁴⁶ According to Balibar, the philosophical conversation out of which Badiou effectively precipitated was dominated by the idea that truth as a philosophical category cannot be understood independently of or isolated from its entanglement within the established conditions of knowledge at some particular time and place, with the consequence that truth has a history. In other words, Badiou’s suspicion that epistemological neutrality is impossible when it comes to truth is something he inherited from a larger French epistemological school. Balibar asserts that it was

⁴⁴ Badiou, *Being and Event*, 327.

⁴⁵ Balibar, “The History of Truth,” 23.

⁴⁶ *Ibid.*, 21. See also Pascal, *The Provincial Letters, Pensées, Scientific Treatises*, 342. Badiou makes this Pascalian phrase the frontispiece of a chapter in *Being and Event*. See Badiou, *Being and Event*, 212.

Canguilhem especially who led this school, inspiring Badiou alongside other major French thinkers, especially Jacques Derrida and Michel Foucault.

In an interview conducted, significantly, by Badiou in 1965—decades before he would write his first substantial works on truth—Canguilhem himself explained the basic philosophical conviction to which Balibar refers here. In response to Badiou’s question, “Can we say that there is truth in the sciences?” Canguilhem responded, “In my sense, there is only one domain in which we can speak of truth, and this is science.”⁴⁷ This might seem at first to be a baldly positivist claim, but it must be understood in the context of Canguilhem’s philosophy. As he explained when Badiou went on to ask what he understood by truth, “scientific coherence is a coherence at the interior of certain conventions, or a coherence at the interior of certain rules of the determination of an object.”⁴⁸ At issue here is what Badiou called in a 1968 lecture Canguilhem’s “experimentalist conception of science,” according to which “the experimental ‘fact’ is itself an artifact,” a product of the scientific apparatus constructed in order to ascertain the fact.⁴⁹ According to Canguilhem, because scientific apparatuses have a history, and because truth is constituted directly by such scientific apparatuses, truth itself has a history.

Importantly, to put a finer point on Canguilhem’s conception of science in his own 1968 exposition of it, Badiou drew directly from unpublished lectures presented in Paris by, of all people, Etienne Balibar: “Balibar has shown that . . . the dialectic of science is thoroughly internal to a process of *production* of knowledge, and that this process is

⁴⁷ Tho and Bianco, *Badiou and the Philosophers*, 24, translation slightly modified.

⁴⁸ *Ibid.*, 25.

⁴⁹ Badiou, *The Concept of Model*, 15. For Canguilhem’s direct treatment of these themes, see Canguilhem, *A Vital Rationalist*, 23–157.

doubly articulated: once according to the *system* of concepts, and again according to the *inscription* of proof.”⁵⁰ In turn, in his more recent article dealing with Badiou and Tarski, Balibar cites Badiou’s own just-mentioned 1968 lectures in which Badiou draws on Balibar.⁵¹ In the mid-1960s, Badiou and Balibar worked side by side under Althusser’s leadership, Althusser himself being deeply and consciously influenced by Canguilhem in his systematic attempt to determine the scientific status of Karl Marx’s materialism.⁵² That Balibar rightly understands the original context from which Badiou launched his philosophical project—at least in its earliest form—thus seems beyond reproach. In setting forth the hypothesis that “Badiou is trying, or at least has tried, to develop a conception of the history of truth (or more specifically, to construct a concept of truth which is at the same time, and in an original manner, the concept of its history),” and that he is doing so or has done so in order, in some way, to improve on Canguilhem’s own approach—as well as on the approaches of others working in Canguilhem’s wake, such as Foucault and Derrida—Balibar knows whereof he speaks.⁵³ The close collusion between Badiou and Balibar in the 1960s makes this quite sure.

Far less sure, however, is Balibar’s claim that the French *uniquely* pursued such an epistemological project in the decades surrounding the 1960s. Given the fact that Alexandre Koyré, after working in France alongside Gaston Bachelard before the Second World War to lay the foundations of the project that Canguilhem would then assume,

⁵⁰ *Ibid.* The lectures to which Badiou referred were delivered between 1967 and 1968 as part of a series, sponsored by Louis Althusser, on philosophy and science. Badiou also contributed to the series, his lectures eventually being published as *The Concept of Model*. For more about the lecture series, see Althusser, “*Philosophy and the Spontaneous Philosophy of the Scientists*” and *Other Essays*, 71–72.

⁵¹ See Balibar, “The History of Truth,” 30.

⁵² See especially Althusser and Balibar, *Reading Capital*, 13–75, a text that must be set alongside Canguilhem, *A Vital Rationalist*, 35–40. To see the close working relationship that linked Badiou and Balibar to Althusser, see Althusser, “*The Humanist Controversy*” and *Other Writings*, 33–37.

⁵³ Balibar, “The History of Truth,” 28. In the course of the essay, Balibar marks specific differences among Badiou’s, Michel Foucault’s, and Jacques Derrida’s respective ways of inheriting Canguilhem’s project.

relocated to the United States where he published in English his widely influential *From the Closed World to the Infinite Universe*, it seems strange to speak of any supposed “relative autonomy” for the French conversation concerning truth and knowledge.⁵⁴ Moreover, however well-policed the borders surrounding the French conversation may or may not have been, one might justifiably wonder why Balibar says nothing of Thomas Kuhn, whose *The Structure of Scientific Revolutions*, published originally in 1962 (its important “Postscript” was published in a second edition in 1969), set forth a conception of scientific practice with striking affinities to that worked out by thinkers like Bachelard and Canguilhem.⁵⁵ This is of course not the place to undertake a comparative study of Canguilhem and Kuhn, but it is worth noting that, whatever the status of the autonomy of the French epistemological tradition in the decades surrounding the 1960s, strikingly similar projects in scientific epistemology appeared elsewhere in the very years in which Canguilhem’s work came to its full maturity.

Still more objectionable than the claim of relative autonomy for French epistemology is the subtle suggestion in Balibar’s claim that thinkers outside of France employed a largely static and therefore essentially naïve conception of truth’s relationship to epistemology. It would seem that Balibar is largely unaware of post-Carnapian—and even late-Carnapian—developments in analytic philosophy of science.⁵⁶ That is, to problematize Balibar’s intimation, one might point not only to Kuhn, already mentioned,

⁵⁴ See, of course, Koyré, *From the Closed World to the Infinite Universe*. It seems particularly strange that Balibar never mentions either Koyré or Bachelard in his discussion of Badiou’s situatedness in the French epistemological tradition. A review of Badiou’s writings in the 1960s makes clear that, like his mentor Althusser, he was arguably more directly influenced by Bachelard than by Canguilhem. It was, at any rate, Bachelard’s notion of “non-Cartesian epistemology” that gave shape to so much of the French conversation to which Balibar refers. See Bachelard, *The New Scientific Spirit*, 135–77.

⁵⁵ It should be noted that Kuhn specifically mentions having read Koyré as he began to develop his own work. See Kuhn, *The Structure of Scientific Revolutions*, vii–viii.

⁵⁶ Of course, it should be noted that, at least in the 1960s, Badiou was himself apparently unaware of such developments in analytic philosophy of science. See, for instance, Badiou, *The Concept of Model*, 5–8.

but also to Rudolf Carnap's late essay, "Empiricism, Semantics, and Ontology," originally published in 1950.⁵⁷ Better yet, one might point directly to Alfred Tarski himself! In his 1944 essay on truth, his most-widely read and cited such essay, Tarski both affirms the epistemological neutrality of his formalizations *and* asks about how direct application of his semantic conception of truth to the philosophy of science might raise important epistemological questions. In a brief discussion of how the truth of sentences might relate to the acceptability of theories, Tarski argues that his semantic conception of truth might help to clarify the manner in which theories change over time.⁵⁸ Far from exhibiting complete naiveté about the complex entanglement of knowledge and truth in theory change, as Balibar suggests, Tarski demonstrates a sophisticated knowledge of at least the basic issues then under discussion in the French conversation, whether or not he was aware of that particular conversation itself.⁵⁹

There are, then, reasons to think that Balibar has profoundly misconstrued the potential relationship between Tarski and Badiou through his problematic insistence that Badiou was reared in a philosophical climate largely—if not entirely—cut off from and incommensurable with that in which Tarski undertook his work on truth. Indeed, the fact that Badiou read Tarski's work on truth in the early 1980s (and likely earlier also) and for a time believed that work to be essential to his own project makes perfectly clear that the

⁵⁷ See Carnap, *Meaning and Necessity*, 205–21. On the relationship between Kuhn and Carnap, see Earman, "Carnap, Kuhn, and the Philosophy of Scientific Methodology."

⁵⁸ See Tarski, "The Semantic Conception of Truth and the Foundations of Semantics," 366–67. I will provide some analysis of this passage in Chapter 4.

⁵⁹ According to an argument I have presented elsewhere (but which remains unpublished), Tarski's brief discussion of theory change in "The Semantic Conception of Truth and the Foundations of Semantics" has more in common with Kuhn's subsequent approach than it does with Karl Popper's approach in *The Logic of Scientific Discovery*, then already a decade old. This is perhaps unsurprising, given the close relationship between Tarski and Carnap. For a helpful account of that relationship, see Coffa, *The Semantic Tradition from Kant to Carnap*, 272–305.

two projects are in some way relevant to one another.⁶⁰

Of course, Balibar's misconstrual of the relationship between the French philosophical scene and the remainder of the Western philosophical conversation in the second half of the twentieth century does not invalidate the substance of his objection. It simply means that he overstates his point, transforming into a claim of incommensurability what might more simply be a point of philosophically interesting difference. Whatever Balibar's failings, at any rate, it must still be asked whether Badiou and Tarski mean to address the same phenomenon when they speak of truth. In the end, I think it is clear that the answer to this question is negative. Badiou and Tarski assume quite distinct pre-theoretic notions of truth in their work, even though they both assign the thinking of truth to the same ontological resources (those of model theory). The virtue of comparative study is that by beginning from a network of similarities (given such a

⁶⁰ In addition to his general analysis of the relationship between Badiou and Tarski, Balibar makes three specific claims about their radical incommensurability. These deserve at least brief mention here, if only in an extended footnote. According to the first point, Tarski understands the basic algebraic operation of satisfaction to be semantic, such that it can be used to construct an adequate definition of truth (for particular languages), while Badiou takes satisfaction to be set theoretical rather than algebraic in nature, stripping it of its supposed semantic character in order to position it at the foundation of the set theoretical enterprise. According to Balibar, this difference allows Badiou to clarify the nature of satisfaction in a way Tarski cannot, organizing its operability according to the ensemble of axioms that govern set theory—axioms that, investigated closely, ground further formal operations that provide a formal schema of truth. Balibar, "The History of Truth," 31. It is not clear to me what Balibar means in making this claim. Presumably, he means to suggest that what Tarski calls satisfaction Badiou regards as the basic set theoretical operation of belonging. At least, I cannot imagine what else Balibar *could* mean. But it makes little sense to suggest that satisfaction and belonging are the same operation, severally construed as algebraic and set theoretical in nature. The second point of incommensurability Balibar identifies is this: Tarski marks both the extrinsic and the intrinsic limitations of truth by excusing natural languages from the need for a formalizable notion of truth and sharply delineating the boundary between the syntactic and the semantic (or between the formal equivalents of concept and intuition), while Badiou insists on fully accounting for the application of truth to natural languages and on troubling any sharp distinction between syntax and semantics through his interest in the decisional operations on offer in set theory (such as the axiom of choice and the operation of forcing). *Ibid.* Although I will agree in Chapter 3 that decision is in important ways central to Badiou's construal of truth, it is not clear from his way of framing the issue that Balibar has adequately grasped the relevant concepts. I therefore leave his critique aside. Finally, the third point of incommensurability: Tarski makes "only an instrumental and weak use of set theory," undertaken after a "watering down of ontology into logical semantics," while Badiou explores the possibility of making "an intrinsic and strong use" of set theory, seeing in it the unmediated science of being. *Ibid.* As my establishment of the ontological ramifications of Tarski's work, clarified both in Chapter 2 and again—more deeply—in Chapter 4, makes clear, this point seems especially misguided.

network of compelling similarities can be established), it allows for the philosophical elucidation of points of real difference. Rather than simply asserting that two thinkers bring incommensurable perspectives to the table, one can determine whether there is a shared site from which philosophical debate might take its basic orientation. My work on Tarski and Badiou over the next several chapters reveals quite clearly, I think, that there is much more shared by these two thinkers than Balibar recognizes. Full elucidation of this point can, of course, only come after the interpretive work of Chapters 2 and 3 has been accomplished and once the shared ontology of Tarski and Badiou has been elucidated in Chapter 4.

Obviously, it is high time to turn directly to the work of engaging with Tarski and Badiou.

TARSKI'S PROJECT

My first task in this chapter is just to give a basic exposition of Alfred Tarski's attempt to provide (in however limited a way) an account of truth by drawing on the resources of set theoretical—and, at least in a preliminary way, model theoretical—mathematics. It is worth noting from the outset that, some eighty years after its first presentation, Tarski's approach to the question of truth remains today the basic starting point for most discussions of the topic in the analytic tradition.¹ Because my focus here is on the use of set theory and model theory in attempting to get clear about truth, I will naturally give my attention first and foremost to the formal apparatus employed in Tarski's proposal. It will, however, be necessary along the way to draw also on Tarski's several less technical discussions of truth, since they help to clarify the philosophical stakes of the recourse he had to things mathematical.

From the beginning it is necessary to note that Tarski's attempt to provide an account of truth was part of a larger attempt to lay the foundations of what he called “scientific semantics,” that is, of a scientifically rigorous account of those notions that pertained to “relations between words and things.”² That this was the larger aim of his work on truth he especially made clear in his most widely read paper on the subject, written a decade and a half after his initial publications on the subject, to which he gave

¹ Tarski's earliest work on truth was, by his own account, undertaken in 1929 and first presented in public lectures in 1930 and 1931. His first publication on the subject dates to 1931.

² Burgess and Burgess, *Truth*, 17. Tarski himself provided a fuller definition of the term “semantics”: “We shall understand by semantics the totality of considerations concerning those concepts which, roughly speaking, express certain connections between the expressions of a language and the objects and states of affairs referred to by these expressions.” Tarski, “The Establishment of Scientific Semantics,” 401.

the telling title, “The Semantic Conception of Truth and the Foundations of Semantics.” But already in his earliest and most important paper on truth from 1931—“The Concept of Truth in Formalized Languages”—he was, like Kurt Gödel at about the same time, working his way toward a clear delineation of syntax and semantics through a close investigation of the latter.³ Indeed, Tarski was among the first to recognize that some kind of relationship could be established between what had to that point developed as two largely independent approaches to logic, one growing out of George Boole’s work and roughly semantic in orientation, the other growing out of Gottlob Frege’s work and essentially syntactic in nature.⁴ Tarski’s work on semantics was successful and influential. He convinced an initially reluctant Rudolf Carnap of the possibility of providing a fully scientific account of semantics, such that Carnap went on to write a full textbook on the subject, as well as to make it a central part of his most influential writings.⁵ It will prove helpful to keep Tarski’s original larger intentions in view throughout the following exposition, particularly in light of the importance Tarski’s semantic commitments will play in the analyses of Chapters 3 and 4. It was this larger project that Tarski pursued throughout his career, even in his more strictly logical and mathematical work, where his semantic interests took the shape, eventually, of model theory.⁶

³ See the summary conclusions (in the form of final theorems) in Tarski, “The Concept of Truth in Formalized Languages,” 266, 273–74.

⁴ See Moore, *Zermelo’s Axiom of Choice*, 250–51.

⁵ See Carnap, *Introduction to Semantics*; and Carnap, *Meaning and Necessity*. For a good account of this influence, see Coffa, *The Semantic Tradition from Kant to Carnap*, 272–305.

⁶ Tarski saw his work on truth as lending itself to work on other philosophical problems as well. In his earliest work on the subject, he stated his “hope” that his work would be of value to “the student of the theory of knowledge above all.” Tarski, “The Concept of Truth in Formalized Languages,” 267. In his later, more philosophical treatment of the subject, he defended at length the applicability of his work to the empirical sciences, to the methodology of the empirical sciences, and to the deductive sciences—some of which has been discussed already in Chapter 1 and will become central to the discussion in Chapter 4. See

Over the course of this chapter, I will argue that set theory plays two chief roles in Tarski's project.⁷ First, it was in working on set theory that he produced the theorem that bears his name, a theorem that directly led him to believe that a semantic (rather than, say, a syntactic) conception of truth was the right one to pursue, and that therefore led him to lay much of the foundational work for what would later become model theory. This consideration is directly—but subtly—connected to the first of the two guiding criteria Tarski takes as essential to his project of providing a satisfactory definition of truth: that of material adequacy. As I will make preliminarily clear in this chapter, but much clearer in the first part of Chapter 4, this first use of set theory in Tarski's project is ontological at its root—ontological in a minimalist sense that will have to be clarified. Second, then, Tarski takes as his task in attempting to clarify the notion of truth to account first and foremost for the truth predicate—specifically, for particular truth predicates constructible for particular formal languages (a limitation demanded by his adherence to a second guiding criterion: that of formal correctness). Because truth in the form of a predicate, once it is constructed in such a way as to avoid paradox or contradiction, picks out an extension or designates a specific class or set, the resources of set theory proved helpful for Tarski's attempt to produce an adequate definition of truth. Of course, there is nothing surprising about this—though it does have certain important implications for philosophical method, characteristic of so much early analytic philosophy. Much more important than the mere fact that Tarski defines truths as sets is the choice he makes in determining the actual formula by which such sets are designated.

Tarski, "The Semantic Conception of Truth and the Foundations of Semantics," 364–69.

⁷ For a Tarskian exposition of the basics of set theory, see Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 68–86. In his early work on truth (before he had written his own textbook on mathematical logic), Tarski refers his readers to the discussions in Whitehead and Russell, *Principia Mathematica*.

This decision, as will be seen, is more a matter, again, of material adequacy than of any other consideration, but its rootedness in set theoretical considerations is undeniable.

The discussion that follows is divided into several parts. In a first section, I provide a brief exposition of the first of Tarski's two main criteria for a satisfactory definition of truth: that of material adequacy. His care in tracking what this criterion required of him led him to insist on providing a specifically semantic definition of truth, a decision that grew out of the first major use to which he put set theory in his work on truth. In a second section, I turn my attention to the second of Tarski's two guiding criteria: that of formal correctness. This criterion led Tarski to exchange the attempt to define truth in general for the program of defining truth for particular formalized languages and drove his insistence that truth could be made the subject of scientific study only if a sharp distinction was maintained between the language in which are found the true sentences whose truth is to be defined and the metalanguage in which the truth of those true sentences is actually defined. Tarski saw, however, that the full formulation of such a definition would have to pass through some technical difficulties, difficulties which Tarski overcame through his careful arrangement of the relationship between truth and satisfaction, which I address in a third and final section.

Material Adequacy

From the outset of his work on truth, Tarski identifies as his chief problem the task of constructing "a materially adequate and formally correct definition of the term 'true sentence.'"⁸ The two criteria of a satisfactory definition provided by this slogan—

⁸ Tarski, "The Concept of Truth in Formalized Languages," 152. Cf. Tarski, "The Semantic Conception of Truth and the Foundations of Semantics," 341: "The main problem is that of giving a satisfactory definition

namely, material adequacy and formal correctness—are crucial to Tarski’s project. Indeed, together they mark the two major differences between his approach to truth and that undertaken by others in the analytic tradition before World War II.⁹ Moreover, as will be seen, they played a central role in determining the direction Tarski took in drawing on the resources of set theoretical mathematics. These criteria, therefore, deserve careful attention. In this first section, I will give my attention only to the first of them (material adequacy), allowing the second (formal correctness) to join the first only in later sections.

The general definition of material adequacy employed in Tarski’s work on truth appears in a number of places in his larger corpus. In a paper from 1930, in connection with an argument concerning definability, Tarski says the following regarding material adequacy: “Now the question arises whether the definitions just constructed (the formal rigor of which raises no objection) are also adequate materially; in other words do they in fact grasp the current meaning of the notion as it is known intuitively?”¹⁰ Here material adequacy is taken to be a certain conformability of formally constructed definitions to pre-reflective intuition. Importantly, this same general conception of material adequacy is echoed in Tarski’s widely-read essay on truth from 1944:

In order to avoid any ambiguity, we must first specify the conditions under which the definition of truth will be considered adequate from the material point of view. The desired definition does not aim to specify the meaning of a familiar word used to denote a novel notion; on the contrary, it aims to

of this notion [truth], i.e., a definition which is materially adequate and formally correct.”

⁹ See the discussions in Sluga, “Truth Before Tarski”; and Diamond, “Truth Before Tarski.” To get a sense in general for how unique Tarski’s approach to truth was to the standard assessments of it in early analytic philosophy, it might be fruitfully compared with an analysis like that in Russell, *The Problems of Philosophy*, 119–30.

¹⁰ Tarski, “On Definable Sets of Real Numbers,” 128–29.

catch hold of the actual meaning of an old notion. We must then characterize this notion precisely enough to enable anyone to determine whether the definition actually fulfills its task.¹¹

As articulated in these quotations, material adequacy might be taken to be a rather straightforward criterion, but it is on display in this form in Tarski's work on truth from the beginning. In his earliest essay on the subject, from 1931, he states as his purpose to grasp "the intentions which are contained in the so-called *classical* conception of truth ('true—corresponding with reality')." ¹²

It should be noted that Tarski was for important reasons increasingly careful, as his work on truth matured, about how he formulated the pre-reflective notion of truth. In his 1944 essay on truth, he remarks that talk of correspondence—as in the quotation just above—might be taken to imply that he means his formalization of truth to capture a very specific epistemological theory (one that, incidentally, Gottlob Frege had already crucially problematized before Tarski undertook his work on truth).¹³ Tarski notes that overly loose formulations, interpreted too strictly, "can lead to various misunderstandings, for none of them is sufficiently precise and clear."¹⁴ Elsewhere in the same essay, he makes fully clear his insistence—already discussed briefly in Chapter 1 above—that his work on truth is epistemologically neutral. His words on this point are worth quoting again: "We may accept the semantic conception of truth without giving up any epistemological attitude we may have had; we may remain naïve realists, critical realists or idealists, empiricists or metaphysicians—whatever we were before. The

¹¹ Tarski, "The Semantic Conception of Truth and the Foundations of Semantics," 341.

¹² Tarski, "The Concept of Truth in Formalized Languages," 153.

¹³ See Frege, "Thoughts," 352–53.

¹⁴ Tarski, "The Semantic Conception of Truth and the Foundations of Semantics," 343.

semantic conception is completely neutral toward all these issues.”¹⁵ Whatever superficial similarities the intuitive or pre-reflective notion of truth Tarski mentions in his work might bear to the specific epistemological “correspondence theory of truth,” it works against Tarski’s explicit purposes to assume any genuine connection between these two. From 1931 onward, Tarski expresses as his intention to have his conception of truth conform simply to “the well-known words of Aristotle: ‘To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true.’”¹⁶ One must be careful not to read more into such formulations than Tarski intends.

In order to satisfy the general criterion of material adequacy in his work on truth, Tarski proposes a schema (his famous “T-schema”) meant to capture the pre-reflective or intuitive notion of truth he draws from Aristotle: “x is a true sentence if and only if p,” where “p” marks the state of affairs asserted to be the case in the sentence “x.” (The linguistic version of the same formal scheme, which is not without its own problems, appears in Tarski’s work as: “a true sentence is one which says that the state of affairs is so and so, and the state of affairs indeed is so and so.”)¹⁷ Each instance of this schema—each “T-sentence”—forms a “partial definition” of truth, since it states the conditions under which *some* sentence (substituted for “x”) can be said to be *true* (namely, when whatever is substituted for “p” is the case).¹⁸ A stricter application of the criterion of

¹⁵ *Ibid.*, 362.

¹⁶ Tarski, “The Concept of Truth in Formalized Languages,” 155. For the original context of Aristotle’s words, see Aristotle, *Metaphysics*, Γ.7 1011b26–29; McKeon, *The Basic Works of Aristotle*, 749. Aristotle introduces his definition in the course of clarifying the nature of the contradictory, claiming that the true and the false present the exemplary contradictory pair, without intermediate.

¹⁷ *Ibid.* Donald Davidson notes the danger of Tarski’s occasional references to states of affairs. I will say more about this further along.

¹⁸ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 344. See also Tarski, “The Concept of Truth in Formalized Languages,” 187.

material adequacy can then be specified in terms of whether a *complete* definition of truth brings together—that is, implies—all relevant *partial* definitions of truth. Tarski puts this point quite clearly in his 1944 paper on truth:

Now at last we are able to put into a precise form the conditions under which we will consider the usage and the definition of the term “*true*” as adequate from the material point of view: we wish to use the term “*true*” in such a way that all equivalences of the form (T) [that is, all T-sentences] can be asserted, and we shall call a definition of truth “adequate” if all these equivalences follow from it.¹⁹

Still clearer is this further formulation, to be found in the same paper: “The definition proves to be . . . materially adequate” if “it implies all equivalences of the form (T).”²⁰ In short, a complete definition of truth is, for Tarski, materially adequate if it “include[s] all partial definitions . . . as special cases”—if it is, “so to speak, their logical product” (that is, the logical conjunction of all the partial definitions).²¹

The importance of ensuring that formal definitions bear some semblance to pre-reflective or intuitive notions seems perfectly obvious. And the manner in which this particular guiding criterion suggests the necessity of pursuing a semantic definition *perhaps* seems equally obvious—at the very least since the reigning notion of truth would seem indeed to be a question of *some* relation between words and things.²² But the way in which Tarski sees material adequacy, particularly in its stricter application specified just

¹⁹ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 344.

