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High performance damper optimization using computer simulation and design of experiments

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HIGH PERFORMANCE DAMPER OPTIMIZATION USING COMPUTER SIMULATION AND DESIGN OF EXPERIMENTS

BY

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B.S., EARTH & PLANETARY SCIENCES, UNIVERSITY OF NEW MEXICO, 1998

THESIS

Submitted in Partial Fulfillment of the
Requirements for the Degree of

Master of Science
Mathematics

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Albuquerque, New Mexico

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Dedication

To my mother, the most generous, supportive, kind, and loving person I know.

Acknowledgements

I would like to thank my mother for providing constant love and support throughout my life. I am particularly thankful for her persistence for me to have options in life, one of the main reasons I have gotten to where I am now. I would like to thank Dr. Ed Bedrick for his contributions to my thesis, and most importantly, for not giving up on me even though it took a long time to get this done. I would like to thank Chris Mower for taking a risk on hiring an unknown and inexperienced engineer from Albuquerque. It is because of this opportunity that I've been able to work at the highest level of motorsport the United States has to offer. Because of the opportunity I've also been able to travel the world and engineer a car with a top-ten finish at the 2006 Indianapolis 500, as well as participate directly in more than 75 major professional racing events, including a race win at the Long Beach Grand Prix, the final Champ Car race in history. I would like to thank all of the people that I've had the opportunity to work with in racing. I have increased my knowledge dramatically because of their help and advice. I would like to give a special thanks to Doug Earick for being a great friend and for all his support with this paper and many other things. I would especially like to thank Stephanie Weber for her patience, love, and support needed to complete this manuscript. Finally, thanks to all my friends and family, for all of their support and who make life a terrific adventure.

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ABSTRACT OF THESIS

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Abstract

Design of experiments has become an increasingly popular tool used by racing teams in professional motorsports. A race car has literally hundreds of adjustable parameters associated with it. Specialized racing engineers are in a constant search to find the optimal combination of settings for these parameters in order to optimize a given car's speed and performance potential for a given driver and race track. Many teams employ a wide variety of computer-based simulations and actual development testing to help optimize aspects of the car's performance, which in most cases is ultimately the minimum lap time for a given race track. Most team development however, requires a large amount of money and time.

The purpose of this study was to develop predictive models and optimization methods for the mean pitch response of a race car, based on a virtual 7-post rig simulation software called Rigsim. In addition, chosen factors

were varied in order to determine the effects and interactions of different factor configurations on mean pitch response. The primary factors of interest were the front and rear, low and high speed bump and rebound damper settings, as well as the front and rear tire stiffness. An initial fractional factorial DOE was generated to study the mean pitch response as a result of selected damper and tire stiffness settings. The results of the DOE were then used to create a model in order to help predict the mean pitch response for a given combination of damper and tire stiffness settings. The initial experiment was then augmented by a D-optimal response surface design in order to further explore the design space and predictive capabilities of the model.

The DOE portion of the study utilized software named Design Expert for the experimental design, data analysis, optimization, and model creation. Optimization routines were employed to optimize the mean pitch response of the virtual Rigsim software, given a range of damper and tire stiffness settings. Optimal solutions were then compared to Rigsim simulations to gauge accuracy and determine the validity of the model.

Design of experiments was shown to help effectively compile a significant amount of information with a relatively small subset of experiments. The model proved to be a fairly reasonable predictor of the simulation’s mean pitch response within limits. Statistical analysis of the data helped determine significant effects and interactions involving mean pitch response, thus providing suggestions in order to focus on factors likely to improve mean pitch response. It appears to be most useful to study trends and comparisons between different damper and tire configurations. Ultimately, the approach to information gathering and modeling used in this study has potential to be highly useful in many aspects of race car engineering.

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Nomenclature

a	Longitudinal distance between sprung mass COG and front axle centerline
b	Longitudinal distance between sprung mass COG and rear axle centerline
c_f	Front suspension damping rate
c_r	Rear suspension damping rate
CF_Y	Lateral coefficient of friction between the tire and road
ct_f	Front tire damping rate
ct_r	Rear tire damping rate
d_i	Individual response desirability function
D	Desirability function
F_Y	Lateral force generating capability of the tire
F_Z	Vertical load on the tire
FRH	Front chassis ride height or ground clearance
I_c	Pitch inertia of the chassis
k_f	Front suspension effective spring stiffness
k_r	Rear suspension effective spring stiffness
K_S	Effective suspension spring stiffness at the wheel
K_{susp}	Combined series spring rate of the tire and effective spring stiffness
K_T	Tire spring stiffness
kt_f	Front tire spring stiffness
kt_r	Rear tire spring stiffness
K_{TREF}	Reference spring rate at the reference tire pressure

m_c Sprung mass

mt_f Front unsprung mass

mt_r Rear unsprung mass

n Number of response measurements

Pitch Relative displacement difference between front and rear ride heights

RRH Rear chassis ride height or ground clearance

TP_R Reference tire pressure

TP_A Actual operating condition tire pressure

TP_{sens} Tire pressure stiffness sensitivity

x Vertical displacement of the sprung mass at the COG

$\dot{x}$ Vertical velocity of the sprung mass at the COG

$\ddot{x}$ Vertical acceleration of the sprung mass at the COG

y_f Vertical displacement of front unsprung mass

$\dot{y}_f$ Vertical velocity of front unsprung mass

$\ddot{y}_f$ Vertical acceleration of front unsprung mass

y_i Individual response measurement

y_r Vertical displacement of rear unsprung mass

$\dot{y}_r$ Vertical velocity of rear unsprung mass

$\ddot{y}_r$ Vertical acceleration of rear unsprung mass

z_f Vertical displacement of front hydraulic ram

$\dot{z}_f$ Vertical velocity of front hydraulic ram

z_r Vertical displacement of rear hydraulic ram

$\dot{z}_r$ Vertical velocity of rear hydraulic ram

Z_f Magnitude of the FFT of rig front wheel pan displacement data

θ Rotational angle of the sprung mass

$\dot{\theta}$ Rotational velocity of the sprung mass

$\ddot{\theta}$ Rotational acceleration of the sprung mass

Θ_p Magnitude of the FFT of rig pitch data

1 Introduction

1.1 Overview

The racing damper a.k.a. "shock absorber" has long been an area of great interest to engineers responsible for optimizing a race car "setup" for a given race track and driver combination. The racecar's suspension damper is a device used to control and dissipate the energy resulting from the car's suspension springs, wheels, and chassis motions. The dampers generate forces directly proportional to the velocity of their movement and are therefore considered to be transient behavior tuning devices. Although there are a variety of high performance racecar dampers available, the overall functionality and tunable characteristics are often very similar.

1.2 Basic Vehicle Dynamics

As a car traverses a race track, there are many road-induced and driver-induced inputs or excitations that energize the car's chassis and suspension. These inputs can involve a large range of frequencies and are usually composed of many frequencies superimposed into a combined input. The resulting wheel and suspension velocities are determined by the input frequency and energy content of the inputs. Some examples of typical road surface inputs include: a fairly smooth road with many small bumps, a rough road with large bumps, dips, curbs, as well as transient accelerations and motions of the car due to driver-induced inputs such as braking, accelerating, and turning. The magnitude of the driver-induced accelerations is governed by the level of mechanical "grip" available between the tire and road. Typically, driver-induced inputs result in low speed suspension velocities in the 0 to 2 inch/sec range, while road surface inputs and curbs tend to involve higher suspension velocities, upwards of 20 in/sec depending on the race track's vertical profile, etc. The car as a whole responds to such inputs in varying ways, but there are four major modes or motions of the sprung mass of interest to the racecar engineer. These modes of motion are referred to as pitch, yaw, roll, and heave. The sprung mass of the racecar includes the car's chassis, transmission and differential, the driver, fluids such as fuel and various oils, and half of the mass of the suspension components. The unsprung mass includes the wheels, tires, wheel hubs or uprights, and the other half of the suspension components. The sprung mass of the car rests on the cars suspension and tires. The suspension and tire at a given corner of the car are equivalent to a series of springs with the damper force in parallel with the effective suspension spring force and tire spring force. The tire spring is in series with the suspension spring. The inherent stiffness of the suspension members, which depends on the material, wall thickness, tube diameter, and so forth, create a third spring in series with the suspension spring and tire spring, but is usually much stiffer, i.e., has a much higher spring rate than the other two [12].

As the car moves along the track surface, the sprung mass moves in relation to the unsprung mass at the four corners and the road in response to driver and road inputs and subsequent accelerations. Pitch is characterized by the angle of the chassis relative to the road in the fore and aft, or longitudinal direction. Roll is characterized by the angle of the chassis relative to the road in the side to side, or lateral direction. Yaw is characterized by horizontal rotation of the car about a vertical axis. Finally, heave is characterized by the displacement of the chassis relative to the ground with equal movements of all four corners. A car will rarely undergo pure heave, pitch, yaw, or roll motions, but understanding the chassis' response to pure inputs can greatly help the understanding of the sprung mass' behavior in general, and as a function of damping.

The loads on the four tires of a racecar statically at rest are determined by the car's weight due to gravity, and the longitudinal location of the car's center of gravity as a whole. Therefore, a 1000 lb car with a 40% front weight distribution will have four hundred lbs on the two front wheels, and 600 lbs on the two rear wheels. If the car is symmetrical in terms of weight distribution relative to its longitudinal centerline and therefore has its center of gravity located along its centerline, then the load across the front and rear wheel pairs will be divided equally. Therefore in this example, the two front tires will carry 200 lbs each, and the rear tires will carry 300 lbs each.

Dynamically, the loads on the four tires are constantly changing in response to accelerations of the racecar due to the engine, brakes, steering inputs, as well as the aerodynamic forces generated and road inputs such as bumps and curbs. Since all forces are transmitted to and from the car via the tire and road surface interface, a moment or torque is created about the car's center of gravity and the ground. Loads are transferred around the four tires in response to the above-mentioned accelerations of the cars inertial components via these moments. Dynamic load transfer is largely dependent on the height of the sprung mass' center of gravity. Often, racecar engineers try to achieve a minimal dynamic ground clearance, a.k.a. ride height, in order to lower the center of gravity and subsequently reduce the amount of dynamic load transfer. Lowering the car's ride height often will increase useful aerodynamic forces and subsequently improve the overall performance capability of the car. Ultimately, for a given acceleration, the absolute amount of weight or load transfer is fixed for a given racecar, but the timing, speed, and characteristics of the load transferred can be changed and controlled primarily via the car's suspension dampers. Since the acceleration of the racecar is determined by the "grip" available from the tires, and the "grip" of the tires on a given track surface is determine by the amount of vertical load on the tires at any given time, then it is obvious that controlling the vertical loading on the tires is of utmost importance for racecar handling and performance optimization.

1.3 Purpose of the Racing Damper

The racing damper serves multiple purposes as a component of a race car's suspension. The most obvious is its ability to dissipate energy stored by the suspension springs. Unlike the suspension springs, which generate force through displacement, dampers are velocity-dependent and are therefore transient tuning devices. Dampers also have a large impact on the overall attitude of the race car as will be discussed later. The other major characteristic of a racing damper is its ability to affect the transient mechanical balance of the car as a whole. For instance, a car that has rear instability during the initial phase of turning quickly into a corner might be improved under these conditions through damper adjustments. Another characteristic of the racing damper is its affect on the variation of vertical loading on a given tire, at a given corner of a race car. Racing engineers tend to refer to this as having an impact on the grip potential of the given tire under various conditions. The damper also has a large influence on keeping the tire in contact with the ground during accelerations and road inputs [3, 4, 8].

1.4 Damper Characteristics and Tunable Parameters

A dampers motion in compression is referred to in the racing industry as the bump direction, and its motion in extension is referred to as rebound. Since the forces generated by the suspension dampers are dependent on the velocity of its shaft and piston, racing dampers have become highly tunable devices, particularly in terms of varying input velocities.

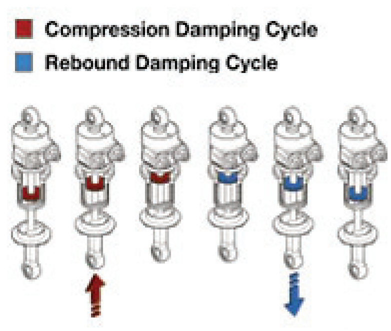

Figure 1.1 Diagram of damper stroke cycle, courtesy of Ohlins

As discussed by Warner [16], a typical racing damper allows adjustment of the forces it generates at low and high speed shaft velocities in the both the bump and rebound directions independently through a series of pressure relief valves and orifices. Many dampers consist of a single tube with an internal piston and shaft assembly, which is filled with the manufacturer or race team’s choice of oil. Valves and bleed holes are typically incorporated into the piston assembly to allow for piston motion through the fluid, although there are quite a few different types of internal damper valving in production. Damper forces are developed by the pressure drop across the piston as it moves through the internal damper fluid. Damper fluid typically flows across the piston through different valving circuits for the bump and rebound directions respectively. Damper forces are also created by internal gas "spring" pressure, and friction.

Since a racing damper's forces can be adjusted independently for high and low shaft speeds in both bump and rebound directions, a given set of adjustments will result in a damper that generates forces at varying shaft velocities that can be graphically displayed as a curve on an x-y plot of damper force versus shaft velocity. The following figure shows an example of such a plot.

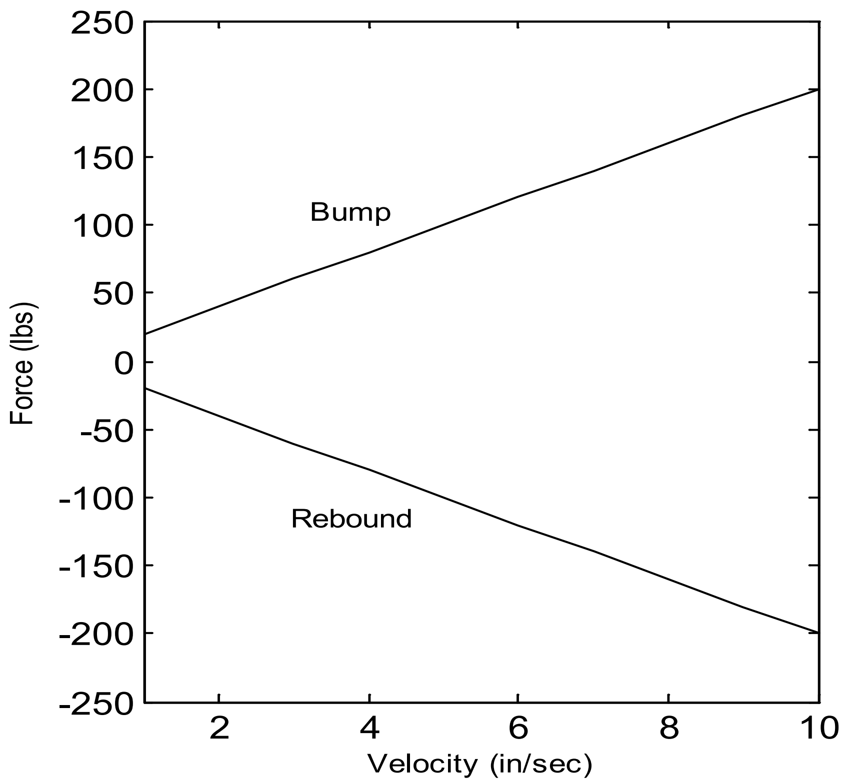

Figure 1.2 Damper force versus velocity

Depending on a given damper's level of adjustability, a wide variety of force versus shaft velocity curves can be achieved, which can directly influence the performance and handling of a racecar on track. A device known as a damper dynamometer and a computer can be used to measure the damper forces at varying shaft velocities to generate these curves and subsequently "characterize" the damper for a given set of adjustments. There are three types of common damper curves usually referred to as linear, progressive, and digressive. A linear damper will generate a linearly increasing force with increasing shaft velocity. A progressive damper will generate an exponentially increasing force relative to shaft velocity. A digressive damper will generate exponentially less force with increasing shaft velocity, and in many cases will reach a near-constant force value after a certain input velocity is reached.

The dynamics of the car are governed by physical phenomena that can be mathematically modeled using differential equations. This is the basis of the field of study known as vehicle dynamics. The dynamic response of the sprung and unsprung masses to road-induced excitations is very important in terms of racecar handling and performance. In particular, target parameters to be optimized usually include heave, pitch, contact patch load variation, and the phase relationships between the front and rear wheel's response to the inputs.

The stability of a racecar, and subsequent handling characteristics, are largely determined by the amount of damping available. If the sprung mass of the car resting on its suspension was subject to a temporary force input and then left to vibrate freely without damping, the sprung mass would oscillate at its natural frequency as a function of its mass, and would continue to oscillate indefinitely in the absence of friction forces. With the presence of damping however, the sprung mass oscillation over time relative to the input would be largely dependent on the level of damping involved at the various velocities of the suspension relative to the sprung mass. Ignoring the effects of friction in the suspension components, the motion would eventually cease due to the removal of energy from the sprung and unsprung mass system via the dampers. How quickly this motion is terminated, and in what fashion is the root of all studies involving suspension damping. If the dampers provide too much damping relative to the masses and suspension springs involved, the system may be over damped, i.e., will return to rest following an input without any overshoot or oscillation about the sprung masses' static position, and will take longer to reach its static position than if critically damped. Critical damping occurs when the system reaches its static position in the shortest time possible without overshoot or oscillation about the static position. If the sprung mass takes less time to return to a steady state position, but with overshoot and oscillations, then the system is said to be under damped [2].

1.5 The Racing Tire

The tires of the racecar are by far the most important component of a racecar in terms of performance and handling. Ultimately, the tire is the point of transmission of all forces to and from the racecar and the road. The part of the tire rubber in contact with the road at any time is usually referred to as the "contact patch". The size of a tire's contact patch is directly related to the tire's internal pressure, which is a tunable performance parameter onto itself. The tire's internal pressure also directly influences the spring rate of the tire [3]. The relationship between tire spring rate and the tire's spring stiffness can be described mathematically as follows:

$$K_T = K_{TREF} + TP_{sens}(TP_A - TP_R) \qquad (1)$$

where K_T is the tire stiffness in lb/in, K_{TREF} is the tire manufacture-specified reference spring rate at the reference tire pressure, TP_R in psi. TP_A is the actual tire pressure under operating conditions, and TP_{sens} is the tire pressure sensitivity given in (lb/in)/psi. The tire pressure sensitivity is a linear measure of tire spring rate change per unit of tire pressure change, and is usually provided by the tire manufacturer.

The tire's spring rate is in series with the suspension spring rate at a given corner of the car [12]. Unlike the suspension damper that provides energy dissipation to the suspension springs, a racing tire has very little internal damping, and is therefore not easily controlled. The combined series spring rate of the spring, resolved to the wheel through an appropriate motion ratio, and tire is calculated as [12]:

$$K_{susp} = \frac{K_T K_S}{K_T + K_S} \tag{2}$$

Where K_{susp} is the combined series spring rate of the tire and effective spring stiffness' respectively. K_T is the spring stiffness of the tire, and K_S is the effective spring stiffness at the wheel modified by the appropriate geometric mechanical advantage. This expression for combined tire and effective spring rate is known as the wheel rate.

The goal of all racecar engineering and subsequent adjustments to the racecar's setup are to optimize the loading on the four tires under any operating conditions, and thus maximizing the mechanical "grip" available from the tires to accelerate the car both longitudinally and laterally. A tire's level of "grip" or ability to generate accelerative forces to the car and driver are due to various phenomena. These include mechanical adhesion and cohesion characteristics, as well as typical frictional characteristics as involved with any two adjacent surfaces [3]. The coefficient of friction is a mathematical representation of the force required to produce relative motion between two adjacent objects. The higher the coefficient of friction between two objects, the more difficult and larger forces necessary to move one object relative to the other. A typical race track surface varies along its length, but assuming a uniform surface for simplicity, a given race tire will have a certain coefficient of friction between the track surface and itself. The coefficient of friction between tire and road is also determined by the amount of vertical force or load on the tire, i.e., how hard the tire is pressed onto the road surface. The industry standard reference to this load is known as contact patch load, or CPL. The coefficient of friction between tire and road while turning laterally is given by [12]:

$$CF_Y = \frac{F_Y}{F_Z} \tag{3}$$

where CF_Y is the coefficient of friction between the tire and the road, F_Y is the lateral force generating capability of the tire, and F_Z is the vertical load on the tire. With most racing tires, this number varies along a given axis direction.

1.5.1 Tire Pressure and Damper Optimization

Finding optimal tire pressures for a given set of running conditions, is as important as optimizing the cars damping levels. The pressure of a given tire is critical due to its affect on the tire’s spring rate, and subsequent dynamic contact patch size and shape. A tire’s spring rate is sensitive to its internal air pressure and rises with increasing internal pressure. The range of vertical spring rate that a tire has for a given range of internal pressures is governed by the tire’s construction, compound, etc [3]. Another aspect of tire pressure is that it influences the static and dynamic diameter of the tire. This occurs statically by expanding the tire carcass with increased air volume, while dynamically it also occurs due to the resulting tire's spring rate, and subsequent deflections under vertical loading. For example, a very stiff tire will not deflect as much across its diameter for a given load, thus maintaining a more constant and larger dynamic diameter as would a tire with a lower internal pressure, and thus spring rate. The tire itself acts as a spring with very little damping, and thus finding optimal damping levels through the dampers is very important, in order to best control the tire's grip potential, and the chassis' platform orientation. This will be discussed in further detail later.

1.6 Aerodynamics and Platform Control

Many racecars, particularly open-wheel cars such as those that participate in the famed Indianapolis 500 or the international Formula 1 racing series, are highly sensitive to the aerodynamic forces applied to the car via its front and rear wings. As discussed by Katz [9], some cars are especially sensitive to the aerodynamic forces generated by the underbody of the car and its venturi-type tunnels, if included. In many cases, the attitude or angle of the underbody with respect to the ground has a dramatic impact on the efficiency and location of application of the underbody's generation of aerodynamic forces. Opposite to the aerodynamic forces generated by vehicles of flight such as airplanes know as lift, aerodynamic racecars generate negative lift forces; referred to in the industry as downforce. The other major type of aerodynamic force experienced by racecars and any object moving through a fluid is drag.

Similar to an object's center of gravity, which is determined by its mass distribution, aerodynamic forces are applied to a racecar at an analogous theoretical point called its center of pressure, henceforth known as the COP. The fore and aft location of the car's COP, is determined by the distribution of aerodynamic forces along the length of the car. Since the underbody typically generates the majority of a racecar's aerodynamic downforce through ground-effect phenomena, the orientation of the underbody relative to the ground has a major effect on the overall level of downforce, and particularly on the location of the car's COP and subsequent handling balance [9]. The car's spring, damper, and tire combination has the most direct influence on controlling the car's platform, i.e., underbody attitude in response to road-inputs and driver-induced inputs and subsequent accelerations. Wind tunnel testing facilities along with varying scale racecar models can be used to determine the distribution of aerodynamic forces along the length of the car and the resulting COP due to a specific aerodynamic configuration and platform attitude.