²⁰ *Ibid.*, 353.

²¹ Tarski, “The Concept of Truth in Formalized Languages,” 187. See also Tarski, “The Establishment of Scientific Semantics,” 404.

²² As Arne Naess’s work already showed in the 1930s, however, philosophers are more prone to assume popular consensus regarding the nature of truth than actual data warrant. See Naess, “*Truth*” *As Conceived by Those who are Not Professional Philosophers*.

above, as demanding the pursuit of a semantic conception of truth runs still deeper in his work. Understanding this point, moreover, is necessary to see the first way in which set theory plays an important role in Tarski's work on truth. It concerns, crucially, what has come to be known as Tarski's theorem—discussed in Chapter 1 in connection with Alain Badiou's work on this and related theorems in the early 1980s. Understanding the role this theorem played in Tarski's thinking already in the late 1920s and early 1930s makes clear that Tarski's strict adherence to the criterion of material adequacy was for him less a matter of following a rather obvious and intuitive requirement, and more of making his work on the semantic conception of truth into a radical departure from the work of his predecessors and contemporaries.

By his own account, Tarski proved his theorem for the first time in the late 1920s and first announced his conclusions in abbreviated form in a presentation in late 1930—that is, shortly before Kurt Gödel first presented and published his similar findings.²³ It was not until later, however, that Tarski actually published his proof of the theorem.²⁴ Although he confessed that Gödel developed the relevant methods “more completely” than he himself had done, he had already in 1931 independently produced the “method of arithmetizing the metalanguage,” the method for which Gödel became rightly famous and which makes the difference between formal languages that produce undecidable statements and those that do not.²⁵ (Tarski provides in his early paper on truth a helpful summary of how this method works, encapsulated in the following words: “It can easily be shown that the axiom system adopted in the metatheory possesses an interpretation in

²³ See Tarski, “The Concept of Truth in Formalized Languages,” 277–78. Regarding Tarski's awareness of Gödel's results, see Feferman and Feferman, *Alfred Tarski*, 81–85.

²⁴ See Tarski, “On Undecidable Statements in Enlarged Systems of Logic and the Concept of Truth.”

²⁵ Tarski, “The Concept of Truth in Formalized Languages,” 277–78.

the arithmetic of the natural numbers. A one-one correspondence can be set up between expressions and natural numbers where operations on numbers having the same formal properties are correlated with the operations on expressions.”)²⁶ But whatever the similarities between Tarski’s theorem and Gödel’s famous theorems, at least one important difference distinguished their respective acts of discovery. Where Gödel rightly saw as the most immediate implication of his work certain consequences for the project of establishing the completeness of the axiomatic system intended to ground arithmetic, Tarski saw as the most immediate implication of his work a certain non-equivalence between two concepts that had been taken, before him, to be equivalent: those of truth and provability.²⁷ In other words, Tarski immediately discerned the implications of his theorem (and, by extension, of Gödel’s theorems) for any subsequent attempt to understand the notion of *truth*.

Tarski’s summary of his theorem in his 1931 essay on truth is somewhat surprising. He presents it in terms of the validity of the law of excluded middle:

It might appear at first sight that . . . this problem [of constructing a definition of “true sentence”] can be solved without further difficulty, that “true sentence” with respect to the language of a formalized deductive science means nothing other than “provable theorem” Closer

²⁶ *Ibid.*, 184.

²⁷ Only a very-brief remark in Gödel’s first paper on the subject noted the connection with truth: “From the remark that $[R(q); q]$ says about itself that it is not provable it follows at once that $[R(q); q]$ is true, for $[R(q); q]$ is indeed unprovable (being undecidable).” Gödel, “On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems I,” 599. For some commentary on this point, see Berto, *There’s Something about Gödel*, 153–57. It should also be noted that the non-equivalence of truth and provability does *not* hold for extremely simple languages, such as the language of “the ordinary sentential calculus” of elementary logic, “enlarged by the introduction of the universal and existential quantifiers.” Tarski, “The Concept of Truth in Formalized Languages,” 221–22. Cf. Gödel, “The Completeness of the Axioms of the Functional Calculus of Logic.” For Tarski’s exposition of the sentential calculus, in no way unconventional, see Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 3–53.

reflection shows, however, that this view must be rejected for the following reason: no definition of true sentence which is in agreement with the ordinary usage of language should have any consequences which contradict the principle of the excluded middle. This principle, however, is not valid in the domain of provable sentences. A simple example of two mutually contradictory sentences (i.e. such that one is the negation of the other) neither of which is provable is provided by Lemma E below [Lemma E is a formal equivalent of the sentence, “This sentence is false,” constructed by Tarski by using the resources of arithmetization]. The extension of the two concepts [of truth and provability] is thus not identical.²⁸

It seems odd at first that in his presentation of his theorem, Tarski takes as equivalent the existence of an undecidable statement (some sentence neither the affirmation nor the negation of which can be proven) and the invalidity of the law of excluded middle for the domain of provable sentences. Yet what he means by this gesture is that any reduction of the law of excluded middle to the idea that either any given sentence that can be constructed through the resources of a given formal system or the negation of that same sentence is *provable* would render that law invalid. The universal validity of the law of excluded middle rests instead on its assertion that either any such sentence or its negation is *true*. There are thus sentences that cannot be proven within a given formal system of sufficient complexity that, given the truth of the law of excluded middle, must

²⁸ Tarski, “The Concept of Truth in Formalized Languages,” 186. Lemma E can be found at *ibid.*, 199. A full exposition of Tarski’s Theorem can be found in Smullyan, *Gödel’s Incompleteness Theorems*, 14–27.

nonetheless be true.²⁹ Because “the class of true sentences contains all provable sentences of the science investigated” even as the classes of true sentences and of provable sentences are not equivalent, the class of provable sentences is a proper subset of the set of true sentences.³⁰

Key to Tarski’s Theorem is the seriousness with which he takes the implications, for founding a scientific semantics, of paradoxical statements like the classic “liar”: “This sentence is false.” Tarski notes in his work a certain tendency among philosophers to treat such sentences—because their truth directly entails their falsity, just as their falsity directly entails their truth—“as jokes or sophistries” and so not to grant them serious philosophical weight.³¹ For his own part, however, Tarski suggests that to make such a dismissive move is “quite wrong and dangerous from the standpoint of scientific progress It is a fact that we are here in the presence of an absurdity.”³² Basically, Tarski’s Theorem draws a general conclusion about the non-equivalence of truth and provability for any language in which sentences like the “liar” can be constructed. Any formal—or, for that matter, non-formal—language that allows statements to refer to themselves (or to refer to other statements that in turn can refer back to the first) allows for the construction of paradoxical sentences like the “liar.” This is the case with any formal language in which it is possible to arithmetize—that is, to assign natural numbers one-to-one to—the terms of that language, as both Tarski and Gödel recognized is the case with basic set

²⁹ Tarski states this finding also in Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 354. See also Tarski, “On Undecidable Statements in Enlarged Systems of Logic and the Concept of Truth.” Note also that Tarski uses the same approach—via the law of excluded middle—in his introductory treatment in Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 134–36.

³⁰ Tarski, “The Concept of Truth in Formalized Languages,” 236. For the definition of “proper subclass,” see Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 74.

³¹ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 348.

³² *Ibid.*

theory. In light of all this, what Tarski says about his findings can be fully clarified.

Tarski's Theorem effectively states that the class of provable statements for any language in which a sentence like the "liar" can be constructed cannot be equivalent to the class of true statements, since (1) the exclusion from the class of provable statements of both the affirmation *and* the negation of the "liar" means that the law of excluded middle does not hold for the class of provable statements, but (2) the law of excluded middle *does* hold for the class of true statements.

What this finding has to do with the criterion of material adequacy is obvious. In light of his theorem, Tarski saw that, if a definition of truth for any formalized language of significant strength were to be materially adequate (that is, were to cover *all* true sentences of a given language), it would be necessary to provide a definition of "true sentence" that would cover all *provable* sentences as well as some "sentences which are *not provable*."³³ It was only in the light of this realization that Tarski recognized the impossibility of providing a merely *syntactic* definition of truth—what at the time, given the then-still-nascent distinction between syntax and semantics, Tarski called a merely "structural" definition of truth.³⁴ The syntax of a given language (that is, the basic rules for correctly concatenating marks and combining well-formed concatenations) provides all but only the resources necessary to define provability. But because, as Tarski showed, "the true" and "the provable" designate non-equivalent classes for certain languages (those, namely, in which sentences like the "liar" can be constructed), and because the

³³ Tarski, "The Concept of Truth in Formalized Languages," 186, emphasis added.

³⁴ See *ibid.*, 163: "I will draw attention here to only *one* such attempt, namely the attempt to construct a *structural definition*. The general scheme of this definition would be somewhat as follows: *a true sentence is a sentence which possesses such and such structural properties* (i.e. properties concerning the form and order of succession of the individual parts of the expression) *or which can be obtained from such and such structurally described expressions by means of such and such structural transformations.*"

class of the true for such languages contains every sentence contained in the class of the provable as well as some other sentences, something extra-syntactic is required to construct a formula that designates the class of the true for such languages. (At least, all this is the case if the law of excluded middle is not to be given up.) In Tarskian terms, for a definition of truth for such languages to be fully adequate from a material point of view, it would therefore have to be semantic in nature.

Often in his writings, as already indicated earlier in this chapter, Tarski states that the basic nature of the semantic concerns a relationship between words and things. In what may be regarded as his canonical statement, he says the following: “We shall understand by semantics the totality of considerations concerning those concepts which, roughly speaking, express certain connections between the expressions of a language and the objects and states of affairs referred to by these expressions.”³⁵ Given this definition, Tarski understands the upshot of his theorem to be that progress can be made on defining truth only if one begins from a recognition that it puts into some kind of relation words (“the expressions of a language”) and things (“the objects and states of affairs referred to by these expressions”), and this—again—is meant to be captured by the T-schema already introduced above. This seems at first to have rather strong ontological and epistemological implications, against the warning, noted before, that Tarski himself issued against controverting the fundamental metaphysical and epistemological neutrality of his formulations. One must be extremely careful at this juncture. Donald Davidson has argued quite forcefully that “one of the strongest arguments for Tarski’s definitions is that in them *nothing* plays the role of facts or states of affairs.”³⁶ Davidson’s convincing

³⁵ Tarski, “The Establishment of Scientific Semantics,” 401.

³⁶ Davidson, “The Folly of Trying to Define Truth,” 22–23.

argument cannot be broached as yet, however, since it depends on the further elaboration of Tarski's formal machinery. Nonetheless, it serves as an early warning against drawing overly hasty conclusions about the ontological implications of Tarski's decision in favor of a semantic approach to truth. An adequate formal definition of truth can only be constructed if it rightly represents the correlation or correspondence between sentences and their truth-conditions, but what is at stake in that correlation or correspondence for Tarski cannot be made fully clear without looking further into Tarski's work.

The first role played by set theory in Tarski's work on truth is nonetheless already clear. His at first apparently unrelated researches into questions of the completeness and consistency of complex deductive systems, especially that of set theory, determined in important ways his overarching approach to the concept of truth. As will become clear in Chapter 4, this development has important ramifications for his relationship to Badiou's work.

The Definition of Truth

Naturally, Tarski's commitment to the semantic approach to truth introduced real complications into his project. Those complications, however, were compounded by his commitment to a second guiding criterion: that of formal correctness. The satisfaction of this criterion he understood, reflecting the orthodoxy among early twentieth-century Polish logicians, to depend on two conditions that together ensure that the introduction of a definition into the language of a formal system will not lead to any inconsistency not already implicit in the system's basic axioms: the conditions of non-creativity ("we cannot permit a formula S introducing a new symbol to make possible the derivation of

some previously unprovable theorem stated wholly in terms of primitive and previously defined symbols”) and eliminability (“any definition introducing a new symbol may be used to eliminate all subsequent meaningful occurrences of the new symbols”).³⁷ These two conditions amount to a simple joint requirement: that definitions must be genuine abbreviations of already-existent terminology. A naïve approach to providing a semantic definition of truth—in other words, one that takes as its task to provide a semantic definition of truth *in general*, for natural (or colloquial) language—cannot satisfy this requirement, as Tarski demonstrates at the outset of his work on truth. “A characteristic feature of colloquial language . . . is its universality,” he explains,

but it is presumably just this universality of everyday language which is the primary source of all semantical antinomies, like the antinomies of the liar or of heterological words. These antinomies seem to provide a proof that every language which is universal . . . , and for which the normal laws of logic hold, must be inconsistent.³⁸

Wherever (1) a sentence and a definite name of that sentence both belong to the same language and (2) semantic properties are taken to exist for well-formed expressions of the language in question, an inconsistency can be easily produced.³⁹ In effect, because natural languages, due to what Tarski calls their universality, allow expressions to be self-reflexive, paradoxes and antinomies result—as in the classic example of the “liar”: “This

³⁷ Suppes, *Introduction to Logic*, 153–54. Tarski clearly embraced these criteria established before him in the Polish logical school by Leśniewski. See Tarski, “Some Methodological Investigations on the Definability of Concepts,” 307. For formal definitions of “system” and “consistency” as used here, see Tarski, “The Concept of Truth in Formalized Languages,” 185.

³⁸ Tarski, “The Concept of Truth in Formalized Languages,” 164–65. Tarski’s analysis of this point reflects that famously provided in Russell, “Mathematical Logic as Based on the Theory of Types,” 59–75. Cf. Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 73–74; and Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 348.

³⁹ See Tarski, “The Concept of Truth in Formalized Languages,” 165, for the explicit statement of this dilemma. Cf. also Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 348.

sentence is false.”⁴⁰

I mentioned before that Tarski’s work on truth marked a departure from most of his contemporaries in two ways, the first being that he took problems of inconsistency to be quite serious. The second aspect of his project that strongly distinguished him from others was his conviction that the problems of inconsistency could be overcome. As for the seriousness of the semantical paradoxes, I have already quoted above Tarski’s claim that “it would be quite wrong and dangerous from the standpoint of scientific progress to depreciate the importance of [the ‘liar’] and other antinomies, and to treat them as jokes or sophistries. It is a fact that we are here in the presence of an absurdity.”⁴¹ Tarski was not entirely alone in *this* conviction, but he *was* more or less entirely alone in both believing that the problem posed by such paradoxes specifically for the concept of truth could be overcome and providing a viable solution. To solve the difficulty, Tarski simply turned his attention to what had already been done to forestall certain structurally similar paradoxes in the formalization of set theory, first set forth in outline by Bertrand Russell in his “theory of types.”⁴² In this way, Tarski discovered a way of achieving formal correctness while providing definitions of semantic concepts for a language that were entirely adequate from a material point of view.

Because the semantic paradoxes can be produced in every language where sentences and their names are both to be found (and where semantic concepts are

⁴⁰ See Tarski, “The Concept of Truth in Formalized Languages,” 157–58.

⁴¹ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 348.

⁴² See, again, Russell, “Mathematical Logic as Based on the Theory of Types.” It is to be noted that Frank P. Ramsey early on influentially distinguished between semantic paradoxes and the structurally similar paradoxes associated with the formalization of set theory—strictly logical or mathematical in nature. As Graham Priest argues, however, “Ramsey’s criterion [of distinction] has the flimsiest basis,” depending solely on “the relatively superficial fact of what vocabulary is used in the paradoxes” when what matters is “the *structure* of the different paradoxes.” Priest, *Beyond the Limits of Thought*, 142, emphasis in original. For Ramsey’s discussion, see Ramsey, “The Foundations of Mathematics.”

operative), and because this is presumably true of every natural language,⁴³ it is impossible to define truth in a satisfactory way for natural languages.⁴⁴ Tarski saw, however, that it *is* possible to construct correct definitions for semantic concepts—and therefore for truth—for particular formalized languages.⁴⁵ This is because, as Tarski explains, “most of these languages possess no terms belonging to the theory of language, i.e., no expressions which denote signs and expressions of the same or another language or which describe the structural connections between them.”⁴⁶ In other words, because formalized languages can be built so as to lack the resources necessary to construct names of sentences that belong to them, and so as to lack relevantly operative semantic concepts, semantic paradoxes need not appear within those languages. It is therefore possible to construct a correct definition of truth *for* such a language but *in* another language. Far from forcing him to despair of scientific semantics and therefore set theory—as, for instance, Gottlob Frege had done before him: “set theory in ruins”⁴⁷—the semantic paradoxes called Tarski to greater rigor.

As just suggested, the key to avoiding the semantic paradoxes, according to Tarski, is not only to avoid natural or colloquial languages in every attempt to define

⁴³ This is because, according to Tarski, “it would not be in harmony with the spirit of this [that is, colloquial] language if in some other language a word occurred which could not be translated into it.” Tarski, “The Concept of Truth in Formalized Languages,” 164.

⁴⁴ Actually, using the resources Tarski developed for defining truth in strictly formal languages, it is possible to construct definitions of truth for natural languages as well, from which definitions no paradox would uniquely follow. Tarski, however, regarded all natural languages as universal in the sense described above and so held them to be inconsistent. This itself meant that they were unsatisfactory by his criteria, and so no definition of truth for them could be satisfactory. It is also worth noting that in the first version of “The Concept of Truth in Formalized Languages,” Tarski held out the possibility of “consistent and correct use of the concept of truth” in languages of infinite order (fully formalized, of course). This, however, he retracted in his subsequent postscript. See *ibid.*, 266, 273.

⁴⁵ Correct definitions can be constructed for *all* formalized languages, as Tarski eventually came to see. See *ibid.*, 271–73. Note that he held, at first, that correct definitions could be constructed only for certain formalized languages of a finite order. See *ibid.*, 247, 254.

⁴⁶ *Ibid.*, 167.

⁴⁷ Frege, “On Schoenfliess,” 176.

semantic concepts. It is also crucial “always [to] distinguish clearly between the language *about* which we speak and the language *in* which we speak, as well as between the science which is the object of our investigation and the science in which the investigation is carried out.”⁴⁸ In other words, unless one takes great care to keep quite distinct (1) a particular formalized language under analysis (which, as explained before, contains no terms belonging to the theory of language) and (2) the language one employs in analyzing that particular formalized language (thus allowing one to refer to the terms of the first language without thereby introducing into it any terms belonging to the theory of language), the semantic paradoxes will appear anew. Operational here is Tarski’s famous distinction between an object-language and its metalanguage, the latter being the language employed when it is necessary to say something about the language to which it is attached or for which it is constructed.⁴⁹ A metalanguage is, of course, essentially richer than the object-language for which it serves as metalanguage, since it can accomplish every linguistic act accomplished by the object-language, but it contains also “specific terms . . . of a structural-descriptive character,” that is, “names of concrete signs or expressions of the language.”⁵⁰ As Tarski summarily puts this double point in his 1944 paper on truth: “the meta-language must contain the object-language as a part,” and “the meta-language must be rich enough to provide possibilities of constructing a name for every sentence of the object-language.”⁵¹ Importantly, while the metalanguage contains terms that could allow for structural-descriptions of its own terms, such structural-descriptions must be introduced only in a meta-metalanguage that takes the metalanguage

⁴⁸ Tarski, “The Concept of Truth in Formalized Languages,” 167.

⁴⁹ For this particular terminology, see Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 350.

⁵⁰ Tarski, “The Concept of Truth in Formalized Languages,” 172.

⁵¹ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 350.

as its object-language.⁵² The metalanguage thus allows for talk of the terms of the object-language under analysis to proceed without producing any formal inconsistency.

It is not difficult to see how this distinction makes it possible to secure formal correctness and therefore to ensure the possibility of proceeding with a scientifically viable semantics. If every definition of a semantic concept is introduced only at the level of the metalanguage, and always and only as an equivalence relation between the defined term and an already-established formulation built from structural-descriptive terms operative in the metalanguage and referring to sentences in the object-language, then the conditions of non-creativity and eliminability (the conditions for the satisfaction of the criterion of formal correctness) are fully met.⁵³ So long as the presence of structural-descriptive terms in the metalanguage does not in itself lead to formal inconsistencies, the introduction of a definition of a semantic concept in terms of those structural-descriptive terms can cause no problems for formal correctness.

Once the construction of both a formalized object-language and its accompanying metalanguage is complete, it is possible to pursue to its end the production of a materially adequate semantic definition of truth without running into problems with formal correctness. It is clear that truth can be defined unproblematically only for particular formalized languages, and only in metalanguages appropriate for such particular formalized languages. Consequently, Tarski clarifies the terms of the T-schema

⁵² Tarski makes this point in Tarski, “The Establishment of Scientific Semantics,” 405; and even more straightforwardly in Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 350: “It should be noticed that these terms ‘object-language’ and ‘meta-language’ have only a relative sense. If, for instance, we become interested in the notion of truth applying to sentences, not of our original object-language, but of its meta-language, the latter becomes automatically the object-language of our discussion; and in order to define truth for this language, we have to go to a new meta-language—so to speak, to a meta-language of a higher level. In this way we arrive at a whole hierarchy of languages.”

⁵³ For a brief overview of why definitions must always be equivalence relations, see Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 32–36.

introduced earlier: “x is a true sentence if and only if p.” Obviously, “x” must be replaced in any instantiation of the scheme by a structural-descriptive name of a sentence in a particular object-language, but constructed in the appropriate metalanguage. In turn, “p” must in the same instantiation of the scheme be replaced by a translation into the metalanguage of the sentence from the object-language designated by the structural-descriptive name that replaces “x.” To illustrate by using Tarski’s now-well-worn example, a particular T-sentence might be “The sentence ‘snow is white’ is true if, and only if, snow is white.”⁵⁴ (Note that the mere use of inverted commas serves to make of “snow is white” a structural-descriptive name in this example.)⁵⁵ Gathered together into a single “logical conjunction” or “logical product,” all the T-sentences that can be generated for a particular object-language in the associated metalanguage constitute the complete or general definition of truth for that object-language.⁵⁶

It is in fully formalizing *this* claim that, subtly but explicitly, Tarski takes up a second use of set theoretical mathematics in his work on truth. He assumes—and not without good reason⁵⁷—that the predicate “true” holds of sentences, which can be gathered by that truth predicate into a determinate set. He therefore produces a convention, called Convention T, that replaces the phrase “is true” or “is a true sentence” with the set theoretical relation of belonging, “ \in Tr” (“Tr” here denotes “the class of all true sentences”). In words rather than symbols, Tarski here replaces “is true” or “is a true

⁵⁴ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 343.

⁵⁵ This is not without its problems. See Davidson, “Theories of Meaning and Learnable Languages,” 9–11.

⁵⁶ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 344; and Tarski, “The Concept of Truth in Formalized Languages,” 187.

⁵⁷ See Tarski, “The Semantic Conception of Truth and the Foundation of Semantics,” 342: “The predicate ‘true’ is sometimes used to refer to psychological phenomena such as judgments or beliefs, sometimes to certain physical objects, namely, linguistic expressions and specifically sentences, and sometimes to certain ideal entities called ‘propositions.’ . . . For several reasons it appears most convenient to apply the term ‘true’ to sentences, and we shall follow this course.” This might be compared, despite other differences from Frege’s conception of truth, with the latter’s similar remarks in Frege, “Thoughts,” 352.

sentence” with the set theoretical relation of belonging (ϵ) and a designation for the set to which true sentences from the object-language belong (“Tr”). To ensure that there belongs to the set Tr only sentences (rather than other things that might be qualified as in some sense “true”), Convention T inscribes also the requirement that whatever belongs to Tr is a sentence (or, more strictly, belongs to the set of all sentences). Convention T also brings together into its formulation the two guiding criteria that have been the principal subjects of this chapter so far: material adequacy and formal correctness.

Convention T, then, might productively be quoted in its full formulation:

A formally correct definition of the symbol “Tr” [associated, of course, with a particular formalized language], formulated in the metalanguage [of that particular formalized language], will be called an *adequate definition of truth* if it has the following consequences:

(α) all sentences which are obtained from the expression “ $x \in \text{Tr}$ if and only if p ” by substituting for the symbol “ x ” a structural-descriptive name of any sentence of the [object-]language in question and for the symbol “ p ” the expression which forms the translation of this sentence into the metalanguage;

(β) the sentence “for any x , if $x \in \text{Tr}$ then $x \in S$ [the class of all sentences].”⁵⁸

What guides the construction of this convention is, of course, Tarski’s guiding criteria for a satisfactory definition of truth. The presence here of the T-schema, recast slightly in set theoretical terms, helps to satisfy in a preliminary way the criterion of material adequacy. The careful description of what is to be substituted for “ x ” and “ p ” ensures that the

⁵⁸ Tarski, “The Concept of Truth in Formalized Languages,” 187–88.

distinction between object-language and metalanguage is maintained—necessary and, here at least, sufficient for the satisfaction of the criterion of formal correctness. Finally, more complexly, the implication relation asserted here between Tr, the complete definition of truth for the object-language in question, and all T-sentences, all partial definitions of truth for that same language, completes the satisfaction of the criterion of material adequacy. (I will come further along to the question of exactly how the T-sentences are actually generated.)

But what interests me at this point is less the way Convention T ensures the satisfaction of Tarski's guiding criteria than the way it recasts his work on truth as a set theoretical gesture more generally—and specifically by turning his attention to identifying a formula by which sets readily and justifiably recognizable as truths might be designated. In Convention T, Tarski uses the set theoretical relation of belonging to limit the domain of the true to sentences (β) and to recast the T-schema in set theoretical terms (α). This gesture makes fully clear the manner in which Tarski's definition of truth (for a particular language) aims at eliminating distracting unnecessary metaphysical questions about the supposed "nature" of truth. As Tarski states this point in some polemical remarks in his 1944 paper on truth, if a definition of truth "states necessary and sufficient conditions for a sentence to be true," then it need not "grasp the [supposed] 'essence' of this concept."⁵⁹ By taking the concept of truth to be adequately defined by constructing a well-formed formula that designates the complete set of true sentences (that is, matches with their truth-conditions sentences) from the language for which truth is being defined—and, of course, by ensuring through the distinction between object-language and metalanguage that no problems of formal correctness arise—Tarski performs a

⁵⁹ Tarski, "The Semantic Conception of Truth and the Foundations of Semantics," 361.

metaphysically demystifying gesture.

Here, then, is the second role played by the set theoretical apparatus in Tarski's work on truth. It allows him to cut away from philosophical reflection on truth so many unnecessary metaphysical accretions, focusing the philosopher just on the rigorous task of constructing a materially adequate and formally correct formula that designates truth for one relatively humble formalized language at a time. Tarski himself explained:

It seems to me obvious that the only rational approach . . . would be the following: We should reconcile ourselves with the fact that we are confronted, not with one concept [of truth], but with several different concepts which are denoted by one word; we should try to make these concepts as clear as possible (by means of definition, or of an axiomatic procedure, or in some other way); to avoid further confusions, we should agree to use different terms for different concepts; and then we may proceed to a quiet and systematic study of all concepts involved, which will exhibit their main properties and mutual relations.⁶⁰

Taking as its focus just the concept of truth as this would be intuitively applied to the sentences of fully formalized languages, and recognizing that that sort of concept of truth is most easily formalized as a simple set of sentences separated out by a well-formed formula, Tarski's work on truth makes serious headway on a question that had mystified philosophers as much as it had in part because every attempt at answering it had tried immaturely to solve problems concerning it that were simply too large. In short, Tarski saw that the supposed problem of truth amounts first and foremost to the clear identification of a set theoretical formula, not to metaphysical speculation regarding the

⁶⁰ *Ibid.*, 355.

nature of some mystified notion of truth.

Tarski thus abandons the metaphysical idea that there is some actual (and ultimately mysterious) relation between particular instances of truth (particular true sentences, within a strictly delimited language) and truth itself (truth always and only for the same strictly delimited language), insisting instead that truth is nothing more than a predicate that holds of a certain set of strictly defined sentences. In doing so, Tarski replicates a move made by his immediate predecessors in logic—already in Frege’s earliest writings, and outlined even earlier in Kant’s philosophy—albeit without direct reference to the concept of truth.⁶¹ Tarski thus sees in the basic operations of set theory, as his predecessors did, the resources for bypassing one of the most ancient philosophical problems: Plato’s so-called “third man” problem, generated by his insistence that *some* relation, given the obscure title of “participation,” allowed predicates to hold of particular objects.⁶² The invention of the predicate calculus, which preliminarily formalized naïve or intuitive set theory before becoming the basic logical apparatus to be used in fully developed set theory, revealed that Plato could be said to have created a nonsensical problem. Because relations can be construed as two-place predicates, and because the places associated with such two-place predicates can be occupied only by self-identical objects (and not with predicates as such), it would seem that Plato’s notion of participation sets forth, nonsensically, a two-place predicate (participation) the places of which are filled, respectively, by a predicate (a Platonic form) and an object (the thing that participates in that form). There is consequently no need to think about the supposed relationship between predicates and objects, but only to ask how predicates can be

⁶¹ See Frege, *Philosophical and Mathematical Correspondence*, 101.

⁶² For an interpretation of Plato’s project in terms of predication, see Davidson, *Truth and Predication*, 76–97.

appropriately defined so as to designate definite and non-problematic extensions. This Tarski did for truth, distributing the supposed general concept of truth among so many particular concepts of truth, each to be rigorously constructed for the particular language to which it is attached.⁶³ Each concept of truth, provided with a formally correct and materially adequate definition, picks out a determinate set of true sentences which, as a set, exhaust whatever questions might need to be asked, philosophically, about the deeper essence of truth. All that needs doing is to identify an appropriate definition or formula that matches sentences to their truth-conditions. Of course, as Donald Davidson notes, there is much that can be said—and more I will say at the end of this chapter—regarding the correspondence or correlation between sentences and their truth-conditions.⁶⁴

This, then, constitutes Tarski's second use of set theory in his work on truth. It provides him with the resources necessary to cut away, in the classic style of early analytic philosophy, a host of problems surrounding the notion of truth that he could only regard as metaphysical nonsense, and it does so by refocusing philosophical attention on constructing well-formed formulae that designate specific sets of sentences rightly called truth.

Truth and Satisfaction

At this point in this exposition, it might seem possible to step back from Tarski's actual work in order to draw general conclusions—and especially to probe more deeply

⁶³ Of course, Tarski can be read as recognizing at least a Wittgensteinian “family resemblance” among the concepts of truth, and one could designate that network of concepts the “general” concept of truth. Some of Donald Davidson's late work on Tarski seems to indicate this possibility. See Davidson, *Truth and Predication*, 7–75. For Wittgenstein's famous discussion of family resemblance, see Wittgenstein, *Philosophical Investigations*, 31–36.

⁶⁴ See, for instance, Davidson, “The Folly of Trying to Define Truth,” 21.

into Tarski's commitment, due to the criterion of material adequacy, to some kind of correspondence or correlation lying at the heart of truth (irreducible, nonetheless, to any epistemological "correspondence theory" of truth). But before turning to such concluding reflections, one further obstacle tackled by Tarski in his work must be considered—especially because it bears rather directly on the relevance of the model theoretical development of set theory that Tarski's work set in motion.

As Tarski explains, it is a relatively simple affair to construct a fully satisfactory definition of truth for a finite language according to the strictures of Convention T. If, for example, one were to attempt to construct such a definition for a language that consisted only of three sentences (say, "One book is on the desk," "One book is on the shelf," and "One book is on the floor"), it would be a matter simply of gathering into a single logical disjunction the appropriate T-sentences: $x \in Tr$ if and only if (1) $x =$ "One book is on the desk" and one book is on the desk; or (2) $x =$ "One book is on the shelf" and one book is on the shelf; or (3) $x =$ "One book is on the floor" and one book is on the floor.⁶⁵ For any language containing a finite number of sentences, a similar definition of truth could be constructed. The difficulty, though, is that few formalized languages are finite—certainly none strong enough to allow for the formulation of the basic operations of arithmetic (addition and multiplication). And languages containing an infinite number of sentences introduce special problems of their own.

Because it is impossible simply to concatenate in a massive logical disjunction the metalinguistic T-sentences for an infinite number of sentences of a particular object-language, it is necessary—if truth is to be defined for such languages—to employ a recursive strategy. Tarski explains:

⁶⁵ See Tarski, "The Concept of Truth in Formalized Languages," 188.

Among the sentences of a language we find expressions of rather varied kinds from the point of view of logical structure, some quite elementary, others more or less complicated. It would thus be a question of first giving all the operations by which simple sentences are combined into complete ones and then determining the way in which the truth or falsity of composite sentences depends on the truth or falsity of the simpler ones contained in them. Moreover, certain elementary sentences could be selected, from which, with the help of the operations mentioned, all the sentences of the language could be constructed; these selected sentences could be explicitly divided into true and false, by means, for example, of partial definitions of the type described above [that is, of T-sentences].⁶⁶

Formalized languages with an infinite number of sentences have so many sentences because the language is constructed through operations that allow basic sentences to be combined into more complex sentences (and for theorems to be derived from such sentences through the rules of derivation). Because this is the case, truth for such languages could be constructed through assembling a logical disjunction of the appropriate T-sentences for the most elementary basic sentences from which other sentences are derived or out of which more complex sentences are constructed (provided, at least, that there are at most finitely many of these basic sentences). If the truth of the most elementary sentences were defined, along with the operations through which other sentences are derived or constructed, truth would be adequately defined for the language in question.

⁶⁶ *Ibid.*, 189. Some of the basic operations of recursivity are explained in Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 38–44.

Unfortunately, however, even this important solution to the problem of languages containing an infinite number of sentences runs into an important difficulty. Complex sentences in formalized languages are constructed not only of simple sentences (with assignable truth-values), but also of what Tarski calls “sentential functions,” that is, expressions containing free variables—expressions that therefore only become actual sentences with assignable truth-values when their variables are either replaced by constants (names of objects) or bound by quantifiers (existential or universal).⁶⁷ And while it is possible to define truth recursively for a language containing an infinite number of sentences if all complex sentences are constructed without variables, it is *not* so possible where variables are present. If a language can produce a hypothetical sentence containing bound variables (in natural language, a sentence like “All human beings have a heart”), it is impossible to define truth for that language recursively, since the basic elements making up certain such complex sentences contain free variables (in this example, the three basic elements would be “x is a human being,” “y is a heart,” and “x has y”—and not, it should be noted, “all human beings” or “all hearts”). The constituent elements of the complex sentence, in other words, are not sentences with truth-values, but sentential functions, which have no truth-values. No T-sentence can be provided for the basic elements from which such complex sentences are constructed, and it is therefore impossible to provide a recursive definition of truth for such a language without finding a way around this difficulty.

Tarski’s insight is to restore to recursivity the potential to solve the problem of

⁶⁷ On sentential functions more generally, see Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 3–13. For the specifics within his work on truth, however, see Tarski, “The Concept of Truth in Formalized Languages,” 176–78. Note that Tarski speaks of *constants* in his more strictly logical work, but generally of *objects* in his work on truth.

defining truth for formalized languages containing variables. He introduces his solution to the problem in the following words: “The possibility suggests itself, however, of introducing a more general concept [more general, that is, than the concept of truth] which is applicable to any sentential function, can be recursively defined, and, when applied to sentences, leads us directly to the concept of truth.”⁶⁸ Here Tarski states three distinguishable conditions that must be met by a “more general concept” by which it might be possible to save recursivity. First, it must be wider than the concept of truth in application, being applicable not only to sentences (as is the concept of truth) but also to sentential functions (as the concept of truth is not). Second, it must, like the concept of truth within languages without free variables, be recursively definable, which is to say that it must be possible to decide whether the concept holds individually of the several parts of a constructible complex sentence with a quantifier (like “all human beings have a heart” from above). Third and finally, it must be possible to construct a materially adequate definition of truth for a language in terms of whether the concept in question holds for that language’s sentences. In sum, Tarski’s basic strategy is to identify a property broader than truth—broad enough to be the property as much of sentential functions as of sentences—and then to define truth in terms of that property, thus securing the power of recursivity to provide a definition of truth for languages with an infinite number of sentences. And the property he nominates to serve such a role is satisfaction.⁶⁹ This property, Tarski explains in a more general essay on semantics, “presents relatively few difficulties” for the construction of a definition, and “the

⁶⁸ Tarski, “The Concept of Truth in Formalized Languages,” 189.

⁶⁹ See *ibid.*, 189; and Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 352.

remaining semantical concepts are easily reducible to it.”⁷⁰

The simplest definition of “satisfaction” provided by Tarski is to be found in his textbook on logic: “The result of the replacement of the variables in a sentential function by constants—equal constants taking the place of equal variables—may lead to a true sentence; in that case, the things denoted by those constants are said to satisfy the given sentential function.”⁷¹ A similar definition appears in Tarski’s 1944 essay on truth: “Satisfaction is a relation between arbitrary objects and certain expressions called ‘sentential functions.’ . . . Given objects satisfy a given function if the latter becomes a true sentence when we replace in it free variables by names of given objects.”⁷² Such definitions make clear the close connection between satisfaction and truth, but they also reveal an important difficulty. It is not uncommon—even within Tarski’s own work!—to define satisfaction in terms of truth. But this would make it impossible to define truth in terms of satisfaction without producing a circle. This Tarski states straightforwardly in the 1944 essay: “However, apart from other difficulties, this method [of defining satisfaction in terms of truth] is not available to us, for we want to use the notion of satisfaction in defining truth.”⁷³ How, then, is truth to be defined in terms of satisfaction?

The solution is to take “satisfaction” as an undefined primitive semantic concept, and then to use it to construct the definition of truth. Tarski explains in his introduction to logic: “When we set out to construct a given discipline, we distinguish, first of all, a

⁷⁰ Tarski, “The Establishment of Scientific Semantics,” 406–407.

⁷¹ Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 5.

⁷² Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 352–53. It should be noticed that these two quotations indicate an essential ambiguity in Tarski’s formulations of satisfaction—in one sentential functions being satisfied by “constants” and in the other by “given objects.” This is a problem that deserves close attention, but to which I cannot dedicate serious attention here. What would need to be undertaken is a smoothing out of Tarski’s language in light, precisely, of his ontology—worked out preliminarily in this chapter and discussed at greater length in Chapter 4.

⁷³ *Ibid.*, 353.

certain small group of expressions of this discipline that seem to us to be immediately understandable; the expressions of this group we call primitive terms or undefined terms, and we employ them without explaining their meanings.”⁷⁴ Other terms can be defined always and only “with the help of primitive terms,” as well as, eventually, “such expressions of the discipline whose meanings have been explained previously.”⁷⁵ Tarski’s strategy is to take “satisfaction” as a primitive term, and then to construct an intelligible definition of “truth” in terms of it. (Only what Tarski calls in his 1944 essay on truth “a general definition” of satisfaction is therefore needed—which is, of course, not a definition in any strict sense, but a kind of formalization of the assumedly immediate understanding of the term.)⁷⁶ And this can be accomplished simply through the construction of a recursive account of satisfaction for the entire object-language in question in the construction of a particular definition of truth.

For this reason Tarski sets out in his earliest essay on truth simply “to make clear by means of some examples the usual meaning of this notion [satisfaction] in its customary linguistic usage,” by which he seems to have reference just to how satisfaction is generally recognized as operating in simple algebra.⁷⁷ He begins, naturally, with the simplest case, “that in which the given sentential function contains only *one* free variable.”⁷⁸ Tarski’s brief explanation deserves to be quoted in full:

We consider the following scheme:

for all a, a satisfies the sentential function x if and only if p

and substitute in this scheme for ‘p’ the given sentential function (after

⁷⁴ Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 118.

⁷⁵ *Ibid.*

⁷⁶ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 353.

⁷⁷ Tarski, “The Concept of Truth in Formalized Languages,” 189.

⁷⁸ *Ibid.*

first replacing the free variable occurring in it by ‘*a*’) and for ‘*x*’ some individual name of this function. Within colloquial language we can in this way obtain, for example, the following formulation:

for all a, a satisfies the sentential function ‘x is white’ if and only if a is white

(and from this conclude, in particular, that snow satisfies the function ‘*x* is white’).⁷⁹

Satisfaction is, in the simplest case, just a matter of replacing the variables of a sentential function with constants (“objects”) and then asking whether the state of affairs indicated by the resulting sentence is indeed the case.⁸⁰ In other words, satisfaction entails the same basic correspondence or correlation assumed to lie at the heart of the pre-reflective notion of truth—a correspondence or correlation between sentences and their truth-conditions—except that it accommodates sentential functions in addition to sentences without open variables.

With this basic move in place, Tarski turns his attention to more complicated examples of satisfaction. It is not difficult to guess what satisfaction looks like for sentential functions with two distinct free variables: “The only difference is that the concept of satisfaction now refers not to single objects but to pairs (more accurately to ordered pairs) of objects.”⁸¹ And from there it is possible to see how the general case would work as well, where sentential functions contain not one or two, but “an arbitrary

⁷⁹ *Ibid.*, 190.

⁸⁰ “A similar construction,” Tarski claims, “will be familiar to the reader from school algebra, where sentential functions of a special type, called *equations*, are considered together with the numbers which satisfy these functions, the so-called *roots* of the equations (e.g. 1 is the only root of the equation ‘ $x+2=3$ ’).” *Ibid.*

⁸¹ *Ibid.*, 190–91.

number of free variables.”⁸² Just as a sentential function with two free variables is satisfied by an appropriate ordered pair, a sentential function with whatever greater number of free variables will be satisfied by an ordered series with that number of appropriate terms. This can even be accomplished if the number of free variables in a sentential function (or set of sentential functions) is infinite. It is then necessary only to employ an infinite ordered sequence of constants or objects. In fact, as Tarski explains, such infinite sequences of constants or objects can be employed to satisfy every sentential function, even finite ones, since one need only use the term located in the appropriate position or positions in an infinite sequence to replace the free variables in sentential functions with a finite number of variables.⁸³ Thus Tarski eventually replaces the scheme of the simple case (“for all a, a satisfies the sentential function x if and only if p”) with a more general scheme: “f satisfies the sentential function x if and only if f is an infinite sequence of classes, and p,”⁸⁴ which allows him to construct a fully formalized definition of satisfaction for a language.⁸⁵ And it should be noted that, even with these complications, the basic idea of satisfaction remains the same. Satisfaction is still a matter of correlating sentences—with or without variables—and their truth-conditions via constants now ordered in an infinite sequence.

In the place of truth, satisfaction can thus be recursively defined, in the appropriate metalanguage, for a particular formalized object language. And this makes clear the possibility of defining at least *one* semantic notion in formal terms. Even more importantly, there is no difficulty about defining truth in terms of satisfaction. Tarski

⁸² *Ibid.*, 191.

⁸³ See *ibid.*, 191–92.

⁸⁴ *Ibid.*, 192.

⁸⁵ See Tarski’s Definition 22, *ibid.*, 193.

explains that, in light of what has been summarized already regarding satisfaction,

it is easy to realize that whether or not a given sequence satisfies a given sentential function depends only on those terms of the sequence which correspond (in their indices) with the free variables of the function. Thus in the extreme case, when the function is a sentence, and so contains no free variable . . . the satisfaction of a function by a sequence does not depend on the properties of the terms of the sequence at all. Only two possibilities then remain: either every infinite sequence of classes satisfies a given sentence, or no sequence satisfies it The sentences of the first kind . . . are the true sentences; those of the second kind can correspondingly be called the false sentences.⁸⁶

Sentences with free variables are, of course, satisfied only by certain sequences of objects or terms—those with the appropriate object or term in the appropriate place in the ordered sequence. A sentence like “ x_4 is my cat” is satisfied only by those infinite ordered sequences in which my cat is indeed in the fourth place (say, an infinite ordered sequence beginning, “my computer, my book, my guitar, my cat, . . .”). No other sequence will satisfy it. A true sentence, however, is a sentence that is satisfied by *every* (pertinent) infinite ordered sequence, just as a false sentence is one that is satisfied by *no* (pertinent) infinite ordered sequence.⁸⁷ Such sentences will be those in which all variables have been bound, but for which it can still be asked whether they are satisfied or not.

⁸⁶ *Ibid.*, 194. Cf. Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 353: “Once the general definition of satisfaction is obtained, we notice that it applies automatically also to those special sentential functions which contain no free variables, i.e., to sentences. It turns out that for a sentence only two cases are possible: a sentence is either satisfied by all objects, or by no objects. Hence we arrive at a definition of truth and falsehood simply by saying that a sentence is true if it is satisfied by all objects, and false otherwise.”

⁸⁷ By “pertinent” here I mean only to indicate that it is only those infinite ordered sequences constructed from the domain of the object-language that can satisfy a sentential function in that language.

This might seem a curious claim at first. One might, for instance, take Tarski to be suggesting that all true sentences are sentences with universal quantifiers, the unquantified sentential functions of which are satisfied by all objects, however they might be ordered in a sequence. In other words, one might think that a true sentence would take the shape of something like “For all x , x is a fish,” which would be false except where the object-language deals only with fish. This, however, would be wrong. The point is much simpler. Tarski means only to say that the T-schema can be constructed through a simple manipulation of the satisfaction schema worked out recursively. In “ f satisfies the sentential function x if and only if f is an infinite sequence of classes, and p ,” “ x ” is to be replaced by a description in the metalanguage of a sentential function drawn from the object-language, and “ p ” is to be replaced by the sentence that results from the substitution for the free variable within x by the appropriate term drawn from the sequence f . An example might help to illustrate: The sequence f , which has “Spot” as its fourth term, satisfies the sentential function “ x_4 is a fish” if and only if f is an infinite sequence of appropriate terms, and Spot is a fish.” Little needs to be done to transform this schema into the T-schema. But rather than sentential functions with free variables, sentential functions *without* free variables are dealt with. (This is possible because Tarski defines sentences as special cases of sentential functions, rather than as a class entirely distinct from sentential functions.) Consequently, the satisfaction schema ends up looking like the following: “ f satisfies the sentence x if and only if f is an infinite sequence of appropriate terms, and p .” Here again “ x ” is to be replaced by a description in the metalanguage, but now of the sentence (rather than the sentential function) drawn from the object-language, and “ p ” is to be replaced simply by the sentence (which is not

produced by any substitution, since there are no free variables in the sentence in question). To borrow from the example used just above: The sequence f satisfies the sentence “Spot is a fish” if and only if f is an infinite sequence of appropriate terms, and Spot is a fish. It is not difficult to see that if Spot is indeed a fish, then “Spot is a fish” will be satisfied by *every* sequence f (so long as f is indeed an infinite sequence of appropriate terms). If, however, Spot is *not* a fish, then no sequence f will satisfy the sentence “Spot is a fish.” Truth is thus easily definable in terms of satisfaction: “ x is a true sentence—in symbols $x \in \text{Tr}$ —if and only if $X \in S$ [the set of all sentences] and every infinite sequence of classes satisfies x .”⁸⁸

Only in light of these last developments is it finally possible to come back, fully prepared, to Davidson’s argument regarding the ontological commitments of Tarski’s work on truth. As noted at the end of the first part of this chapter, there underlies all this brilliant formal work—so much solving of problems, one after another—a basic and not yet fully explored set of commitments to a certain correspondence or correlation lying at the heart of truth. But it is only in the course of his discussion of satisfaction and the manner in which truth can be defined in terms of satisfaction that the nature of this correspondence or correlation is fully clarified—and its distance from any epistemologically and metaphysically weighty correspondence theory of truth can be adequately recognized. I conclude this chapter, then, with a few remarks about this point, clarifying what I want to call the minimalist ontological interpretation of Tarski’s work on truth.

In “The Folly of Trying to Define Truth,” as already noted, Davidson states that “one of the strongest arguments for Tarski’s definitions is that in them *nothing* plays the

⁸⁸ Tarski, “The Concept of Truth in Formalized Languages,” 195.

role of facts or states of affairs.”⁸⁹ This might seem odd, given all the references Tarski makes to states of affairs in his attempts at clarifying both the notion of semantics and the requirements of the criterion of material adequacy. And yet Davidson seems quite right. He justifies his claim by referring to “a persuasive argument, usually traced to Frege . . . , to the effect that there can be at most one fact or state of affairs.”⁹⁰ That Tarski shares this Fregean intuition seems to follow from the way he builds a definition of truth out of the primitive notion of satisfaction, as described just above. Davidson points out that

the sequences which satisfy sentences [in Tarski’s proposal] are nothing like the “facts” or “states of affairs” of correspondence theorists, since if one of Tarski’s sequences satisfies a closed sentence [that is, a sentence where all variables are bound], thus making it true, then that same sequence also satisfies every other true sentence, and thus also makes it true, and if any sequence satisfies a closed sentence, every sequence does.⁹¹

Davidson makes two crucial points here. First, the infinite ordered sequences that satisfy sentences—in terms of which satisfaction the definition of truth is to be constructed—can in no way be said to be equivalent to facts or objects or states of affairs. Second, all true sentences are satisfied by the same sequences, and all sequences satisfy the same true sentences. The first point serves to problematize Tarski’s imprecise references to states of affairs, to the correlation of words *and things*. The second point serves to note the manner

⁸⁹ Davidson, “The Folly of Trying to Define Truth,” 22–23.

⁹⁰ *Ibid.*, 23. Davidson has reference here to Frege’s notion that “the True” is the singular referent of every true sentence. For Davidson’s interpretation of Frege more generally, see Davidson, *Truth and Predication*, 130–40. For Frege’s earliest discussion of the True as the referent of true sentences, see Frege, “Function and Concept,” 144–45.

⁹¹ Davidson, “The Folly of Trying to Define Truth,” 25.

in which Tarski appears to concede the Fregean notion that there is at most one fact or state of affairs.

But if all this clarifies that Tarski's conception of truth—as he himself argues—need make no concessions to the correspondence (or any other epistemological) theory of truth, with all its (or their) metaphysical commitments, it still must be asked what sort of correspondence or correlation Tarski's criterion of material adequacy *does* commit him to. Actually, the answer to this question is not in any way difficult. Tarski's criterion of material adequacy commits him only to the idea that truth presupposes some kind of correlation of sentences with their truth-conditions. What the T-schema formalizes is just this correlation. It in no way *determines* the truth-conditions of any particular sentence; it merely correlates the truth-conditions of the sentences of a given (formal) language with those sentences. That is all Tarski assumes about the supposed “nature” of truth in his work.

And yet Tarski's decision to construct truth definitions along the lines just outlined *by drawing on the more-primitive semantic notion of satisfaction* complicates things in an important way. The Tarskian clarification of satisfaction itself would go on to lay the groundwork for the full development of model theory out of set theory.⁹² I will have more to say about model theory in Chapter 4, but already it must be noted that models are defined in terms, precisely, of satisfaction. The criterion for deciding what constitutes a model of a particular formal language is just whether it *satisfies* the formal language in question. At the bottom of truth, according to Tarski, lies (the embryonic

⁹² See Vaught, “Alfred Tarski's Work in Model Theory.” As Feferman and Feferman note, “Tarski did not create this field,” namely that of model theory, but “his influence was decisive.” Importantly, moreover, “the name for this emerging field seems” in fact “to have been first used in print in several of Tarski's articles of the mid-1950s.” Feferman and Feferman, *Alfred Tarski*, 280, 282.

form of) model theory. And model theory concerns a certain entanglement of or commerce between the formal and the material, the abstract and the concrete, to which Tarski's work on truth is subtly committed. With Frege, I will argue, Tarski recognizes that strictly formal considerations force a recognition of structures and constraints that are "founded deep in the nature of things."⁹³

To make these commitments fully clear, however, it is necessary first to provide a preliminary analysis of Badiou's work. But even before turning to this task, it is necessary to note that the commitments I mean to unearth in Tarski's work are in no way at odds with the *non*-commitments of his work defended in Davidson's name just above. The ontological implications of Tarski's use of satisfaction—and therefore of a nascent form of model theory—are what I want to call *minimalist*. They commit him to no more, in epistemological and metaphysical terms, than Davidson believes. According to an interpretation that, again, I can only spell out fully after having worked through a basic exposition of Badiou's work, Tarski's approach commits him only to a minimalist ontology organized around what might be called the material excess of the formal over itself.