For cars largely influenced by aerodynamic forces, the pitch attitude of the car tends to have the greatest impact on the overall aerodynamic downforce and drag, as well as the COP of the car at any given moment on the race track. If the chassis motion in pitch is not well controlled, and therefore allowed to oscillate quickly with large amplitudes, the aerodynamic forces will be constantly changing, while the car's COP migrates forward and backward with the pitch motions. This can result in a highly unpredictable and unstable racecar under various operating conditions [10]. This paper focuses primarily on optimizing the pitch control of the sprung mass of highly aerodynamic racecars, particularly those that are very pitch sensitive, as determined by wind tunnel testing.

Chassis pitch motions arise for a variety of reasons. As a driver applies braking forces to the car via the brake pedal, the car naturally pitches forward due to the inertial resistance of the sprung mass and the moment generated between the sprung masses' center of gravity and the tire and road interface. Likewise, as the driver accelerates the car via the throttle pedal and engine, the car naturally pitches towards the rear of the car for the same reasons. The development of aerodynamic downforce at both the front and rear of the car at different levels for a given road speed and suspension setup also induces pitch motions of the sprung mass.

The other important contributions to pitch motions are irregularities and bumps in the road surface and curbs. Driver's have a tendency to drive over curbs on the race track in order to shorten the distance traversed through a given corner and subsequently the lap. Driving over curbs also allows the driver to negotiate a given corner at a larger radius than if they didn't use the curb. Due to the relationship between lateral acceleration, tangential path velocity, and path radius, this means that the driver can maintain a higher cornering speed and corner exit speed by using the curb. Corner exit speed can be critical if the corner leads onto a long straight portion of the track. The downside to curbs is their influence on the stability, behavior and attitude of the sprung mass in response to the curb input.

Irregularities and bumps in the track also generate chassis pitch motions. Consider the situation of a car driving along a straight line approaching a bump in the road that has equal height and profile across the width of the cars wheels, also known as the track. As the car passes over the bump, the front wheels hit the bump first, followed by the rear wheels. The front wheels are thus forced into a damped harmonic oscillation just prior to the rear wheels, which follow the same fate. Since the whole car is ultimately connected at both ends via the sprung mass, the sprung mass must react to the inputs at both ends in a complex oscillatory motion that is a combination of pitch and heave motions. To further complicate the situation, the weight distribution of the car, the weight of the rear unsprung mass versus the weight of the front unsprung mass, the difference in spring rates of the front and rear suspensions and tires can create very different harmonic motions and response at the front and rear to the exact same input. Again, one must consider that the whole car is connected via the sprung mass, which creates a complicated sprung mass oscillation in relation to the road bump input considered. Likely, it will result in a series of irregular pitch motions whose control and duration is determined by the size of the input, as well as the damping levels of the four corner dampers.

2 7 Post Rig and Damper Optimization

2.1 The 7 Post Rig

Optimal damper settings can be very difficult to quantify and is usually largely subjective. Dampers are characterized by their force versus velocity curves. Determining what the best damper curve is for a given track and driver is usually based on driver feedback and trial and error tests at the race track. Some teams will also create simple linear dynamics models based on a quarter car. Driver-developed dampers may or may not be applicable to other drivers, while simple models do not incorporate the many non-linearity's found in vehicle and suspension dynamics [10].

The typical measurement to determine the effectiveness of a damper change at the race track is ultimate lap time. In order to really gauge the effectiveness of a change based on lap time, the driver has to be incredibly consistent from lap to lap on any given outing. The other problem that arises from lap time measurements is that the fuel load and tire wear is constantly changing as the driver and car are testing.

Unlike many adjustable racecar setup parameters that a driver can feel, damper changes are often not felt by the driver, but can have dramatic outcomes on racecar performance and handling, ultimately resulting in reduced lap times. Since the winner of any motorsports race is determined by the driver and car that completes the race distance in the least amount of time, lowering lap times is always the absolute goal of a racecar driver and their engineer. Particularly for qualifying, where the team has a chance to optimize their starting position for the race. Due to the inherent lack of feel from a driver for many damper adjustments, and other reasons to be discussed, methods other than track testing with a driver have been developed to explore damper optimization. One such method is the so-called "7 post-rig". There are many 7-post rig facilities around the world available to race teams for testing. One such example is the

Auto Research Center (ARC) in Indianapolis, Indiana. Many open-wheel and Nascar-series race teams utilize the ARC 7 post rig on a regular basis. ARC features a 7 post shaker unit manufactured by Servotest [10].

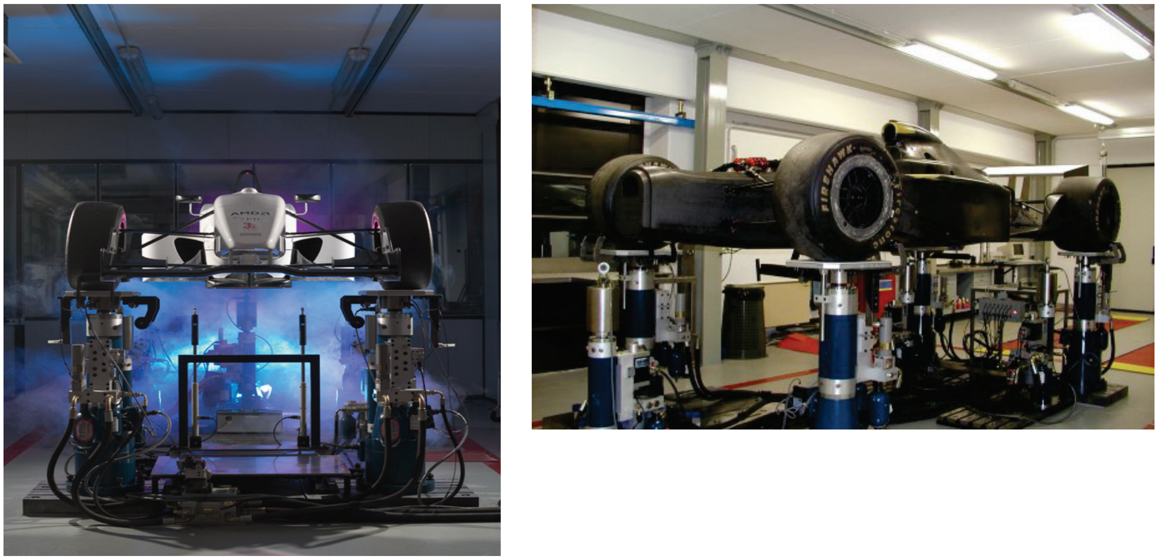

Figure 2.1 Seven-post rig photos, courtesy of Dallara Automobili

As discussed by Kowalczyk [10], the 7-Post rig, a.k.a the “shaker” or simply the “rig”, is a test apparatus that is composed of four hydrodynamic posts, a.k.a. wheel pans, on which the racecar’s wheels rest, along with up to three additional hydrodynamic rams mounted at various locations on the car. These are used to simulate aerodynamic forces and inertial loadings due to the front and rear wings, as well as the underbody of the car. The amount of induced aerodynamic load is usually based on actual values measured while cornering for the car being tested. The “rig” is controlled via a computer-based system that provides inputs to the 7 rams, as well as measurements using sensors of various components of the “rig” itself and the racecar. Some of the measurements include: wheel pan displacements and accelerations, car suspension displacements and wheel accelerations, suspension loads, and wheel pan loads [10].

There are different methodologies employed by various “rig” facilities and their engineers, but the general concept of the “rig” is to provide a computer-controlled excitation input to the racecar’s four wheels via the wheel pans, and then determine the response of the car’s sprung and unsprung masses, to the inputs. The input signals can consist of time-varying sinusoidal waveforms, white noise, random vibrations, and even race track surface profiles based on actual on-track measurements. The “aerodynamic”-simulating rams are used to hold the car down during the excitation period, while applying a pre-determined amount of “aerodynamic” force to the car. As discussed by Kowalczyk [10], the ARC testing methodology for an Indy-type car is to excite the car with a swept sine wave with all wheel pans in phase with one another. The excitation input is a pure heave input, with the sine wave frequencies swept between 0.5 and 20 Hz. The excitation input profile is generated for the sine frequency sweep in order to maintain a constant maximum velocity of 100 mm/s.

2.2 7 Post Rig Testing and Optimization

Pure lateral acceleration of the car in a turn produces a rolling motion of the chassis, but most actual dynamic chassis motions are a combination of roll, pitch, yaw, and heave. For simplicity sake, only pure pitch motions are considered in this study. For various reasons, a race engineer may have developed a specific front and rear spring package along with front and rear tire pressures that is felt to be the best compromise for a given race track. The next goal for the race engineer is to determine optimal damper characteristics and levels of damping to accompany the spring package of choice.

In the case of 7-post rig testing, the race engineer will set the car up with the given spring package, and then create a matrix of damper valvings and settings to test on the rig. For each test, the rig provides excitation inputs or signals to the wheels via the wheel pans and the sensors measure the accelerations and general response of the sprung and unsprung masses to the inputs. Transfer or frequency response functions are then created in order to characterize or measure the vehicle's response to the input. One such transfer function is the pitch transfer function, which provides a metric for determining the quality and magnitude of a car's pitch response to a given input [1, 10]. Once the testing is over and the matrix runs completed, the race engineer can analyze the results to determine the best pitch response for the given spring package.

2.3 Cost and Limitations

Rig testing can be difficult and inconvenient for two primary reasons. The first is the high cost for test time at the rig. It can range from five to ten thousand dollars per day of testing. The other problem arises from the busy testing and racing season schedule, which limits the amount of time a team has available for such testing. The other drawback to rig testing is the limited amount of tests that can be done in a day at the rig. Damper behavior and effects on the racecar setup is largely associated with the suspension spring and tire combinations used in conjunction with a given damper setup. It is therefore only possible to test a very limited amount of combinations during a single test day. Damper, suspension, and tire setup combinations do not have linear effects on racecar performance, thus making it virtually impossible to use the limited data obtained from the rig to extrapolate or deduce potential outcomes of using combinations not tested on the rig. It is because of these reasons that many high level racing teams have begun to create virtual "7-Post Rigs" to save testing and development costs, eliminate scheduling problems, and have a track-side tool that can be used to model any suspension and tire combination on the fly at any time.

Often the test matrix used at the actual rig is a very small subset of the actual possible combinations of front and rear damper settings available due to cost and time availability. It is often simply an engineer's educated guess as to what combination of damper settings to try given the limited availability of testing, and in many cases will barely cover all the potential combinations available to them. Usually they can only work on one spring package as well due to rig cost and availability. The recent trend of high level race teams developing virtual half or full car dynamic simulation car models in conjunction with a virtual shaker rig, has allowed race engineers to run as many tests as desired on the fly, with any combination of springs, tire pressures, suspension geometries, and other important setup parameters. As is typical with actual rig testing, analysis of post test data helps guide damper setup choices to try at actual track testing and race events, as well as provide a variety of alternate setups to try as well. As discussed by Boisson et al. [1], it is not always the case that optimal rig damper settings are ideal for an actual track test or event, but quite often they are close and can help provide a window of settings to work in. The trends found on the rig also tend to closely resemble the trends found at the race track, and therefore provide a powerful tool for understanding the effects of certain damper changes on the car's performance and handling characteristics, even before actually trying them at the track. Further, if a virtual model can be validated against actual rig test data for the same overall setup and inputs, then the need for actual rig testing becomes much less important or critical to the success of the engineer and driver combination, as well as the team.

2.4 The Virtual 7 Post Rig

A couple of past colleagues of mine have developed a very practical "in-house" virtual rig software based on the dynamics of a half car model, and is called "Rigsim" [1]. The software simulation results have been compared by the developers to actual rig test data and have proven to be quite accurate in many cases in terms of the trends and responses [1]. The numbers have not always been the same, but damper optimization seems to be more about trends and response behaviors than about absolute numbers. Many successful damper changes at the actual race track have been suggested by analysis of this software's results and thus has become the tool of choice for my study.

2.4.1 Model and Simulation Parameters and Inputs

Rigsim is a 7-Post Rig simulation software that is based on a differential system of equations, mathematically used to describe the dynamics of a half car. In other words, the model simplifies a real 4-corner car model by focusing on the dynamics and interactions of the sprung and unsprung masses of the front and rear of the car, as well as the chassis itself [1]. Many times in the industry this is referred to as a “bicycle model” [12]. In order to create an accurate mathematical model of a real half-car, there are many important input parameters required.

Some of these parameters used in the model include the sprung and unsprung masses of the car, the moment of inertia of the sprung mass in pitch, the wheelbase, the front and rear tire spring and damping rates, the damper model, the front and rear effective suspension spring rates at the wheel, and some geometric characteristics of the front and rear suspension geometry [1]. All simulations used the same parameter inputs other than those adjusted for the purpose of this study. The parameter choices were based on an actual road course-type setup used on an Indy Car. For proprietary reasons, the actual values of these parameters used in this study have been omitted.

After the model parameters are chosen, the simulation parameters must be set. One of the primary simulation settings is the input signal. This can be a swept sine wave, white noise, a square wave or saw tooth-type signal, or a signal based on an actual race track’s road profile. The user must also set a simulation time, i.e., the duration of which the signal is generated in order to run the simulation. For this study I have chosen a swept sine wave signal with a duration of 105 seconds.

2.4.2 Damper Characterization

Dampers are typically characterized in the racing industry by the shape of its force versus velocity graph. Damper force is a function of its shaft velocity and can vary in both bump and rebound directions across the damper’s operating speed range. In its simplest form, a damper can be characterized as a linear damper, whose damping force varies linearly with shaft velocity, as shown in the following figure.

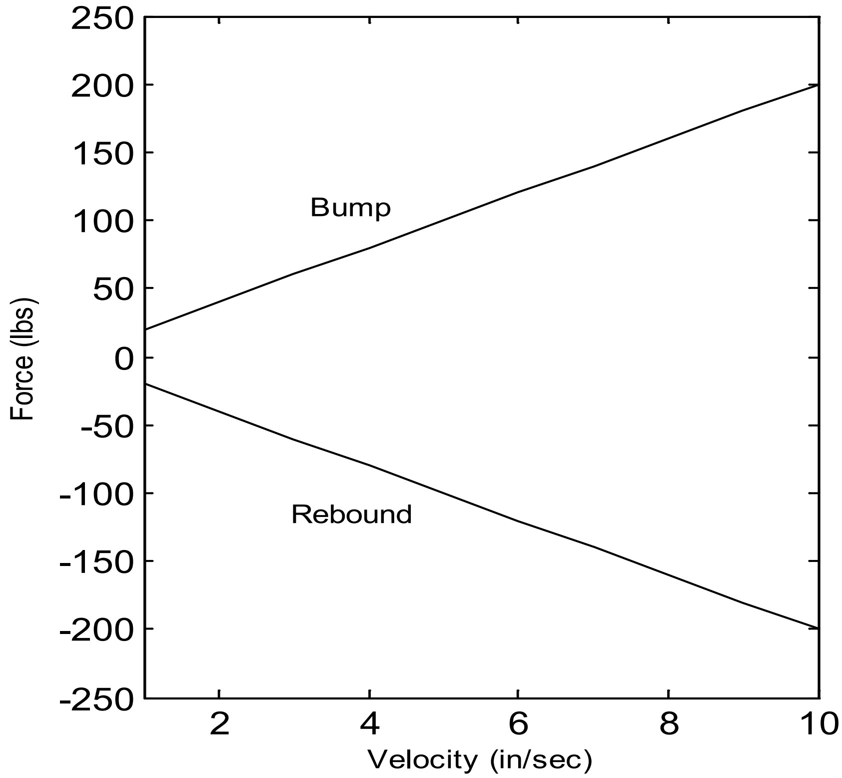

Figure 2.2 Typical linear damper

This type of damping force versus shaft velocity curve can be obtained for an actual damper in multiple ways. There are two primary methods for generating a damper's force versus velocity profile or curve. The first is to test the damper on a machine known in the industry as a damper dynamometer. This machine employs a very powerful motor to cycle the damper in the bump and rebound direction with a swept sine wave input signal. The signal can be varied by the operator to measure the force generated by the damper at various shaft velocities. The damper force is measured directly by an onboard calibrated load cell. The damper dynamometer then produces a logged data file that includes the damper forces generated at the desired range of shaft velocities. Usually the dynamometer is supplied with its own software that can be used to analyze and plot damper test data.

The other common method of obtaining a damper's force versus velocity profile is by using a piece of software usually supplied to the customer by the damper manufacturer. The software typically allows the user to create a damper curve by selecting the desired valves in both bump and rebound, as well as the specifying the settings for the high and low speed adjustments in both bump and rebound. The software is then able to produce a damper force versus velocity graph similar to the one above, based on the manufacturer's dynamometer data which is included in the software.

Rigsim allows the user to input a linear damper by simply specifying a constant for the slope of damping force versus velocity, e.g., 20 lbs/(in/sec). This implies that the damper generates 20 lbs of force per inch per second of velocity increase. This is the description of the damper shown in Figure 2.2.

For this study, hundreds of damper curves were generated that correspond to a wide variety of damper setting combinations. These curves and their associated raw data were accessed using a well known racing damper manufacture's software. This company's dampers are found on many different types of vehicles in the world of motorsports, in this case from dampers used on an Indy Car. The following figure shows a few of the damper curves used in this study.

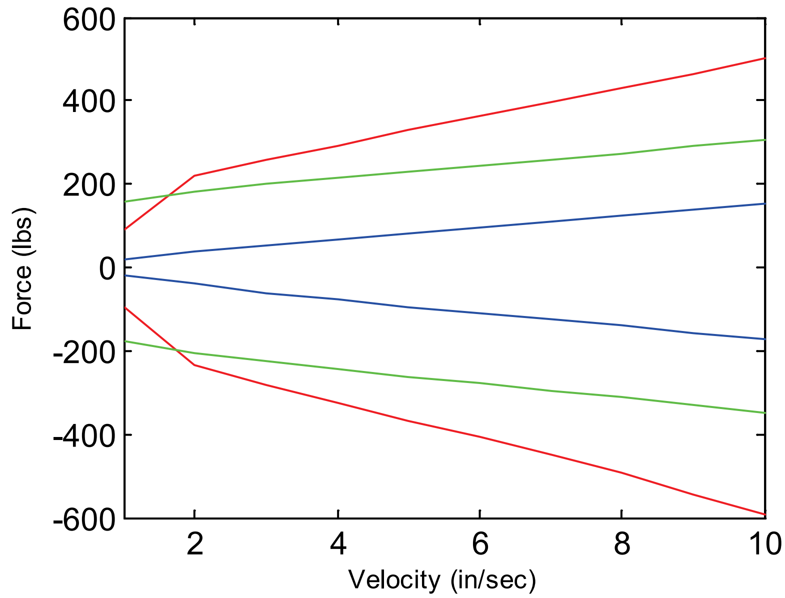

Figure 2.3 Plots of different damper curves based on settings

2.4.3 Half Car Model and Dynamics

Rigsim utilizes a system of differential equations mathematically describing the equations of motion for the masses of a half-car or "bicycle" dynamics model [1]. The excitation inputs are provided by mathematically modeled hydraulic rams as would be found on an actual 7-post rig. The fundamental model is essentially the same as that presented by Kasprzak [8], and is shown schematically in figure 2.4. The actual mathematic model and engine employed by Rigsim is significantly more involved however, in that it incorporates actual non-linear damper curves, mechanical leverages, etc.

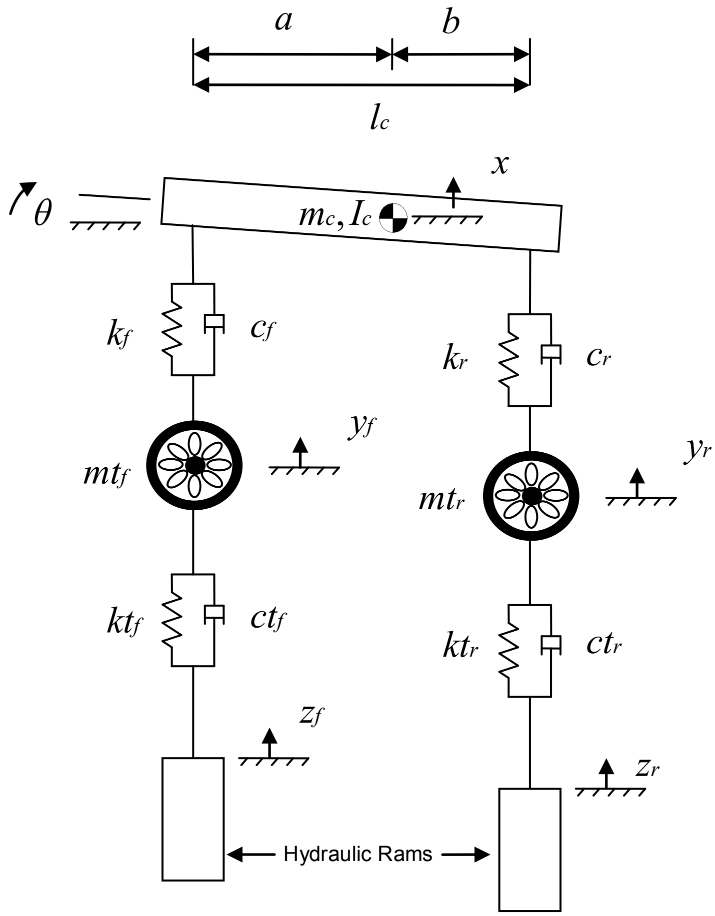

Figure 2.4 Half car model schematic

The sprung mass of the modeled vehicle, m_c, is represented as a ridged beam that is free to translate vertically and rotate in pitch. The beams pitch inertia is given as I_c. The vertical and rotational motions of the sprung mass at the center of gravity are denoted as x and θ respectively. The distance from the sprung mass CG and the front suspension mounting point is a, while b represents the distance from the sprung mass CG to the rear suspension mounting point. The front and rear unsprung masses are denoted mt_f and mt_r, and are attached to the sprung mass by a spring and damper in parallel at each end of the vehicle. The unsprung masses are free to translate vertically, which is represented by the terms y_f and y_r. The suspension's effective spring stiffness is denoted k_f and k_r for the front and rear ends of the vehicle, while the damping rates are denoted c_f and c_r respectively. At each end of the vehicle, the unsprung mass is connected to the virtual rig's hydraulic rams through the tire's spring stiffness and damping rate in parallel, denoted as ct_f and ct_r for front and rear tire damping rates, and kt_f and kt_r for front and rear tire spring stiffness. The vertical displacement of the ram inputs at each end is denoted z_f and z_r respectively.