Obviously, this formula will have to be clarified. And it is through Badiou's work, I believe, that it can be clarified best.

⁹³ Frege, "Function and Concept," 156.

BADIOU'S PROJECT

In this chapter, I will largely keep my focus just on doing with Alain Badiou what I did in the preceding chapter with Alfred Tarski. That is, I mean here first and foremost to provide a basic exposition of Badiou's attempt to account for truth by drawing on the resources of set theoretical mathematics. Badiou's project that eventually culminated in his work on truth began in earnest only in the late 1970s and came to full fruition only in the late 1980s, with the obvious consequence that his writings on the subject have not yet had the kind of impact on the field that Tarski's have had.¹ They have, nonetheless, already been recognized for their importance, and they have unquestionably helped to focus a number of continental philosophers anew on the question of truth. As in Chapter 2, my focus here will be on the use of set theory in attempting to get clear about truth. Consequently, I will give my attention primarily to the formal apparatus of Badiou's philosophical account of truth. As with Tarski, however, it will prove necessary to draw on Badiou's occasional non-formal discussions of truth, since they help to clarify the philosophical stakes of Badiou's interest in the mathematical.

The political commitments that lay behind and largely inspired Badiou's work on truth are widely recognized—and are often enough, unfortunately, used as an excuse to dismiss his account (or, just as commonly, to ignore its formal precision in favor of more immediately “applicable” formulations). The political bearings of his project are perhaps

¹ Badiou's *Theory of the Subject* has already been discussed in Chapter 1. It contains five years of Badiou's annual seminar, from 1975 to 1979, and traces the pathway by which he came to the central problems that would occupy him during the 1980s. Many of the themes central to *Being and Event* appear already in *Theory of the Subject*.

most clearly on display in the lectures published as *Theory of the Subject*, an unmistakable turning point in Badiou's thinking about truth, discussed briefly in Chapter 1. Those lectures, begun shortly before the death of Mao Zedong but published just after the election of Francois Mitterrand, make Badiou's 1970s Maoism more than apparent. Although the 1980s saw a transformation of Badiou's relationship to Mao, his emphatic commitment to communism has never flagged,² and even the far less politically charged *Being and Event* of 1988 is probably best interpreted as largely motivated by the need to provide radical leftist politics with a workable ontology.³ Badiou himself makes this point in the introduction to *Being and Event* by noting that the "new departure in the doctrine of truth" he means to trace is reflected in "the modern theories of the subject," theories that are the focus of his overtly Maoist *Theory of the Subject*.⁴ The years since the publication of *Being and Event* have only made Badiou's political commitments clearer, and there is today no mistaking that his philosophical work on truth is fundamentally meant to pave the way toward a workable political ontology—toward accounting for what he called "communist invariants" already in the 1970s.⁵ It is crucial to keep these larger commitments on Badiou's part in view when approaching his work on the use of set theoretical mathematics to think truth. As will become clear, Badiou's larger aims have much to do with what he sees as the virtues of set theory.

The principal focus here, however, will be on Badiou's use of set theoretical mathematics to arrive at the possibility of understanding truth. What role does the set

² This has been made particularly clear in recent circumstantial works by Badiou, most notably Badiou, *The Meaning of Sarkozy*; and Badiou, *The Communist Hypothesis*.

³ On Badiou's reworked relationship to communism in the late 1980s, see Badiou, *Conditions*, 147–76. For an excellent discussion of the political bearings of Badiou's work over the course of his work, see Bosteels, *Badiou and Politics*. For a distinct approach to Badiou's development, see Hallward, *Badiou*, 49–78.

⁴ Badiou, *Being and Event*, 3.

⁵ Badiou and Balmès, *De l'ideologie*, 67.

theoretical apparatus play in his project? The answer to this question will prove to be, as for Tarski, twofold, and in a roughly parallel fashion. First, Badiou turned his attention to set theory because he found in it a means to answer what Martin Heidegger called the question of being. Because Badiou takes Heidegger to mean that this question demands the thinking of pure multiplicity, of the many without the one, and because he interprets set theory as the thought of the pure multiple, he argues that set theory constitutes the science of being. This argument is not unrelated to questions of truth, since Badiou takes his bearings regarding the concept of truth not only from thinkers like Gaston Bachelard and Georges Canguilhem (as noted in Chapter 1), but also and perhaps more profoundly from Heidegger, whose account of truth's relation to being is visible in Badiou's work. The first role played in Badiou's project by set theory is thus rather obviously ontological in nature. The second role set theory plays in Badiou's work concerns the continuum hypothesis, first proposed by Georg Cantor at the outset of the set theoretical enterprise. Badiou (as also discussed preliminarily in Chapter 1) has insisted since the 1970s that the only way to account for the dynamic relationship between truth and being is to provide a robust theory of the subject, and that the subject is to be associated with some kind of decision to be made in confronting the undecidable. A major purpose of *Being and Event* is to show that truth becomes clear only when the undecidable is itself clarified in terms of the undecidability of the continuum hypothesis. By deciding against the undecidable continuum hypothesis in favor of the existence of what mathematician Paul Cohen calls "generic sets," one allows for the full set theoretical articulation of the concept of truth. For Badiou this decision amounts to an operational decision, a favoring of the forcing operation developed by Cohen in his work on the continuum hypothesis. As with Tarski,

then, exposition of Badiou's project requires in turn ontological considerations and an investigation of the formulae or, really, operations by which truths (of specific languages) as sets are designated—or, in Badiou's case, constructed.

The discussion that follows is divided into several parts. In a first section, I outline the Heideggerian background of Badiou's project. Because Badiou takes seriously the Heideggerian question of being, but also because he takes himself to disagree with the classic Heideggerian approach to that same question, it is necessary to make clear how Badiou's work is situated with respect to the tradition he inherits—especially so that the nature of his ontological commitments can be fully clarified in Chapter 4. In a second section, I develop more fully a specific feature of Badiou's set theoretical ontology, the void, by addressing the distinction between the void proper (the empty set, a matter of “nature,” according to Badiou) and the situational void (the void specifically relevant in what Badiou calls “historical” situations). This clarification lays the groundwork for understanding Badiou's notion of the event, which brings to a first formulation the sort of procedure that Badiou takes to be operative in the construction of sets identifiable as truths. Finally, in the third and concluding section, I turn from ontological concerns to definitional concerns, giving my attention to the continuum hypothesis and spelling out more fully Badiou's set theoretical conception of truths as Cohenian generic sets.

The Question of Being

In a review of Badiou's impassioned little book, *Ethics: An Essay on the Understanding of Evil*, Terry Eagleton states regarding Badiou that “scarcely any other moral thinker of our day is as . . . prepared to put notions of truth and universality back

on the agenda.”⁶ In a philosophical tradition for which, as Badiou himself puts it, “truth is a new word,” Badiou’s insistence on providing a philosophical defense and elucidation of invariant truth is novel and largely unique.⁷ It is in no way, however, entirely without precedent. In lectures throughout the 1920s and 1930s and in published books from 1927 and 1929, Martin Heidegger made truth a central focus of his work, and Badiou explicitly notes his indebtedness to Heidegger.⁸ But it must be noted from the outset that Badiou ties his work to Heidegger’s less in terms of a shared investment in or a common conception of truth than in terms of an inherited investigation of being. A detailed study of the relationship between Badiou’s and Heidegger’s respective conceptions of truth is beyond my purposes in this dissertation—indeed, it would require at least another dissertation!—but even passing familiarity with the two thinkers reveals that Badiou in no way aims simply to reproduce Heidegger’s work on truth.⁹ Much more central to Badiou’s interest in Heidegger is ontology, rather classical questions regarding what it means to speak of what is. Thus, when Badiou describes Heidegger as “the last universally recognizable philosopher,” he means first and foremost to note the importance of maintaining Heidegger’s insight that “philosophy as such can only be re-assigned on the basis of the ontological question.”¹⁰

Despite certain manifest tendencies among philosophers to be mystified by

⁶ Eagleton, *Figures of Dissent*, 253.

⁷ Badiou, *Being and Event*, 3.

⁸ The most important of these can be found in Heidegger, *Logic*; Heidegger, *Being and Time*; Heidegger, *Kant and the Problem of Metaphysics*; and Heidegger, *The Essence of Truth*. Certainly, Heidegger in no way abandoned the theme of truth in his later work, though Badiou is more directly critical of Heidegger’s later developments than of his earlier work.

⁹ For a good overview of Heidegger’s approach to truth, see Wrathall, *Heidegger and Unconcealment*, 11–39.

¹⁰ Badiou, *Being and Event*, 1–2. It should be noted that Badiou expresses in a note his distaste for thinkers who follow Heidegger without explicitly denouncing his political commitments. As a result, he signals his ontological indebtedness in particular to thinkers like Philippe Lacoue-Labarthe and Jean-Luc Nancy. See *ibid.*, 482–83.

Heideggerian talk of being, the point of such talk is relatively straightforward. Mark Wrathall nicely summarizes Heidegger's basic point: "Heidegger argued that entities are constituted as the entities they are by the relationships they bear to things, people, activities, and so on. Nothing is what it is without these relationships."¹¹ Being concerns "the meaningful relational structure within which entities can show up as what they are."¹² When Heidegger speaks of "ontological difference," then, of the difference between being and beings, he has reference to the difference and the complex conditioning relationship between, on the one hand, entities as they show up in a given world and, on the other hand, what governs the bearing of those entities in their showing up. On Heidegger's account, what governs the bearing of entities in their showing up actually changes over time, passing through successive "constellations of intelligibility," with the consequence that entities are, as it were, determined by changing existential categories (or simply "existentials").¹³ Badiou, along with many of Heidegger's French heirs, expresses less interest in the specifically existential cast of the Heideggerian project than in its structure, in the dynamic entanglement of being and beings as such.¹⁴ A first major consequence of the Heideggerian picture for someone like Badiou, then, is that work in ontology must amount to a close investigation of what lies behind and occasionally interrupts regimes of appearance, punctuating the history of intelligibility.

¹¹ Wrathall, *Heidegger and Unconcealment*, 13.

¹² *Ibid.*

¹³ Thomson, *Heidegger on Ontotheology*, 8. In a rough Anglo-American parallel, Thomas Kuhn writes pithily of "a sort of post-Darwinian Kantianism," a Kantianism "with moveable categories." Kuhn, *The Road Since Structure*, 104, 264. The relationship between Heidegger and Kant, however, is complicated, making even the assimilation of the former's existentials to the latter's categories complicated. For Heidegger's comments on the nature of the "existentials" and their relationship to Kantian categories, see Heidegger, *Being and Time*, 28–35.

¹⁴ In a note to *Being and Event*, Badiou cites as Heidegger's three most important heirs in France Jacques Derrida, Jean-Luc Nancy, and Philippe Lacoue-Labarthe. See Badiou, *Being and Event*, 482–83. Perhaps the best single representative work of this cadre of French Heideggerians is Nancy, *Being Singular Plural*.

For a Heideggerian of Badiou's sort, what constitutes "metaphysics" as opposed to ontology is the gesture of mystifying the entanglement of being and beings by fixing it in one way or another. In Iain Thomson's words, "metaphysics establishes both the most basic conceptual parameters and the ultimate standards of legitimacy for history's successive epochs of unified intelligibility."¹⁵ For his part, Badiou puts this point in classical philosophical terms, that is, in terms of the "one" and the "many." In a late-1990s study meant to pave the way from *Being and Event* to its sequel, *Logics of Worlds*, Badiou explains regarding what he takes to be the upshot of Heidegger's project: "Metaphysics can be defined as follows: the *enframing* of Being by the One. . . . [Therefore] the question from which I began speculating," culled from Heidegger's work, "can now be formulated as follows: Can the One be unsealed from Being?"¹⁶ This question is essential for Badiou, since the possibility of *change*, of *revolution*, is in his view predicated on the plurality or multiplicity of being. At any rate, there is no question that Badiou's primary motivation in utilizing set theory is to attempt to address what he takes to be the Heideggerian question of being: How can being be thought without the Parmenidean One? As Badiou states in a study of Gilles Deleuze also produced in the late 1990s, twentieth-century philosophy has "been stamped and signed . . . by the return of the question of Being," and "this is why it is dominated by Heidegger."¹⁷ Badiou's entire project makes clear that, so far as he is concerned, the Heideggerian question of being—the task, that is, of thinking being as pure multiplicity—must be answered adequately if

¹⁵ Thomson, *Heidegger on Ontotheology*, 8.

¹⁶ Badiou, *Briefings on Existence*, 34. It is again worth noting that this interpretation of Heidegger is controversial—and particularly French. Badiou's Heidegger is the Heidegger of thinkers like Nancy and Lacoue-Labarthe. For this reason, Badiou is occasionally careful to specify that he comments only on "the 'current' Heidegger," that is, the Heidegger "who organizes [French philosophical] opinion." Badiou, *Manifesto for Philosophy*, 47.

¹⁷ Badiou, *Deleuze*, 19.

the question of truth is to be asked and answered in a productive way.

It is thus that Badiou finds himself, at the opening of *Being and Event*, “on the brink of a decision,” which “can take no other form than the following: the one *is not*.”¹⁸ Of course, Badiou does not deny that “there is oneness” (that is, that things appear consistently rather than chaotically in the world), but he insists that “the one, which is not, solely exists as *operation*” (rather than as some kind of metaphysical reality with an ontologically positive status).¹⁹ This Badiou presents as an axiomatic decision, a decision wagered in “attempting the passage to a second modernity,” a second attempt at secularization.²⁰ The one thus serves for Badiou, according to this decision, as the operation by which the consistency of what is presented is produced—consistency not yet in the strict sense utilized in the formal sciences (the lack of any contradiction between sentences contained in a given system), but at this point simply in the sense of matter cohering “so as to ‘stand together’ or retain its form.”²¹ What is presented thanks to this operation is, of course, multiple, but consistently so—that is, as so many countable objects making up a given situation. In effect, the operation of the one secures the numerability of beings, their appearance as discrete individuals. As Badiou puts it, what

¹⁸ Badiou, *Being and Event*, 23, emphasis in original. For Badiou, the decision in favor of multiplicity (over the one) is demanded by the sheer fact that God—metaphysical figure of the one—is dead. I will come back to this question of the death of God.

¹⁹ *Ibid.*, 24, emphasis in original. One should be careful not to read too much into Badiou’s talk of “operation” here. He certainly does not mean to suggest that there is some subject (or, worse, Subject) who stands outside of being and imposes structure and consistency on it. Rather, by “operation” he seems to mean something as simple as process, and he could be said to have reference to the sort of thing Louis Althusser found in Marxism, as already in Hegel (in a problematic form) and in Spinoza: “a process without a subject.” See Althusser, “*Lenin and Philosophy*” and *Other Essays*, 107–25; and Althusser, *Essays in Self-Criticism*, 126–41.

²⁰ Badiou, *Number and Numbers*, 13.

²¹ I borrow these words from the Oxford English Dictionary. See “consistency, n.,” *OED Online*, <http://www.oed.com.libproxy.unm.edu/view/Entry/39644?redirectedFrom=consistency> (accessed February 3, 2015). Adrian Johnston seems to me ultimately correct when he claims that Badiou deploys, in *Being and Event*, “an equivocal use of the word ‘inconsistency,’” or at least of the word “consistency.” Johnston, “Phantom of Consistency,” 354. Whether this equivocal use is ultimately problematic, however, seems unclear to me.

the one produces is “the multiplicity of composition which is that of number and the effect of structure,” given realms of presentation within which discrete and therefore countable individual beings appear as such.²² Arithmetic and its derivatives are the proper sciences of presented multiplicity, disciplines that take the measure and allow for the manipulation of the consistently presented—that is, more simply put, of entities. But because consistent multiplicity, as product of the one-as-operation, is only what *is presented*, Badiou determines that the (consistent) multiple, quite as much as the one, ultimately is not: “being is neither one (because only presentation itself is pertinent to the count-as-one), nor multiple (because the multiple is *solely* the regime of presentation).”²³ Strictly speaking, being is, according to Badiou, nothing—founded on and woven from nothing that can be said to exist.

If being is neither one nor multiple (but in fact nothing), how is the Heideggerian question of being to be addressed? Badiou’s answer is that there are two sorts of multiplicity. In addition to consistent multiplicity (the result of the one as operation), there is “inconsistent multiplicity.”²⁴ Here, again, Badiou seems clearly to mean with his talk of inconsistency not the sort of inconsistency central to the formal sciences, but the sort of inconsistency that would be attributable to matter that fails to cohere or “stand together” or retain any intelligible form. Being, according to Badiou, is thus pure or inconsistent multiplicity, while beings are consistently multiple due to the operation of the one. Put another way, while beings are entirely structured, “there is no structure of being.”²⁵ This directly implies that ontology, the science of being, if it exists, will be “the

²² Badiou, *Being and Event*, 25.

²³ *Ibid.*, 24.

²⁴ *Ibid.*, 25.

²⁵ *Ibid.*, 26.

science of the multiple qua multiple . . . , subtracted from the one in its being.”²⁶ Badiou further explains: “What is required is that the operational structure of ontology discern the multiple without having to make a one out of it, and therefore without possessing a definition of the multiple. . . . The prescription . . . operates such that it is only ever a matter of pure multiples, yet there is no defined concept of the multiple to be encountered anywhere.”²⁷

To this point, Badiou might be said to be following Heidegger, at least as he understands him. The Heideggerian project is (at least on one interpretation) similarly to think being without the Parmenidean One, and so to think inconsistent multiplicity. And yet Badiou is insistent that at this point there is a “Great Temptation” that must be avoided: the temptation to claim that being, in its inconsistency, cannot be thought consistently. In Badiou’s terms, the temptation is to assume that “ontology is not actually a situation,” that being as such (in its inconsistent multiplicity) cannot be thought.²⁸ On Badiou’s account, there are three common or historically traceable ways of giving in to this temptation: (1) by rendering being so radically other that it cannot be thought, as in negative theology; (2) by making being what can only be experienced rather than thought, as in certain traditions of experiential religion; or (3) by taking being to be thinkable only through the resources of the poem. These ways of giving in to temptation correspond to three distinct Gods, the deaths of which Badiou announces consistently in his work.²⁹ The last of these Gods—“the nostalgic God of the return”—Badiou associates

²⁶ *Ibid.*, 28.

²⁷ *Ibid.*, 29.

²⁸ *Ibid.*, 26.

²⁹ For the most detailed account of this, see Badiou, *Briefings on Existence*, 21–32.

with Heidegger.³⁰ Although Badiou follows Heidegger on the nature of the question of being, as well as on the basic outline of what it would mean to provide a satisfactory answer to the question of being, he parts ways with him on exactly what resources must be summoned in order to address it adequately. In the place of Heidegger's recourse to the poem, a move that traps Heidegger within the third form of theism, Badiou proposes an ontology beginning from set theoretical mathematics.³¹

As Badiou tells the story, the foundations of set theory were laid only when several points of naiveté bound up with Georg Cantor's original presentation of set theory were overcome. And for Badiou, these were overcome principally in the wake of the discovery, by Bertrand Russell, of the paradox that follows directly from Gottlob Frege's "Basic Law V." The original set theoretical intuition was that a set is a total grouping of intuited or intuitable objects. Frege went on to formulate this intuition as follows: Every thinkable predicate picks out a real class or set of objects (even if, of course, that class or set is empty). In order to secure the usefulness of this formulation, Frege found it necessary to assume his Basic Law V, which effectively guarantees the possibility of exchanging predicates and sets.³² Frege saw that this law was necessary for his larger project, which was to trace the basic operations of arithmetic to the irrefutable laws of logic. Russell famously discovered, however, that Frege's Basic Law V leads directly to a paradox—one that, in Frege's view, spelled disaster for set theory as such.³³

³⁰ *Ibid.*, 29. Badiou clarifies elsewhere his reasons for calling this God "nostalgic": "Essentially, the relationship to the poetic God . . . is a nostalgic relationship. It melancholically envisages a chance to re-enchant the world through the gods' improbable return." Badiou, *Briefings on Existence*, 28. A certain confirmation of at least this *tendency* among Heidegger's heirs can be witnessed in Dreyfus and Kelly, *All Things Shining*.

³¹ For more on Badiou's relationship to Heidegger, see Badiou, *Manifesto for Philosophy*, 47–52, 69–77.

³² For the basic outline of this law and its role in Frege's project, see Frege, *The Basic Laws of Arithmetic*.

³³ This is too roughly stated. Russell's original letter to Frege located the paradox in Frege's basic intuitions, but Frege himself quickly recognized the connection between the paradox and his Basic Law V.

Frege's axiom did not, however, necessarily have the disastrous implications Frege felt it to have. In the wake of Russell's discovery, a number of logicians and mathematicians attempted to revitalize Frege's basic logicist approach to arithmetic—even retaining his basic definitions of the numbers—but they did so by attempting to provide a set of axioms that would allow for a consistent deployment of Basic Law V. The strategy was to establish a series of “types” (as Russell originally called them), that is, a differentiated hierarchy of languages each of which can refer only to a language of a type “lower” than itself.³⁴ (The same general strategy has already been witnessed, of course, in Tarski's distinction between object-language and metalanguage, described in Chapter 2.) What eventually took shape, fully formalized by Zermelo and Fraenkel, was the basic axiom system that regulates the use of a single relation—that of “belonging” (of belonging to a set)—that avoided any of the naïve notions concerning sets that were originally espoused in Cantor's founding gesture.³⁵ Set theory as it was formulated by Zermelo and Fraenkel allowed for a tracing of all the basic operations of arithmetic to just a handful of axioms, only two of which were in any way simply or purely existential: (1) the axiomatic affirmation that there is an empty set or a set of nothing, and (2) the axiomatic affirmation that there are some infinite sets. The other axioms allow for the systematic manipulation of these axiomatically existent sets such that it is possible to produce other sets that can be defined unproblematically as the numbers used in

See the appropriate correspondence between Russell and Frege in Frege, *Philosophical and Mathematical Correspondence*, 130–33.

³⁴ For the basic view here, already articulated in many ways in connection with Tarski, see Russell, “Mathematical Logic as Based on the Theory of Types.”

³⁵ There are important differences between type theory and set theory that I have overlooked here in order to draw on the broad similarity of their strategies with respect to the paradox resulting from Frege's Basic Law V. The most important of these differences is that the basic axioms of Zermelo-Fraenkel set theory do not establish in advance a series of carefully defined types, to some one of which every determinate element must appear (as in type theory); rather, they simply forestall the sort of self-relation that leads to the undesirable paradox.

arithmetic.

It is this picture of set theory that interests Badiou, and it is this picture of set theory on which he draws in arguing that mathematics can provide a thinking of pure or inconsistent multiplicity. Zermelo-Fraenkel set theory provides an account of pure multiplicity because it allows for the construction of multiples whose constituent elements are all themselves constructed from (or simply are) the empty set. The empty set can be said to be a pure multiple inasmuch as it is a multiple of nothing, a set that does not group together any thing or things. Classical set theory (except in its wager that there are some infinite sets) presupposes as existent only that pure multiple, although it provides the axioms necessary to regulate the construction of other multiples out of that single pure multiple. The result is that set theory regulates the infinite proliferation of pure multiplicity (this is true even of the axiom of infinity, which affirms the existence of some infinite sets), the indefinite construction of multiples of multiples of multiples—with no constituent element of any multiple being made up of anything but manipulations of the purest multiple, the empty set. In the axioms of set theory, this purest multiplicity is counted, but the empty set itself remains as the inconsistent excess of being over its consistent presentation.

Badiou sets forth this basic intuition in terms again reminiscent of Heidegger. In a famous (and infamous) 1929 lecture, “What Is Metaphysics?” Heidegger speaks of the realm of presentation—the realm of consistent beings—and of what exceeds that realm:

That to which the relation to the world refers are beings themselves—and nothing besides. That from which every attitude takes its guidance are beings themselves—and nothing further. That with which the scientific

confrontation in the irruption occurs are beings themselves—and beyond that nothing. But what is remarkable is that, precisely in the way scientific man secures to himself what is most properly his, he speaks of something different. What should be examined are beings only, and besides that—nothing; beings alone, and further—nothing; solely beings, and beyond that—nothing. *What about this nothing?*³⁶

Asking whether such talk of “nothing” is “only a manner of speaking,” whether it is only “an accident that we talk this way so automatically,”³⁷ Heidegger goes on to argue that it is precisely through the exclusion of the nothing that the consistency of the scientifically investigatable realm (the realm of consistent presentation, of empirical experience) is secured—that is, in Heidegger’s language, that “beings . . . are revealed somehow as a whole.”³⁸ The nothing, Heidegger contends, can be and is encountered, but only in “a fundamental experience” to which science does not give its attention—the experience, for instance, of boredom (but also, Heidegger notes in passing, of love).³⁹ Outside or in excess of consistent presentation, excluded from the world of entities, the nothing operates. As Heidegger puts it, “the nothing nothings [*das Nichts nichtet*].”⁴⁰ Or again: “In the Being of beings the nihilation of the nothing occurs.”⁴¹

Clearly echoing Heidegger’s formulation—albeit without the existentialist overtones—Badiou describes his interest in the empty set as follows: “To put it more

³⁶ Heidegger, “What Is Metaphysics?” 95, emphasis added.

³⁷ *Ibid.*

³⁸ *Ibid.*, 99. Note here that I, like Badiou, use the word “consistency” without having reference to its more strict use in the formal sciences.

³⁹ *Ibid.* For a fuller exposition by Heidegger of boredom, see Heidegger, *The Fundamental Concepts of Metaphysics*, 59–167. For an exposition of love’s relationship to the nothing, see Badiou, *Conditions*, 179–98.

⁴⁰ Heidegger, “What Is Metaphysics?” 103, translation modified.

⁴¹ *Ibid.*, 104.

clearly, once the entirety of a situation is subject to the law of the one and consistency, it is necessary, from the standpoint of immanence to the situation, that the pure multiple, absolutely unrepresentable according to the count, be *nothing*. But being-nothing is as distinct from non-being as the ‘there is’ is distinct from being.”⁴² Just as Heidegger argues that “the nothing makes possible the openedness of beings as such,”⁴³ Badiou argues that the “being-nothing” marked by the empty set “is the base of there being the ‘whole’ of the compositions of ones in which presentation takes place.”⁴⁴ Pure or inconsistent being, as opposed to apparent and therefore consistent beings or entities, is what exceeds presentation, according to Badiou. It is nothing—“not a-nothing” but simply “‘nothing,’ phantom of inconsistency,” the “name of unrepresentation in presentation.”⁴⁵ For Badiou as much as for Heidegger, the nothing is an operation, a nihilation or a “nothinging”: “It comes down to exactly the same thing to say that the nothing is the operation of the count . . . and to say that the nothing is the pure multiple upon which the count operates.”⁴⁶

It might be possible to put a finer point on Badiou’s gesture here by remembering that it was Heidegger’s 1929 lecture that drew the interest—and the philosophical scorn—of Rudolf Carnap.⁴⁷ In a famous 1930 paper, Carnap took the very passage from Heidegger quoted above—representative, he said, of “that metaphysical school which at present exerts the strongest influence in Germany”—as an example with which “to show that the possibility of forming [metaphysical] pseudo-statements is based on a logical

⁴² Badiou, *Being and Event*, 53.

⁴³ Heidegger, “What Is Metaphysics?” 104.

⁴⁴ Badiou, *Being and Event*, 54.

⁴⁵ *Ibid.*, 55.

⁴⁶ *Ibid.*

⁴⁷ On Carnap’s interest in Heidegger in 1929–30, see Friedman, *A Parting of the Ways*, 7–9.

defect of language.”⁴⁸ Carnap’s simple critique amounts to the claim that Heidegger is fooled by the grammatical similarity between sentences like “Rain is outside” and “Nothing is outside.”⁴⁹ Heidegger’s metaphysics, Carnap says, “is simply based on the mistake of employing the word ‘nothing’ as a noun, because it is customary in ordinary language to use it in this form in order to construct a negative existential statement.”⁵⁰ Thus where “Nothing is outside” appears *grammatically* to make of “nothing” a name that refers to some identifiable object (and which can therefore be spoken of meaningfully), it *logically* does no more than position a negative existential quantifier before a predicate: “there does not exist an x such that x is outside,” $\sim(\exists x)(O_x)$. Consequently, Carnap asserts that it is entirely impossible to construct in logically correct language Heidegger’s series of questions: “What about this nothing?”⁵¹

Badiou might be said to be, in effect, providing a direct response to Carnap’s criticisms of Heidegger. Rather than assuming that Heidegger means simply to draw on the grammatical similarity between sentences like “Rain is outside” and “Nothing is outside,” Badiou can be interpreted as finding in Heidegger a kind of set theoretical gesture—although, of course, Heidegger himself never made any explicit connection between his thinking and set theory. To speak of the nothing is, Badiou suggests, to speak of the empty set, and there is an unmistakably logically correct way of formulating the empty set. Badiou explains: “In its technical formulation—the most suitable for conceptual exposition—the axiom of the void-set will begin with an existential quantifier (thereby declaring that being invests the Ideas), and continue with a negation of existence

⁴⁸ Carnap, “The Elimination of Metaphysics through Logical Analysis of Language,” 69.

⁴⁹ *Ibid.*, 70.

⁵⁰ *Ibid.*

⁵¹ *Ibid.*

(thereby un-presenting being) . . . : $(\exists\beta) [\sim(\exists\alpha)(\alpha \in \beta)]$. This reads: there exists β such that there does not exist any α which belongs to it.”⁵² On Badiou’s account, this logical (and set-theoretical) formulation— $(\exists\beta) [\sim(\exists\alpha)(\alpha \in \beta)]$ —captures quite precisely the Heideggerian conception of the nothing that operates outside the realm of consistent presentation. So far as Badiou is concerned, then, Heidegger’s questions concerning the nothing are questions about what role the empty set plays in ensuring that the several axioms of Zermelo-Fraenkel set theory have a founding pure multiple from which to weave other multiples. Heidegger’s insight concerns the rigorous thinking of being, not the mere play of grammatically deceptive language. Heidegger’s mistake, according to Badiou, lay *only* in privileging poetry among possible resources for thinking the nothing. Heidegger failed only to see the resources of the mathematical for the thinking of being.

Voids and Events

If it is Heidegger whom Badiou takes as a kind of beginning point for his philosophical work, guiding him toward the possibility of drawing on the resources of set theory for doing work in ontology, it must nonetheless be said that Badiou leaves Heidegger more or less behind once he turns his attention to set theory. If a full-blooded ontology is to be discovered at work in the thinking of those who founded set theory, then it will be necessary to pay close attention to the details of the mathematical theory, and it is in proceeding to the details of set theory that Badiou begins to wind his way toward nominating a specific operation by which sets rightly designatable as truths might be constructed. Consequently, a major portion of Badiou’s *Being and Event* is a massive

⁵² Badiou, *Being and Event*, 68. When, between parentheses, Badiou refers here to “the Ideas,” he has reference to the several axioms of set theory.

philosophical exegesis of or commentary on the several axioms of set theory: their motivations, their development, their formulations, their implications, and the impasses to which they lead. A handful of these axioms are, of course, relatively straightforward, and Badiou deals with them in short order. Others among the axioms, however, are a good deal more complex, and Badiou develops their exposition over the course of many chapters.

The first set theoretical axiom to which Badiou gives detailed attention is that of the powerset. At the heart of this axiom is the difference between two set theoretical relations—one of which can be entirely defined in terms of the other—namely, what Badiou (following Tarski) calls “belonging” or “being an element” and “inclusion” or “being a part/subset.”⁵³ Badiou summarizes the difference straightforwardly: “There is the originary relation, *belonging*, . . . which indicates that a multiple is counted as element in the presentation of another multiple. But there is also the relation of *inclusion*, . . . which indicates that a multiple is a sub-multiple of another multiple.”⁵⁴ Even more simply put, to say that a multiple *b* is included in (rather than belongs to) another multiple *a* is to say that every multiple that belongs to *b* also belongs to *a*.⁵⁵ This last way of putting the difference is important because it makes clear that inclusion is defined entirely in terms of belonging. Consequently, as Badiou explains, “these determinations—element and subset—do not allow one to think anything intrinsic.”⁵⁶ The being of a multiple *a* remains what it is regardless of whether it is considered in terms of its elements or in terms of its parts. As Badiou says: “In one case . . . , the multiple falls under the count-as-

⁵³ For Tarski’s use of these same terms, see Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 68–69, 74–77.

⁵⁴ Badiou, *Being and Event*, 81.

⁵⁵ See *ibid.*, 62.

⁵⁶ *Ibid.*, 82.

one which is the other multiple. In the other case . . . , every element presented by the first multiple is also presented by the second. But being-multiple remains completely unaffected by these distinctions of relative position.”⁵⁷

What makes the distinction between belonging and inclusion important for Badiou is the role it plays in the powerset axiom. This axiom states simply that the existence of a set directly entails the existence of another set made up of all of the subsets of the first. The subsets or parts of a set themselves form an independent set. This might be best illustrated by having recourse to naïve set theory. Imagine I have a set with three elements: (1) my watch, (2) my wallet, and (3) my keys. Given the definition of inclusion outlined above, it is possible also to say that there are eight parts of this set, eight subsets of this set: (1) the set of my watch; (2) the set of my wallet; (3) the set of my keys; (4) the set of my watch and my wallet; (5) the set of my watch and my keys; (6) the set of my wallet and my keys; (7) the set of my watch, my wallet, and my keys; and, finally and perhaps surprisingly, (8) the empty set. What the powerset axiom states is that the existence of this set—with its three elements and its eight parts—directly entails the existence of a set of its parts. In other words, if there exists the above set with its three elements, then there exists also a distinct set of its eight parts, a set whose eight elements are the subsets or parts of the first set. (That second set, now with eight elements, would in turn have subsets or parts—two hundred and fifty-six of them, in fact!—and those subsets or parts would make up the elements of still another set, according to the powerset axiom.)

Badiou strongly emphasizes the fact that a set and its powerset are two distinct sets:

⁵⁷ *Ibid.*

The set $p(\alpha)$ of all the subsets of the set α is *a multiple essentially distinct from α itself*. This crucial point indicates how false it is to sometimes think of α as forming a one out of its elements (belonging) and sometimes as the whole of its parts (inclusion). . . . The gap between α (which counts-as-one the belongings, or elements) and $p(\alpha)$ (which counts-as-one the inclusions, or subsets) is, as we shall see, the point in which the impasse of being resides.⁵⁸

At issue in the difference between a set and its powerset, on Badiou's interpretation, is the difference between what he calls a set's structure and what he calls its metastructure. The powerset of a given set, he explains, is "a metastructure, another count, which 'completes' the first in that it gathers together all the sub-compositions of internal multiples, all the inclusions."⁵⁹ In effect, the powerset axiom prescribes "that every count-as-one be doubled by a count of the count, that every structure call upon a metastructure."⁶⁰ According to what at first appears to be a simple metaphor, Badiou claims that the relationship between a given set and the powerset of that set should be understood in terms of the relationship between a *situation* and the *state* of that situation.⁶¹

As I have already indicated above, Badiou follows Heidegger in taking the

⁵⁸ *Ibid.*, 83.

⁵⁹ *Ibid.*

⁶⁰ *Ibid.*, 84.

⁶¹ Badiou often sounds in *Being and Event* as if he were employing a simple political metaphor: powersets operate like states, at the level of representation, attempting to secure the consistency of the situation over which they hold power. More strictly, however, Badiou does not see the relationship between powersets and political states as metaphorical. They are structurally parallel because they appear in parallel but distinct truth procedures, and similarly parallel structures can be found in the truth procedures associated with art and love. For full clarification of the relationships among the several truth procedures, see Badiou, *Manifesto for Philosophy*, 27–39; and Badiou, *Conditions*, 3–25.

consistency of every situation to be linked to the excess of the nothing over it,⁶² just as he therefore follows Heidegger in suggesting that the nothing haunts the consistent situation from which it has been excluded as a kind of ghostly trace. What constitutes the consistency of a situation is, precisely, the fact that the nothing remains, inconsistently, unrepresented. In an unmistakable allusion to Heidegger, then, Badiou speaks of “the situational anxiety of the void,” the implicit recognition that every situation is in a way threatened by the operation of the nothing that lies at its foundation.⁶³ In order to ward off the threat that underlies such “situational anxiety,” according to Badiou, “it is necessary to prohibit that catastrophe of presentation which would be its encounter with its own void, the presentational occurrence of inconsistency as such.”⁶⁴ But in order for such a prohibition to be fully operative, “*it is necessary that structure be structured.*”⁶⁵ This is the ontological motivation for the powerset axiom. The axiom is necessary because it protects the consistency of an original situation (that is, of a particular set) by inscribing within a second consistent situation (the powerset of the original set) the operation by which that original situation was produced: the count-as-one.

This move on Badiou’s part must be understood carefully. I have already cited an important passage in which Badiou claims that “it comes down to exactly the same thing to say that the nothing is the operation of the count . . . and to say that the nothing is the pure multiple upon which the count operates.” This, it turns out, is essential to his

⁶² That Heidegger understands such a link to exist seems clear from his three formulae introducing the nothing: “That to which the relation to the world refers are beings themselves—and nothing besides. That from which every attitude takes its guidance are beings themselves—and nothing further. That with which the scientific confrontation in the irruption occurs are beings themselves—and beyond that nothing.” Heidegger, “What Is Metaphysics?” 95.

⁶³ Badiou, *Being and Event*, 93. Anxiety is, of course, the chief experience of the nothing on Heidegger’s account. See Heidegger, “What Is Metaphysics?” 100–101; as well as Heidegger, *Being and Time*, 225–35.

⁶⁴ Badiou, *Being and Event*, 93.

⁶⁵ *Ibid.*

understanding of the powerset axiom. Here is the larger passage in which the line just quoted appears, containing the full argument for this point:

By itself, the nothing is no more than the name of unpresentation in presentation. Its status of being results from the following: one has to admit that if the one results, then “something”—which is not an in-situation term, and which is thus nothing—has not been counted, this “something” being that it was necessary that the operation of the count-as-one operate. Thus it comes down to exactly the same thing to say that the nothing is the operation of the count—which, as source of the one, is not itself counted—and to say that the nothing is the pure multiple upon which the count operates—which “in-itself,” as non-counted, is quite distinct from how it turns out according to the count.⁶⁶

On Badiou’s account, what Heidegger called “the nothinging of the nothing” is the count-as-one, producer of consistency through its own exclusion from the realm of presentation. But that same “nothinging of the nothing” is that upon which the count operates, the radically unrepresented. The operation of the count-as-one is entirely equivalent to the nothing as such, pure inconsistent being.

All this bears on the powerset axiom because the powerset of any given set has as one of its elements the empty set. This means that the nothing, Badiouian pure or inconsistent being, at once does *not* belong to and yet *is* included in the consistently presented set. (This rigorous formulation allows for a clarification of what I just above called the “exclusion” of the nothing from consistent presentation. Exclusion *here* means precisely inclusion without belonging. Similarly, anxiety as Badiou understands it and

⁶⁶ *Ibid.*, 55.

appropriates it from Heidegger can be formulated as the ontological mood associated with the inclusion but non-belonging of the void.) The inclusion of the nothing—equivalent to the very operation of the count-as-one—in a set to which it nonetheless does not belong as an element, even while its unpresentation secures the consistency of the presentation of that set, makes clear that the powerset counts the very count-as-one, presenting as an element in itself what in the original set is the unrepresented but included nothing. In this sense, the powerset or state effectively *counts the count*, and it thus, on Badiou’s interpretation, guarantees the consistency of the set from whose parts it draws its elements. If what is counted in an original situation can be regarded as what is *presented*, then, Badiou suggests, what shows up in the count of the count, in the state of the situation, can be regarded as what is *represented*. “There is always presentation and representation.”⁶⁷

The difference between presentation and representation allows Badiou to outline a brief typology. He calls “normal” whatever is both presented in a situation *and* represented in the state of that situation, “excrement” whatever is *not* presented in a situation but nonetheless *is* represented in the state of that situation, and “singular” whatever *is* presented in a situation but is *not* represented in the state of that situation. On Badiou’s account, nature is entirely normal. This is glimpsed in the operations that allow for the construction of the ordinal numbers in set theory. Any set where every element is also a part, where whatever belongs to the set is also included in the set, is strictly normal, representative of “the maximum correlation between belonging and inclusion.”⁶⁸ Each of the ordinal numbers is normal in this way. Badiou explains this by reference to

⁶⁷ *Ibid.*, 94.

⁶⁸ *Ibid.*, 131.

the set theoretical definition of the number two:

Let's consider the set of subsets of this $\{\emptyset\}$ [that is, of the set of the empty set], that is, $p(\{\emptyset\})$ [this is the notation Badiou employs for the powerset ($p(x)$) of the set ($\{x\}$) of the empty set (\emptyset)] What would the parts of $\{\emptyset\}$ be? Doubtless there is $\{\emptyset\}$ itself, which is, after all, the "total part." There is also \emptyset , because the void is universally included in every multiple It is evident that there are no other parts. The multiple $p(\{\emptyset\})$, set of parts of the singleton $\{\emptyset\}$, is thus a multiple which has *two* elements, \emptyset and $\{\emptyset\}$. Here, woven from nothing apart from the void, we have the ontological schema of the Two, which can be written: $\{\emptyset, \{\emptyset\}\}$ Consequently, the two elements of the Two are also two parts of the Two and the Two is transitive insofar as it makes a one solely out of multiples that are also parts.⁶⁹

But not only is the number two itself normal, all of its elements are normal. And the same turns out to be true for every ordinal number. Not only can an analysis like Badiou's above be carried out for each ordinal, it is possible to provide a proof that all ordinals are normal in this way.⁷⁰

Nature—as what is captured by the operations of mathematics and physics—is thus, for Badiou, entirely normal.⁷¹ Quite different in essence, of course, is history. According to Badiou, history is what trades in the "abnormal," and more specifically in the singular, those multiples that, while failing to be represented (by the state of a

⁶⁹ *Ibid.*, 132; I have corrected the notation in the translation, which contains some errors.

⁷⁰ Badiou provides the proof in *ibid.*, 133.

⁷¹ Badiou includes in *Being and Event* a polemic against Heidegger's interpretation of nature (of Greek *physis*) as part of his exposition of this point. See *ibid.*, 123–29.

situation), are nonetheless presented (in that situation). Badiou explains: “That a presented multiple is not at the same time a subset of the situation necessarily means that certain multiples from which this multiple is composed do not, themselves, belong to the situation.”⁷² A singularity hides within or beneath it, so to speak, something foreign to the situation to which the singularity itself belongs. Badiou illustrates the idea as follows:

Here is an image (which in truth is merely approximate): a family of people is a presented multiple of the social situation (in the sense that they live together in the same apartment, or go on holiday together, etc.), and it is also a represented multiple, a part, in the sense that *each* of its members is registered by the registry office, possesses French nationality, and so on. If, however, one of the members of the family, physically tied to it, is not registered and remains clandestine, and due to this fact never goes out alone, or only in disguise, and so on, it can be said that this family, despite being presented, is not represented. It is thus *singular*. In fact, one of the members of the presented multiple that this family is, remains, himself, un-presented *within the situation*.⁷³

Even as the family from this illustration is presented in the situation, one member of the family is not, with the consequence that, while most of the family’s several members are individually represented by the state, *one* member of the family is not so represented and, as a result, neither is the family as a whole.

Among possible singularities, Badiou focuses on a unique sort—the *entirely* abnormal multiple or the multiple *none* of whose elements are presented in the situation

⁷² *Ibid.*, 174.

⁷³ *Ibid.*, emphases in original.

even as the multiple of those un-presented elements is itself presented. Badiou again illustrates: “To employ the image used above, it would be a case of a concrete family, *all* of whose members were clandestine or non-declared, and which presents itself (manifests itself publicly) *uniquely* in the group form of family outings.”⁷⁴ So far as the situation in which such a multiple presents itself would be concerned, it would have as its elements *nothing*—not necessarily making it the unique empty set already discussed, but making it a kind of void *for the situation in which it is presented*. For this reason, Badiou describes the entirely abnormal multiple as “on the edge of the void.”⁷⁵ As Badiou says, such a multiple bears a consistency that is “composed solely from what, with respect to the situation, in-consists.”⁷⁶

The distinction between the void *proper* (the actually empty set) and what might be called the *situational* void (just described) secures the distinction between nature and history. According to the axiom of foundation, every given set has belonging to it some multiple that either is the empty set itself or is, in the sense described above, on the edge of the void. That guaranteed void—the void proper or the situational void—serves as the situation’s foundation, the “halting point” beneath which, so to speak, there is nothing that belongs to the situation.⁷⁷ When a given situation has the actual empty set as its foundation (as with, for example, the ordinals), nature is what presents itself. When, on the other hand, a given situation takes some entirely abnormal set as its foundation, history is what presents itself. As Badiou summarily puts this point, “what is natural is

⁷⁴ *Ibid.*, 175, emphases in original.

⁷⁵ *Ibid.*

⁷⁶ *Ibid.*

⁷⁷ *Ibid.*, 186.

what is founded solely by the void; all the rest schematizes the historical.”⁷⁸ Historical situations present some multiple that is on the edge of the void, marking a kind of border (Badiou speaks of a “border effect”) across which nothing can pass if the consistency of the historical situation is to remain intact—that is, if the well-foundedness of the historical situation is not to be compromised.⁷⁹ But, of course, what interests Badiou precisely is the set of circumstances in which the well-foundedness of a historical situation *is* compromised. To such circumstances Badiou gives the name of the event.

The event is perhaps the most crucial concept Badiou introduces in *Being and Event*. According to his proposal, an event is an ill-founded multiple, ill-founded because it breaks the ontological law expressed by the axiom of foundation. Badiou states that “the event is a one-multiple made up of, on the one hand, all the multiples which belong to its site [that is, all the elements of a multiple positioned on the edge of the void for a particular situation], and on the other hand, the event itself.”⁸⁰ (The basic schema for the event is thus as follows, where X is the evental site and e is the event: $e_x = \{ x \in X, e_x \}$). In words, this says that an event is a set [1] to which belong all the elements of a situation all of whose elements are in turn unrepresented in that situation, and [2] to which the event itself belongs as well.)⁸¹ In short, an event is a set that augments a situational void by adding to its clandestine elements one further element: the event itself. How is this to be understood? First, it must be remembered that what guarantees the consistency of a historical situation is the non-presentation within it of the elements of its (situational) void. But if there is within a historical situation some set to which belong all those non-

⁷⁸ *Ibid.*, 188.

⁷⁹ *Ibid.*, 175.

⁸⁰ *Ibid.*, 179.

⁸¹ See *ibid.*

presented elements making up the historical situation's void, *and* to which belongs some element that rules out the possibility of that set serving as the historical situation's founding void, then the historical situation in question must be said to be ill-founded. The set that would guarantee its consistency, that would serve as its foundation, effectively breaks the law captured by the set theoretical axiom of foundation. That excess element that troubles the well-foundedness of a historical situation is the event itself: at once the set that would (but cannot) found the historical situation *and* an element of that same set.

At first glance, there seems to be something confused about what has just been presented. That, however, is intentional. What Badiou suggests is that an event is *self-belonging*. A historical site—the founding situational void of a historical situation—gives way to an event when it in effect becomes an element of itself. Paradoxical as this proposal sounds, Badiou insists that it matches the “intuitive” notion of an event. He defends this point by citing the example of the French Revolution. Two sorts of things go into the make-up of the event of the French Revolution, according to Badiou. On the one hand, there belongs to that event “everything delivered by the [revolutionary] epoch as traces and facts,” “everything which makes up its site.”⁸² There is no surprise about this first sort of thing, of course, but Badiou insists that a full inventory of this first sort of thing would be insufficient as an account of the French Revolution; it “may well lead to the unity of the event being undone to the point of being no more than the forever infinite numbering of the gestures, things, and words that co-existed with it.”⁸³ What *other* element is necessary, then? Badiou's answer is that “the halting point for this dissemination is *the mode in which the Revolution is a central term of the Revolution*

⁸² *Ibid.*, 180.

⁸³ *Ibid.*, translation slightly modified.

itself.”⁸⁴ This, then, is the second sort of thing that goes into the make-up of the French Revolution: the Revolution itself. In a crucial way, what makes an event an event is the peculiar way in which it belongs to itself, the way in which it is itself an element of its own make-up.⁸⁵

This intuitive notion of the event is captured in set theoretical terms by taking an entirely abnormal set, on the edge of the void for a particular historical situation, and transforming it into a self-belonging set. Doing this, however, introduces into a historical situation a problematic foundation. The axiom of foundation prohibits every self-belonging set. Indeed, the original motivation for the creation of the axiom of foundation was to ensure that self-belonging sets were prohibited, since it was the thinkability of the self-belonging that led directly to Russell’s paradox (the set of all sets that do not belong to themselves).⁸⁶ At any rate, it is clear that a self-belonging set cannot serve as a genuine halting point—and therefore as a foundation—for a historical situation, guaranteeing its consistency, since something that is presented (the would-be foundational set itself) turns out to be among elements supposedly non-presented in the situation (the elements belonging to the entirely abnormal set). A self-belonging situational void both is and is not presented, and one of its elements (in fact, itself) both is and is not presented.

In light of the paradoxical nature of the event (apparently both presented and non-presented in a given historical situation), Badiou determines to take it as an

⁸⁴ *Ibid.*, emphasis in original.

⁸⁵ Badiou defends this point in *Being and Event* in part by citing the manner in which important figures during the French Revolution cited the Revolution itself, in the form of a name, in their attempts to understand the event. In more recent years, Badiou has recognized the difficulties associated with such an approach, with the slippery equivalence of events and their names. In *Logics of Worlds*, as a result, he has formulated a theory of the event that entirely abandons this problematic formulation. See Badiou, *Logics of Worlds*, 357–96.