Rigsim models the front and rear effective suspension damping rate, c_f and c_r, based on user input. The damping force can be linear and thus modeled as a simple constant slope of damping force versus damper velocity, or Rigsim will model damping force as a function of instantaneous damper velocity based on an actual damper curve. Damper models used in this study are based on actual damper force versus velocity curves.

The free body diagrams of the sprung and unsprung masses with the forces and moments acting on them are shown in the following figures [8].

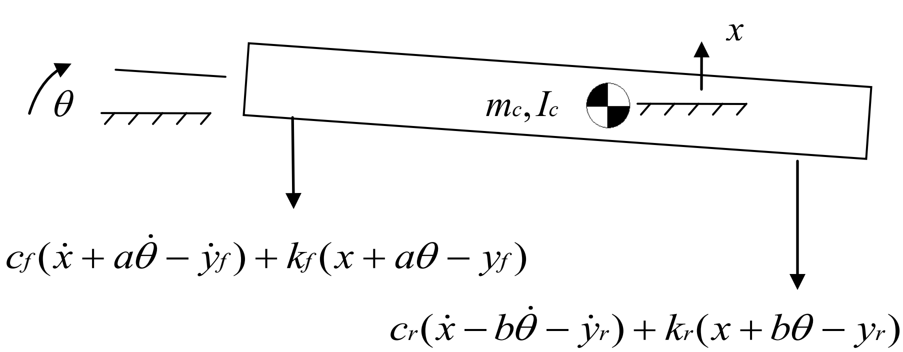

Figure 2.5 Sprung mass free body diagram

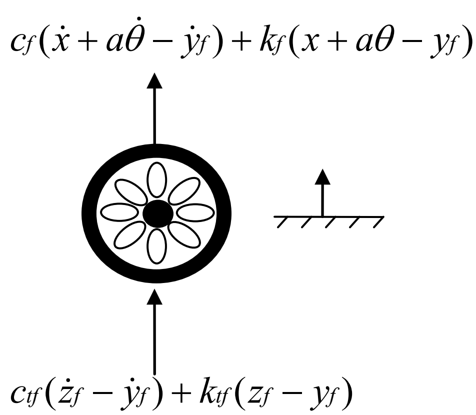

Figure 2.6 Front unsprung mass free body diagram

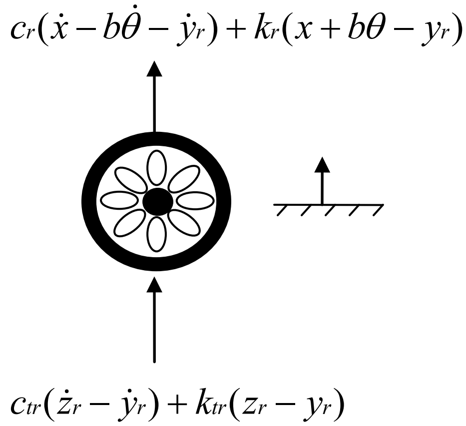

Figure 2.7 Rear unsprung mass free body diagram

The heave equation of motion for the sprung mass results from summation of forces in the vertical direction on the sprung mass, and is given by,

$$\begin{aligned} m_c\ddot{x} = &-c_f(\dot{x} + a\dot{\theta} - \dot{y}_f) - c_r(\dot{x} - b\dot{\theta} - \dot{y}_r) \\ &-k_f(x + a\theta - y_f) - k_r(x - b\theta - y_r) \end{aligned} \tag{4}$$

It follows that the rotational pitch equation of motion is found by summation of moments acting in the sprung mass about the vehicle CG, and is given by,

$$\begin{aligned} I_c\ddot{\theta} = &-ac_f(\dot{x} + a\dot{\theta} - \dot{y}_f) - bc_r(\dot{x} - b\dot{\theta} - \dot{y}_r) \\ &-ak_f(x + a\theta - y_f) - bk_r(x - b\theta - y_r) \end{aligned} \tag{5}$$

Summation of the vertical forces acting on the front unsprung mass results in the following equation of motion,

$$mt_f\ddot{y}_f = c_f(\dot{x} + a\dot{\theta} - \dot{y}_f) + k_f(x + a\theta - y_f) + ct_f(\dot{z}_f - \dot{y}_f) + kt_f(z_f - y_f) \qquad (6)$$

Likewise, the equation of motion for the rear unsprung mass is given by,

$$mt_r\ddot{y}_r = c_r(\dot{x} - b\dot{\theta} - \dot{y}_r) + k_r(x - b\theta - y_r) + ct_r(\dot{z}_r - \dot{y}_r) + kt_r(z_r - y_r) \qquad (7)$$

2.4.4 Model Outputs and Frequency Response

Actual 7-Post rigs generate data that is then further analyzed and manipulated to provide useful information. As mentioned earlier, a transfer function is often used with the data in order to provide a measurement known to vehicle dynamics engineers as the "pitch" response [1, 10]. The pitch data from a simulation is usually analyzed in two ways. The first is to examine the mean pitch variation with the time of the simulation. The second is to analyze the mean pitch variation with frequency. The actual value of the pitch output is also very important, as it is ultimately what many people use as the measure to be optimized.

In the case of Rigsim, "pitch" is calculated from the wheel pan input data and the resulting pitch behavior of the model, which is based on the difference in front and rear ride heights as given by,

$$RRH - FRH \qquad (8)$$

where RRH and FRH are rear ride height and front ride height respectively, measured in units consistent with wheel pan displacement [1]. The pitch response, or pitch frequency response function, is then calculated by taking the magnitudes of the Fast Fourier Transform (FFT), or Discrete Fourier Transform (DFT) in this case, of both the input wheel pan displacement data and the pitch data as given by equation 8 [1]. The resulting pitch frequency response function is given as,

$$Pitch = \frac{\Theta_p}{Z_{fl}} \tag{9}$$

where Θ_p is the magnitude of the FFT of the pitch data, and Z_f is the magnitude of the FFT of the front left wheel pan displacement input data [1, 10]. Since all four corner wheel pans move in phase with one another, the front left wheel pan displacement data can be used as the input data for the analysis [10].

2.5 Damper Optimization

Damper optimization is a somewhat subjective concept since it depends on what is to be optimized, as well as the race car driver's perception of whether or not a damper is at its optimal settings. For this study, the goal is to optimize the mean pitch response for a given car setup, as a result of varying the front and rear tire pressures, and front and rear damper settings, including low and high speed bump and rebound. The smallest value of the pitch response is considered to result from the optimal combination of tire pressure and damper settings.

3 Experimental Design and Simulation

3.1 Design of Experiments and Motorsports

All racing cars and car setups are not created equal. There are many factors determining the ultimate performance and speed potential of a race car.

There are also many factors that are uncontrollable, including ambient conditions such as air temperature, humidity, track temperature, rain, wind, barometric pressure, track surface consistency and irregularities, mistakes made by the car's crew or engineers, and most importantly, the drivers themselves. Factors that can be changed however, include tire pressures, aerodynamic bodywork and component configuration, suspension geometries, spring rates, roll bar rates, damper settings, motor cooling, suspension alignment settings, static ride heights, driveline differential settings, tire compounds, brake system components, transmission gearing ratios, etc. The combination of these factors with a given setting for each is known as the car's "setup".

A car setup can easily involve hundreds of adjustable parameters, each with many setting options. The typical approach to racecar setup optimization, i.e., reduced lap times by making the car and driver combination go faster, is to test one factor at a time. This is a very tedious method which provides limited understanding within a given amount of test time available.

Most racecar and component testing is done through actual track testing, computer simulation, 7 post rig testing, wind tunnel testing, and various other development projects for individual components. Many of these testing techniques and facilities incur a large financial cost to the team and require a lot of time and team resources to test just a small number of possible settings. Often, a team will leave a test session with more questions than when they started, due to the information acquired throughout the test session. This information might suggest additional and alternate tests to further increase the team's understanding of the subject being tested. It is usually not possible to explore these additional tests because of cost and scheduling constraints. Another problem faced by many racing teams is the limitations imposed by the racing series sanctioning bodies on allowable test days. Many series drastically limit the number of track test days that a team may conduct per year, if any, thus greatly reducing the opportunity to understand and optimize their racecar and driver combination. Some racing series will actually ban certain types of testing, wind tunnel for example.

In the search of increased testing efficiency and reducing testing costs, more and more teams have begun exploring design of experiments (DOE) as an additional tool to use to achieve these goals [5, 6, 7, 13]. DOE can be used to test many factors at once and gain an understanding of the effects that each factor has on another factor, as well as the overall response. The response can vary depending on the engineer's optimization goals. The most obvious goal is to reduce lap times for a given car, driver, and track. In this case, a DOE can be created to determine the best settings to achieve the lowest possible lap times.

3.2 Damper Optimization and DOE

As previously discussed, the parameter to be optimized in this study was the mean pitch response as determined by Rigsim simulations for the given car model. In order to do this, Rigsim was used in conjunction with software called Design Expert, a product of Stat-Ease [11]. Design Expert is a very powerful statistical package that specializes in design of experiments and related data analysis. With Design Expert, an initial Fractional Factorial experiment was created in which to study and model the simulation's pitch response as a function of front and rear tire pressure and damper settings. The initial fractional factorial design was then augmented by a D-Optimal design in order to further fill and explore the design space.

3.3 Model Parameters and Response

The experiment chosen for this study is a fractional factorial DOE with 10 factors each incorporating 2 levels. The factors and level ranges are shown in the following table.

Factor	Description	Lower Level	High Level
FLSB	Front Low Speed Bump	10	40
FHSB	Front High Speed Bump	0	50
FLSR	Front Low Speed Rebound	10	40
FHSR	Front High Speed Rebound	0	50
RLSB	Rear Low Speed Bump	10	40
RHSB	Rear High Speed Bump	0	50
RLSR	Rear Low Speed Rebound	10	40
RHSR	Rear High Speed Rebound	0	50
F Tire	Front Tire Stiffness (lb/in)	1730	1960
R Tire	Rear Tire Stiffness (lb/in)	1685	1880

Table 3.1 DOE factors

The dampers used in this study are based on actual dampers that have adjustment knobs with detents to select settings between 0 and 50 clicks. 0 represents the stiffest setting, i.e., greatest damping force for a given velocity, whereas 50 represents the least amount of damping force for a given setting. In reality, the low speed settings affect the high speed damper characteristics and vice versa. The upper and lower limits of the damper settings were chosen based on actual ranges that are typically used on the particular type of race car and damper combination that this study is based [1].

The Tire stiffness settings are also based on a realistic range of stiffness' that would be used on the actual car of interest. The stiffness is determined directly from the target hot tire pressures as determined by the team's track engineers.

3.4 Other DOE Considerations

Runs in a DOE are typically placed in random order due to the potential influences of uncontrollable environmental and experimental variables and conditions [14]. In this case, all simulations were performed using Rigsim, a mathematically-driven model-based software that will produce the exact same result for each run, as long as the initial setup and parameter settings are the same. Because of consistency of simulation results, the goal of this study was to explore the design space and possibly develop a predictive model, rather than concentrate on replication to study experimental error. The simulations used in this experiment were therefore performed in the non-randomized order determined by Design Expert [11]. Appendix A contains the initial fractional factorial DOE matrix used in this study. Appendix B contains an augmented DOE matrix that was used to further populate the design space.

A full factorial would have required 1024 runs given the 10 factors involved, each evaluated at 2 distinct levels. Design Expert allows a maximum of 512 runs for an experiment involving 10 factors, thus the initial experiment was a fractional factorial, or more specifically, a one-half fraction of a 2^{10} design [11]. Design Expert was used to determine that the design was of resolution X, and no aliases were found for the reduced five-factor interaction model. Thus, assuming all three-factor and higher interactions are negligible, the design used allows for complete estimation of all main effects and two-factor interactions [11, 14]. The following is a Design Expert table with a summary of the initial DOE.

Study Type		Factorial			**Runs**	512	
Initial Design		2 Level Factorial			**Blocks**	No Blocks	
Center Points		0			**Model**	RMain effects	
Design Model		Reduced 5FI					
Factor	**Name**	**Units**	**Type**	**Low**	**High**	**Mean**	**Std. Dev.**
A	FLSB		Numeric	10	40	25.000	15.000
B	FHSB		Numeric	0	50	25.000	25.000
C	FLSR		Numeric	10	40	25.000	15.000
D	FHSR		Numeric	0	50	25.000	25.000
E	RLSB		Numeric	10	40	25.000	15.000
F	RHSB		Numeric	0	50	25.000	25.000
G	RLSR		Numeric	10	40	25.000	15.000
H	RHSR		Numeric	0	50	25.000	25.000
J	F Tire Stiff	lb/in	Numeric	1730	1960	1845.000	115.000
K	R Tire Stiff	lb/in	Numeric	1680	1880	1780.000	100.000
Response	**Name**	**Obs**	**Min**	**Max**	**Mean**	**Std. Dev.**	**Ratio**
Y1	Pitch	512	0.253	1.510	0.696	0.267	5.959

Table 3.2 Pitch response initial DOE summary

Following the initial DOE experiment and analysis of the results, as yet to be discussed, a further matrix was created in order to augment the initial matrix. The new matrix was generated using a D-Optimal Response Surface Method, and was intended to further populate and analyze the design space, with the goal of potentially generating a more accurate model representation of the Rigsim simulations. In particular, the model could be used for optimization purposes, as well as a substitute for Rigsim if the results of this study indicated that the model was accurate enough to predict Rigsim outcomes. The secondary study was used to create a cubic model of the simulation's pitch response. The Design Expert summary of the secondary augmented DOE is as follows:

Study Type	Response Surface				**Runs**	734	
Initial Design	D-optimal	Point Exchange			**Blocks**	2	
Design Model	Cubic				**Model**	Cubic	
Factor	**Name**	**Units**	**Type**	**Low**	**High**	**Mean**	**Std. Dev.**
A	FLSB		Numeric	10	40	24.943	14.514
B	FHSB		Numeric	0	50	25.016	24.230
C	FLSR		Numeric	10	40	25.025	14.560
D	FHSR		Numeric	0	50	25.040	24.151
E	RLSB		Numeric	10	40	24.916	14.523
F	RHSB		Numeric	0	50	24.960	24.229
G	RLSR		Numeric	10	40	25.120	14.523
H	RHSR		Numeric	0	50	25.165	24.208
J	F Tire Stiff	lb/in	Numeric	1730	1960	1844.924	111.333
K	R Tire Stiff	lb/in	Numeric	1680	1880	1780.751	96.738
Response	**Name**	**Obs**	**Min**	**Max**	**Mean**	**Std. Dev.**	**Ratio**
Y1	Pitch	734	0.245	1.510	0.687	0.258	6.165

Table 3.3 Pitch response final design summary

Table 3.3 simply shows that the augmented DOE involved 734 total runs, and was a response surface type of study. The 10 factors were kept the same as were the range of settings for each. Interestingly, in comparison to table 3.2, the range or limits of observed pitch response, as well as the mean and standard deviation was very similar for the augmented response surface study as compared to the initial half factorial study.

4 Results and Analysis

4.1 Initial DOE Results and Analysis

The initial part of the DOE project was to run the simulation iterations in Rigsim as determined by the initial one-half fraction experimental design. The details of the individual runs and responses are shown in Appendix A.

4.1.1 Effects and Interactions

Design Expert was first used to create a Box-Cox plot as shown in Figure 4.1. As discussed by Design Expert [11], the Box-Cox plot is used to plot the model-based residual sum of squares, a measure of model error, versus a power transformation exponent used to transform the response data. The model is based on a chosen subset of factors and interactions as described in the following section. The bottom of the curve is depicted with a green line, and represents the power transformation resulting in the minimal error, thus optimizing the model. The blue line represents the current power transformation exponent used with the response data, in this plot the exponent is 1, which is the same as not applying any transformation to the data. The red lines indicate a confidence interval, in which Design Expert calculates that any power transformation using an exponent whose value falls between the red lines, will produce essentially the same results [11]. In this case, Design Expert recommends that the response data is transformed by raising the response to the 0.5 power, thus taking the square root of the response. Doing this should allow minimal error in the analysis of the data and model, i.e., improving the fit of the model to the data, stabilization of response variance, and helping the distribution of the response variable become closer to a normal distribution [11].

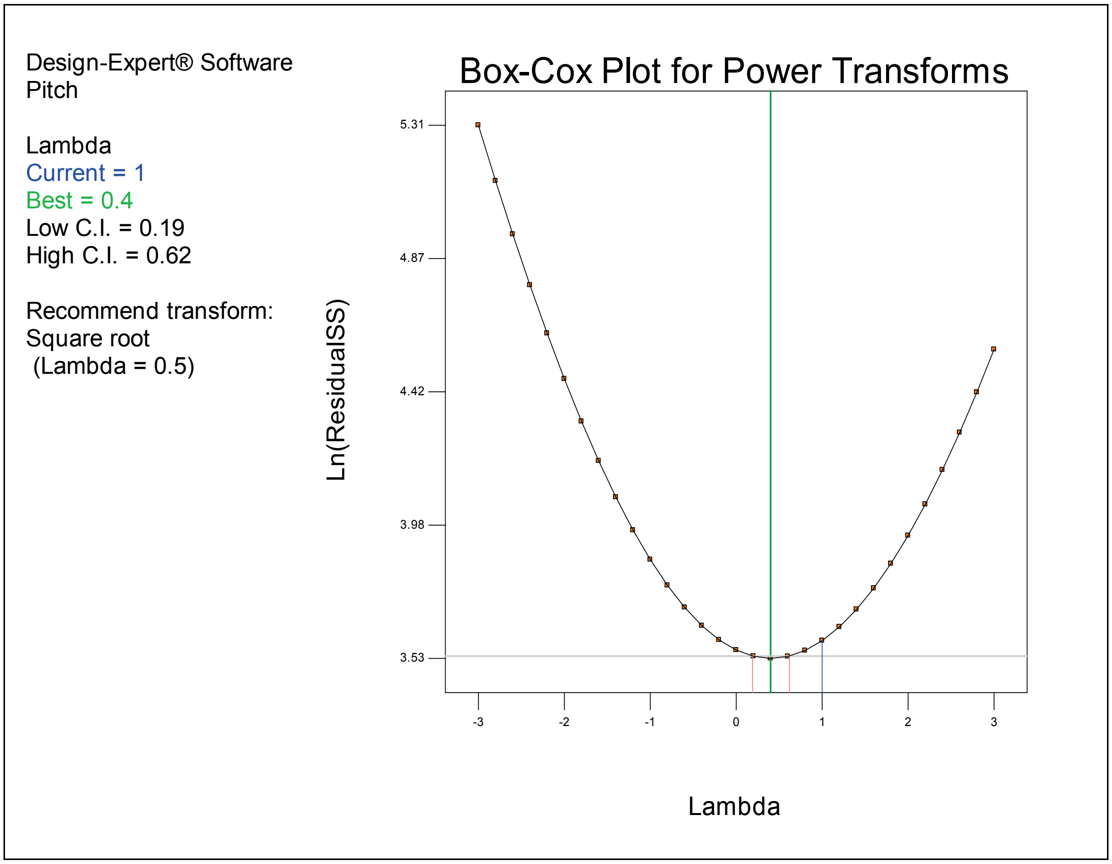

Figure 4.1 Box-Cox plot for initial DOE

As suggested by Design-Expert, a square root transform was applied to the response data and model with a lambda of 0.5. As shown by Figure 4.2, after applying the square root transform, the Box-Cox plot of the transformed model residuals was generated. The plot shows that the square root power transformation is closer to the minimum error as expected, and now the power used for the transformation falls within the red line-depicted confidence interval.

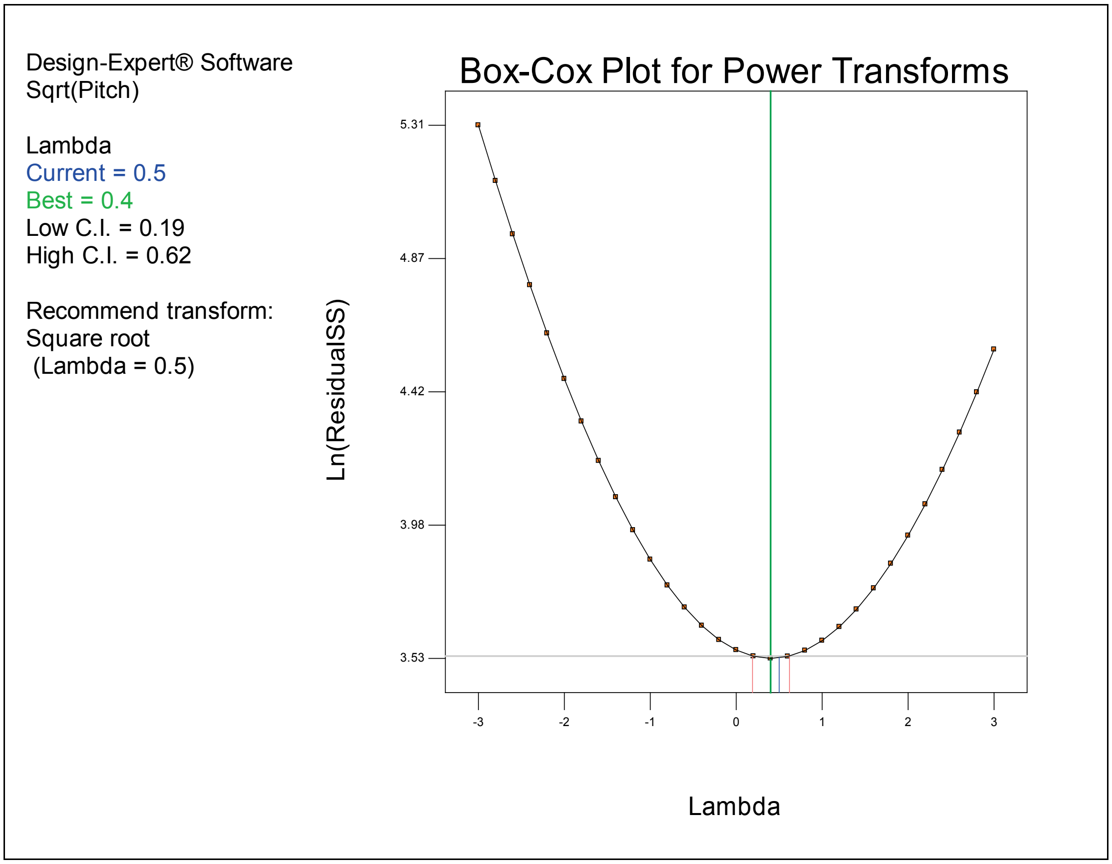

Figure 4.2 Box-Cox plot after square root transform

A half-normal probability plot (Figure 4.3) was then generated in order to visualize the effect of different parameters and parameter combinations, or interactions, on the pitch response. As discussed by Montgomery [14] and Design Expert [11], this is a plot of the absolute value of each main and interaction effect estimates, ordered from largest to smallest, against their cumulative normal probabilities. This type of plot uses the term “Half” to indicate that it is showing the absolute value of effects, regardless of whether the effects are positive or negative, i.e., whether the effects increase or decrease the value of the response for a given change of the factor or interaction producing the effect. Design Expert [11] uses a proprietary method for calculating the standardized effects as shown on the x-axis, but it essentially involves normalizing the effects by adjusting for each effect's standard error. If the design is balanced, i.e., all effects have the same standard error, then nothing is done other than taking the effects absolute value. The y-axis is generated by calculating the cumulative frequency for each effect and then converting it to a standard normal z-score [11].

The plot is shown below with certain parameters and interactions highlighted and labeled. The highlighted selection of effects and interactions were chosen as initial factors to include in the predictive model of pitch response, and will be discussed further.

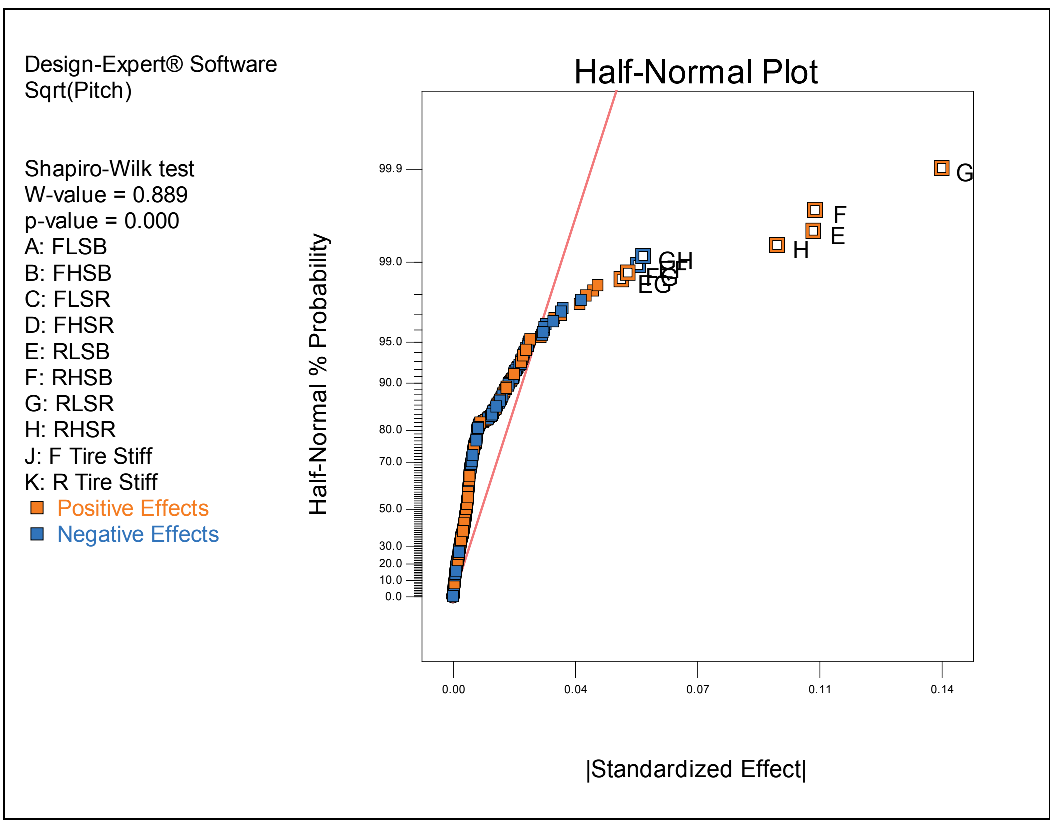

Figure 4.3 Half-normal probability plot of main effects and interactions

Figure 4.4 is a pareto plot of main effects and interactions that was then generated as another visualization tool of the impact of the most significant effects. The calculated t-value of the main effects and interaction effects are organized in the chart from largest to smallest according to their rank, or impact on the pitch response. In many cases, DOE-related Pareto charts are simply an ordered plot of effects based on the calculated size or estimate of each effect. Design Expert's Pareto chart produces an ordered plot of t-values, since t-values are adjusted by the standard errors of the effects, whereas the pure effect estimation calculation can be influenced by differing standard errors [11].

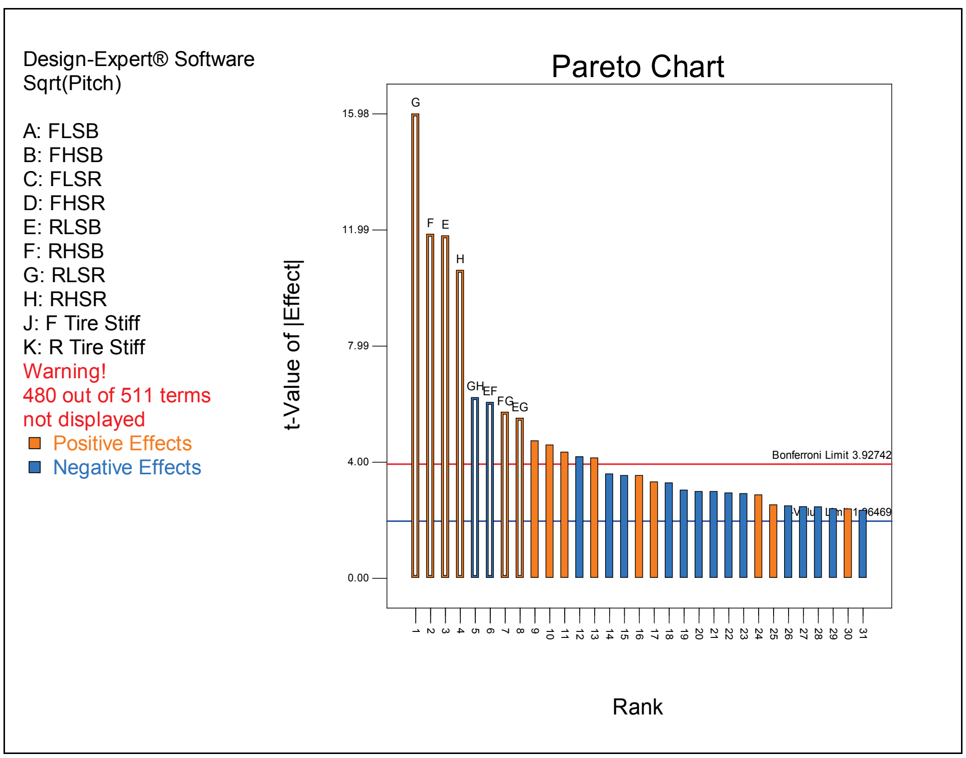

Figure 4.4 Pareto chart of significant effects

In addition to the main effect and interaction effect's t-values being plotted in order of size, the chart also shows both a blue and red line, which are used to display calculated t limits in order to gauge statistical significance. The blue line represents a standard t limit, while the red line represents the Bonferroni corrected t limit, both of which are approximations to the 5% risk level [11]. This implies that any effects above the standard t limit are potentially significant, whereas the effect t-values that are above the Bonferroni limit are almost certainly significant, since effects large enough to pass the Bonferroni limit assure a system-wide 5% maximum error rate [11].

The next step in the analysis was to choose which effects are thought to be the most statistically significant in influencing the pitch response, and thus which effects are incorporated into the statistical model. The decision of which factors and subsequent interactions to include was based on each effect and interaction's statistical significance estimate as determined by its t-value, as well as the combination of information shown on the pareto and half-normal plots. Consideration was also given to actual experience and opinions of other professional engineers in the racing industry. Figure 4.3 shows four obvious outliers which are extremely likely to be statistically important. The effects represented by these points are RLSB, RHSB, RLSR, and RHSR respectively. The next closest four points which are also highlighted in the figure, are four interactions that include these four factors. These effects and interactions are also highlighted on the pareto chart. Based on actual 7-post rig tests, experience, and logical analysis, it is not unreasonable that the rear damper settings would have the greatest influence on pitch. The rear end of the car used in the simulation model, as well as most actual open-wheel type cars for which the model is based, have most of the weight of the car distributed in the rear. Further, the rear end of many open-wheeled race cars tends to be significantly softer in terms of spring and wheel rates, largely due to traction considerations. Therefore the front of the car tends to not move as much as the rear due to it being lighter and much stiffer sprung.

Eventually however, a decision is required to determine when to stop incorporating more effects and interactions into the model. The experimental design chosen consisted of 512 runs, which is large enough to create a potentially oversensitive design, and thus may put too much statistical significance into smaller effects and interactions [11]. Referring to Figure 4.4, the first non-highlighted effect shown is for factor J, the front tire stiffness. In practical applications, front tire pressure stiffness is not usually regarded by many professionals as a major factor influencing the pitch response of a race car. The following figure is a plot of the effect of front tire stiffness on the pitch response.

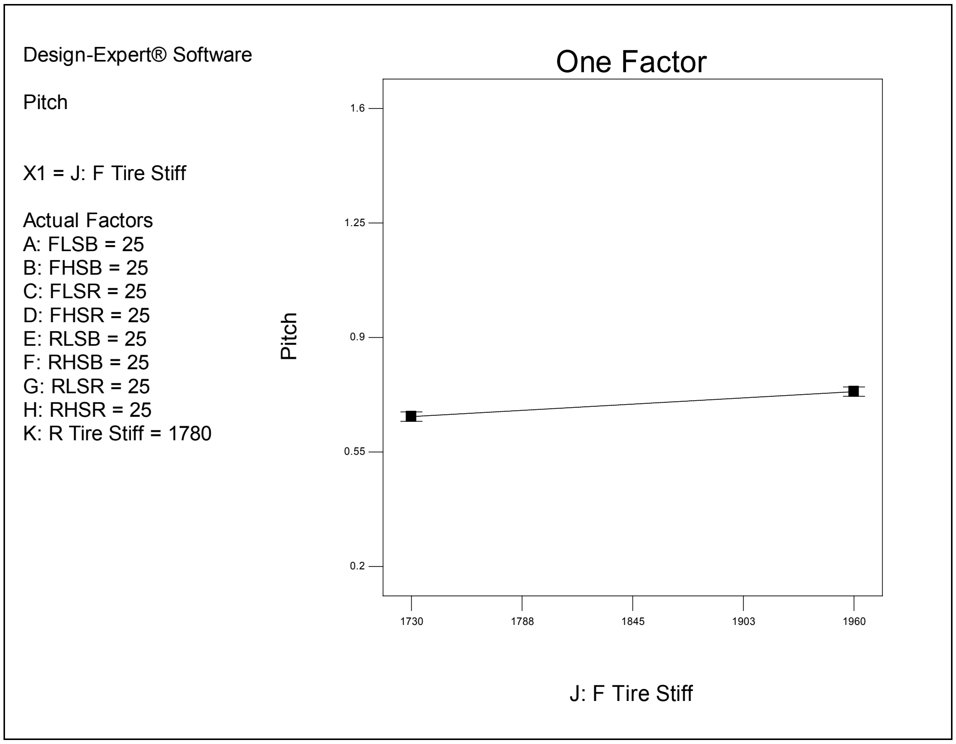

Figure 4.5 Effect plot for front tire stiffness

Figure 4.5 indicates that the pitch response changed by 0.076 over the entire range of adjustment for the front tire stiffness. In terms of actual 7-post rig testing, engineers are typically looking for settings that result in the lower range of observed values of pitch response. Typically settings are usually not chosen simply because they may result in the lowest pitch response, but rather that they have a pitch response in the lower range of values tested, while also meeting other requirements other than pitch response. Typically changes to pitch response on the order of 0.20 or more are considered large by engineers in the industry. The effect of front tire stiffness therefore is not practically as large as the design analysis might suggest and is therefore omitted as a model factor. Further, introducing this factor into the model, will also introduce other effects and interactions into the model [11]. The few other factors shown above the

Bonferroni limit in Figure 4.4 were omitted by the same logic, experience, and analysis. Based on actual 7-post rig testing and analysis of collected data and results, only considering the rear damper settings as suggested by their higher t-values, should suffice for this study.

The rear damper setting effects chosen as model factors were all involved in significant interactions that were also included as model factors. Since the effect of each of these factors is determined to be largely dependent on the effects of other factors, the interaction effects on pitch response are of primary interest. The following are interaction plots of the most significant interactions.

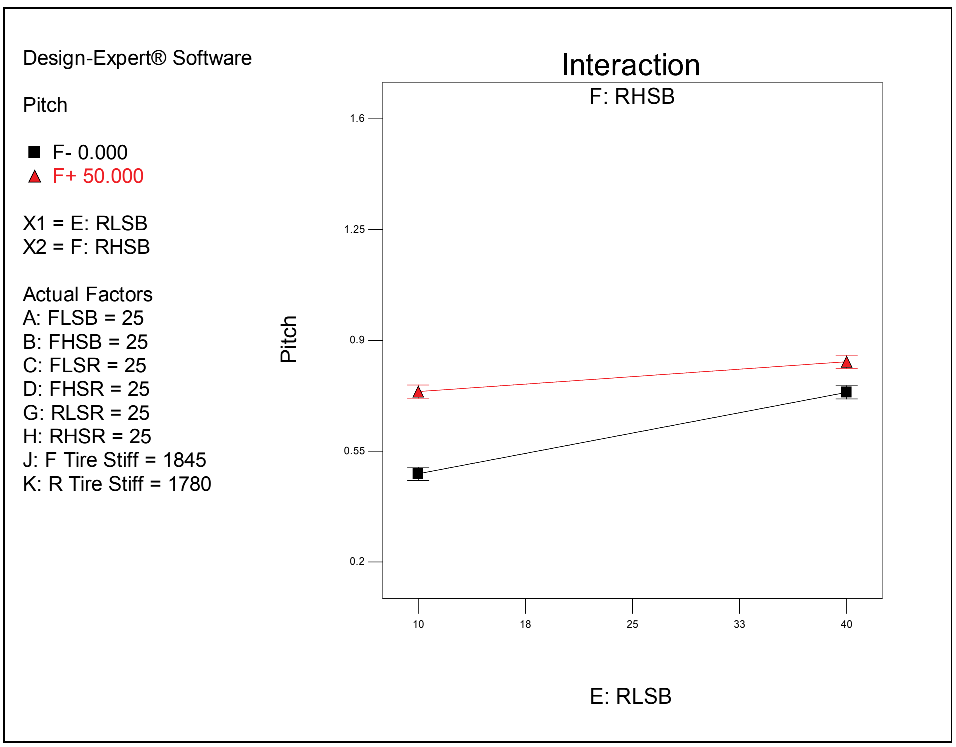

Figure 4.6 Interaction plot of RHSB and RLSB

Figure 4.6 shows that the greatest change in pitch results when factor F, RHSB, is set at its lowest, i.e., stiffest setting, while the RLSB is varied. This result would agree with practical experience, track testing, and 7-post rig testing since the race car platform tends to be more stable and demonstrates less movement with stiffer damper settings. Sometimes however, very stiff damper settings can cause other problems on track, but in this case only pitch optimization is of concern. Further, the plot also indicates that the best pitch response occurs when both RHSB and RLSB are at their lowest settings. It is interesting to note that there appears to be relatively little change in pitch for the softest setting of RHSB, regardless of the setting chosen for RLSB. It is also obvious from the plot that any setting of RLSB with the softest setting of RHSB results in a larger, and hence less desirable pitch response, this suggesting that the RHSB should always be set at the stiffer end of its range, while varying RLSB accordingly. Low speed damper settings are often chosen based on driver perception and feedback about the car’s platform. It is also interesting to note that the two lines in the interaction plot are not parallel, and therefore indicate a significant interaction effect. RHSB and RLSB also appear to be coupled as indicated by the slope of the lower setting RHSB line.

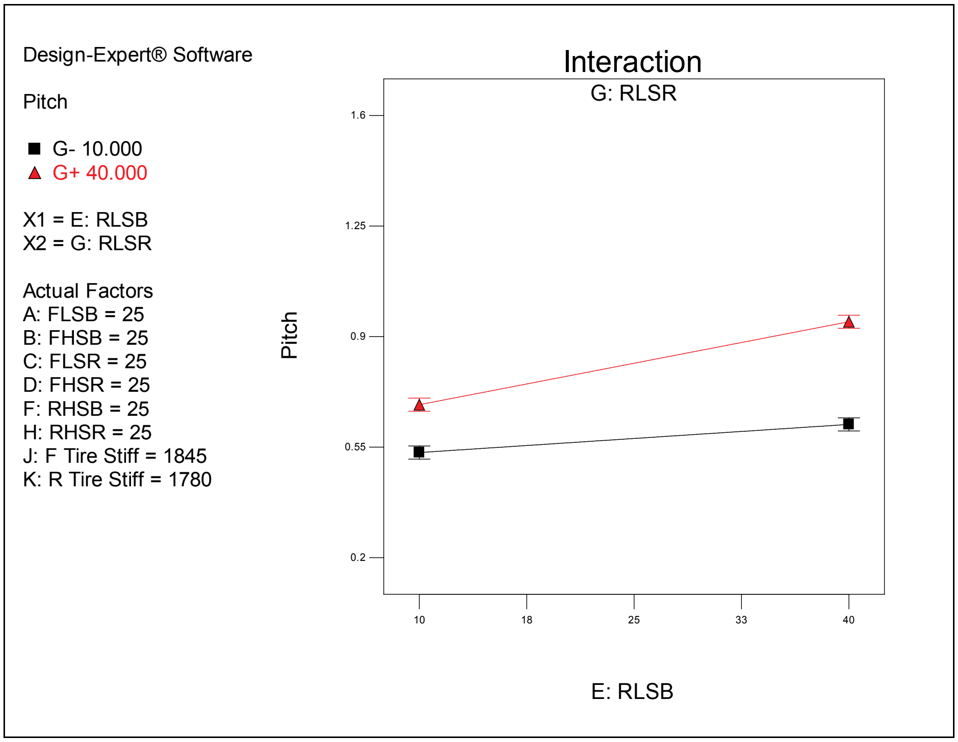

Figure 4.7 Interaction plot of RLSR and RLSB

Figure 4.7 is the interaction plot of RLSR and RLSB. The plot indicates that the stiffest settings for RLSR produce the best pitch response, although varying RLSB does not appear to have a large effect on pitch response for the stiffest setting of RLSR. Combining the information in figure 4.7 and figure 4.6 would suggest that the best pitch response would come from stiff RHSB and RLSR, while still having flexibility to change RLSB for the driver, without having a major impact on the pitch response.

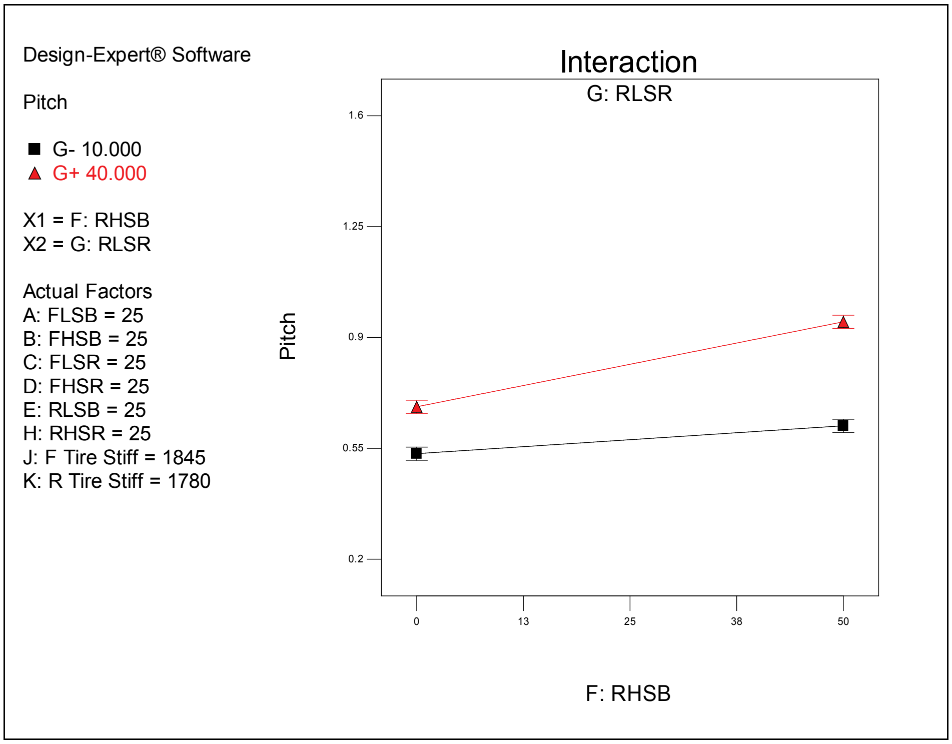

Figure 4.8 Interaction plot of RLSR and RHSB

Figure 4.8 also suggests in conjunction with figure 4.7 that the best pitch response results in the stiffest setting of RLSR. This plot also indicates that RHSB has minimal effect on the pitch response for the stiffest setting of RLSR, similar to that shown with RLSB in figure 4.7. Unlike low speed damper settings such as RLSB, high speed dampers settings in practice tend to not be perceivable to the driver as much as low speed settings. Typically, high speed settings are largely chosen based on track surface irregularities and bumps, as well as with settings suggested by actual rig testing that may help optimize pitch for example. It is interesting to note from figure 4.8 however, that the pitch response varies a lot more when the RLSR is set at its softest setting. Again, this is in agreement with practical experience, although it is interesting in this plot that very similar pitch response results can be obtained with the softest RLSR setting combined with the stiffest setting for RHSB. This would then suggest an alternate approach to optimizing pitch, where perhaps it is worth a slightly less desirable pitch response if this particular combination of settings helps to improve other parameters or even driver opinion of the car's balance and behavior.