⁸⁶ Badiou provides a brief proof showing that if the axiom of foundation and a generic self-belonging set (a ϵ a) are both taken to exist, a contradiction results. See Badiou, *Being and Event*, 190.

undecidable—or, more strictly, he determines to take the statement “the event belongs to the situation” as an undecidable statement. If it is decided that the event does not belong to the situation, then everything in its make-up is foreign to the situation—since neither the event itself nor the several elements of the situational void can be said to be presented in the situation. To such a decision Badiou attaches the Mallarmean formula “nothing will have taken place but the place” (that is, the historical site).⁸⁷ If, however, it is decided that the event does indeed belong to the situation, then the event remains singular (some of its elements are not presented in the situation—namely those elements that belong also to the situational void) but it cannot be equated with the situational void (it is not entirely abnormal, since one of its elements—itsself—is actually presented in the situation). For this reason, Badiou claims that the event, if its pertinence to the situation is decided upon, separates itself from the void of the situation, and it does so by itself (by the presentation of the event). If this latter decision is made, on Badiou’s account, there follows the possibility of a procedure of deductive investigation—an infinite series of inquiries about other elements in a historical situation, each time asking whether the element in question is or is not “connected” to the event.⁸⁸ In this way, a decision regarding the undecidable can, if its deductive consequences are followed out, lead to the production of a set among other sets, one that can be said to be generic because of its orientation to what the laws of being foreclose from the situation in which it is constructed.

In order, however, to see more clearly what is at stake in this last-mentioned procedure, it is necessary to turn from more general ontological considerations to the more specific operation to which Badiou gives extensive attention in the last part of

⁸⁷ *Ibid.*, 182, 191–98.

⁸⁸ *Ibid.*, 234.

Being and Event—namely, the model theoretical operation of forcing. This, moreover, allows us finally to come to the question of how Badiou defines truth.

Set Theory and Truth

In Chapter 1, in addressing Badiou's apparent dismissal of Tarski's theorem as irrelevant to his own work on truth, I discussed briefly the fact that, already in the mid-1970s, Badiou saw the possibility of tying the politically expedient concept of the subject to the decision on the undecidable. What Badiou apparently spent the first part of the 1980s searching for in work on the theorems of Gödel, Tarski, and Löwenheim-Skolem was some way of clarifying for himself the nature of the undecidable and what it means to decide on the undecidable. As I already made clear in Chapter 1, however, what marked the break between Badiou's meanderings in the early 1980s and his subsequent focused production of *Being and Event* was his decision that it was not the generalizable impasse indicated by the production of the Gödelian or Tarskian undecidable that should draw his attention, but the specific impasse for set theory of the undecidability of the continuum hypothesis. And, significantly, it was only as Badiou turned his attention to the continuum hypothesis that he saw that set theory might give him a way to provide a genuinely robust philosophical clarification of the notion of truth. Every decision on the undecidable constitutes a trace of the subject for Badiou, and the deductive process of transformation that follows a decision on an event's belonging to a situation illustrates the quintessential work of a subject. But it is only the decision on the undecidable continuum hypothesis—specifically, *against* the hypothesis—that clarifies the nature of truth.

The continuum hypothesis concerns the consequences of the powerset axiom,

already discussed in some detail above. The respective cardinalities of a given set and its powerset can be determined without any difficulty for finite sets. A set of three elements (I used the example of the set of my watch, my wallet, and my keys above) corresponds to a powerset of exactly eight elements. According to a simple proof, the number of elements in any powerset of some finite set will be 2^n , where n equals the number of elements in the “original” set.⁸⁹ But this simple formula does not tell us anything about the respective cardinalities of an infinite set and its powerset. Cantor famously introduced into mathematics the possibility of distinguishing among transfinite numbers, each assignable to a unique infinite set. These can be constructed in the same way that finite sets assigned to specific numbers are constructed, which I have summarized above in connection with Badiou’s concept of normal sets. What orders all normal sets is the way in which each succeeds another and is succeeded by yet another according to a determinate successor function, which gathers into a set all the numbers it succeeds. The numbers associated with such normal sets, arranged in order of their succession, are referred to as ordinals. Transfinite numbers are ordered among themselves in the same way as finite numbers, according to the same successor function.

What complicates matters when it comes to infinite sets is that the ordinal number assigned to an infinite set does not actually tell us how (comparatively) big it is. The size of a set is determined not by its ordinality (its place in a well-ordered succession of constructed sets), but by its cardinality, which can be defined in terms of one-to-one correspondence. The cardinality of an ordinal set determines a class of all those ordinals whose members can be placed in a one-to-one correspondence.⁹⁰ Every finite ordinal is

⁸⁹ A simple sketch of the proof can be found in Stoll, *Set Theory and Logic*, 11.

⁹⁰ This is, of course, the simple definition of number set forth by Frege in *The Foundations of Arithmetic*.

of unique cardinality, since no two differing finite ordinals can be placed in a one-to-one correspondence. But mathematicians even before Cantor showed that, however paradoxical it might seem, certain infinite sets of distinct ordinality can be shown to have the same cardinality—that is, the members of certain (in fact, infinitely many) infinite sets can be put into a one-to-one correspondence with the members of the infinite sets that immediately succeed them ordinally. Thus while every finite ordinal is of a different cardinality than every other finite ordinal, many transfinite ordinals are of the same size or share the same cardinality. Cantor was also able to show, however, that this was *not* the case for *all* infinite sets. In particular, it was not the case for an infinite set and its powerset; the powerset of an infinite set is always larger (is always of a larger cardinality) than that original set.⁹¹

What Cantor desperately sought to determine but could not was what exactly the difference was between the cardinalities of an infinite set and its (similarly infinite) powerset. His *hypothesis*—the continuum hypothesis—was that the cardinality of the powerset of an infinite set was the next infinite size up (in a series of well-ordered cardinals), that is, the cardinal immediately succeeding the cardinal assignable to the original set. Cantor was himself unable to demonstrate the truth of his hypothesis, and subsequent work in mathematical logic—and especially in model theory—has shown that Cantor’s hypothesis is actually undecidable with respect to the basic axioms of set theory. Kurt Gödel constructed a model of the Zermelo-Fraenkel axioms in which the continuum hypothesis holds true, which demonstrated that it is impossible to prove the hypothesis false from the axioms of set theory alone. And then Paul Cohen constructed a model of

⁹¹ On Cantor’s proof for this point, see the helpful discussion in Priest, *Beyond the Limits of Thought*, 117–19.

the same axiom system in which the continuum hypothesis does not hold, demonstrating that it is equally impossible to prove the hypothesis true from the axioms of set theory alone. Strictly speaking, the continuum hypothesis is undecidable.

Badiou connects the undecidability of Cantor's continuum hypothesis to the pre-Cantorian philosophical debate over "the dialectic of the discontinuous and the continuous."⁹² What the undecidability of the continuum hypothesis marks in the realm of mathematical logic, the interrogation of "the abyss which separates numerical discretion from the geometrical continuum" marks in the history of philosophy.⁹³ Badiou thus finds it possible to line up thinkers from the history of thought behind the representatives of major logico-mathematical approaches to the undecidability of Cantor's hypothesis. Badiou discerns three such approaches, but only two of them seem especially pertinent to his project: those represented respectively by Gödel and Cohen.⁹⁴ Gödel receives his place in this basic typology because of his having produced the model of the set theoretical axioms in which the continuum hypothesis holds.⁹⁵ Whatever Gödel's actual intentions in producing this model (Gödel did not mean to make any particular ontological claims, but just hoped to determine the truth or falsity of the continuum hypothesis), Badiou finds embodied in the model a certain guiding intuition: "whatever is not distinguishable by a well-made language is not."⁹⁶ If the powerset of an infinite set of a determinate cardinality bears the immediately succeeding cardinality, then

⁹² Badiou, *Being and Event*, 281.

⁹³ *Ibid.*

⁹⁴ Paul Livingston has argued that Badiou overlooks a fourth possible orientation, what he calls the "paradoxico-critical" orientation of thought. Livingston depends heavily on the recasting of the three orientations in Badiou's later *Briefings on Existence*, rather than strictly on their presentation in *Being and Event*. That difference sets Livingston's approach at some distance from my concerns here. See Livingston, *The Politics of Logic*, 51–60; Badiou, *Being and Event*, 282–83; Badiou, *Briefings on Existence*, 51–56.

⁹⁵ For Gödel's original findings, see Gödel and Brown, *The Consistency of the Axiom of Choice and of the Generalised Continuum-Hypothesis*.

⁹⁶ Badiou, *Being and Event*, 283.

the indiscernible is not and “there is no place for an event to take place.”⁹⁷ Cohen in turn receives his place in this basic typology because of his having produced the model of the set theoretical axioms in which the continuum hypothesis does not hold.⁹⁸ Again, regardless of Cohen’s own intentions in constructing his model of the axioms, Badiou finds in the model itself a guiding intuition: “what is representative of a situation is not what distinctly belongs to it, but what is evasively included in it.”⁹⁹ If the powerset of an infinite set of a determinate cardinality does not bear the immediately succeeding cardinality, then “there exists an ontological concept of the indiscernible multiple.”¹⁰⁰ That indiscernible multiple Badiou proposes to call “truth.”

To make clear what is at stake here, it is necessary to outline what Badiou understands Gödel’s demonstration to amount to. To build a model of the set theoretical axioms in which the continuum hypothesis holds is to envision a universe in which every subset of an infinite set must be constructible. In other words, it is to envision a universe that “will only recognize as ‘part’ [or subset] a grouping of presented multiples which have a property in common, or which all maintain a defined relationship to terms of the situation which are themselves univocally named.”¹⁰¹ In the constructible universe, in short, every subset, to be a subset, must be constructible by virtue of some knowable formula in the language.¹⁰² It directly follows from the axioms that underlie the constructible universe that the cardinality of the powerset of an infinite set is that immediately succeeding the cardinality of that original set. On Badiou’s interpretation,

⁹⁷ *Ibid.*, 289.

⁹⁸ For Cohen’s original findings, see Cohen, *Set Theory and the Continuum Hypothesis*.

⁹⁹ Badiou, *Being and Event*, 283.

¹⁰⁰ *Ibid.*, 355.

¹⁰¹ *Ibid.*, 287.

¹⁰² See Devlin, “Constructibility,” 454, 462–63. Badiou puts it this way in a related article: “A subset of the universe U will be called *constructible* if in the language there exists a formula $F(x)$ that constructs it.” Badiou, *Conditions*, 117.

this also implies the reduction of all truth to the knowable, since in the constructible universe there only *is* the knowable. (Badiou defines “knowledge” as “the capacity to discern multiples within the situation which possess this or that property; properties that can be indicated by explicit phrases of the language, or sets of phrases. The rule of knowledge is always a criterion of exact nomination.” Thus, for Badiou, to say that something is knowable is in no way to say anything about the limits of perception or empirical experience.)¹⁰³ Thus any decision in favor of the continuum hypothesis, materialized in a model of the axioms of set theory,¹⁰⁴ turns out to bear within it a decision regarding the nature of truth; it is a decision for the idea that truth just is what can be correctly said of the knowable.

From Badiou’s perspective, it is this last implication that makes the constructible universe unacceptable. Badiou insists that truth is subtracted from sense, drawing on a commitment to the idea that truth must be distinguished from fact because of the *passion*—more strictly, the *affect*—that attends it.¹⁰⁵ (Consequently, in *Being and Event*, Badiou distinguishes between the terms “truth” and “veridicity,” leaving the predicate “veridical” to serve the functions for which the predicate “true” generally serves in philosophical discussions of truth.)¹⁰⁶ That passion or affect manifests itself as a rupture in the fabric of sense, a hole in the weave of meaning that supports the consistency of the world of appearing. Badiou often states that truth punches a hole in knowledge. In a surprising book for a militant atheist to have written, Badiou even defends this point by

¹⁰³ Badiou, *Being and Event*, 328.

¹⁰⁴ See Badiou, *The Concept of Model*, 42–43.

¹⁰⁵ See especially, in this regard, Badiou, *Wittgenstein’s Antiphilosophy*, 107–59. Badiou has stated that this topic will be the chief focus of the third and concluding volume of *Being and Event*. See the discussion in Badiou and Tarby, *Philosophy and the Event*, 105–18.

¹⁰⁶ See Badiou, *Being and Event*, 331–34.

reading closely the letters of Saint Paul.¹⁰⁷ Truth for Badiou has more to do with faith—albeit with faith entirely repositioned outside of the realm of religious dogma—than with knowledge.¹⁰⁸

For this reason Badiou finds more to like in Cohen’s model in which the undecidable continuum hypothesis is excluded than in Gödel’s model where it holds. And just as Gödel’s model requires first that there are only constructible subsets of infinite sets, Cohen’s model requires first that there are *non-constructible* subsets of infinite sets—or, more simply, a commitment to the idea that there are non-constructible sets *at all*, sets for which “no one predicative trait can group the terms that make [them] up.”¹⁰⁹ To such subsets Cohen gave the name of “generic” sets, generic because—as Badiou puts it—they contain “a bit of everything, such that no predicate can ever group all the terms.”¹¹⁰ A generic set is the product of a procedure which Badiou outlines in some detail, in which a series of enquiries is gathered into a set, enquiries specifically concerning the deductive relationship between the terms of a situation and an event, positioned on the edge of the void of that situation. Each of these enquiries is a “minimal report,” a statement concerning the relationship or the connection (or non-connection) of some situated multiple to the unsituated event.¹¹¹ The generic set, which gathers such enquiries together, is non-constructible with respect to the situation precisely because its construction begins from or is organized by the unsituated event. What makes the set

¹⁰⁷ See Badiou, *Saint Paul*.

¹⁰⁸ See Badiou, *Second Manifesto for Philosophy*, 140–41: “I like the great metaphors hailing from religion: Miracle, Grace, Salvation, Glorious Body, Conversion . . . This has, predictably enough, led to the conclusion that my philosophy is a disguised Christianity. The book I published on St Paul in 1997 did not help matters That said, all in all I would rather be a revolutionary atheist cloaked in a religious vocabulary than a Western ‘democrat’-cum-persecutor of Muslim men and women, disguised as a secular feminist.”

¹⁰⁹ Badiou, *Conditions*, 117.

¹¹⁰ *Ibid.*

¹¹¹ Badiou, *Being and Event*, 330.

generic is the fact that, for every constructible set to which it might be compared or with which it might be identified, there will be some enquiry that belongs to it that cannot be found in that constructible set. The genericity of the generic set is the direct consequence of the fact that the procedure by which it is produced follows a formula foreign to the situation.¹¹²

The procedure by which the construction of a non-constructible set is accomplished Cohen called “forcing.” Its name is actually quite appropriate. To construct his model of the Zermelo-Fraenkel set theory axioms in which the continuum hypothesis is false, Cohen began with Gödel’s model of the axioms in which the continuum hypothesis is true, but then he “forced” into that model certain generic subsets that, according to Gödel’s model, do not exist. As Badiou says, “the truth of a situation . . . forces the situation to accommodate it: to extend itself to the point at which this truth—primitively no more than a part, a representation—attains belonging, thereby becoming a presentation.”¹¹³ In its details, forcing is a notoriously difficult procedure, and it is obviously beyond the scope of this chapter to provide a detailed exposition of it.¹¹⁴ Rather helpful, however, for understanding what Badiou draws from its resources is the following passage:

The decisive point . . . is the following: it is certain that the elements of a generic subset cannot be (definitely) named, since a generic subset is simultaneously incomplete in its infinite composition, and subtracted from

¹¹² See *ibid.*, 338–39.

¹¹³ *Ibid.*, 342.

¹¹⁴ See Badiou, *Being and Event*, 355–71. A decent summary of the concept of forcing can be found in Hallward, *Badiou*, 323–48. For Cohen’s original presentation of the strategy, see Cohen, *Set Theory and the Continuum Hypothesis*, 113–29. A general account of forcing as a strategy for use in a number of mathematical situations can be found in Burgess, “Forcing.” A very-brief but helpful summary of the strategy in connection with an analysis of Badiou can be found in Livingston, *Politics of Logic*, 195–97.

every predicate that would, in language, identify it in a single blow. But it can be maintained that *if* such and such an element of the situation *will have been* in the hypothetically completed generic subset under consideration, *then* such a statement, as rationally connectable to the element in question, is, or rather will have been, factual.¹¹⁵

This, it seems to me, is a sufficient description of what is at stake in forcing for present purposes. Although, as it turns out, actually completing the work of constructing an infinite generic subset is impossible, the factuality of certain statements on condition of the (hypothetical) completion of the procedure *can* be ascertained through the operation of forcing. It is in this sense that one can, on the strength of the incommunicable truth of a situation, state that certain propositions *will have been* factual, although they are not determinably factual as yet.

In the set theoretical operation of forcing—essential to constructing a model in which the undecidable continuum hypothesis is decided against—Badiou finds, at last, the complete figure of the subject. The subject, he asserts, is to be found “‘between’ the terms that the [forcing] procedure groups together,”¹¹⁶ in something loosely like the way that Jacques Lacan located the Freudian subject “between” the elements of the linguistic structure of the unconscious. The subject is for Badiou the figure of chance encounter, the chance link between the unsituated event and some situated term whose deductive relation to the event must be determined. In performing that deductive function, the subject begins to produce a language built up of names that “do not, in general, have a referent in the situation,” since they “designate terms . . . which ‘will have been’

¹¹⁵ Badiou, *Conditions*, 138, translation slightly modified.

¹¹⁶ Badiou, *Being and Event*, 396.

presented in a new situation.”¹¹⁷ Badiou constructs a conditional to capture the subject’s function: “*if* this or that term, when it will have been encountered, turns out to be positively connected to the event, *then* this or that name will probably have such a referent, because the generic part, which remains indiscernible in the situation, will have this or that configuration, or partial property.”¹¹⁸ In a word, subjects “displace established significations and leave the referent void: this void will have been filled if truth comes to pass as a new situation.”¹¹⁹ Or better: “A subject emptily names the universe to-come which is obtained by the supplementation of the situation with an indiscernible truth,” in the way that Cohen supplements the Gödelian model with non-constructible subsets.¹²⁰ With such a supplementation fully in place, a decision on the undecidable has been made. Having deployed the strategy of forcing, Cohen constructs a model in which the undecidable continuum hypothesis does not hold. And there Badiou finds the operation he nominates as that which allows for the construction of sets he takes to be rightly definable as truths.

Whatever the far-ranging functions of the subject turn out to be, therefore, truth for Badiou is captured by the notion of the generic set to the construction of which the subject contributes. Truth is the set of enquiries regarding the implications, for the terms of a particular situation organized by a particular language, of an unsituated event, on the edge of that situation’s void. Each enquiry is an element in a set that, once completely constructed, constitutes the truth of the situation—the set theoretical concept of truth for a particular situation or its language. That set, because its make-up does not coincide with

¹¹⁷ *Ibid.*, 398.

¹¹⁸ *Ibid.*, 399.

¹¹⁹ *Ibid.*

¹²⁰ *Ibid.*

any set that can be constructed through a formula native to the situation in question, is marked by a kind of universality—a classic feature of truth according to most conceptions.

In what other senses Badiou's conception of truth as modeled by Cohenian generic sets plays into classical conceptions of truth remains to be seen. Given Tarski's strict commitment to the material adequacy of his own strategy for defining truth, it will have to be asked to what extent Badiou's conception of truth looks like Tarski's, both in its ontological bearings and in its operational decisions. That, of course, is the subject of Chapter 4. It is necessary, at last, to ask what can be said in a comparative vein about Badiou's and Tarski's respective attempts at using set theoretical mathematics to clarify the notion of truth.

TRUTH, SET THEORY, AND MODELS

In this concluding chapter, I will finally replace the largely expository and interpretive approach of Chapters 2 and 3 with a more directly comparative approach, allowing such comparative work, moreover, to open directly onto philosophical reflection. In a certain way—as becomes clear, moreover, in the course of the present chapter—the preceding chapters have already outlined comparative findings in an anticipatory way. Just to give to the exposition of Alfred Tarski's or of Alain Badiou's thought the structure I do is to begin to recognize important ways in which the two thinkers can be brought into productive philosophical conversation—each thinker addressing in turn a set of (at least minimalist) ontological considerations and a set of considerations bearing on providing an adequate formula for designating truth for particular languages. It is nonetheless necessary to bring out with full clarity the manifold points of similarity and the fewer but philosophically essential points of difference between the two projects, allowing comparative work to ripen. It will then be possible, beyond comparison, to defend at last my central thesis: that any adequate conception of the relation that lies at the heart of truth beings with an investigation of the entanglement of the formal and the material as this is set forth in the model theoretical development of set theoretical mathematics. This is my task in this final chapter.

Of course, in light of much of the expository work undertaken in the preceding two chapters, it might seem easier to mark real points of *divergence* between Tarski and Badiou than to discern genuine points of *convergence*. The unmistakably distinct

philosophical milieus from which they arise, highlighted by the now-classic (if nonetheless increasingly tired) antagonism between the continental and analytic traditions more generally, would seem to suggest that there is still something suspicious about any attempt to bring these two thinkers into conversation. And I have in no way attempted to hide the fact that Tarski and Badiou begin from rather distinct intuitions regarding the basic notion of truth in their respective attempts to draw from set theory. Some might well be skeptical from the outset, therefore, worrying that the differences between Badiou and Tarski are many times more significant than any apparent similarities. But regardless of whether other attempts to work across the divide between the two philosophical traditions have been successful or in any way convincing—and I might note that I share the suspicions of others with regard to much synthetic work!—I argue in this chapter that Tarski’s and Badiou’s projects are deeply related and strikingly similar. Their very real differences are ultimately significant only in the light of certain profound similarities. In short, their similarities help to reveal the manner in which their differences outline the contours of a central philosophical problematic.

The first section of this final chapter examines—in order to dismiss—a number of apparent dissimilarities between Tarski’s several essays on truth and Badiou’s *Being and Event*, dissimilarities that one might too easily see arising from the preceding expository or exegetical chapters. These dissimilarities, deceptive and distracting on my argument, go beyond the very real and irreducible difference between the basic intuitive conceptions of truth presupposed by Tarski and Badiou respectively. Two particularly misleading ways of seeing the two projects as incommensurable I find especially worth highlighting, because they attempt to cover what in each thinker speaks quite directly to the more

obvious features of the other's project. Removing these obstacles to comparison opens the way to a double comparative study, focused in turn on the ontological investments in set theory that—on my argument—set both Tarski's and Badiou's projects in motion and on the specific formulas or operations drawn from set theory that organize each thinker's attempt to isolate sets rightly designatable as truths of or proper to particular languages.

The second and third sections of this chapter are thus dedicated in turn to the double comparative study just mentioned. The aim in the second section is to identify crucial points of similarity between Tarski's ontological commitments consequent to his rigid adherence to the criterion of material adequacy and Badiou's ontological employment of set theory in pursuing the possibility of formalizing the Heideggerian conception of being. In the third section, I turn my attention to the real point of difference between Tarski's and Badiou's work on truth. There, finally, I develop quite fully the distinct beginning points for each thinker and clarify what these beginning points have to do with their distinct uses of model theory in making philosophical sense of truth. In the fourth and final section of this chapter, I briefly leave comparative work—and the specific projects of Badiou and Tarski—largely to one side in order to draw some preliminary general conclusions about the problematic these efforts in comparative philosophy help to reveal: a model theoretical conception of the entanglement of the formal and the material that clarifies the basic stakes of thinking philosophically about the notion of truth. What comparative work ultimately reveals is the manner in which truth may be thought—both in terms of its basic ontological bearings and in terms of the possibility of determining what kind of formula rightly designates what can be called truth—according to the framework provided by model theory.

Divergence, Convergence

As I have already noted above, in light of the expositions of Tarski and Badiou provided in Chapters 2 and 3, it might seem at first blush a good deal easier to mark real points of divergence between the two projects than to discern genuine points of convergence. The most obvious such point of apparent divergence concerns ontology. While it must be said—as argued in Chapter 2—that Tarski’s project is not without certain minimalist ontological investments (which remain to be clarified), there might seem to be little reason to think that his recognition of the ontological implications of his early work in set theory is in any real way comparable to Badiou’s heavily Heideggerian interpretation of the ontological import of set theoretical mathematics. Indeed, where Tarski’s principal aim in outlining a means of constructing definitions for particular truth predicates is to cut metaphysical accretions away from standard philosophical work on truth, one might well be inclined to say that Badiou’s principal aim in building up a philosophical account of truth is to continue resolutely in the metaphysical tradition that Rudolf Carnap rightly criticized.

In this vein, it is well worth noting that Tarski’s “polemical remarks” in his 1944 paper on truth contest the very existence of a “philosophical problem of truth”:

I have heard it remarked that the formal definition of truth has nothing to do with “the philosophical problem of truth.” However, nobody has ever pointed out to me in an intelligible way just what this problem is. I have been informed in this connection that my definition, though it states necessary and sufficient conditions for a sentence to be true, does not

really grasp the “essence” of this concept. Since I have never been able to understand what the “essence” of a concept is, I must be excused from discussing this point any longer.¹

It is difficult not to hear in these words a direct echo of Carnap:

The development of *modern logic* has made it possible to give a new and sharper answer to the question of the validity and justification of metaphysics. . . . In the domain of *metaphysics*, including all philosophy of value and normative theory, logical analysis yields the negative result *that the alleged statements in this domain are entirely meaningless*.

Therewith a radical elimination of metaphysics is attained, which was not yet possible from the earlier antimetaphysical standpoints.²

To the extent that Tarski’s work, despite what he arguably sees as the ontological implications of his efforts in set theory, aims at eliminating certain metaphysical excesses, one might suggest that his project must be regarded as fundamentally distinct from Badiou’s.

Certainly it is of importance that—as argued in Chapter 3—Badiou’s chief project in *Being and Event* can be justly interpreted as both a direct continuation of the very Heideggerian project Carnap contested in the essay just cited and, in fact, as a subtle attempt at arguing against Carnap’s essay itself. Although it would of course be anachronistic to do so, one *could* in the above vein take Tarski’s work on truth as constituting a polemic against Badiou, an updating of the Carnapian polemic against Heidegger, now specifically focused on the notion of truth rather than metaphysics

¹ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 361.