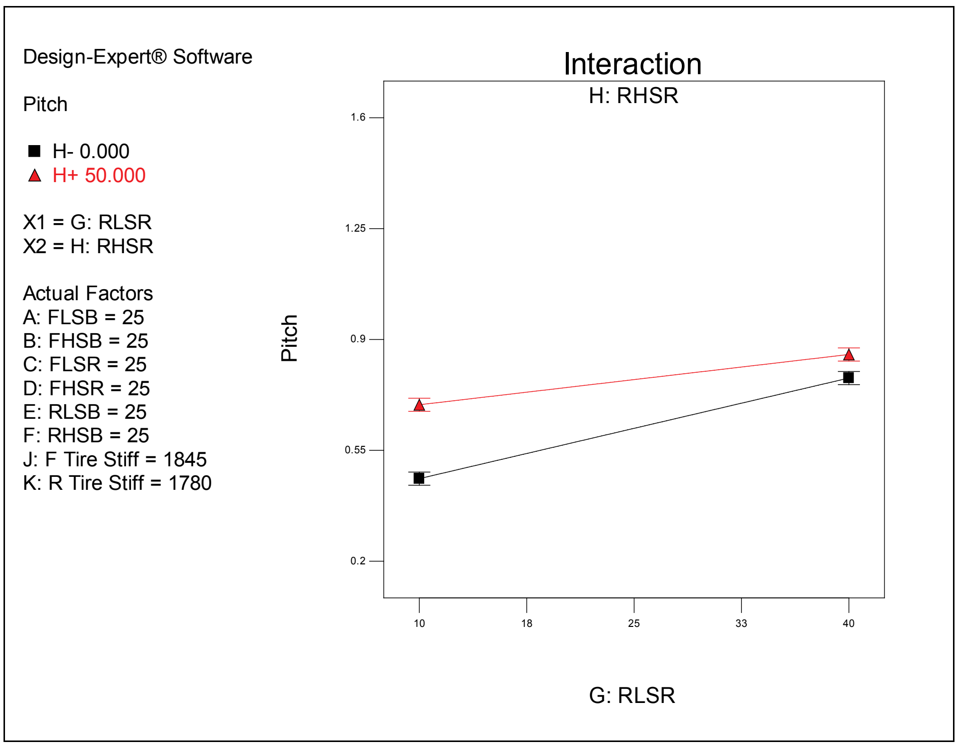

Figure 4.9 Interaction plot of RHSR and RLSR

Figure 4.9 is the last interaction plot of the effect interactions chosen to model pitch response with this design. Clearly the plot indicates that the stiffest setting of RHSR results in the best pitch response, although varies largely with RLSR. The plot shows that for the stiffest setting of RHSR, the stiffest setting of RLSR produces the best pitch response. The previous two interaction plots also suggested that RLSR produced the best pitch response when set at its stiffest. The fact that three of the plots all suggest the best pitch response results from the stiffest setting of RLSR, would make that a likely choice to try in actual rig or track testing. It would then need to be determined whether or not a very stiff RLSR setting produces other undesirable effects to ultimate car performance or the driver's perception, which may not be a good compromise to keep that setting.

4.1.2 ANOVA and model characteristics

Table 4.1 gives a summary generated by Design Expert of ANOVA results for the initial fractional factorial DOE along with a table of model statistics at the bottom.

Analysis of variance table [Partial sum of squares - Type III]					
Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**
Model	8.13	8	1.02	98.19	< 0.0001
E-RLSB	1.44	1	1.44	138.93	< 0.0001
F-RHSB	1.45	1	1.45	140.28	< 0.0001
G-RLSR	2.65	1	2.65	255.47	< 0.0001
H-RHSR	1.16	1	1.16	112.39	< 0.0001
EF	0.38	1	0.38	36.69	< 0.0001
EG	0.31	1	0.31	30.37	< 0.0001
FG	0.34	1	0.34	32.69	< 0.0001
GH	0.40	1	0.40	38.70	< 0.0001
Residual	5.21	503	0.01		
Cor Total	13.34	511			

Std. Dev.	0.10		**R-Squared**	0.6096	
Mean	0.82		**Adj R-Squared**	0.6034	
C.V. %	12.43		**Pred R-Squared**	0.5955	
PRESS	5.40		**Adeq Precision**	33.4773	

Table 4.1 ANOVA and model statistics for initial DOE

Examination of this table reveals some important information about the DOE, choices of effects and interactions, and the regression model generated by the initial DOE. As discussed by Design Expert [11], the F value of 98.19 for the model indicates that the model is statistically significant, and that there is only a

0.01% chance that this F value is due to noise in the experiment. The F values and p-values for the effects and interactions chosen for the model also indicate that they are statistically significant model terms. This information does not however indicate if other terms that were not included in the model are significant or not. The predicted versus adjusted R-Squared values are in close agreement which indicates that there are not any major problems with the data or the model chosen. The high Adequate Precision value indicates that the model should provide reasonable prediction capabilities [11, 14].

4.1.3 Response optimization and further simulation

The next step in this project was to use Design Expert's optimization methods, which is based on a numerical optimization algorithm with constraints, to find the factor combinations that would produce the lowest and thus most desirable pitch response [11]. The optimization criteria were set to minimize the pitch response within the range of 0 and 1.229. Each factor was allowed to vary within the range originally specified when the DOE was created.

Design expert found 80 solutions which it ranked in terms of their desirability factor, or measurement of how close the solution was to the optimization goal. The desirability multiple response method and calculations used by Design Expert are based on the work of Myers and Montgomery [15]. The desirability method involves a desirability function, also known as an objective function, D. This function is maximized by Design Expert using the direct search method of optimization. As outlined by Montgomery [15], each response, y_i, is converted into a desirability function, d_i, such that each d_i is a normalized value between 0 and 1. Further, d_i is equal to 0 if the response used to calculate it is outside of a predefined acceptable region, or d_i is equal to 1 if y_i is at the goal or target. The design factors or variable are then chosen in order to maximize the overall desirability function, which is given as:

$$D = (d_1 \times d_2 \times .. \times d_n)^{\frac{1}{n}} = (\prod_{i=1}^{n} d_i)^{\frac{1}{n}} \tag{10}$$

where there are *n* responses [15].

Based on the solutions given, the top 10 were chosen as an additional set of simulations to perform in Rigsim in order to measure the accuracy of the predictive capability of the model created from the original DOE. Microsoft Excel's "randbetween" function was also used to generate hundreds of randomly chosen parameter settings for additional runs for randomized model validation. 10 of the randomly generated runs were randomly chosen to include as additional simulations. The following table lists the 20 additional runs that were used to validate the model. The first 10 runs are the suggested top ten optimal settings from Design Expert, while the last 10 are randomly generated settings used to explore the design space and validity of the model. The actual mean pitch response from each simulation run in Rigsim is also included.

Run	FLSB	FHSB	FLSR	FHSR	RLSB	RHSB	RLSR	RHSR	F Tire	R Tire	Pitch
1	37	49	10	1	10	0	10	0	1734	1683	0.3413
2	37	1	37	1	10	0	10	0	1734	1683	0.5096
3	37	1	10	1	10	0	10	0	1734	1877	0.4020
4	10	46	10	45	10	0	10	0	1936	1747	0.4587
5	15	32	35	31	10	0	10	0	1945	1731	0.4439
6	10	46	10	50	10	0	10	0	1823	1802	0.5570
7	11	49	40	15	10	0	10	0	1949	1773	0.5163
8	35	42	35	43	10	0	10	0	1769	1814	0.7643

9	21	22	40	2	10	0	10	0	1734	1683	0.4539
10	10	1	17	33	10	0	10	0	1859	1811	0.3351
11	24	18	37	22	20	44	22	47	1802	1791	0.6712
12	10	1	33	29	32	39	30	34	1824	1834	0.8736
13	18	21	17	44	36	19	17	4	1878	1790	0.4301
14	14	43	39	32	33	47	29	44	1929	1876	0.8164
15	31	48	26	16	12	33	23	36	1845	1878	0.4966
16	30	30	32	36	16	11	30	23	1928	1803	0.4418
17	15	0	25	28	25	0	21	3	1795	1716	0.5297
18	34	22	18	20	25	27	21	46	1852	1756	0.7028
19	16	10	27	35	37	19	26	11	1909	1870	0.7025
20	39	6	40	38	25	41	35	1	1826	1831	0.6929

Table 4.2 DOE model validation matrix with pitch response

Runs 1 through 10, the top ten optimal solutions as determined by Design Expert, all have the same rear damper settings. These damper settings correspond to all rear damper adjustments set to full stiff. Again this is likely based on experience to produce minimal pitch, but is unlikely to produce a well behaved optimal race car in reality. Design Expert randomly assigned values to the insignificant factors that were not included in the model such as FLSB, etc [11]. The value of the factors not included should not have any impact on the predicted pitch response since they are omitted from the model.

The actual simulation mean pitch response values are then compared to the model-predicted values in the following table.

Run	Pitch	Pred. Pitch	Residual	% Diff
1	0.3413	0.3313	-0.0100	-2.9%
2	0.5096	0.3313	-0.1784	-35.0%
3	0.4020	0.3313	-0.0707	-17.6%
4	0.4587	0.3313	-0.1275	-27.8%
5	0.4439	0.3313	-0.1127	-25.4%
6	0.5570	0.3313	-0.2257	-40.5%
7	0.5163	0.3313	-0.1850	-35.8%
8	0.7643	0.3313	-0.4330	-56.7%
9	0.4539	0.3313	-0.1224	-27.0%
10	0.3351	0.3313	-0.0035	-1.0%
11	0.6712	0.7919	0.1207	18.0%
12	0.8736	0.8543	-0.0193	-2.2%
13	0.4301	0.5893	0.1592	37.0%
14	0.8164	0.9014	0.0849	10.4%
15	0.4966	0.6833	0.1866	37.6%
16	0.4418	0.5974	0.1557	35.2%
17	0.5297	0.5104	-0.0194	-3.7%
18	0.7028	0.7442	0.0414	5.9%
19	0.7025	0.7209	0.0184	2.6%
20	0.6929	0.8035	0.1106	16.0%
		Mean	-0.0315	-5.6%
		Median	-0.0147	-2.6%
		Std Dev	0.1547	27.1%
		Variance	0.0239	7.3%
		Min	-0.4330	-56.7%
		Max	0.1866	37.6%

Table 4.3 Statistical analysis of predicted versus actual pitch response

Table 4.3 shows a large range of residuals with a mean residual error of -5.6%. The results of this additional set of validation-oriented runs suggests that the initial model struggles to predict the mean pitch response within 0.10 of the Rigsim-generated actual pitch value, particularly with factor treatments in between the actual values used in the initial DOE. Further, the predicted model-based pitch value is the same for the first 10 runs since the model factors, rear damper settings in this case, are held constant. Adjusting the front damper settings however did produce changes in the simulation pitch response as shown by Table 4.3, thus suggesting that perhaps the front damper settings have some significant influence on pitch response not shown by the initial design. This could be attributed to the predominant non linearity of dampers. This also suggests further study, particularly throughout the design space in order to reveal nonlinearities, other effects and interactions, etc.

4.2 Secondary DOE Results and Analysis

Given the results of the initial fractional factorial experiment, a D-optimal response surface experiment was designed to augment the initial experiment. Design Expert was used to generate a matrix of an additional 222 runs to fill in the gaps of the original design space, thus allowing for further investigation of any potential underlying nonlinear effects and interactions, and perhaps devise a better and more robust predictive model. Further, the damper settings used in the initial experiment were tested at their extremes. Since Rigsim employs models of actual damper curves which can be highly nonlinear, it is reasonable to expect that the various damper settings can also have a nonlinear effect on the mean pitch response of the simulation. Appendix B contains a complete table of the additional runs used to augment the initial experiment. The table also includes the pitch response for each run.

4.2.1 Effects and Interactions

For this part of the project, a transform was not applied to the model. Combining the original factorial experiment data with the additional data, a response surface cubic model was generated. As suggested by Design Expert [11], the next step was to perform a backward stepwise elimination regression on the model in order to isolate statistically significant effects or terms of the model. The alpha criterion chosen was 0.001 which was used to eliminate potential factors that represent experimental noise. In particular, the chosen alpha value was used to remove terms where the term's p-value is greater than 0.001 [11].

4.2.2 ANOVA and Model Characteristics

Table 4.4 gives a Design Expert-generated summary of ANOVA results for the reduced cubic model DOE along with a table of model statistics at the bottom.

Analysis of variance table [Partial sum of squares - Type III]					
Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	46.42	90	0.52	143.45	< 0.0001
A-FLSB	0.00	1	0.00	0.11	0.7449
B-FHSB	0.00	1	0.00	0.03	0.8726
C-FLSR	0.46	1	0.46	126.97	< 0.0001
D-FHSR	0.00	1	0.00	0.36	0.5466
E-RLSB	5.43	1	5.43	1511.24	< 0.0001
F-RHSB	5.31	1	5.31	1477.27	< 0.0001
G-RLSR	9.86	1	9.86	2741.14	< 0.0001
H-RHSR	3.87	1	3.87	1076.03	< 0.0001
J-F Tire Stiff	1.00	1	1.00	278.97	< 0.0001
K-R Tire Stiff	0.21	1	0.21	57.42	< 0.0001
AB	0.61	1	0.61	170.54	< 0.0001
AC	0.49	1	0.49	135.88	< 0.0001
AD	0.06	1	0.06	15.77	< 0.0001
AE	0.19	1	0.19	51.81	< 0.0001
AF	0.18	1	0.18	50.07	< 0.0001
AG	0.26	1	0.26	71.18	< 0.0001
AH	0.12	1	0.12	33.97	< 0.0001
AJ	0.09	1	0.09	24.78	< 0.0001
AK	0.10	1	0.10	28.78	< 0.0001
BC	0.33	1	0.33	93.05	< 0.0001
BD	0.08	1	0.08	21.43	< 0.0001
BE	0.19	1	0.19	53.97	< 0.0001

BF	0.19	1	0.19	52.55	< 0.0001
BG	0.27	1	0.27	75.36	< 0.0001
BH	0.19	1	0.19	53.05	< 0.0001
BJ	0.07	1	0.07	20.32	< 0.0001
BK	0.09	1	0.09	26.11	< 0.0001
CD	0.90	1	0.90	250.11	< 0.0001
CE	0.31	1	0.31	85.65	< 0.0001
CF	0.41	1	0.41	113.36	< 0.0001
CG	0.52	1	0.52	145.56	< 0.0001
CH	0.32	1	0.32	89.23	< 0.0001
CJ	0.20	1	0.20	55.86	< 0.0001
CK	0.18	1	0.18	50.30	< 0.0001
DE	0.16	1	0.16	45.87	< 0.0001
DF	0.21	1	0.21	57.58	< 0.0001
DG	0.27	1	0.27	75.60	< 0.0001
DH	0.16	1	0.16	45.42	< 0.0001
DJ	0.09	1	0.09	23.96	< 0.0001
DK	0.12	1	0.12	34.07	< 0.0001
EF	1.07	1	1.07	298.63	< 0.0001
EG	1.25	1	1.25	346.33	< 0.0001
EH	0.68	1	0.68	188.03	< 0.0001
EJ	0.16	1	0.16	44.42	< 0.0001
EK	0.14	1	0.14	38.73	< 0.0001
FG	1.39	1	1.39	385.71	< 0.0001
FH	0.84	1	0.84	233.58	< 0.0001
FJ	0.15	1	0.15	41.44	< 0.0001
FK	0.12	1	0.12	32.92	< 0.0001
GH	1.05	1	1.05	293.13	< 0.0001
GJ	0.25	1	0.25	69.55	< 0.0001
GK	0.20	1	0.20	56.79	< 0.0001
HJ	0.18	1	0.18	51.23	< 0.0001
HK	0.10	1	0.10	28.76	< 0.0001
JK	0.10	1	0.10	27.49	< 0.0001
B^2	0.05	1	0.05	15.17	0.0001
C^2	0.02	1	0.02	5.99	0.0146
F^2	0.05	1	0.05	15.28	0.0001
ABC	0.08	1	0.08	21.86	< 0.0001
ABE	0.06	1	0.06	17.05	< 0.0001
ABF	0.07	1	0.07	19.49	< 0.0001
ABG	0.09	1	0.09	26.25	< 0.0001
ABH	0.06	1	0.06	17.30	< 0.0001
ABJ	0.07	1	0.07	18.43	< 0.0001
ABK	0.05	1	0.05	13.71	0.0002
AEF	0.05	1	0.05	13.73	0.0002
AGH	0.09	1	0.09	25.83	< 0.0001
BEF	0.12	1	0.12	33.06	< 0.0001
BGH	0.11	1	0.11	30.44	< 0.0001

CDE	0.07	1	0.07	18.21	< 0.0001
CDF	0.08	1	0.08	23.19	< 0.0001
CDG	0.11	1	0.11	31.41	< 0.0001
CDH	0.09	1	0.09	24.14	< 0.0001
CDJ	0.10	1	0.10	27.15	< 0.0001
CDK	0.07	1	0.07	20.22	< 0.0001
CEF	0.20	1	0.20	56.53	< 0.0001
CGH	0.19	1	0.19	53.10	< 0.0001
DEF	0.10	1	0.10	27.66	< 0.0001
DGH	0.10	1	0.10	27.76	< 0.0001
EFG	0.54	1	0.54	151.08	< 0.0001
EFH	0.21	1	0.21	59.76	< 0.0001
EFJ	0.11	1	0.11	30.10	< 0.0001
EFK	0.09	1	0.09	25.72	< 0.0001
EGH	0.36	1	0.36	100.06	< 0.0001
FGH	0.27	1	0.27	76.08	< 0.0001
GHJ	0.10	1	0.10	27.67	< 0.0001
GHK	0.11	1	0.11	30.74	< 0.0001
AB^2	0.02	1	0.02	5.10	0.0242
C^2D	0.02	1	0.02	6.84	0.0091
B^3	0.00	1	0.00	0.04	0.8442
Residual	2.31	642	0.00		
Cor Total	48.87	733			

Std. Dev.	0.06		**R-Squared**	0.9526	
Mean	0.69		**Adj R-Squared**	0.9460	
C.V. %	8.73		**Pred R-Squared**	0.9379	
PRESS	3.03		**Adeq Precision**	71.4868	

Table 4.4 ANOVA for response surface reduced cubic model

As indicated by Design Expert [11], the model f-value of 143.45 implies that this model is statistically significant. The adjusted and predicted r-squared values are in close agreement and the adequate precision value is well above 4, thus indicating that the model is reasonable to explore the design space [14]. With the addition of data points in the design space along with the increased order of the model, it is also notable that the r-squared values increased significantly from the original model r-values. This would be an indication of a model that is a better fit of the data for the given experiment [11].

It is also interesting to note that using the backward stepwise elimination regression, with the given criteria, generated a model that includes all 10 linear effects, all 45 linear two factor interactions, and 35 third order effects. The fact that so many effects and interactions are deemed to be significant could be due to the large number of runs and oversensitivity of the DOE [11].

4.2.3 Model Diagnostics

In order to study the fit and accuracy of the model created, a normal plot of residuals was produced.

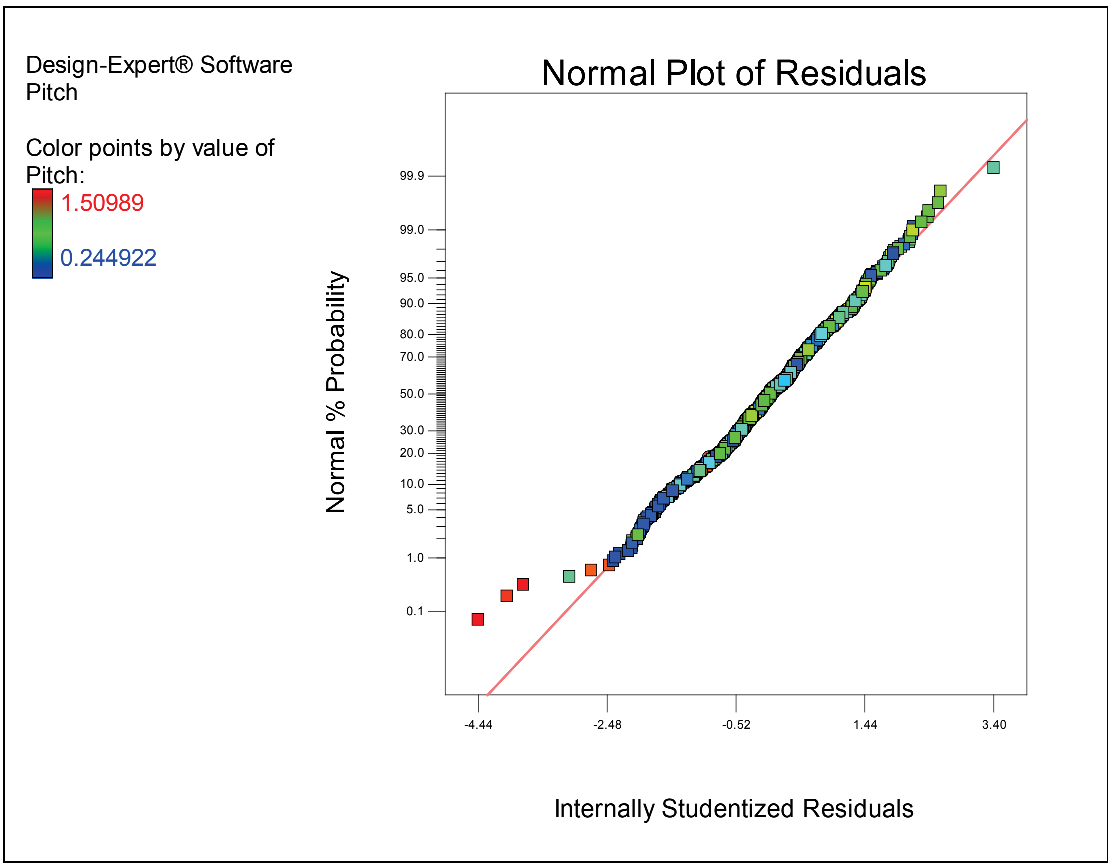

Figure 4.10 Normal plot of residuals

Inspection of this plot reveals 5 data points that appear to be outliers, relative to the rest of the data. The apparent outlier data correspond with runs

16, 369, 465, 481, and 561, with response values of 0.6638, 1.4674, 1.4288, 1.4685, and 0.6430 respectively. Out of 734 runs, the pitch response values ranged from 0.245 to 1.510, with a mean of 0.687 as shown in Table 3.3. Most of the data however fits well to the straight line shown, thus indicating that the residuals follow a normal distribution.

Further examination of another diagnostic plot reveals the same outliers.

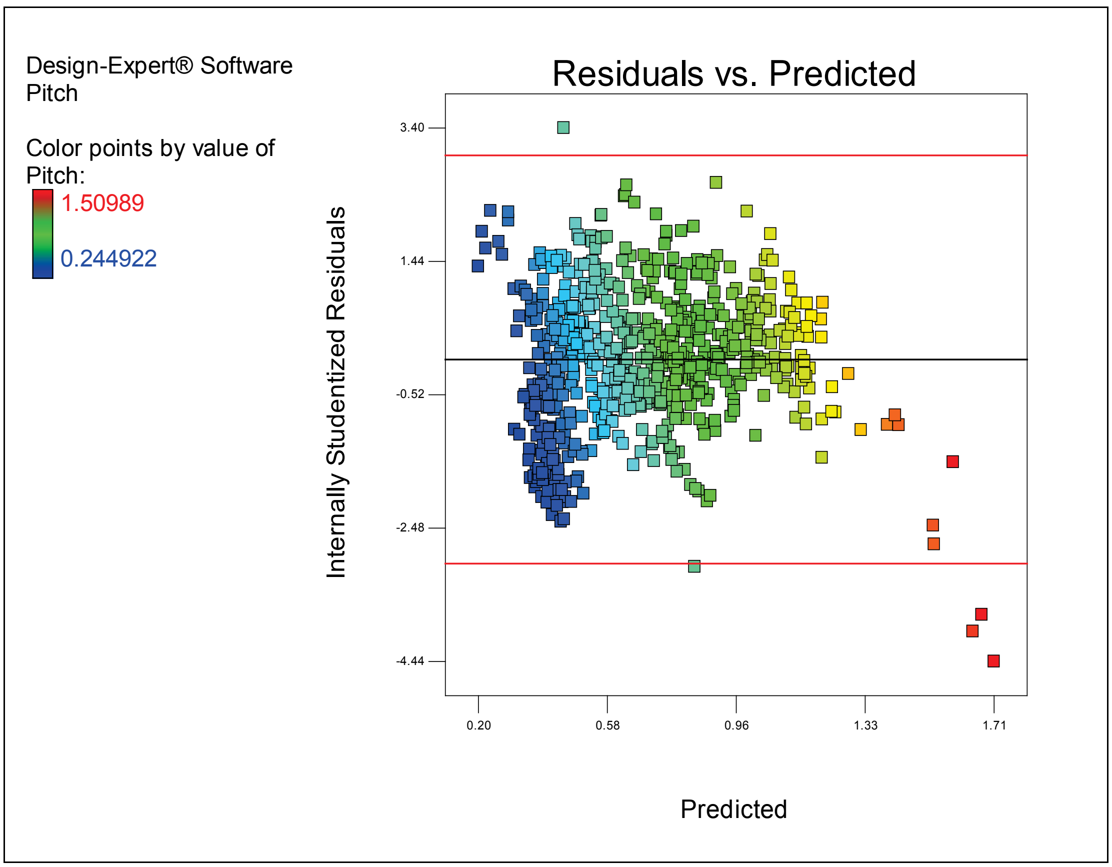

Figure 4.11 Plot of residuals versus model predicted values

The 5 points in Figure 4.11 located outside of the 2 red lines, are the same outliers shown in Figure 4.10. The plot is otherwise fairly well behaved. A very interesting plot reveals intrinsic characteristics of the model.

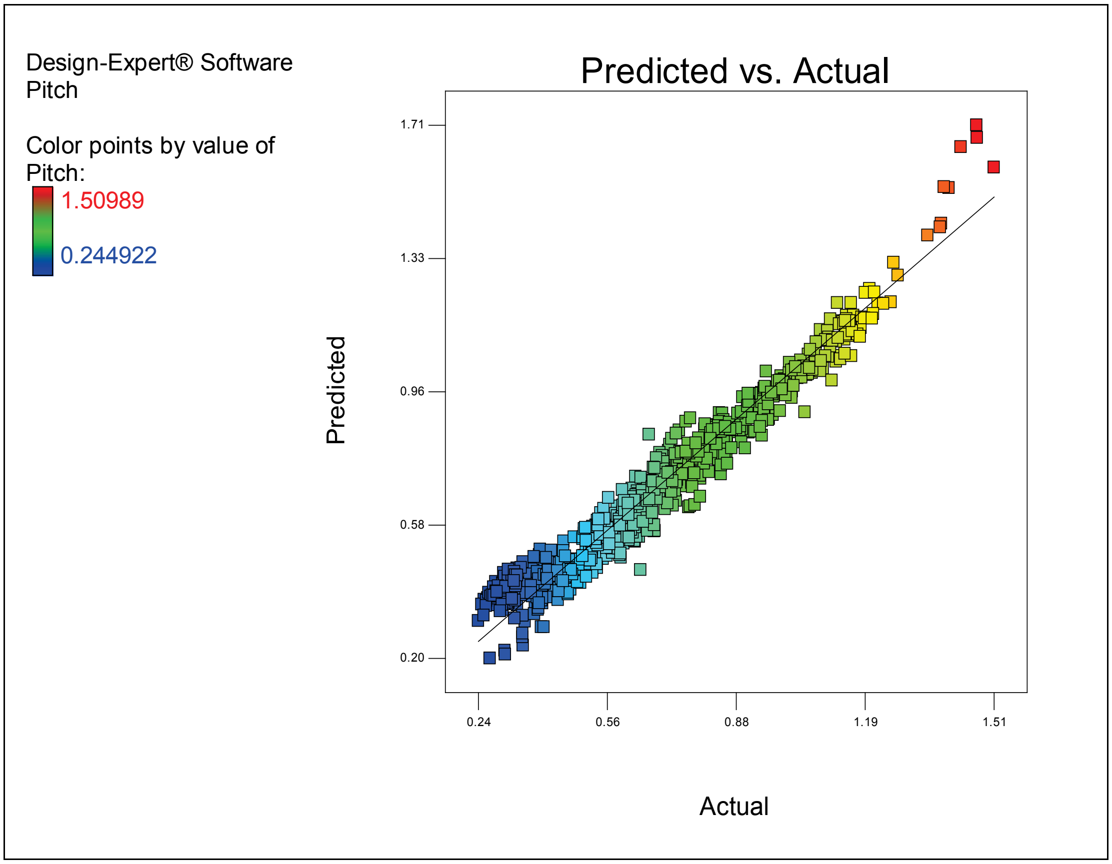

Figure 4.12 Predicted vs. actual response values

Figure 4.12 is a plot of predicted versus actual response values. The plot shows that up to a response value of approximately 1.20, the model provides fairly reasonable prediction capabilities. Some of the noticeable outliers at the higher values of pitch correspond to the same data points discussed previously. Since minimization of the pitch response is the goal of this study, the model appears to be well behaved relative to the values of pitch that are of interest.

4.2.4 Response Optimization and Further Simulation

Employing the same methodology as in the initial experiment, Design Expert was used to optimize the simulation's mean pitch response by varying the 10 factors included in the DOE. Taking the top 10 solutions, and incorporating the same random runs as generated previously, the following 20 validation runs were performed. In this case, all 10 factors are included in the response surface model and are therefore not assigned random settings in the optimization process by Design Expert [11].

Run	FLSB	FHSB	FLSR	FHSR	RLSB	RHSB	RLSR	RHSR	F Tire	R Tire	Pitch
1	10	11	10	2	10	0	10	0	1730	1684	0.2808
2	10	13	19	0	10	0	10	4	1798	1726	0.2653
3	10	15	10	0	10	0	14	0	1743	1693	0.2861
4	10	16	24	0	10	21	10	0	1859	1771	0.2761
5	10	12	10	5	10	4	10	0	1863	1880	0.2824
6	10	19	10	0	10	24	10	0	1775	1880	0.2111
7	10	19	14	6	13	14	10	0	1838	1880	0.2652
8	10	0	27	0	11	0	10	2	1858	1796	0.2950
9	10	19	15	18	10	0	12	2	1942	1685	0.3878
10	10	19	23	1	37	25	10	0	1739	1879	0.2834
11	24	18	37	22	20	44	22	47	1802	1791	0.6712
12	10	1	33	29	32	39	30	34	1824	1834	0.8736
13	18	21	17	44	36	19	17	4	1878	1790	0.4301
14	14	43	39	32	33	47	29	44	1929	1876	0.8164
15	31	48	26	16	12	33	23	36	1845	1878	0.4966
16	30	30	32	36	16	11	30	23	1928	1803	0.4418
17	15	0	25	28	25	0	21	3	1795	1716	0.5297
18	34	22	18	20	25	27	21	46	1852	1756	0.7028
19	16	10	27	35	37	19	26	11	1909	1870	0.7025
20	39	6	40	38	25	41	35	1	1826	1831	0.6929

Table 4.5 Secondary DOE model validation runs with pitch response

Runs 1 through 10 are the optimization solutions runs with the goal of minimizing the simulation’s pitch response. Runs 11 through 20 are the same random runs from the initial experiment.

A comparison of the residuals between the predicted model response and the actual simulation response follows. Although the last 10 runs are the same random runs as used previously, the predicted values have changed due to the change in the model used.

Run	Pitch	Pred. Pitch	Residual	% Diff
1	0.2808	0.1750	-0.1058	-37.7%
2	0.2653	0.1891	-0.0762	-28.7%
3	0.2861	0.2052	-0.0808	-28.3%
4	0.2761	0.2064	-0.0697	-25.2%
5	0.2824	0.2071	-0.0753	-26.7%
6	0.2111	0.2176	0.0065	3.1%
7	0.2652	0.2274	-0.0378	-14.3%
8	0.2950	0.2329	-0.0621	-21.0%
9	0.3878	0.2330	-0.1548	-39.9%
10	0.2834	0.2332	-0.0501	-17.7%
11	0.6712	0.6709	-0.0002	0.0%
12	0.8736	0.8186	-0.0550	-6.3%
13	0.4301	0.4634	0.0333	7.7%
14	0.8164	0.7970	-0.0194	-2.4%
15	0.4966	0.5769	0.0802	16.2%
16	0.4418	0.5388	0.0971	22.0%
17	0.5297	0.4796	-0.0502	-9.5%
18	0.7028	0.6613	-0.0415	-5.9%
19	0.7025	0.6219	-0.0807	-11.5%
20	0.6929	0.6702	-0.0227	-3.3%
		Mean	-0.0383	-11.5%
		Median	-0.0501	-10.5%
		Std Dev	0.0599	16.9%
		Variance	0.0036	2.9%
		Min	-0.1548	-39.9%
		Max	0.0971	22.0%

Table 4.6 Statistical analysis of predicted versus actual pitch response, RSM DOE

Table 4.6 indicates that the errors are rather large in terms of percentages. The percent errors are large due to the fact that the response value is very small. In terms of the magnitude of the residuals however, the absolute value of the mean residual is only 0.0383. The fact that the actual response values are within a relatively narrow window around the model predicted response values indicates that the model can be used to predict the pitch response within 0.20 of the actual value, for example, with a high degree of confidence. As expected, some of the lowest and therefore best pitch response values are due to the suggested optimal settings from Design Expert, runs 1-10. This was true for both the prediction and the actual validation simulations. Again, these damper settings correspond to the stiffer possibilities, which would be expected to produce better platform stability and pitch control, but may not be practically better in reality. Actual rig and track testing based on these results would be useful to determine their validity, as well as find potential compromise's between pitch control via damper settings in conjunction with driver perception and lap time potential.

The next step is to compare the initial model and augmented model results, in order to gauge whether an improvement in the predictive capability of the model was made.

		Initial DOE			Augmented DOE		
	Run	Pitch	Pred. Pitch	Residual	Pitch	Pred. Pitch	Residual
Optimal Solution Runs	1	0.3413	0.3313	-0.0100	0.2808	0.1750	-0.1058
	2	0.5096	0.3313	-0.1784	0.2653	0.1891	-0.0762
	3	0.4020	0.3313	-0.0707	0.2861	0.2052	-0.0808
	4	0.4587	0.3313	-0.1275	0.2761	0.2064	-0.0697
	5	0.4439	0.3313	-0.1127	0.2824	0.2071	-0.0753
	6	0.5570	0.3313	-0.2257	0.2111	0.2176	0.0065
	7	0.5163	0.3313	-0.1850	0.2652	0.2274	-0.0378
	8	0.7643	0.3313	-0.4330	0.2950	0.2329	-0.0621
	9	0.4539	0.3314	-0.1224	0.3878	0.2330	-0.1548
	10	0.3351	0.3316	-0.0035	0.2834	0.2332	-0.0501
Randomly generated runs	11	0.6712	0.7919	0.1207	0.6712	0.6709	-0.0002
	12	0.8736	0.8543	-0.0193	0.8736	0.8186	-0.0550
	13	0.4301	0.5893	0.1592	0.4301	0.4634	0.0333
	14	0.8164	0.9014	0.0849	0.8164	0.7970	-0.0194
	15	0.4966	0.6833	0.1866	0.4966	0.5769	0.0802
	16	0.4418	0.5974	0.1557	0.4418	0.5388	0.0971
	17	0.5297	0.5104	-0.0194	0.5297	0.4796	-0.0502
	18	0.7028	0.7442	0.0414	0.7028	0.6613	-0.0415
	19	0.7025	0.7209	0.0184	0.7025	0.6219	-0.0807
	20	0.6929	0.8035	0.1106	0.6929	0.6702	-0.0227
			Mean	-0.0315		**Mean**	-0.0383
			Median	-0.0147		**Median**	-0.0501
			Std Dev	0.1547		**Std Dev**	0.0599
			Variance	0.0239		**Variance**	0.0036
			Min	-0.4330		**Min**	-0.1548
			Max	0.1866		**Max**	0.0971

Table 4.7 Comparison of model predictive capability

Table 4.7 is a comparison of the predicted versus actual residuals of the initial and augmented DOE runs. Runs 1 through 10 are the runs suggested by Design Expert's optimization routine as the best choices to minimize the simulation's pitch response. Runs 11 through 20 are the same runs for each DOE, with the factors chosen randomly in order to explore the predictive capability of the models within the design space. Since a direct comparison can be made between the models for the randomly generated runs, the smallest residuals for each run are highlighted. The augmented DOE model was closer in its prediction of the simulation response in 6 of the 10 runs, whereas the initial DOE was closer in the other 4. The mean residual is slightly lower in magnitude for the initial model, but the standard deviation, variance, minimum, and maximum residuals were better for the augmented model. Based on these numbers, it appears that the augmented model is slightly better at predicting pitch response, likely due to the further explored design space, particularly where non linearity's are involved. The augmented model also predicts response values within a smaller numerical proximity to their actual values, as shown by the summary statistics.

5 Conclusions and Future Work

From actual track and 7-post rig testing experience, it is not surprising that the most important factors in controlling the pitch response revealed by this study were the rear damper settings [1]. What was perhaps less obvious was the interaction effects of the rear damper settings as discussed earlier. In particular, RLSR clearly produced better pitch results at its stiffest setting in three of the interactions that it was involved. RHSB and RLSR appeared to be best at the stiffest setting, while allowing flexibility in adjusting RLSB, a good tuning tool for driver feel and perception. The data suggested that RHSB can be varied with minimal impact as long as RLSR is set at its stiffest setting. RHSR produced the best pitch results when set at its stiffest setting as well, although appears highly coupled with RLSR. These results would suggest further study and tests of these parameters on an actual race car, either on a 7-post rig or at a race track.

Typically, very stiff damper settings can produce other highly undesirable results, such as wheel hop, resonance, etc [2, 4]. The goal is always to find the best compromise. Performing validation testing of these results on an actual 7-post rig based on these interactions would be educational as well. This would help determine if Rigsim is able to correctly model the real effects and interactions, and their magnitudes as demonstrated by a real car and 7-post rig testing. Also, although front and rear tire stiffness' were included as factors in this study, they were omitted in the model due to the seemingly low impact on pitch response. Actual 7-post rig validation would be useful to test the effects and interactions involving tire stiffness, particularly since actual tires have intrinsic damping and other characteristics that are difficult to measure and model correctly.

In terms of predictive modeling, from the comparison of the DOE model predicted mean pitch response values and the Rigsim simulation pitch response values, it is not surprising that the values differ to some degree. The damper models used in Rigsim, which are based on software curves determined by clicker settings for the low and high speed bump and rebound valves of an actual damper, are highly nonlinear and can be very irregular in shape. Typically low speed adjustments directly affect the damper's high speed characteristics and vice versa, in both bump and rebound directions. Based on experience and actual rig testing however, the absolute value of the pitch response is typically not as important to racing engineers in comparison to the range of its value, and the comparison of values for different damper settings. In other words, a complete set of damper settings that are found to have a lower pitch number than another set of settings on an actual rig for instance, are usually viewed as being better, insofar as only the pitch response is concerned [1].

The pitch number is just one of many response outputs of an actual 7-post rig and the Rigsim virtual rig. A given race track may favor damper settings that optimize contact patch load variation, known in the industry as CPL. Further, driver feedback and comfort level is an important parameter in setting up a race car as well. Optimal damper settings for one driver may be undesirable to another driver.

Rigsim and other simulation platforms like it can be very powerful and informative tools. Testing at an actual 7-post rig can also be very useful in finding optimum damper settings for a race car. Quite often however, testing costs, scheduling restrictions, personnel availability, and other circumstances necessitate the need for reliable methods from which to generate useful information in the shortest amount of time, and with the least amount of cost and work. The results of this study have shown that simulation results from Rigsim can be modeled to provide predicted values of pitch for a given array of damper and tire pressure settings, at least to a certain degree of accuracy. The model is not perfect, but for many engineers in the industry would be adequate for predicting trends, as well as comparing one set of parameters to another. The predictions and solutions of the experimental model can then be used to guide future validation studies, both at the rig and on track. In particular, some future studies could include taking simulation results and validating them against actual

7-post rig results, and actual track testing with many of the run setups on an actual car, while gauging which are more effective in reducing lap times and appeasing the driver.

Further studies can also include developing the DOE and creating new ones with other optimization and analytical purposes. Target response parameters such as front and rear CPL could also be included in the model. The model could then be used to predict and optimize all three or more target response values simultaneously. It would be interesting as well to see if the model in this study can be improved in order to predict the actual pitch response value more closely, if a much larger set of runs were added to even further map the design space. This would perhaps reveal and model the complex interactions between damper characteristics and setting interactions more closely.

Many professional racing teams use tools like Rigsim and actual 7-post testing, but many also rely on areas of development such as wind tunnel testing, straight line aerodynamic testing, differential and gearbox development and testing, and computer software-based dynamic lap time simulation. With the vast number of adjustable parameters on an actual racecar, and the development time and cost to test a relatively small number of combinations, DOE can provide a very useful tool in most other areas of racing as well. The methodologies used in this study can be applied to other aspects of race car engineering and development, perhaps with much greater success.

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Appendix A Factorial DOE matrix with response

Run	FLSB	FHSB	FLSR	FHSR	RLSB	RHSB	RLSR	RHSR	F Tire	R Tire	Pitch
1	10	0	10	0	10	0	10	0	1730	1680	0.274
2	40	0	10	0	10	0	10	0	1730	1880	0.403
3	10	50	10	0	10	0	10	0	1730	1880	0.419
4	40	50	10	0	10	0	10	0	1730	1680	0.337
5	10	0	40	0	10	0	10	0	1730	1880	0.479
6	40	0	40	0	10	0	10	0	1730	1680	0.511
7	10	50	40	0	10	0	10	0	1730	1680	0.536
8	40	50	40	0	10	0	10	0	1730	1880	0.836
9	10	0	10	50	10	0	10	0	1730	1880	0.420
10	40	0	10	50	10	0	10	0	1730	1680	0.458
11	10	50	10	50	10	0	10	0	1730	1680	0.492
12	40	50	10	50	10	0	10	0	1730	1880	0.781
13	10	0	40	50	10	0	10	0	1730	1680	0.374
14	40	0	40	50	10	0	10	0	1730	1880	0.808
15	10	50	40	50	10	0	10	0	1730	1880	0.844
16	40	50	40	50	10	0	10	0	1730	1680	0.664
17	10	0	10	0	40	0	10	0	1730	1880	0.341
18	40	0	10	0	40	0	10	0	1730	1680	0.290
19	10	50	10	0	40	0	10	0	1730	1680	0.328
20	40	50	10	0	40	0	10	0	1730	1880	0.322
21	10	0	40	0	40	0	10	0	1730	1680	0.259
22	40	0	40	0	40	0	10	0	1730	1880	0.589
23	10	50	40	0	40	0	10	0	1730	1880	0.617
24	40	50	40	0	40	0	10	0	1730	1680	0.522
25	10	0	10	50	40	0	10	0	1730	1680	0.320
26	40	0	10	50	40	0	10	0	1730	1880	0.514
27	10	50	10	50	40	0	10	0	1730	1880	0.541
28	40	50	10	50	40	0	10	0	1730	1680	0.443
29	10	0	40	50	40	0	10	0	1730	1880	0.352
30	40	0	40	50	40	0	10	0	1730	1680	0.477
31	10	50	40	50	40	0	10	0	1730	1680	0.504
32	40	50	40	50	40	0	10	0	1730	1880	0.756
33	10	0	10	0	10	50	10	0	1730	1880	0.311
34	40	0	10	0	10	50	10	0	1730	1680	0.268
35	10	50	10	0	10	50	10	0	1730	1680	0.304
36	40	50	10	0	10	50	10	0	1730	1880	0.318
37	10	0	40	0	10	50	10	0	1730	1680	0.253
38	40	0	40	0	10	50	10	0	1730	1880	0.602
39	10	50	40	0	10	50	10	0	1730	1880	0.630
40	40	50	40	0	10	50	10	0	1730	1680	0.529
41	10	0	10	50	10	50	10	0	1730	1680	0.295
42	40	0	10	50	10	50	10	0	1730	1880	0.525
43	10	50	10	50	10	50	10	0	1730	1880	0.551
44	40	50	10	50	10	50	10	0	1730	1680	0.448
45	10	0	40	50	10	50	10	0	1730	1880	0.356