² Carnap, “The Elimination of Metaphysics Through Logical Analysis of Language,” 60–61.

generally and directed at one of Heidegger's philosophical heirs rather than at Heidegger himself. At any rate, it seems most responsible, in general terms and at first blush, to understand Tarski and Badiou to be direct opponents, representatives of two fundamentally different philosophical traditions with fundamentally different philosophical sensibilities. One might indeed conclude that what is to be gained from bringing their two projects into conversation is first and foremost a strong sense for the insuperable difficulties of overcoming the differences between the analytic and continental philosophical traditions. How could two projects so obviously distinct in their basic aims and strategies be made to work together synthetically?

This first point would seem to be confirmed by a second, related point. What interests Tarski in set theory when he turns his attention to truth is, quite clearly, the way that its basic apparatus might allow a definition of a particular truth predicate to be constructed as a set of determinate sentences. That is, the sets that most interest Tarski in his work on truth are, first and foremost, *sets of sentences*. Over against this, it might be said that what interests Badiou in set theory is primarily the way it can be interpreted as accounting for the consistency of appearance or experience, as providing an account of how things take the shape of discretely countable objects. Using set theory to get back behind things, to think the inconsistent being from which they supposedly issue in their consistency, Badiou gives his attention first and foremost to sets woven entirely from the void, *sets of sets of sets of . . . nothing*. Where Tarski finds in set theory the means to group sentences according to their definable truth, Badiou finds in set theory the means to provide a consistent account of how what is is woven from the nothing. Tarski thus attempts the rather humble project of constructing definitions of truth predicates for

particular (formalized) languages, while Badiou attempts the grandiose—if not philosophically hubristic—project of viewing truth in light of a massive ontological account of absolutely everything that is. Here again, it seems a little too obvious that Tarski’s and Badiou’s respective projects begin from such radically distinct interests and intentions that little more than stark contrast is to be gained by bringing them into conversation.

As it turns out, however, these *apparent* points of divergence are actually misleading. Two crucial ways in which the facile characterizations of Tarski’s and Badiou’s projects laid out in the preceding paragraphs mislead must be overtly identified.

First, it is misleading to suggest that Tarski’s work on truth consists more or less entirely in the metaphysical pruning his set theoretical approach to defining truth allows him to do. That is, while Tarski’s rigorous, formally correct construction of particular definitions of the truth predicate for determinate formal languages indeed allows him to pursue a metaphysically modest task, formal correctness constitutes only *one* of the two guiding criteria from which he takes his orientation. Equally important is the guiding criterion of material adequacy. As I showed in Chapter 2, this criterion demands, according to Tarski, that truth be regarded as consisting at the very least of a certain correlation or correspondence of some sort: minimally, between sentences and their truth-conditions, organized by the basic semantic operation of satisfaction. Although it is necessary to restrict any interpretation of the ontological import of this commitment on Tarski’s part, there undeniably *are* ontological implications at work here, implications I will spell out further along in terms of a certain entanglement of the formal and the material. Thus, although Tarski eventually produces definitions of actual truth predicates

by attempting to undercut needless metaphysical speculation, his project begins by recognizing that even a formal consideration of truth as such demands a semantic project.

Second, it is misleading to suggest that Badiou's work on truth grows more or less entirely out of the ontological work to which he puts set theory in his attempt to improve on Heidegger's thought. That is, while Badiou's Heideggerian commitments are unquestionably on display from the outset of his work on set theory, tying him to what is too often regarded as a deeply metaphysical project, set theoretical ontology is only *a part* of Badiou's larger project in *Being and Event*. When he turns his attention from the details of accounting for the Heideggerian question of being to considerations related to Georg Cantor's continuum hypothesis, Badiou gives himself to the question of how truth might be constructed through a rigorous and formally correct operation. Indeed, strictly speaking, Badiou's project comes to its ultimate fruition in *Being and Event* only when he makes clear that a truth is a set of inquiries—formalized as sentences that mark relationships—entirely localized to a particular situation that can be defined as a self-contained language. In other words, a generic set is, rightly understood, a set of sentences, each of which says something about certain sentences within a determinate language. Whatever the ontological commitments may be that set Badiou's entire project in motion and provide him with a basic orientation for his work on truth, when he turns to the detailed work of providing an actual account of truth, he turns his attention to operations that gather sentences about sentences into determinate (if nonetheless locally indiscernible) sets. In this regard at least, Badiou's work looks more like Tarski's than one would guess at first.

Thus, what might seem the most natural way of distinguishing Badiou's and

Tarski's projects is ultimately quite deceptive. It is too easy just to suggest that Badiou gets trapped in unnecessary metaphysical worries or that Tarski attempts a positivistic reduction of truth to straightforward truth predicates for fully lucid languages, and then to claim that the projects lie in incommensurable traditions that have nothing to say to one another. Quite the opposite seems to be the case, actually. Both Tarski and Badiou orient themselves by a set of preliminary ontological considerations, internal to the development of set theory itself, and then take seriously the limitations they find those considerations to impose on the search for a formula or an operation by which sets rightly recognizable as truths for particular languages can be either designated or constructed. Both clearly regard the difference between and the entanglement of syntax and semantics to lie at the heart of the ontological questions that bear on truth. And both clearly find the resources for formulating definitions or designations of truths in the set theoretical apparatus. Far from being deeply incommensurable, it is quite startling to see in the end how fundamentally *similar* Tarski's and Badiou's projects are, even if their respective intuitive conceptions of truth must be distinguished.

These similarities are, moreover, productive. That so much is ultimately similar in the two projects allows for their real differences to be fixed with greater clarity—and in fact to be raised to the level of a genuine philosophical problematic. To see this, however, it will be necessary to look quite closely at just *how* similar the two projects are. So far, I have only outlined the broadest points of similarity between the two projects, and I have left some essential points overly vague at this point. What do things look like closer up? What further and more rigorous similarities can be discerned when the details come into view? And how do *those* similarities in turn help to clarify the genuine points of

difference between Tarski's and Badiou's respective projects, raising them to the level of a genuine philosophical problematic?

I propose, as noted above, to divide the following comparative discussion into two major parts. First, I turn my attention to ontological questions, those concerning the relationship between syntax and semantics. Second, I turn my attention to definitional or operational questions, those concerning the actual designation or construction of sets that can be called truths (for particular languages). Details of ontological parity allow for divergence in definitions or operations to emerge with real clarity. On the other side of these two discussions, I will draw more general conclusions.

Model Theory and Ontology

My aim in this section is to show that despite certain appearances, Tarski and Badiou share a set of ontological commitments—that, in effect and despite the surprise that likely attends this announcement, their respective ontologies are *basically the same*. Because this shared ontology establishes the site of their real disagreements, making these disagreements speak to one another must be done as fully as possible. It will be easiest to begin with Tarski, finally clarifying fully the ontological bearings of his criterion of material adequacy.

In Chapter 2, I examined the manner in which Tarski's independent discovery of Kurt Gödel's incompleteness theorems (at least in a somewhat rougher form) revealed to him what might be called the limits of the formal or of the syntactic. The theorem that still bears Tarski's name, according to which truth and provability are non-equivalent for any formal system in which it is possible to arithmetize the language, indicates that truth

cannot be adequately defined without building into the formula that would designate it some sort of correlation or correspondence—at the very least, between sentences and their truth-conditions. As I have already described at some length, Tarski mobilizes the basic semantic operation—left undefined or primitive—of satisfaction in order to secure the consequently necessary material adequacy of his truth definitions. The question that arises now concerns the ontological commitments bound up with these moves made by Tarski. I have already indicated that I regard these ontological commitments to be minimal, but it remains for me to clarify exactly what it means to speak of these minimal ontological commitments.

Even to take a first step here, it is necessary first to summarize briefly Tarski's work in model theory, noted only in general terms in Chapter 2. As Robert Vaught notes, Tarski's work on truth grew directly out of his work on the related findings of Leopold Löwenheim and Thoralf Skolem, which he took as the focus of his lectures at Warsaw University in 1927–29.³ What took shape over the course of those lectures and in Tarski's published works over the next decade and a half was the laying of the foundations of model theory, the branch of mathematics over which Tarski would come to rule. In his canonical presentation of the basics of model theory, Tarski provides an implicit outline of the ontological ramifications of his work on these questions. He begins his discussion of models with the following words:

[A] deductive theory rests upon a suitably selected system of primitive terms and axioms. Our knowledge of the things denoted by the primitive terms . . . is very comprehensive and is by no means exhausted by the adopted axioms. But this knowledge is, so to speak, our private concern

³ Vaught, "Alfred Tarski's Work in Model Theory," 869–70.

which does not exert the least influence on the construction of our theory. In particular, in deriving theorems from the axioms, we make no use whatsoever of this knowledge, and behave as though we did not understand the content of the concepts involved in our considerations, and as if we knew nothing about them that had not been expressly asserted in the axioms. We disregard, as it is commonly put, the meaning of the primitive terms adopted by us, and direct our attention exclusively to the form of the axioms in which these terms occur.⁴

In strictly formal considerations, everything materially known about the real that some formal system is meant to capture must be put in suspension, as it were. Formal work, in other words, begins with a kind of *epochē*, a bracketing of the real that is necessary to the rigorous work of determining the consequences of the basic sentences and rules of inference that organize the formal system in question. This bracketing is to be accomplished through the replacement of any material or real terms with variables.⁵

A model is then “a realization of the axiom system of our theory,” that is, what is produced when certain material terms can be said to satisfy the variables built into a strictly formal theory.⁶ Here as elsewhere in Tarski, satisfaction is of capital importance. Satisfaction is for him the basic semantic notion, the primitive and therefore undefined concept on which the definitions of all other semantic concepts might be constructed. Here it is possible to say that every model of a formal system can be identified by the fact that it satisfies the axioms at the foundation of that formal system. A model is a concretization or materialization—via satisfaction—of what otherwise would remain

⁴ Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 121–22.

⁵ See *ibid.*, 122. A fuller discussion of sentential functions can be found in Chapter 2.

⁶ *Ibid.*, 123.

strictly formal. What Tarski made clear in his work on models is the importance of using them in what he called “metamathematical” work. He developed model theory beyond its Löwenheimian-Skolemian roots in order to show that there lay within it the resources for determining—without waiting around for someone to produce, often enough by sheer luck, a definitive proof—whether the axioms of a given theory directly entail some particular statement.⁷ By undertaking a formalizing *epochē* for the theory in question, and then by producing models of that fully formalized theory in which the (equivalent of the) statement in question is negated and/or affirmed, it is possible to determine whether the statement is independent (that is, undecidable with respect to) or already implied by (that is, decided by) the axioms of the given theory.⁸ The network of models of a fully formalized axiom system allow, in their mutually revealing light, for a recognition of exactly what *cannot* be deduced from the axioms of that system.

Model theory in this form grew directly out of Tarski’s work in 1929–30, the work in which he independently produced Gödel’s basic findings regarding incompleteness. At that early point, as I have already noted with help from Donald Davidson, Tarski too often or too easily employed language that suggested that his findings in still-nascent model theory bore rather robust metaphysical implications—speaking, for instance, of facts or things or objects or states of affairs. As Davidson points out, however, “one of the strongest arguments for Tarski’s definitions is that in them *nothing* plays the role of facts or states of affairs,” making clear that such talk in Tarski is misleading.⁹ If scientific

⁷ On the earliest roots of model theory, beginning with Löwenheim’s work, see Badesa, *The Birth of Model Theory*.

⁸ Tarski develops this methodology in detail in Tarski, Mostowski, and Robinson, *Undecidable Theories*. It is presented in summary form in Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 124–40.

⁹ Davidson, “The Folly of Trying to Define Truth,” 22–23.

semantics amounts to an investigation of the relation between words and things, between the strictly formal and the strictly material or real, then it does so in a far more minimalist manner than that in which Tarski was at first wont to suggest in his writings. The basic correlation or correspondence at the heart of Tarskian truth concerns not the relation between statements and epistemologically knowable states of affairs, but just that between certain fully formal languages and the network of materializations or realizations—*models*—that satisfy them.

Tarski's work on truth thus begins from his work that was undertaken, as it were, at the border between the strictly formal and the minimally material. His clarity about the boundaries between the formal and the material allowed him to recognize that the concept of truth for a particular language, even if that concept is to be fully formalized in a formally correct definition, must operate across that same border. Indeed, in light of Tarski's work on model theory, it is possible to define "true sentence" in his work on truth as *any model of the formalized T-schema* (for a particular language).¹⁰ Although he takes the basic notion of satisfaction to remain undefined or primitive, Tarski recognizes in that notion a certain commerce between the minimally real or material and the strictly formal—and truth he of course defines entirely in terms of satisfaction. Obviously, the full construction of a formally satisfying definition of truth for some particular language requires much more than just this basic recognition, but any such work of construction *must* take its bearings (such is the constraint of the criterion of material adequacy) from this fully recognized but strictly minimal commerce between the formal and the real.

Importantly, in his first philosophical monograph, *The Concept of Model*, Badiou

¹⁰ This is, in effect, how two of Tarski's former doctoral students present the notion of truth. See Chang and Keisler, *Model Theory*, 1–4.

gave his attention to this same point. In a loosely Davidsonian vein, Badiou took as his aim in that book—originally a series of public lectures, interrupted by the events of May 1968—to draw what Louis Althusser often called (following Lenin) a line of demarcation between a strictly scientific concept of model and its ideological appropriation by certain philosophers—especially Carnap.¹¹ Although Tarski never becomes the focus of Badiou’s work in the book, his criticisms of Carnap can quite easily be seen as Davidson-like criticisms of Tarski’s occasional lapses into metaphysically robust descriptions of semantics in his early work on truth. But Badiou’s approbation of the strictly scientific concept of model can just as easily be seen as Davidson-like approval for what in Tarski’s overarching project resists such lapses into metaphysics. (This is clearest when Badiou provides a list of the most essential theorems at work in model theory, namely those two he elsewhere couples with Tarski’s theorem: Gödel’s incompleteness theorems and the Löwenheim-Skolem theorem.)¹² According to Badiou, what Carnap and, by implication, Tarski-at-his-less-careful-moments get wrong lies in their readiness to assimilate the strictly scientific distinction between syntax and semantics, between the formal and the material, to “a presupposed distinction between empirical reality and theoretical form.”¹³ In the place of such an assimilation, Badiou (following Gaston Bachelard) proposes a notion of “practical immanence,” where “the distinction between syntax and semantics has the fragility of the distinction between the *existence* and the *use* of an experimental apparatus.”¹⁴ In the place of any metaphysically fraught correspondence between words

¹¹ For the fullest articulation of Althusser’s project and his attribution of its basic orientation to Lenin, see Althusser, “*Philosophy and the Spontaneous Philosophy of the Scientists*” and *Other Essays*. Badiou himself uses the language of drawing a line of demarcation in Badiou, *The Concept of Model*, 22.

¹² See Badiou, *The Concept of Model*, 18.

¹³ *Ibid.*, 5. Badiou’s fullest articulation of this conception is found at *ibid.*, 19.

¹⁴ *Ibid.*, 44, emphases in original.

and things, between realms which “confront one another as two heterogeneous regions,” Badiou finds in the scientific concept of model—which in turn lies at the heart of the Tarskian project—a minimalist correlation between “intra-mathematical” entities, the supposed “outside” always deployed “within a mathematical envelopment” of the supposed “inside.”¹⁵

What secures the immanence of the semantic to the syntactic, even as a real distinction between the two must be recognized, is the manner in which the existence or construction of various semantic models is demanded solely by the impasse proper to (correlated) syntaxes of sufficient complexity (that is, those for which it is possible to produce undecidable statements). This immanence makes all the clearer in which sense it must be said that Tarski’s ontological commitments are quite minimal. Far from committing him to the existence of any particular objects in the world, or to any particular conception of what the notion of truth requires at the epistemological level, it binds his work only to the minimal recognition that there is a certain (plural) materialization immanent to the strictly formal. To the extent that there can be said to be a set of ontological commitments operative in Tarski’s work on truth, he would have to say that ontology limits itself to a recognition of the complex manner in which the network of material models bears certain necessary and informative consequences for the purely formal language out of which it grows, just as the purely formal language itself bears within its construction a host of direct implications concerning any possible materialization of its formal structure.

If this much is clear, it must now be asked what all this has to do with Badiou’s

¹⁵ *Ibid.*, 46, 42. For Badiou’s strict definition of this correlation, which he calls “a function of correspondence,” see *ibid.*, 30.

Being and Event, which certainly seems at first blush to bear much stronger ontological commitments than the minimal or minimalist ones that, I have argued, should be attributed to Tarski. I argued in Chapter 3 that the first motivation for Badiou's interest in set theory is the way that it might be said to account for the countability of what is. The objects populating any given regime of appearance—so many discrete, self-identical entities or beings—can be immediately distinguished by their consistent *numerosity*.¹⁶ That is, each being, as an apparent being, is one, something that can be counted. The reason it is possible to grant to mathematics a kind of foundational and therefore privileged status among the sciences is because all that appears appears proximally and for the most part as what can be counted, what can be added together or subtracted from each other, what can be multiplied or divided up, what can be measured and calculated and otherwise placed under the regime of number.¹⁷ What Badiou sees in set theoretical work almost from its outset—certainly by the time Gottlob Frege published his *Foundations of Arithmetic* in the late nineteenth century, seeking to provide a clear definition of number—is an attempt to account for the consistency of the countable.¹⁸ In Badiou's perspective, in other words, the set theoretical enterprise is a systematic effort to provide an axiom system that will prove to be materially adequate to everyday uses of numbers, to everyday manipulations, in terms of the countability of beings, of what is. What, he finds mathematical logicians asking, *lies behind* the consistently countable realm of everyday presentation?

This certainly seems like a more robustly ontological project than Tarski's, a

¹⁶ It should be remembered that in connection with Badiou's earliest meditation in *Being and Event*, the words "consistent" and "inconsistent" are used in a more general rather than in a strictly formal sense.

¹⁷ See Badiou, *Number and Numbers*, 1–4.

¹⁸ See *ibid.*, 16–23.

project with more robust ontological commitments than Tarski's. It should be remembered, however, that ontology remains as much for Badiou as for Tarski a strictly formal affair, all of its material implications always and only bound up with the specifiable models of the formal system Badiou employs in attempting to account for the basic consistency of what appears. And, at least apart from the axiomatic decision regarding the existence of the empty set, Badiou understands all the most important ontological decisions—that regarding the existence of infinite sets, and especially that regarding the existence of generic sets—to concern questions of the commerce between the formal and the material in the strict model theoretical sense. To the extent that set theoretical mathematics itself constitutes the thinking of being, ontology as such, Badiou has to admit that model theory, an inescapably necessary development of and internal to set theory, lies at the heart of ontology. Indeed, not only does he have to admit this, he argues this point overtly. What concerns Badiou above all else in his work on truth is the set of implications to be drawn from the model theoretical findings of two mathematicians: those of Gödel first, in his work on the so-called “constructible universe,” and those of Paul Cohen second, in his production of “generic” sets. Although Badiou consistently speaks of his set theoretical ontology, it would be more correct, strictly speaking, to refer instead to his set theoretical *and* model theoretical ontology, to his equation of ontology with set theory *as it develops itself into model theory*.

Nonetheless, it must be said that there is at least one point where Badiou's ontological commitments go beyond Tarski's: his ontological investment of the empty set. Tarski nowhere attempts to map set theory's empty set onto the Heideggerian nothing, with the consequence that he never ventures a strict ontological interpretation of

the basic axioms of classical set theory that regulate the manipulation of the presupposed-existent empty set into a proliferation of other sets. In this regard at least, Badiou's conception of ontology outstrips Tarski's. And yet it must be noted that Badiou never suggests in *Being and Event* that his investigation of set theory is supposed in any way to determine in advance what the actual furniture of any particular realm of appearance really is. Even as he invests the set theoretical empty set with an ontological significance, he presupposes nothing in particular about the metaphysical make-up of any particular existents. (Even in *Logics of Worlds*, Badiou posits nothing in particular about particular existents, limiting his work just to outlining the abstract formal apparatus by which to understand the network of relations organizing any particular regime of appearance.) Despite certain appearances, then, it should be said that Badiou's own ontological commitments are in important ways more minimalist than they at first appear.

Of course, one might well raise the objection that, however minimal Badiou's ontological commitments can be said to be, his investment in set theory's empty set—the move that allows him to see in set theory an account of the consistency of what is—finds its basic motivations in an intuitive conception of truth (and its relationship to knowledge) that is fundamentally distinct from that underlying Tarski's work. In other words, it seems it cannot be said that Tarski's ontology is strictly consistent with Badiou's ontology. This is because Badiou refuses to envisage the basic concept of truth as being entirely extricable from certain epistemological considerations addressed in Chapter 1: what philosophers of science call theory change. Much of what drives Badiou's interest in Heidegger, as well as in the possibility of equating set theory quite generally with ontology, is the manner in which doing so allows him to construct a philosophical

account of theory change. For this reason, while Tarski simply *assumes* the consistency of what appears, Badiou *interrogates* it.

Nonetheless, this one (unquestionably essential) point of difference between the basic conceptions of ontology operative in Tarski's and Badiou's respective projects does not seem to me to be enough to render their projects incommensurable. Rather, I want to suggest that this one point of difference is chiefly—if not solely—an effect *at the ontological level* of what is really the two thinkers' major point of difference *at the definitional or operational level*. Because Badiou and Tarski begin from distinct intuitive conceptions of truth to begin with, they formulate quite distinct notions of what it means to locate truth within the machinery made available in model theory. That point of difference leads Badiou *to augment*—but only retroactively—his ontological commitments, which would otherwise remain effectively equivalent to Tarski's, with a certain *additional* ontological consideration: his set theoretical development of Heidegger's basic ontological picture. The only point of strict difference between Badiou and Tarski at the ontological level can thus be taken to be only a retroactive effect of a difference whose real cause is located elsewhere—and which remains to be discussed in the next section. If this one point of difference in conceptions of ontology bears within it any implications regarding the possibility of bringing Tarski and Badiou to bear on one another, the possibility of glimpsing it depends, somewhat paradoxically, only on recognizing the manner in which their other ontological commitments bring their projects enough into proximity to allow for fruitful philosophical comparison.

In the end, then, Tarski's and Badiou's respective work on truth can be brought into a philosophically productive comparative framework because of what they share in

terms of basic ontological commitments. I have argued, in essence, that Tarski's project bears within it stronger ontological commitments than are usually recognized, although these commitments never go so far as to bind Tarski to any particular metaphysical picture of the world; and that Badiou's project bears within it weaker ontological commitments than are usually recognized, although these commitments do not obviate certain other ontological considerations in Badiou's work that derive from the pre-theoretic conception of truth from which he begins. With Tarski's stronger-than-recognized and Badiou's weaker-than-recognized ontological commitments, the model theoretical focus of their respective conceptions of ontology turns out to be a shared one. The basic commerce between the formal and the material conceptualized by model theory constitutes a kind of commensurable space within which the real—and philosophically informative—debate between Tarski and Badiou regarding the nature of truth takes place. It is only when Badiou turns his attention from the question of the *general shape* of set theory and model theory to the question of those axioms' *specific models* that his essential differences from Tarski begin to emerge. But for both Tarski and Badiou, it can be said that ontology in its most basic form is a matter just of recognizing the complex entanglement between the formal and the real, and of recognizing that that entanglement is figured most rigorously (and according to an appropriate minimalism) in model theory. The basic impasses of historical set theory forced its model theoretical development, and that development gives strict shape to the most basic of ontological commitments: the formal bears on the real in determinate ways, and the real in turn bears on the formal in determinate ways.

The fact that Tarski and Badiou share a set of ontological commitments is enough

to secure their relevance to each other when they turn their attention to specifying the formula or operation by which to designate or construct sets that might rightly be defined as truths (for particular languages). Both clearly recognize that truth concerns the model theoretical development within set theory, the necessary entanglement of certain formal syntaxes with any number of appropriate semantic materializations. Where they disagree is in their definitional or operational nominations—in fact, in the very difference between the one’s attempting just to fix a definition and the other’s investment in a transformative operation. This is the subject of the next section.

The Debate

Although it is necessary to identify real points of difference between Tarski and Badiou when it comes to the formulae or operations they identify as allowing for the construction of truth definitions, it must first be noted how similar their respective conclusions are in many ways. Only in the light of the projects’ remarkable similarity can their essential and philosophically instructive differences be clearly identified. In each thinker’s case, truth for a particular language is conceived as a grouping of sentences into a set, designatable or constructible by an appropriate formula or operation. Moreover, in each case, the formula or operation in question asks, so to speak, a kind of yes-or-no question, producing a set of those sentences for which a certain affirmative answer to the question can be provided—for Tarski, “Do this sentence and its translation into the metalanguage together satisfy the T-schema?” and for Badiou, “Does this sentence rightly affirm a deductive connection between an event and some point in the situation?” It should be further noted that the formula or operation elected by each thinker designates

as truth something specific or local rather than something general or global: truths in the plural, and always for particular languages (or situations, in Badiou's terminology). It is perhaps only when *these* particular points of similarity between Tarski's and Badiou's respective projects are glimpsed that the real viability of contrasting them comes fully into focus. There is nothing superficial about these similarities. Nonetheless, the real differences between the two projects at this level run even deeper, as it is now possible to show.

For both Badiou and Tarski, philosophical consideration of truth requires recognition of the limits of the strictly formal, of the strictly syntactic, but each thinker marks that limit in a distinct fashion. For Tarski, the limit of the syntactic concerns the implicit presence, in most syntaxes, of certain undecidable statements—that is, of sentences that can be constructed according to the syntax's rules of construction but the truth or falsity of which cannot be demonstrated through any combination of the syntax's basic sentences and combinatorial operations. Where it can be shown that a language or a theory or a syntax is indeed undecidable—that is, that it contains at least one undecidable statement—the strictly formal considerations of the syntax call for a model theoretical extension from syntax into semantics, possible materializations of the merely formal that lie at the limit of the syntactical as such. Because truth, according to Tarski's theorem, cannot be reduced for undecidable formal systems to provability, it must be a semantic notion and therefore must be constructed only in light of the complex limit that distinguishes the syntactic from the semantic.

All of these Tarskian details Badiou takes over into his own project, it seems. He too marks the boundary between the formal or the syntactic and the material or the

semantic. But truth cannot be fully captured on his account simply by attending to the entanglement of the syntactic and the semantic in working out truth definitions for particular formalized languages. Rather, according to Badiou, the full conceptualization of truth requires also *a decision among particular models* of the formal system specifically of set theory—and especially, for him, a decision between the models of set theory provided in turn by Gödel and Cohen. It is not just that truth is a semantic concept for Badiou. Whether a notion of truth captured by set or model theoretical formalisms will be genuinely adequate will *further* depend, according to Badiou, on what decisions have been made about the materialization of set theory. In certain materializations or models, truth is conceived to be one sort of thing, while in other materializations or models, truth is conceived to be something else instead. In each case, the sort of set or model theoretical formalisms on which one will draw will differ. Thus while for Tarski, it might be said to be sufficient, for the adequate construction of truth definitions, just to recognize in full rigor the undecidable boundary between the syntactic and the semantic, for Badiou it must be said that it is necessary also to consider the differences among various semantic materializations of a given (very specific) syntactical system if truth is to be defined adequately.

There is thus in Badiou's project a certain *further* material consideration that governs his selection of the set or model theoretical operation by which to construct truths as sets of sentences. (This is, moreover, presumably what he means to signal by claiming that it was only as he finally moved beyond the theorems of Gödel, Tarski, and Löwenheim-Skolem that he found his way to *Being and Event*.)¹⁹ So far as Tarski is concerned, the criterion of material adequacy is exhausted by the model theoretical

¹⁹ See, again, Badiou, *Being and Event*, 5, as well as the discussion in Chapter 1 of this dissertation.

demand that truth be regarded as a semantic notion and by the construction of a set of all the sentences that satisfy the T-schema for a particular language. For Badiou, it might be said that the same criterion of material adequacy requires further consideration of the variety of possible materializations of a formal language. If a selected model of the syntax allows for the construction of truth definitions that do not conform to what Badiou takes to be the material processes involved in the production of truths (scientists, for instance, at work on scientific problems), then one should—for material reasons—decide against that model and for another that can be said to be more adequate, materially, to what is known of truth. Here at last, the relevance of the distinct notions of truth to which Tarski and Badiou respectively give their attention registers as a concrete difference in their work with set theory and model theory.

Thus, a first way of construing the difference between Tarski's and Badiou's attempts at defining truths or the processes involved in the production of truths is to root it in distinct considerations of what is required by the Tarskian criterion of material adequacy. In Chapter 2, I explained that this criterion, in its most general application, is one Tarski himself took to concern whether a definition does or does not “in fact grasp the current meaning of the notion as it is known intuitively” or pre-reflectively, pre-theoretically.²⁰ As Tarski puts this point in his 1944 essay on truth, “the desired [that is, materially adequate] definition does not aim to specify the meaning of a familiar word used to denote a novel notion; on the contrary, it aims to catch hold of the actual meaning of an old notion.”²¹ For a definition of truth to be materially adequate, it must account for the use of the notion of truth operative in concrete everyday situations. For Badiou, who

²⁰ Tarski, “On Definable Sets of Real Numbers,” 128–29.

²¹ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 341.

insists that “we are contemporaries of a *new departure* in the doctrine of truth, following the dissolution of its relation of organic connection to knowledge,”²² the criterion of material adequacy requires, in addition to the constraints Tarski seems to have felt it to impose, that a definition of truth capture something of what has come to be recognized as lying at the heart of the material production of truths and as constituting in some way the dissolution of the once credited organic connection between truth and knowledge.

This key difference between Tarski’s and Badiou’s approaches to truth in the first place—this difference between them as regards what the criterion of material adequacy requires of philosophical work on truth—leads to their ultimately distinct ways of drawing on set and model theoretical machinery in their respective formalizations. Tarski’s insistence that truth can be thought entirely independently of the role its pursuit plays in theory change leads him to fix on the usefulness of the semantic notion of satisfaction, while Badiou’s insistence that truth cannot be thought independently of the role its pursuit plays in theory change leads him to fix on the usefulness of the Cohenian operation of forcing. These points need some exposition.

Although Tarski had not yet developed model theory enough in the 1930s to describe things this way, it should be said that satisfaction is the basic operation of model theory. What allows a model of a particular (fully formalized) theory to be identified as that model is the fact that it satisfies the theory in question.²³ Satisfaction is the primitive, undefined semantic notion because it is the basic relation between words and things that organizes (or can be used to organize) the materialization of otherwise strictly formal systems. But it must be asked exactly *how* satisfaction is to serve, as it does for Tarski, as

²² Badiou, *Being and Event*, 3.

²³ This is explained in some detail in Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 120–25.

the basic organizing principle of all materialization—and therefore of all commerce across the border between the formal and the material, the syntactic and the semantic. As Tarski himself puts it in an early paper, “it has been found useful, in defining the semantical concepts, to deal first with the concept of satisfaction; both because a definition of this concept presents relatively few difficulties, and because the remaining semantical concepts are easily reducible to it.”²⁴ But *how* is it that satisfaction regulates the relation between the formal and the material? And does the manner in which it does so bear in any way on the connection—organic or not, dissolved or not—between truth and knowledge?

A too-facile approach to this last question might begin by guessing that Badiou’s lack of interest in satisfaction is rooted in an implicit conviction on his part that satisfaction somehow smuggles knowledge into the Tarskian definition of truth. After all, satisfaction is, as discussed in Chapter 2, just a matter of replacing the variables of a sentential function with constants (“objects”) and then asking whether the state of affairs indicated by the resulting sentences is indeed the case.²⁵ Perhaps one could argue that some sort of presupposed knowledge lies at the heart of asking whether the state of affairs indicated by a particular sentence is actually the case. This, however, would be a mistake, as the warnings already heeded above from Donald Davidson help to indicate. If anything, the problem with Tarski’s notion of satisfaction from Badiou’s perspective is that its elucidation seems to be possible only by having recourse to the notion of *truth*. To

²⁴ Tarski, “The Establishment of Scientific Semantics,” 406–407.

²⁵ Again, to illustrate this operation, Tarski turns to simple algebra: “A similar construction will be familiar to the reader from school algebra, where sentential functions of a special type, called *equations*, are considered together with the numbers which satisfy these functions, the so-called *roots* of the equations (e.g. 1 is the only root of the equation ‘ $x + 2 = 3$ ’).” Tarski, “The Concept of Truth in Formalized Languages,” 190. Other simple algebraic examples can be found in Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences*, 5: “For example, the numbers 1, 2 and $2\frac{1}{2}$ satisfy the sentential function: $x < 3$, but the numbers 3, 4, and $4\frac{1}{2}$ do not.”

say that satisfaction is just a matter of replacing the variables of a sentential function with objects and then asking whether the state of affairs indicated by the resulting sentences is indeed the case is, it seems, just to say that satisfaction is a matter of replacing appropriate variables and then *asking whether the resulting sentences are true*. Does Tarski not end up providing an ultimately circular definition of truth, such that truth is defined in terms of satisfaction, which is in turn defined in terms of truth? As discussed before, it is in order to avoid this problem of circularity that Tarski insists on leaving the notion of satisfaction undefined, despite both his actual clarifications of the notion directly in his work on truth and his talk elsewhere of encountering few difficulties in providing a definition of the notion. It would seem that Tarski means to relegate all clarity regarding the notion of satisfaction to the *practical* realm, retaining the concept as strictly undefined *only in the abstract*. And *in practice*, it would seem that satisfaction is in fact dependent on the notion of truth. Truth and satisfaction are, in practice, interdefined, entangled at their roots.

Whatever other implications may be bound up with the interdefinability of truth and satisfaction in Tarski's work,²⁶ it at least helps to clarify this: that satisfaction does not in any way smuggle knowledge into the definition of truth. At the heart of satisfaction there lies not a question of knowledge but a question of being. Tarski understands satisfaction to be a matter of asking whether certain states of affairs (described in would-be true sentences) *are* the case, not at all of asking whether they are *known* to be the case. Here as elsewhere in his work on truth, Tarski rightly insists on the epistemological neutrality of his truth definitions. They say nothing of the known or even of the

²⁶ This is, of course, the beginning point of Donald Davidson's development of Tarski's work on truth. See especially Davidson, *Truth and Predication*.

knowable, even or especially in virtue of their being constructed from the semantic notion of satisfaction. True sentences might be either contingently or even necessarily unknowable without their truth suffering in any way. Tarski's construction of truth definitions, it would seem, *exemplarily* accommodate any and every supposed dissolution of the organic connection between truth and knowledge, freeing truth as a concept so that it can be thought in its independence from specific epistemologies.

And yet, one might well argue that it is precisely the epistemological neutrality of Tarski's work on truth that implicitly worries Badiou. What ultimately interests Badiou in the Cohenian forcing operation is the way it resists what he takes to be the reduction of all truth to the knowable (that is, discernible) in the Gödelian constructible universe. The operation of satisfaction, in this way embodying the epistemological neutrality of Tarski's entire project, fails to discriminate between the knowable and the unknowable. Consequently, Tarskian definitions of truth, built up from the non-discriminating notion of satisfaction, fail to distinguish between the sort of truth that is knowable and the sort of truth that is, strictly speaking, unknowable. At the heart of Badiou's work on truth is a distinction between what he calls the veridical and what he calls truth, the former being effectively *knowable* truth and the latter *unknowable* truth—or, better, truth *unknowable without a major change in the relevant epistemological frame* (without, that is, theory change). Tarskian satisfaction does not discern this difference in its epistemological neutrality, bracketing not only the knowing subject inherent in so much of modern philosophy but also the revolutionary subject operative in the kinds of thought to which Badiou gives so much of his attention (Freudianism, Marxism, and so on).

Thus, despite appearances, Badiou's talk of the dissolution of an organic

connection between truth and knowledge is not meant to gesture toward the Tarskian insistence that truth can be regarded in strict independence of any epistemological entanglements (even if epistemology cannot, in turn, pursue its work without close attention to the notion of truth).²⁷ Rather, it is meant to summarize his conviction that the material production of knowledge relates to truth in two fundamentally distinct ways. First, the material production of knowledge takes place within a concrete setting in which certain truths are taken to be known and other truths are taken to be, if not directly or already known, to be already *knowable* within the present epistemological frame. Second, the material production of knowledge involves not only coming to know truths already knowable within the present epistemological frame (for example, what Thomas Kuhn refers to as “problem solving” within the scientific enterprise), but also coming to know truths constitutively unknowable within the present epistemological frame (through what Kuhn, for example, calls “scientific revolutions”).²⁸ The dissolution of the organic connection between truth and knowledge is not, for Badiou, the recognition of the possibility of considering truth independently of knowledge in order to become clear about its concrete definability; it is rather the recognition that the production of truths involves transformation of the basic categories of thought in addition to and alongside straightforward use of any presently normative categories of thought in the pursuit of knowledge. As Badiou puts this point in his earlier *Theory of the Subject*, there are both structural *and* historical considerations at issue in truth.²⁹

From a Badiouian perspective, the notion of satisfaction would seem to be a

²⁷ On this parenthetical point, see Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 366–69.

²⁸ See again, of course, Kuhn, *The Structure of Scientific Revolutions*.

²⁹ See Badiou, *Theory of the Subject*, 3–50.

materially inadequate base from which to construct a definition of truth because its direct recourse to what *is* (rather than to what is *known* to be) fails to distinguish between these two sorts of considerations. Badiou after all turned his attention eventually from the question of the undecidable as such to the specific undecidability of Cantor's continuum hypothesis. Only where there is a question of deciding between two materializations of the formal, one of which (on Badiou's interpretation) recognizes both the structural and the historical and the other of which (again, on Badiou's interpretation) recognizes only the structural, is it possible to begin constructing a materially adequate definition of truth. Satisfaction *as such* does not distinguish among models, merely providing for their identification as models. What Badiou regards as necessary to his own work on truth is the strategy of forcing that decides among models of set theory in favor of the non-discernible or generic, beginning thus from a notion of truth riven at its heart in accordance with what can be learned from the material production of actual knowledge.

Of course, as I already made clear in Chapter 1, Tarski was in no way unaware of the general problem of theory change, nor of the necessity of interlacing truth and knowledge in important ways in any attempt to account for such historical matters. I before alluded to a crucial section of his 1944 paper on truth, in which he distinguishes between truth, properly speaking, and what he calls "acceptability":

One of the main problems of the methodology of empirical science consists in establishing conditions under which an empirical theory or hypothesis should be regarded as acceptable. This notion of acceptability must be relativized to a given stage of the development of a science (or to a given amount of presupposed knowledge). In other words, we may

consider it as provided with a time coefficient; for a theory which is acceptable today may become untenable tomorrow as a result of new scientific discoveries.³⁰

Tarski here clearly recognizes the temporal or historical aspect of truth that must be considered in addition to the merely structural aspect of truth, to which he gives primary place in his work. *And yet*, as he goes on, he makes clear that his interest in the question of theory change does not involve much of the epistemological complexity Badiou finds there. Tarski reduces the relationship between truth and acceptability to two postulates. According to the first, “as soon as we succeed in showing that an empirical theory contains (or implies) false sentences, it cannot be any longer considered acceptable.”³¹ According to the second, which, Tarski states, “could be at most regarded as the expression of an ideal limit for successively more adequate theories in a given field of research,” every genuinely “acceptable theory cannot contain (or imply) any false sentences.”³² These two postulates reveal that Tarski even here does not distinguish between knowable and unknowable truths. Clearly, he presupposes that theory change is a matter not of encountering the unknowable in a manner that calls for a revolution in theory, but simply of recognizing where the knowable forces the relinquishment of certain sentences built into a theory.

What seems to be at issue for Badiou, then, is the idea that the operation of satisfaction—even when it is used to define truth and then truth is taken to have historical epistemological entanglements—is too weak to account for theory change as it actually unfolds in concrete settings. As the operation that determines no more and no less than

³⁰ Tarski, “The Semantic Conception of Truth and the Foundations of Semantics,” 366.

³¹ *Ibid.*, 367.

³² *Ibid.*

that models of a particular formal system *are* models for that system, satisfaction constitutively *cannot* explain the process of change that takes place when what is unknowable (due to the structure of a given epistemological frame) becomes knowable (through the transformation of the structure of the given epistemological frame). Only *specific* models—or really *a* specific model of classical set theory—provide the formal resources necessary to figure adequately the nature of theory change, from Badiou’s perspective. Although Tarski recognizes that there is theory change, from a Badiouian perspective he seems not to recognize that only certain materializations of set theory provide some kind of genuine account of it, and so he sees no problem with making satisfaction the foundation of every truth definition.

Badiou turns his attention from satisfaction to the Cohenian operation of forcing because he there finds a direct capture of the process of theory change, a kind of subjective decision on the undecidable that directly and materially transforms (by force!) another model or materialization of formal set theory. Tarski ultimately defines truth as whatever holds across all models, whatever is universally satisfied regardless of which materialization one considers. (It will be remembered that, according to Tarski, “a sentence is true if it is satisfied by all objects, and false otherwise.”)³³ Badiou, however, defines truth as what is produced as one decides on a certain materialization and follows out the consequences of that decision through the work of forcing necessary to bring that materialization to quasi-completeness (at least in the form of the future anterior). Even if Tarski recognizes that there is theory change, he does not capture it directly in the formalisms of his definition of truth. Badiou, however, believes that he does so.

This difference is of real importance. It is clear that, so far as basic ontological

³³ *Ibid.*, 353.

matters are concerned, Tarski and Badiou effectively agree that truth must be regarded as a model theoretical question, as a question of what happens in the complex entanglement of the strictly formal and the strictly material, rigorously construed in terms laid out in model theory. But when it comes to determining the exact formula or operation that allows for the designation or construction of particular truths for particular languages, the two thinkers differ strongly, and in a surprising way.³⁴ Tarski takes model theoretical considerations to provide the basic shape for truth definitions, such that the basic operation by which models are determined as models—satisfaction—is used in the construction of such truth definitions. For his part, Badiou understands the material production of truths to be captured only by a specific model of the basic axioms of set theory, the Cohenian model in which the operation of forcing allows for the construction of generic sets. Both thinkers understand truth to be definable only for particular languages, and each regards the production of that definition to be a matter of designating or constructing sets of determinate sentences. But what leads them to look to radically distinct set theoretical operations—satisfaction and forcing, respectively—is the fact that each understands the importance of model theory to the definition of truth to be distinct.

Material Adequacy and Model Theory

It is time, at last and rather briefly, to step back from strictly comparative work to add a few words of general philosophical reflection on what has been gained through this study. In the broadest terms, what has emerged in this dialogue of sorts between Tarski and Badiou is a set of motivations for believing that truth does indeed deserve to be

³⁴ And, as I have already noted, this point of difference has certain retroactive effects on the two thinkers' approaches to ontology.

closely linked with set theory—and more specifically with the model theoretical development of set theory largely accomplished in its initial form by Tarski himself. Inasmuch as truth is a matter of the entanglement between the formal and the material, and inasmuch as model theory figures that entanglement in a particularly rigorous fashion, independently of strictly metaphysical considerations, model theory provides a particularly fruitful beginning place for thinking through the basic commitments of every conception of truth. What remains unclear at the conclusion of comparative work, however, is exactly what is to be said about actual definitions of truth in light of model theory. Is the basic gesture of model theory—captured operationally in satisfaction—enough to secure a satisfactory definition of truth? Or is it necessary to consider more specific findings of model theoretical work—for example the strategy of forcing useful in determining, model theoretically, the undecidability of certain hypotheses—to provide a fully satisfying definition of truth? Is truth to be found in the model theoretical *as such*, or in *specific employments of model theory*?

What has emerged in the course of comparison, it seems to me, is the centrality to this question of Tarski's criterion of material adequacy, whether understood in its more general or in its more specific formulation. Does the criterion demand just that the material excess of certain formal systems over themselves be taken into consideration (so that truth is not mistakenly taken to be equivalent to provability)? Or does the criterion demand, further, that the variety of materializations of a given formalism be considered for the adequacy to what is known of the material production of knowledge? Must an adequate definition of truth be able to discern between the knowable and the unknowable, or is such a distinction fundamentally irrelevant to the strict notion of truth? What does

the criterion of material adequacy demand in this regard?

Fixing the set theoretical problematic of truth on Tarski's criterion of material adequacy is helpful because it allows that criterion to fix both the ontological site of the debate between Tarski and Badiou and to specify the stakes of their real disagreement. It is the strictly *material* considerations of truth that bring thinkers as different as Tarski and Badiou to the same ontological position, where the fact is glimpsed that truth cannot be thought through to its conclusion independently of a fully rigorous investigation of the model theoretical entanglement of the formal and the material. But it is the essential ambiguity of the term "material" itself that gives rise to the debate between the two thinkers. Should material considerations be exhausted by a bare accounting for the semantic nature of truth? Or must they extend into adjudication among materializations, some of which suggest rather different relations between truth and knowledge?

Here, at last, it seems, is the real point of debate. The question that set theoretical work on truth brings out with unique force is whether an account of truth must, or whether it need not, say something about the relationship between truth and knowledge, right in its definition. Badiou's work reveals that it is possible to provide formalized definitions of truth that concede or inveigh against the reducibility of truth to the knowable. Tarski's work reveals that it is also possible to construct formal definitions of truth that relegate such distinctions or concerns to irrelevant epistemological consideration. But whatever of these approaches one might take, they are all to be seen as gathered about a model theoretical (and therefore set theoretical) site. Whether truth should be defined solely in terms of what is found in every model, or whether it should be defined more strictly in terms of what can be produced only within certain

philosophically suggestive models (in fact, through a forced transformation of one model into another), truth unmistakably lies at the heart of model theory.

For my purposes in this dissertation, it is enough just to make clear that there *is* the problematic that all this comparative work reveals. Tarski and Badiou are not isolated philosophical oddballs, coincidentally interested in both truth and set theory. Rather, they jointly establish the site whereon the philosophical problem of truth remains to be worked out in the most formal way. Only in their convergences and divergences, moreover, are the basic stakes of the problem made clear. How one will decide to move forward in pursuing the problem and its solutions remains, of course, to be determined.

CONCLUSION

Close comparison of Alfred Tarski's and Alain Badiou's respective attempts to formalize the notion of truth reveals a central problematic. What secures the consistency or identity of this problematic is a set of shared ontological considerations. For as long as truth has been investigated philosophically, it has been recognized that there must be some kind of *relation* at work in truth, whether—as I noted in the Introduction—that relation has been taken to be a matter of correspondence between minds and objects in the world, of coherence among terms or claims made regardless of inaccessible states of affairs, or of fulfillment of intentions and aims in practical encounters with the world. Set theoretical work on the problem of truth suggests that what lies behind every attempt at fixing this relation is a presupposed entanglement between the formal and the material. Before deciding among distinct construals of that relation—as correspondence, as coherence, as fulfillment—set theoretical work on truth suggests that it is necessary first just to become clear about the question of the entanglement of the formal and the material. Developments internal to set theory further reveal that there are important resources for clarifying that entanglement in full rigor. The development of model theory has made it possible to see in a way previously indiscernible exactly how to mark the boundary between the formal and the material, between the ideal and the real. And the nature of that boundary is made fully clear by model theoretical work—not only *where* it lies, but also *how* it operates—both in terms of what a materialization of the formal genuinely amounts to and in terms of how the variety of possible materializations of the

formal constrains and clarifies the nature of the formal.

Regardless of any particular metaphysics, the model theoretical regulation of the relation between the formal and the material constitutes a set of minimal ontological considerations. That is, set theory's necessary development into model theory makes fully clear that set theory bears within itself deeply (yet minimal) ontological implications, where ontology is to be understood just as a matter of the necessary constraints the formal and the material impose on one another, regardless of any actual metaphysical claims that might be made about what, if anything, "exists" in either the formal or the material realms. And truth, it seems, can be understood to be rooted in these ontological considerations. If, as Aristotle said, to speak truly is to say of what is that it is, while to speak falsely is to say of what is that it is not, then any adequate conception of truth must begin from a close investigation of the ontological implications of the entanglement of the formal and the material, and these as they are set forth in the model theoretical developments of the set theoretical enterprise.

This ontological matter, however, is more the *site* than the *struggle* of serious philosophical work on truth. What further demands set theoretical attention in every attempt to think truth concerns what resources might be used to provide a full elucidation of the concept of truth. If the basic entanglement of the formal and the material is mapped first and foremost in set theory and model theory, there is at least good reason to think that truths themselves are sets. There is therefore good reason to believe that there lies at the heart of truth some identifiable formula or operation by which those sets that can be reasonably defined or constructed. Given the basic model theoretical bearings of the problem of truth, it is clear that this question concerns deciding exactly which resources

to draw on, made available within formal work on models. And here, comparison between relevant thinkers reveals that matters are far from settled. If it is clear that truth cannot be said to be merely formal, at the very least because of Tarski's theorem and others like it (Gödel's theorems, the Löwenheim-Skolem theorem, and so forth), it nonetheless remains unclear exactly how to decide which, among available options, is the materially-relevant operation by which truths are to be determined as the sets they are.

Comparative work, in other words, reveals a genuine struggle, and a struggle that concerns the relevance of epistemological considerations to the philosophical investigation of truth. Can truth be considered entirely independent of epistemological considerations—such as, for instance, the question of theory change—without producing an account of truth that appears materially inadequate? Or can truth be considered in its deep epistemological entanglements—again, as with considerations relevant to theory change—without forcing truth to become a largely subjective matter? It seems that the very first question that must be asked is whether truth must be considered in connection with or independently of its epistemological entanglements when it is considered in connection with set theory and model theory. Is truth inherently riven at its heart by the distinction between knowable and unknowable truths, or is truth a fundamental unity regardless of any such fine epistemological distinctions?

But these are only the questions that seem to emerge in a contrived conversation between Tarski and Badiou. What other considerations might prove useful or even necessary to identifying in a materially adequate way the set theoretical or model theoretical operation that would allow for the fixing of appropriate definitions of truth? A closer investigation of model theory is, it seems to me, necessary to such an investigation.

And while set theory has received its share of philosophical interpreters—beginning already with Georg Cantor and Gottlob Frege—model theory seems largely to have been ignored by philosophers, taken to be chiefly of logical or mathematical concern. What light might be shed on truth by closer investigation of model theory? This remains to be seen.

I have nonetheless, I believe, identified here the problematic site of a problematic struggle, largely still to take place, over the philosophical investigation of truth. Truth is closely enmeshed in a very specific context, if it is considered rightly. It is, I believe, focused indelibly on model theoretical questions, questions that have grown out of set theoretical questions. This Tarski has made clear for the analytic philosophical tradition, even if his work has been taken as a beginning point for reflection on truth in rather different and less mathematically rigorous ways. And this Badiou has made clear for the continental philosophical tradition, even if his work only formalizes what has been implicit for a long time and has come increasingly to serve only as a side attraction. The two philosophical traditions converge on questions of the formal and the material, as points of historical intersection have long revealed. That they converge in a particularly forceful way on the question of models has, however, seldom if ever been glimpsed. It is here that one of the longest-standing questions of philosophy must be addressed and the struggle for truth must continue.

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