46	40	0	40	50	10	50	10	0	1730	1680	0.482
47	10	50	40	50	10	50	10	0	1730	1680	0.508
48	40	50	40	50	10	50	10	0	1730	1880	0.766
49	10	0	10	0	40	50	10	0	1730	1680	0.578
50	40	0	10	0	40	50	10	0	1730	1880	0.273
51	10	50	10	0	40	50	10	0	1730	1880	0.296
52	40	50	10	0	40	50	10	0	1730	1680	0.328
53	10	0	40	0	40	50	10	0	1730	1880	0.307
54	40	0	40	0	40	50	10	0	1730	1680	0.440
55	10	50	40	0	40	50	10	0	1730	1680	0.454
56	40	50	40	0	40	50	10	0	1730	1880	0.676
57	10	0	10	50	40	50	10	0	1730	1880	0.287
58	40	0	10	50	40	50	10	0	1730	1680	0.359
59	10	50	10	50	40	50	10	0	1730	1680	0.385
60	40	50	10	50	40	50	10	0	1730	1880	0.595
61	10	0	40	50	40	50	10	0	1730	1680	0.294
62	40	0	40	50	40	50	10	0	1730	1880	0.628
63	10	50	40	50	40	50	10	0	1730	1880	0.661
64	40	50	40	50	40	50	10	0	1730	1680	0.577
65	10	0	10	0	10	0	40	0	1730	1880	0.427
66	40	0	10	0	10	0	40	0	1730	1680	0.354
67	10	50	10	0	10	0	40	0	1730	1680	0.390
68	40	50	10	0	10	0	40	0	1730	1880	0.325
69	10	0	40	0	10	0	40	0	1730	1680	0.309
70	40	0	40	0	10	0	40	0	1730	1880	0.586
71	10	50	40	0	10	0	40	0	1730	1880	0.608
72	40	50	40	0	10	0	40	0	1730	1680	0.535
73	10	0	10	50	10	0	40	0	1730	1680	0.382
74	40	0	10	50	10	0	40	0	1730	1880	0.504
75	10	50	10	50	10	0	40	0	1730	1880	0.524
76	40	50	10	50	10	0	40	0	1730	1680	0.448
77	10	0	40	50	10	0	40	0	1730	1880	0.351
78	40	0	40	50	10	0	40	0	1730	1680	0.490
79	10	50	40	50	10	0	40	0	1730	1680	0.510
80	40	50	40	50	10	0	40	0	1730	1880	0.743
81	10	0	10	0	40	0	40	0	1730	1680	1.148
82	40	0	10	0	40	0	40	0	1730	1880	0.815
83	10	50	10	0	40	0	40	0	1730	1880	0.848
84	40	50	10	0	40	0	40	0	1730	1680	0.909
85	10	0	40	0	40	0	40	0	1730	1880	0.754
86	40	0	40	0	40	0	40	0	1730	1680	0.714
87	10	50	40	0	40	0	40	0	1730	1680	0.716
88	40	50	40	0	40	0	40	0	1730	1880	0.668
89	10	0	10	50	40	0	40	0	1730	1880	0.840
90	40	0	10	50	40	0	40	0	1730	1680	0.729
91	10	50	10	50	40	0	40	0	1730	1680	0.753
92	40	50	10	50	40	0	40	0	1730	1880	0.657

93	10	0	40	50	40	0	40	0	1730	1680	0.854
94	40	0	40	50	40	0	40	0	1730	1880	0.651
95	10	50	40	50	40	0	40	0	1730	1880	0.656
96	40	50	40	50	40	0	40	0	1730	1680	0.745
97	10	0	10	0	10	50	40	0	1730	1680	1.165
98	40	0	10	0	10	50	40	0	1730	1880	0.818
99	10	50	10	0	10	50	40	0	1730	1880	0.856
100	40	50	10	0	10	50	40	0	1730	1680	0.914
101	10	0	40	0	10	50	40	0	1730	1880	0.750
102	40	0	40	0	10	50	40	0	1730	1680	0.692
103	10	50	40	0	10	50	40	0	1730	1680	0.694
104	40	50	40	0	10	50	40	0	1730	1880	0.660
105	10	0	10	50	10	50	40	0	1730	1880	0.848
106	40	0	10	50	10	50	40	0	1730	1680	0.706
107	10	50	10	50	10	50	40	0	1730	1680	0.733
108	40	50	10	50	10	50	40	0	1730	1880	0.634
109	10	0	40	50	10	50	40	0	1730	1680	0.852
110	40	0	40	50	10	50	40	0	1730	1880	0.636
111	10	50	40	50	10	50	40	0	1730	1880	0.637
112	40	50	40	50	10	50	40	0	1730	1680	0.724
113	10	0	10	0	40	50	40	0	1730	1880	1.186
114	40	0	10	0	40	50	40	0	1730	1680	1.054
115	10	50	10	0	40	50	40	0	1730	1680	1.092
116	40	50	10	0	40	50	40	0	1730	1880	0.940
117	10	0	40	0	40	50	40	0	1730	1680	0.987
118	40	0	40	0	40	50	40	0	1730	1880	0.711
119	10	50	40	0	40	50	40	0	1730	1880	0.714
120	40	50	40	0	40	50	40	0	1730	1680	0.807
121	10	0	10	50	40	50	40	0	1730	1680	1.087
122	40	0	10	50	40	50	40	0	1730	1880	0.730
123	10	50	10	50	40	50	40	0	1730	1880	0.760
124	40	50	10	50	40	50	40	0	1730	1680	0.821
125	10	0	40	50	40	50	40	0	1730	1880	0.880
126	40	0	40	50	40	50	40	0	1730	1680	0.798
127	10	50	40	50	40	50	40	0	1730	1680	0.802
128	40	50	40	50	40	50	40	0	1730	1880	0.738
129	10	0	10	0	10	0	10	50	1730	1880	0.320
130	40	0	10	0	10	0	10	50	1730	1680	0.277
131	10	50	10	0	10	0	10	50	1730	1680	0.312
132	40	50	10	0	10	0	10	50	1730	1880	0.318
133	10	0	40	0	10	0	10	50	1730	1680	0.265
134	40	0	40	0	10	0	10	50	1730	1880	0.614
135	10	50	40	0	10	0	10	50	1730	1880	0.639
136	40	50	40	0	10	0	10	50	1730	1680	0.540
137	10	0	10	50	10	0	10	50	1730	1680	0.305
138	40	0	10	50	10	0	10	50	1730	1880	0.537
139	10	50	10	50	10	0	10	50	1730	1880	0.560

140	40	50	10	50	10	0	10	50	1730	1680	0.458
141	10	0	40	50	10	0	10	50	1730	1880	0.368
142	40	0	40	50	10	0	10	50	1730	1680	0.497
143	10	50	40	50	10	0	10	50	1730	1680	0.521
144	40	50	40	50	10	0	10	50	1730	1880	0.778
145	10	0	10	0	40	0	10	50	1730	1680	1.050
146	40	0	10	0	40	0	10	50	1730	1880	0.691
147	10	50	10	0	40	0	10	50	1730	1880	0.725
148	40	50	10	0	40	0	10	50	1730	1680	0.790
149	10	0	40	0	40	0	10	50	1730	1880	0.625
150	40	0	40	0	40	0	10	50	1730	1680	0.607
151	10	50	40	0	40	0	10	50	1730	1680	0.606
152	40	50	40	0	40	0	10	50	1730	1880	0.617
153	10	0	10	50	40	0	10	50	1730	1880	0.716
154	40	0	10	50	40	0	10	50	1730	1680	0.607
155	10	50	10	50	40	0	10	50	1730	1680	0.630
156	40	50	10	50	40	0	10	50	1730	1880	0.564
157	10	0	40	50	40	0	10	50	1730	1680	0.730
158	40	0	40	50	40	0	10	50	1730	1880	0.588
159	10	50	40	50	40	0	10	50	1730	1880	0.586
160	40	50	40	50	40	0	10	50	1730	1680	0.657
161	10	0	10	0	10	50	10	50	1730	1680	1.064
162	40	0	10	0	10	50	10	50	1730	1880	0.689
163	10	50	10	0	10	50	10	50	1730	1880	0.728
164	40	50	10	0	10	50	10	50	1730	1680	0.789
165	10	0	40	0	10	50	10	50	1730	1880	0.617
166	40	0	40	0	10	50	10	50	1730	1680	0.601
167	10	50	40	0	10	50	10	50	1730	1680	0.596
168	40	50	40	0	10	50	10	50	1730	1880	0.649
169	10	0	10	50	10	50	10	50	1730	1880	0.717
170	40	0	10	50	10	50	10	50	1730	1680	0.587
171	10	50	10	50	10	50	10	50	1730	1680	0.610
172	40	50	10	50	10	50	10	50	1730	1880	0.567
173	10	0	40	50	10	50	10	50	1730	1680	0.722
174	40	0	40	50	10	50	10	50	1730	1880	0.612
175	10	50	40	50	10	50	10	50	1730	1880	0.606
176	40	50	40	50	10	50	10	50	1730	1680	0.657
177	10	0	10	0	40	50	10	50	1730	1880	1.091
178	40	0	10	0	40	50	10	50	1730	1680	0.941
179	10	50	10	0	40	50	10	50	1730	1680	0.983
180	40	50	10	0	40	50	10	50	1730	1880	0.819
181	10	0	40	0	40	50	10	50	1730	1680	0.864
182	40	0	40	0	40	50	10	50	1730	1880	0.638
183	10	50	40	0	40	50	10	50	1730	1880	0.631
184	40	50	40	0	40	50	10	50	1730	1680	0.712
185	10	0	10	50	40	50	10	50	1730	1680	0.976
186	40	0	10	50	40	50	10	50	1730	1880	0.622

187	10	50	10	50	40	50	10	50	1730	1880	0.644
188	40	50	10	50	40	50	10	50	1730	1680	0.704
189	10	0	40	50	40	50	10	50	1730	1880	0.753
190	40	0	40	50	40	50	10	50	1730	1680	0.696
191	10	50	40	50	40	50	10	50	1730	1680	0.695
192	40	50	40	50	40	50	10	50	1730	1880	0.690
193	10	0	10	0	10	0	40	50	1730	1680	0.649
194	40	0	10	0	10	0	40	50	1730	1880	0.306
195	10	50	10	0	10	0	40	50	1730	1880	0.329
196	40	50	10	0	10	0	40	50	1730	1680	0.381
197	10	0	40	0	10	0	40	50	1730	1880	0.329
198	40	0	40	0	10	0	40	50	1730	1680	0.466
199	10	50	40	0	10	0	40	50	1730	1680	0.471
200	40	50	40	0	10	0	40	50	1730	1880	0.678
201	10	0	10	50	10	0	40	50	1730	1880	0.324
202	40	0	10	50	10	0	40	50	1730	1680	0.382
203	10	50	10	50	10	0	40	50	1730	1680	0.401
204	40	50	10	50	10	0	40	50	1730	1880	0.592
205	10	0	40	50	10	0	40	50	1730	1680	0.341
206	40	0	40	50	10	0	40	50	1730	1880	0.633
207	10	50	40	50	10	0	40	50	1730	1880	0.660
208	40	50	40	50	10	0	40	50	1730	1680	0.592
209	10	0	10	0	40	0	40	50	1730	1880	1.143
210	40	0	10	0	40	0	40	50	1730	1680	1.011
211	10	50	10	0	40	0	40	50	1730	1680	1.048
212	40	50	10	0	40	0	40	50	1730	1880	0.893
213	10	0	40	0	40	0	40	50	1730	1680	0.942
214	40	0	40	0	40	0	40	50	1730	1880	0.688
215	10	50	40	0	40	0	40	50	1730	1880	0.685
216	40	50	40	0	40	0	40	50	1730	1680	0.779
217	10	0	10	50	40	0	40	50	1730	1680	1.043
218	40	0	10	50	40	0	40	50	1730	1880	0.701
219	10	50	10	50	40	0	40	50	1730	1880	0.724
220	40	50	10	50	40	0	40	50	1730	1680	0.789
221	10	0	40	50	40	0	40	50	1730	1880	0.835
222	40	0	40	50	40	0	40	50	1730	1680	0.774
223	10	50	40	50	40	0	40	50	1730	1680	0.773
224	40	50	40	50	40	0	40	50	1730	1880	0.719
225	10	0	10	0	10	50	40	50	1730	1880	1.183
226	40	0	10	0	10	50	40	50	1730	1680	1.036
227	10	50	10	0	10	50	40	50	1730	1680	1.076
228	40	50	10	0	10	50	40	50	1730	1880	0.921
229	10	0	40	0	10	50	40	50	1730	1680	0.963
230	40	0	40	0	10	50	40	50	1730	1880	0.696
231	10	50	40	0	10	50	40	50	1730	1880	0.692
232	40	50	40	0	10	50	40	50	1730	1680	0.780
233	10	0	10	50	10	50	40	50	1730	1680	1.072

234	40	0	10	50	10	50	40	50	1730	1880	0.702
235	10	50	10	50	10	50	40	50	1730	1880	0.728
236	40	50	10	50	10	50	40	50	1730	1680	0.787
237	10	0	40	50	10	50	40	50	1730	1880	0.856
238	40	0	40	50	10	50	40	50	1730	1680	0.771
239	10	50	40	50	10	50	40	50	1730	1680	0.770
240	40	50	40	50	10	50	40	50	1730	1880	0.731
241	10	0	10	0	40	50	40	50	1730	1680	1.399
242	40	0	10	0	40	50	40	50	1730	1880	1.084
243	10	50	10	0	40	50	40	50	1730	1880	1.123
244	40	50	10	0	40	50	40	50	1730	1680	1.160
245	10	0	40	0	40	50	40	50	1730	1880	1.015
246	40	0	40	0	40	50	40	50	1730	1680	0.887
247	10	50	40	0	40	50	40	50	1730	1680	0.888
248	40	50	40	0	40	50	40	50	1730	1880	0.806
249	10	0	10	50	40	50	40	50	1730	1880	1.120
250	40	0	10	50	40	50	40	50	1730	1680	0.923
251	10	50	10	50	40	50	40	50	1730	1680	0.958
252	40	50	10	50	40	50	40	50	1730	1880	0.824
253	10	0	40	50	40	50	40	50	1730	1680	1.104
254	40	0	40	50	40	50	40	50	1730	1880	0.800
255	10	50	40	50	40	50	40	50	1730	1880	0.801
256	40	50	40	50	40	50	40	50	1730	1680	0.894
257	10	0	10	0	10	0	10	0	1960	1880	0.310
258	40	0	10	0	10	0	10	0	1960	1680	0.355
259	10	50	10	0	10	0	10	0	1960	1680	0.399
260	40	50	10	0	10	0	10	0	1960	1880	0.400
261	10	0	40	0	10	0	10	0	1960	1680	0.354
262	40	0	40	0	10	0	10	0	1960	1880	0.567
263	10	50	40	0	10	0	10	0	1960	1880	0.607
264	40	50	40	0	10	0	10	0	1960	1680	0.455
265	10	0	10	50	10	0	10	0	1960	1680	0.406
266	40	0	10	50	10	0	10	0	1960	1880	0.525
267	10	50	10	50	10	0	10	0	1960	1880	0.574
268	40	50	10	50	10	0	10	0	1960	1680	0.443
269	10	0	40	50	10	0	10	0	1960	1880	0.443
270	40	0	40	50	10	0	10	0	1960	1680	0.426
271	10	50	40	50	10	0	10	0	1960	1680	0.480
272	40	50	40	50	10	0	10	0	1960	1880	0.754
273	10	0	10	0	40	0	10	0	1960	1680	0.761
274	40	0	10	0	40	0	10	0	1960	1880	0.300
275	10	50	10	0	40	0	10	0	1960	1880	0.339
276	40	50	10	0	40	0	10	0	1960	1680	0.468
277	10	0	40	0	40	0	10	0	1960	1880	0.271
278	40	0	40	0	40	0	10	0	1960	1680	0.349
279	10	50	40	0	40	0	10	0	1960	1680	0.377
280	40	50	40	0	40	0	10	0	1960	1880	0.569

281	10	0	10	50	40	0	10	0	1960	1880	0.329
282	40	0	10	50	40	0	10	0	1960	1680	0.312
283	10	50	10	50	40	0	10	0	1960	1680	0.360
284	40	50	10	50	40	0	10	0	1960	1880	0.500
285	10	0	40	50	40	0	10	0	1960	1680	0.406
286	40	0	40	50	40	0	10	0	1960	1880	0.518
287	10	50	40	50	40	0	10	0	1960	1880	0.566
288	40	50	40	50	40	0	10	0	1960	1680	0.505
289	10	0	10	0	10	50	10	0	1960	1680	0.765
290	40	0	10	0	10	50	10	0	1960	1880	0.282
291	10	50	10	0	10	50	10	0	1960	1880	0.317
292	40	50	10	0	10	50	10	0	1960	1680	0.457
293	10	0	40	0	10	50	10	0	1960	1880	0.275
294	40	0	40	0	10	50	10	0	1960	1680	0.352
295	10	50	40	0	10	50	10	0	1960	1680	0.381
296	40	50	40	0	10	50	10	0	1960	1880	0.577
297	10	0	10	50	10	50	10	0	1960	1880	0.307
298	40	0	10	50	10	50	10	0	1960	1680	0.317
299	10	50	10	50	10	50	10	0	1960	1680	0.363
300	40	50	10	50	10	50	10	0	1960	1880	0.508
301	10	0	40	50	10	50	10	0	1960	1680	0.393
302	40	0	40	50	10	50	10	0	1960	1880	0.524
303	10	50	40	50	10	50	10	0	1960	1880	0.572
304	40	50	40	50	10	50	10	0	1960	1680	0.501
305	10	0	10	0	40	50	10	0	1960	1880	0.624
306	40	0	10	0	40	50	10	0	1960	1680	0.536
307	10	50	10	0	40	50	10	0	1960	1680	0.573
308	40	50	10	0	40	50	10	0	1960	1880	0.340
309	10	0	40	0	40	50	10	0	1960	1680	0.450
310	40	0	40	0	40	50	10	0	1960	1880	0.465
311	10	50	40	0	40	50	10	0	1960	1880	0.496
312	40	50	40	0	40	50	10	0	1960	1680	0.460
313	10	0	10	50	40	50	10	0	1960	1680	0.560
314	40	0	10	50	40	50	10	0	1960	1880	0.392
315	10	50	10	50	40	50	10	0	1960	1880	0.432
316	40	50	10	50	40	50	10	0	1960	1680	0.402
317	10	0	40	50	40	50	10	0	1960	1880	0.309
318	40	0	40	50	40	50	10	0	1960	1680	0.409
319	10	50	40	50	40	50	10	0	1960	1680	0.442
320	40	50	40	50	40	50	10	0	1960	1880	0.640
321	10	0	10	0	10	0	40	0	1960	1680	0.840
322	40	0	10	0	10	0	40	0	1960	1880	0.371
323	10	50	10	0	10	0	40	0	1960	1880	0.404
324	40	50	10	0	10	0	40	0	1960	1680	0.529
325	10	0	40	0	10	0	40	0	1960	1880	0.318
326	40	0	40	0	10	0	40	0	1960	1680	0.389
327	10	50	40	0	10	0	40	0	1960	1680	0.408

328	40	50	40	0	10	0	40	0	1960	1880	0.577
329	10	0	10	50	10	0	40	0	1960	1880	0.395
330	40	0	10	50	10	0	40	0	1960	1680	0.352
331	10	50	10	50	10	0	40	0	1960	1680	0.390
332	40	50	10	50	10	0	40	0	1960	1880	0.499
333	10	0	40	50	10	0	40	0	1960	1680	0.469
334	40	0	40	50	10	0	40	0	1960	1880	0.525
335	10	50	40	50	10	0	40	0	1960	1880	0.565
336	40	50	40	50	10	0	40	0	1960	1680	0.528
337	10	0	10	0	40	0	40	0	1960	1880	1.225
338	40	0	10	0	40	0	40	0	1960	1680	1.084
339	10	50	10	0	40	0	40	0	1960	1680	1.117
340	40	50	10	0	40	0	40	0	1960	1880	0.950
341	10	0	40	0	40	0	40	0	1960	1680	1.008
342	40	0	40	0	40	0	40	0	1960	1880	0.748
343	10	50	40	0	40	0	40	0	1960	1880	0.751
344	40	50	40	0	40	0	40	0	1960	1680	0.848
345	10	0	10	50	40	0	40	0	1960	1680	1.109
346	40	0	10	50	40	0	40	0	1960	1880	0.764
347	10	50	10	50	40	0	40	0	1960	1880	0.783
348	40	50	10	50	40	0	40	0	1960	1680	0.855
349	10	0	40	50	40	0	40	0	1960	1880	0.891
350	40	0	40	50	40	0	40	0	1960	1680	0.841
351	10	50	40	50	40	0	40	0	1960	1680	0.843
352	40	50	40	50	40	0	40	0	1960	1880	0.785
353	10	0	10	0	10	50	40	0	1960	1880	1.257
354	40	0	10	0	10	50	40	0	1960	1680	1.102
355	10	50	10	0	10	50	40	0	1960	1680	1.139
356	40	50	10	0	10	50	40	0	1960	1880	0.968
357	10	0	40	0	10	50	40	0	1960	1680	1.020
358	40	0	40	0	10	50	40	0	1960	1880	0.731
359	10	50	40	0	10	50	40	0	1960	1880	0.734
360	40	50	40	0	10	50	40	0	1960	1680	0.831
361	10	0	10	50	10	50	40	0	1960	1680	1.131
362	40	0	10	50	10	50	40	0	1960	1880	0.748
363	10	50	10	50	10	50	40	0	1960	1880	0.770
364	40	50	10	50	10	50	40	0	1960	1680	0.839
365	10	0	40	50	10	50	40	0	1960	1880	0.900
366	40	0	40	50	10	50	40	0	1960	1680	0.821
367	10	50	40	50	10	50	40	0	1960	1680	0.823
368	40	50	40	50	10	50	40	0	1960	1880	0.767
369	10	0	10	0	40	50	40	0	1960	1680	1.467
370	40	0	10	0	40	50	40	0	1960	1880	1.123
371	10	50	10	0	40	50	40	0	1960	1880	1.158
372	40	50	10	0	40	50	40	0	1960	1680	1.205
373	10	0	40	0	40	50	40	0	1960	1880	1.044
374	40	0	40	0	40	50	40	0	1960	1680	0.939

375	10	50	40	0	40	50	40	0	1960	1680	0.940
376	40	50	40	0	40	50	40	0	1960	1880	0.854
377	10	0	10	50	40	50	40	0	1960	1880	1.151
378	40	0	10	50	40	50	40	0	1960	1680	0.970
379	10	50	10	50	40	50	40	0	1960	1680	0.995
380	40	50	10	50	40	50	40	0	1960	1880	0.864
381	10	0	40	50	40	50	40	0	1960	1680	1.141
382	40	0	40	50	40	50	40	0	1960	1880	0.844
383	10	50	40	50	40	50	40	0	1960	1880	0.847
384	40	50	40	50	40	50	40	0	1960	1680	0.946
385	10	0	10	0	10	0	10	50	1960	1680	0.776
386	40	0	10	0	10	0	10	50	1960	1880	0.294
387	10	50	10	0	10	0	10	50	1960	1880	0.327
388	40	50	10	0	10	0	10	50	1960	1680	0.465
389	10	0	40	0	10	0	10	50	1960	1880	0.288
390	40	0	40	0	10	0	10	50	1960	1680	0.367
391	10	50	40	0	10	0	10	50	1960	1680	0.393
392	40	50	40	0	10	0	10	50	1960	1880	0.588
393	10	0	10	50	10	0	10	50	1960	1880	0.319
394	40	0	10	50	10	0	10	50	1960	1680	0.332
395	10	50	10	50	10	0	10	50	1960	1680	0.375
396	40	50	10	50	10	0	10	50	1960	1880	0.518
397	10	0	40	50	10	0	10	50	1960	1680	0.407
398	40	0	40	50	10	0	10	50	1960	1880	0.539
399	10	50	40	50	10	0	10	50	1960	1880	0.584
400	40	50	40	50	10	0	10	50	1960	1680	0.512
401	10	0	10	0	40	0	10	50	1960	1880	1.134
402	40	0	10	0	40	0	10	50	1960	1680	0.979
403	10	50	10	0	40	0	10	50	1960	1680	1.017
404	40	50	10	0	40	0	10	50	1960	1880	0.835
405	10	0	40	0	40	0	10	50	1960	1680	0.896
406	40	0	40	0	40	0	10	50	1960	1880	0.635
407	10	50	40	0	40	0	10	50	1960	1880	0.638
408	40	50	40	0	40	0	10	50	1960	1680	0.732
409	10	0	10	50	40	0	10	50	1960	1680	1.008
410	40	0	10	50	40	0	10	50	1960	1880	0.641
411	10	50	10	50	40	0	10	50	1960	1880	0.660
412	40	50	10	50	40	0	10	50	1960	1680	0.733
413	10	0	40	50	40	0	10	50	1960	1880	0.768
414	40	0	40	50	40	0	10	50	1960	1680	0.721
415	10	50	40	50	40	0	10	50	1960	1680	0.724
416	40	50	40	50	40	0	10	50	1960	1880	0.696
417	10	0	10	0	10	50	10	50	1960	1880	1.160
418	40	0	10	0	10	50	10	50	1960	1680	0.990
419	10	50	10	0	10	50	10	50	1960	1680	1.033
420	40	50	10	0	10	50	10	50	1960	1880	0.844
421	10	0	40	0	10	50	10	50	1960	1680	0.901

422	40	0	40	0	10	50	10	50	1960	1880	0.632
423	10	50	40	0	10	50	10	50	1960	1880	0.633
424	40	50	40	0	10	50	10	50	1960	1680	0.719
425	10	0	10	50	10	50	10	50	1960	1680	1.023
426	40	0	10	50	10	50	10	50	1960	1880	0.626
427	10	50	10	50	10	50	10	50	1960	1880	0.647
428	40	50	10	50	10	50	10	50	1960	1680	0.716
429	10	0	40	50	10	50	10	50	1960	1880	0.770
430	40	0	40	50	10	50	10	50	1960	1680	0.703
431	10	50	40	50	10	50	10	50	1960	1680	0.706
432	40	50	40	50	10	50	10	50	1960	1880	0.701
433	10	0	10	0	40	50	10	50	1960	1680	1.387
434	40	0	10	0	40	50	10	50	1960	1880	1.012
435	10	50	10	0	40	50	10	50	1960	1880	1.054
436	40	50	10	0	40	50	10	50	1960	1680	1.104
437	10	0	40	0	40	50	10	50	1960	1880	0.924
438	40	0	40	0	40	50	10	50	1960	1680	0.818
439	10	50	40	0	40	50	10	50	1960	1680	0.818
440	40	50	40	0	40	50	10	50	1960	1880	0.756
441	10	0	10	50	40	50	10	50	1960	1880	1.044
442	40	0	10	50	40	50	10	50	1960	1680	0.848
443	10	50	10	50	40	50	10	50	1960	1680	0.874
444	40	50	10	50	40	50	10	50	1960	1880	0.747
445	10	0	40	50	40	50	10	50	1960	1680	1.031
446	40	0	40	50	40	50	10	50	1960	1880	0.738
447	10	50	40	50	40	50	10	50	1960	1880	0.739
448	40	50	40	50	40	50	10	50	1960	1680	0.836
449	10	0	10	0	10	0	40	50	1960	1880	0.703
450	40	0	10	0	10	0	40	50	1960	1680	0.591
451	10	50	10	0	10	0	40	50	1960	1680	0.630
452	40	50	10	0	10	0	40	50	1960	1880	0.397
453	10	0	40	0	10	0	40	50	1960	1680	0.508
454	40	0	40	0	10	0	40	50	1960	1880	0.488
455	10	50	40	0	10	0	40	50	1960	1880	0.510
456	40	50	40	0	10	0	40	50	1960	1680	0.491
457	10	0	10	50	10	0	40	50	1960	1680	0.620
458	40	0	10	50	10	0	40	50	1960	1880	0.412
459	10	50	10	50	10	0	40	50	1960	1880	0.443
460	40	50	10	50	10	0	40	50	1960	1680	0.431
461	10	0	40	50	10	0	40	50	1960	1880	0.354
462	40	0	40	50	10	0	40	50	1960	1680	0.447
463	10	50	40	50	10	0	40	50	1960	1680	0.471
464	40	50	40	50	10	0	40	50	1960	1880	0.651
465	10	0	10	0	40	0	40	50	1960	1680	1.429
466	40	0	10	0	40	0	40	50	1960	1880	1.073
467	10	50	10	0	40	0	40	50	1960	1880	1.108
468	40	50	10	0	40	0	40	50	1960	1680	1.159

469	10	0	40	0	40	0	40	50	1960	1880	0.993
470	40	0	40	0	40	0	40	50	1960	1680	0.905
471	10	50	40	0	40	0	40	50	1960	1680	0.902
472	40	50	40	0	40	0	40	50	1960	1880	0.821
473	10	0	10	50	40	0	40	50	1960	1880	1.101
474	40	0	10	50	40	0	40	50	1960	1680	0.935
475	10	50	10	50	40	0	40	50	1960	1680	0.955
476	40	50	10	50	40	0	40	50	1960	1880	0.828
477	10	0	40	50	40	0	40	50	1960	1680	1.097
478	40	0	40	50	40	0	40	50	1960	1880	0.816
479	10	50	40	50	40	0	40	50	1960	1880	0.815
480	40	50	40	50	40	0	40	50	1960	1680	0.916
481	10	0	10	0	10	50	40	50	1960	1680	1.469
482	40	0	10	0	10	50	40	50	1960	1880	1.114
483	10	50	10	0	10	50	40	50	1960	1880	1.153
484	40	50	10	0	10	50	40	50	1960	1680	1.194
485	10	0	40	0	10	50	40	50	1960	1880	1.028
486	40	0	40	0	10	50	40	50	1960	1680	0.907
487	10	50	40	0	10	50	40	50	1960	1680	0.906
488	40	50	40	0	10	50	40	50	1960	1880	0.829
489	10	0	10	50	10	50	40	50	1960	1880	1.146
490	40	0	10	50	10	50	40	50	1960	1680	0.941
491	10	50	10	50	10	50	40	50	1960	1680	0.967
492	40	50	10	50	10	50	40	50	1960	1880	0.835
493	10	0	40	50	10	50	40	50	1960	1680	1.128
494	40	0	40	50	10	50	40	50	1960	1880	0.820
495	10	50	40	50	10	50	40	50	1960	1880	0.819
496	40	50	40	50	10	50	40	50	1960	1680	0.917
497	10	0	10	0	40	50	40	50	1960	1880	1.510
498	40	0	10	0	40	50	40	50	1960	1680	1.347
499	10	50	10	0	40	50	40	50	1960	1680	1.381
500	40	50	10	0	40	50	40	50	1960	1880	1.239
501	10	0	40	0	40	50	40	50	1960	1680	1.274
502	40	0	40	0	40	50	40	50	1960	1880	0.948
503	10	50	40	0	40	50	40	50	1960	1880	0.946
504	40	50	40	0	40	50	40	50	1960	1680	1.037
505	10	0	10	50	40	50	40	50	1960	1680	1.378
506	40	0	10	50	40	50	40	50	1960	1880	0.985
507	10	50	10	50	40	50	40	50	1960	1880	1.014
508	40	50	10	50	40	50	40	50	1960	1680	1.065
509	10	0	40	50	40	50	40	50	1960	1880	1.175
510	40	0	40	50	40	50	40	50	1960	1680	1.038
511	10	50	40	50	40	50	40	50	1960	1680	1.037
512	40	50	40	50	40	50	40	50	1960	1880	0.951

Appendix B Augmented DOE matrix

Run	FLSB	FHSB	FLSR	FHSR	RLSB	RHSB	RLSR	RHSR	F Tire	R Tire	Pitch
1	25	25	25	50	25	25	25	25	1845	1780	0.572
2	25	25	10	25	25	25	25	25	1845	1780	0.654
3	25	0	25	25	25	25	25	25	1845	1780	0.585
4	25	25	25	25	25	25	25	25	1960	1780	0.639
5	25	25	25	25	10	25	25	25	1845	1780	0.382
6	25	25	25	25	25	50	25	25	1845	1780	0.713
7	40	25	25	25	25	25	25	25	1845	1780	0.562
8	25	25	25	25	25	25	10	25	1845	1780	0.35
9	10	0	10	0	10	0	10	33	1730	1747	0.305
10	40	0	10	0	10	0	10	25	1730	1680	0.245
11	10	50	10	0	40	50	30	0	1883	1680	1.112
12	40	50	40	0	40	17	40	50	1883	1880	0.802
13	10	0	10	0	40	17	10	0	1807	1880	0.402
14	40	50	40	0	40	33	10	33	1730	1880	0.637
15	10	0	40	0	30	50	40	50	1960	1747	1.214
16	40	0	10	50	40	0	30	50	1807	1880	0.704
17	10	0	10	50	40	0	40	25	1960	1680	1.126
18	40	0	40	50	40	50	10	17	1730	1747	0.539
19	10	0	40	0	10	17	40	0	1883	1880	0.424
20	10	50	10	50	40	0	30	0	1730	1747	0.648
21	10	17	40	50	10	50	10	17	1960	1680	0.424
22	40	0	40	50	30	0	10	50	1883	1680	0.634
23	40	50	10	0	30	50	40	50	1807	1680	1.192
24	40	17	10	50	10	0	40	0	1807	1880	0.476
25	10	33	40	0	10	0	40	0	1807	1680	0.362
26	40	50	20	50	10	0	30	50	1960	1680	0.449
27	20	0	10	0	10	0	40	50	1807	1880	0.416
28	10	50	20	50	10	50	40	17	1960	1680	0.861
29	20	50	10	0	10	0	30	50	1730	1880	0.313
30	20	0	10	50	40	50	40	50	1807	1880	1.014
31	40	50	10	50	20	0	10	33	1730	1880	0.556
32	40	17	10	50	10	50	40	0	1730	1813	0.648
33	40	50	30	0	10	0	10	0	1807	1880	0.709
34	10	0	10	33	10	50	10	33	1730	1880	0.545
35	40	17	10	0	40	50	10	0	1730	1813	0.264
36	10	0	40	33	40	0	40	0	1883	1880	0.85
37	10	33	10	50	10	0	20	50	1960	1880	0.378
38	10	0	40	33	40	50	30	50	1730	1880	0.956
39	10	50	30	50	10	50	10	50	1883	1880	0.614
40	10	50	40	50	30	50	30	50	1960	1880	0.868
41	30	0	40	0	10	0	20	50	1960	1680	0.336
42	10	0	40	0	20	0	40	17	1960	1880	0.626
43	40	0	20	50	40	50	10	33	1960	1880	0.553
44	40	17	10	50	10	50	30	0	1730	1880	0.554

45	40	50	10	17	10	50	40	0	1883	1880	0.817
46	40	0	10	33	40	17	40	0	1730	1680	0.778
47	40	50	40	0	30	0	10	17	1730	1680	0.53
48	30	50	20	0	40	0	40	0	1730	1680	0.781
49	10	0	10	0	30	0	10	0	1883	1680	0.643
50	20	0	40	0	40	0	20	0	1960	1880	0.502
51	40	33	10	0	10	50	40	17	1960	1880	0.999
52	40	0	20	0	30	0	40	50	1960	1880	0.841
53	10	0	30	50	10	50	40	50	1730	1813	0.917
54	10	50	40	50	40	33	30	50	1730	1880	0.699
55	40	50	40	33	10	0	40	50	1883	1680	0.522
56	40	0	40	50	20	50	10	0	1960	1813	0.47
57	40	50	20	50	40	0	40	50	1960	1747	0.881
58	40	33	10	0	10	50	30	50	1960	1680	1.162
59	10	50	20	0	10	33	10	50	1960	1880	0.505
60	10	17	40	50	40	0	10	0	1730	1747	0.309
61	10	33	40	50	10	33	10	0	1960	1680	0.393
62	10	50	10	17	40	0	30	50	1960	1880	0.956
63	10	50	10	33	40	0	10	0	1883	1680	0.338
64	30	50	10	50	10	33	40	0	1730	1880	0.534
65	30	0	20	0	10	0	10	0	1960	1880	0.354
66	10	0	10	0	30	17	40	50	1730	1680	1.211
67	10	0	10	33	10	50	10	50	1960	1813	1.019
68	20	50	40	0	40	0	40	50	1960	1813	0.84
69	10	0	40	33	30	0	40	0	1730	1680	0.789
70	40	0	40	17	20	50	10	50	1960	1680	0.748
71	10	50	40	17	10	0	40	17	1730	1680	0.466
72	40	50	40	17	10	0	10	17	1960	1880	0.671
73	40	0	10	50	40	17	10	0	1960	1813	0.354
74	30	50	30	50	40	50	10	0	1730	1880	0.677
75	10	0	10	0	20	0	40	0	1960	1747	1.046
76	10	0	10	17	20	50	40	50	1730	1880	1.182
77	20	33	10	0	10	50	40	50	1730	1680	1.097
78	20	50	40	50	10	0	30	0	1730	1680	0.537
79	30	50	40	0	10	50	40	0	1807	1880	0.681
80	40	0	40	17	40	17	10	50	1730	1880	0.581
81	40	50	30	0	20	50	10	50	1960	1680	0.763
82	40	50	10	33	30	50	40	0	1960	1680	0.984
83	40	17	40	50	10	17	10	50	1730	1680	0.523
84	30	50	40	0	40	50	20	50	1730	1680	0.767
85	10	0	30	50	40	0	30	0	1960	1680	0.941
86	10	0	40	17	20	50	10	0	1960	1880	0.279
87	20	33	40	0	40	50	40	0	1960	1680	0.955
88	10	50	40	0	10	17	10	33	1960	1680	0.419
89	10	50	10	50	40	0	40	50	1807	1813	0.799
90	10	33	20	0	40	0	10	50	1730	1680	0.719
91	10	33	40	0	10	50	20	50	1730	1880	0.586

92	10	0	40	0	40	50	30	0	1730	1813	0.811
93	20	17	40	0	40	0	10	50	1730	1880	0.539
94	10	50	10	50	40	0	10	17	1730	1813	0.454
95	40	0	10	50	20	17	40	0	1960	1880	0.574
96	30	50	40	33	10	0	40	0	1730	1680	0.535
97	40	50	20	50	10	50	10	50	1730	1747	0.597
98	40	0	40	0	20	33	10	0	1730	1680	0.442
99	10	50	40	17	10	33	40	50	1730	1680	0.599
100	10	33	20	50	40	50	40	0	1730	1680	0.878
101	40	50	40	0	20	0	40	33	1960	1680	0.659
102	40	0	30	50	10	33	40	50	1730	1880	0.6
103	40	50	40	17	10	33	40	0	1960	1880	0.636
104	40	50	10	0	10	0	30	17	1960	1880	0.333
105	10	50	10	50	20	0	20	50	1730	1680	0.496
106	40	50	40	50	10	17	40	0	1730	1813	0.672
107	40	50	10	50	40	50	20	0	1807	1880	0.526
108	10	50	10	0	10	33	10	0	1960	1813	0.316
109	10	17	10	33	10	0	40	50	1960	1880	0.413
110	10	17	30	0	40	50	40	50	1960	1680	1.217
111	20	50	10	50	40	33	10	50	1960	1680	0.79
112	40	50	40	0	10	0	10	50	1807	1813	0.61
113	20	0	10	17	10	0	40	0	1960	1680	0.59
114	20	50	10	0	40	0	10	0	1730	1747	0.283
115	40	33	40	0	40	0	30	0	1960	1880	0.683
116	30	33	40	50	40	50	10	50	1730	1680	0.696
117	33	38	33	38	33	13	33	38	1788	1830	0.678
118	20	17	40	50	10	50	40	50	1960	1880	0.863
119	40	17	40	0	40	50	40	50	1807	1680	0.941
120	40	0	30	17	40	50	10	50	1960	1680	0.835
121	40	17	30	0	10	0	40	0	1730	1680	0.389
122	20	0	40	50	10	50	10	0	1807	1680	0.291
123	10	50	10	0	40	50	10	33	1960	1747	0.89
124	40	0	40	0	30	50	40	33	1730	1880	0.728
125	20	0	40	50	40	33	40	0	1960	1680	0.948
126	20	0	10	50	30	0	10	50	1960	1680	0.799
127	10	50	10	0	40	33	40	17	1730	1680	1.023
128	10	50	30	50	30	0	10	0	1960	1880	0.533
129	40	0	10	0	40	50	10	50	1883	1813	0.989
130	40	0	10	0	30	33	10	50	1960	1880	0.866
131	20	0	20	50	40	0	10	50	1730	1680	0.674
132	40	50	10	17	10	33	10	0	1730	1680	0.335
133	40	50	10	50	10	50	20	17	1960	1680	0.604
134	10	50	40	17	40	50	40	0	1960	1747	0.906
135	30	17	10	50	40	0	40	50	1960	1680	0.964
136	10	17	10	0	40	50	40	17	1730	1880	1.144
137	30	50	10	0	30	50	10	0	1960	1880	0.336
138	10	0	20	0	10	50	20	0	1960	1680	0.856

139	40	0	10	0	40	0	20	50	1730	1813	0.806
140	20	50	40	50	10	0	10	50	1883	1880	0.649
141	10	50	30	0	40	0	40	33	1960	1880	0.785
142	10	0	40	0	20	50	10	50	1807	1880	0.731
143	40	50	10	0	40	17	40	50	1960	1747	1.148
144	40	0	20	33	40	50	40	0	1730	1880	0.743
145	40	0	40	50	10	50	40	33	1807	1680	0.774
146	10	50	10	50	20	17	10	0	1730	1880	0.588
147	40	0	40	50	40	0	30	50	1960	1747	0.83
148	10	0	30	0	10	0	40	50	1807	1680	0.427
149	10	33	40	50	20	0	40	50	1730	1880	0.509
150	40	50	10	17	40	50	10	33	1730	1680	0.598
151	40	0	40	0	40	50	10	17	1883	1680	0.459
152	10	50	40	50	30	50	10	0	1807	1680	0.473
153	40	33	40	50	40	0	40	17	1960	1880	0.772
154	10	0	10	50	10	50	20	0	1883	1880	0.591
155	40	50	40	0	40	17	20	0	1960	1680	0.615
156	40	0	30	50	10	0	40	0	1960	1747	0.417
157	30	50	10	33	10	50	10	50	1960	1680	0.773
158	40	50	40	33	10	0	20	0	1730	1880	0.765
159	40	0	10	0	10	17	10	50	1883	1680	0.544
160	30	0	40	0	10	0	10	50	1730	1747	0.449
161	40	50	20	0	30	50	40	0	1730	1880	0.761
162	40	33	10	17	40	0	40	50	1960	1680	1.074
163	20	0	10	0	10	50	30	0	1730	1680	0.936
164	40	0	10	33	30	0	10	0	1730	1880	0.439
165	10	50	40	50	10	0	10	33	1960	1813	0.567
166	10	0	20	0	10	0	30	50	1730	1880	0.304
167	10	50	10	50	20	50	10	50	1730	1747	0.629
168	30	0	10	50	40	0	40	33	1730	1880	0.682
169	10	33	10	33	40	50	10	0	1960	1880	0.328
170	10	0	10	50	40	33	20	50	1960	1680	1.156
171	10	17	10	17	10	0	10	0	1730	1680	0.311
172	40	50	40	0	10	50	30	50	1730	1747	0.72
173	10	0	40	33	40	0	10	33	1960	1680	0.696
174	40	33	40	0	20	0	10	0	1960	1880	0.539
175	10	50	30	33	40	0	10	50	1730	1880	0.555
176	30	0	40	50	10	50	20	50	1730	1880	0.603
177	40	0	10	50	10	50	30	0	1807	1680	0.672
178	10	0	40	50	10	17	10	17	1730	1880	0.453
179	10	0	10	0	10	33	40	33	1960	1880	1.086
180	30	0	40	50	40	0	40	0	1883	1880	0.719
181	40	33	10	50	40	0	10	0	1807	1680	0.375
182	10	50	40	17	10	50	30	0	1960	1880	0.659
183	40	33	40	50	20	50	40	0	1730	1680	0.745
184	10	17	40	0	40	33	40	0	1730	1880	0.722
185	20	0	30	0	40	50	40	50	1730	1880	0.911

186	30	50	40	50	30	0	40	50	1960	1880	0.771
187	10	0	10	17	40	0	40	50	1730	1747	1.145
188	30	0	10	0	40	50	10	17	1960	1880	0.564
189	10	50	20	50	40	17	40	0	1960	1880	0.781
190	40	0	20	50	30	50	10	0	1730	1680	0.394
191	40	50	30	0	40	50	40	0	1883	1680	0.908
192	40	0	10	17	10	0	40	0	1960	1813	0.341
193	10	0	10	33	10	50	40	50	1883	1680	1.264
194	10	50	40	0	10	50	40	33	1730	1813	0.674
195	40	17	40	50	40	50	10	50	1960	1813	0.763
196	10	50	40	50	40	0	40	33	1807	1680	0.786
197	40	50	10	50	20	33	40	50	1960	1680	0.754
198	10	0	40	0	10	0	20	0	1960	1747	0.299
199	10	0	30	33	10	0	10	0	1730	1680	0.334
200	10	33	10	50	40	17	10	50	1730	1880	0.593
201	10	50	10	17	10	0	10	50	1807	1880	0.333
202	10	33	20	0	10	50	10	0	1960	1880	0.258
203	40	0	10	0	40	33	20	0	1730	1880	0.52
204	40	50	40	0	30	0	20	50	1730	1880	0.613
205	20	50	20	50	10	0	40	50	1730	1880	0.594
206	40	17	10	0	10	0	10	50	1960	1747	0.414
207	10	0	40	50	40	33	10	50	1883	1880	0.775
208	40	33	40	33	10	50	10	50	1730	1880	0.614
209	10	50	40	0	20	0	30	0	1960	1680	0.567
210	40	0	40	50	10	50	30	33	1960	1880	0.71
211	10	0	10	50	10	17	40	50	1730	1747	0.54
212	30	0	40	33	40	50	40	50	1960	1680	1.059
213	30	50	10	0	10	50	40	0	1960	1747	1.057
214	20	50	10	33	40	0	40	0	1960	1880	0.815
215	40	0	40	50	40	0	30	17	1730	1680	0.665
216	40	50	10	50	10	50	10	33	1883	1680	0.508
217	10	50	40	50	20	50	40	17	1730	1880	0.678
218	40	50	30	33	10	50	40	50	1730	1680	0.779
219	10	0	40	0	10	17	10	50	1960	1813	0.412
220	10	50	40	17	40	50	40	50	1807	1880	0.851
221	20	50	40	50	40	50	10	0	1960	1747	0.501
222	20	0	10	50	30	50	40	0	1730	1680	0.907