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LQ Robust Synthesis With Non-fragile Controllers: The Static State Feedback Case

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Abstract

This paper describes the synthesis of Non-fragile or *Resilient* regulators for linear systems. The general framework for fragility is described using state-space methodologies, and the LQ/$\mathcal{H}_2$ static state-feedback case is examined in detail. We discuss the multiplicative structured uncertainties case, and propose remedies of the fragility problem. The benchmark problem is taken as example to show how an "uncertain" or resilient static state feedback controller can affect the performance of the system.

1 Introduction

The purpose of this paper is to address and understand the effects of uncertainties in the implementation of robust regulators which optimize a performance index in linear systems. In the literature, there are different algorithms that give an answer to the classical problem shown in Figure 1:

Given a linear plant P with some additive uncertainties ΔP find a feedback controller K which internally stabilizes the family $P + \Delta P$ and satisfies some performance requirements.

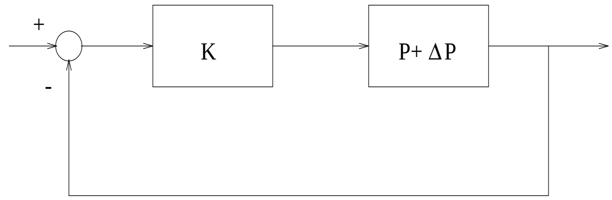

Figure 1: Robust Control Scheme

In this paper we will consider structured uncertainties in the plant, to represent the effect of (generally) slowly time-varying parameters whose exact values are unknown but which are known to belong to a given set [1]. Most control algorithms proposed in the literature do not consider the problems introduced by implementing uncertain controllers. We first remark that it is reasonable to consider only structured uncertainties in the controller since by design, one can choose its exact structure even though the designer may not be able to exactly implement that nominal configuration. The controllers obtained using most robust design approaches are thus *optimal* if implemented exactly. There are however many reasons to believe that one can never exactly implement a compensator which theoretically meets all objectives (see [2] for an example of a compensator that cannot be implemented). Moreover, it is easy to argue that even when exact implementation is possible, some tuning by the control engineer is required on the actual controller in order to achieve a "safety" margin with respect to sampling procedures, roundoff errors etc. We thus consider the more realistic block diagram in Figure 2.

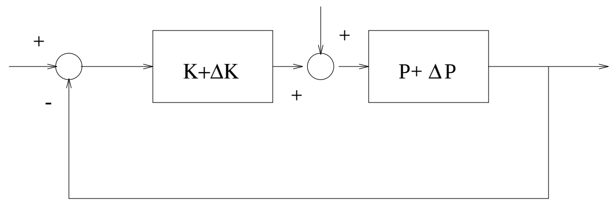

Figure 2: Robust Fragility Control Scheme

In a recent paper, Keel and Bhattacharyya [3] have shown that, in the case of unstructured uncertainties in the plant, and using $\mathcal{H}_\infty$, $\mathcal{H}_2$ or l_1 synthesis techniques, the resulting controllers exhibit a poor stability margin if not implemented exactly! This so-called "fragility" is displayed despite of, or because of, the fact that these controllers are optimal when implemented using their nominal parameters. Paper [3] gives the following suggestions to overcome the fragility problem:

1. Developing synthesis algorithms which take into account some structured uncertainties in the controllers and searching for the "best" solution that guarantees a compromise between optimality and fragility,

2. Examining the structure of the controller in order to parameterize it in a useful way (lower-order or fixed-structure controller).

In a following paper, Haddad and Corrado [4] address and solve a special case the fragility problem by considering a *structured uncertain* dynamic compensator for a noise-driven linear plant. They obtain sufficient conditions by bounding the uncertainties in the controller using classical quadratic Lyapunov bounds [5]. The resulting controllers are proven to be "resilient" in the sense that even when they are not exactly implemented, stability and some measure of performance are maintained.

It is true that other authors have hinted at the problem of fragility [6] and that many critics have dismissed the issue, since robust controllers are not designed to be resilient. On the other hand, the problem is reminiscent of the Linear Quadratic Gaussian (LQG) optimal controllers which were only useful when implemented on the exact plant, and had no robustness margins if the plant was uncertain. This lack of robustness was corrected using Linear Quadratic Gaussian synthesis with Loop Transfer Recovery (LQG/LTR) [7]. In addition, even robust controllers will eventually have to be implemented on an actual system using digital hardware and should be resilient both to implementation errors and to tuning [6].

The aim of this paper is to extend the ideas in the two papers [3, 4] and to analyze the robust fragility problem by considering the combined effect of structured uncertainties in the plant and in the compensator. The basic idea is that, instead of computing the controller as a single point in the parameters space, we look for a controller set using an *a priori* information. This is reminiscent of the designs of Ackermann [6] and of those in [8].

This paper is organized as follows. In section 2, we present the synthesis of static state-feedback controllers for linear systems while allowing structured uncertainties in the feedback gain matrix. We then further restrict our study to the multiplicative structured uncertainties schemes in the plant. In section 3, a numerical example using Linear Matrix Inequalities as a computational tool is given. Our conclusions and directions for future research are finally given in section 4.

2 Outline of the problem

Let us consider the following time-varying linear system

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & A(t)x(t) + B(t)u(t) \\ y(t) & = & C(t)x(t) \end{array} \right. \tag{1}$$

where

- $x(t) \in \mathbb{R}^n$, is the state vector,
- $u(t) \in \mathbb{R}^m$ is the control input,
- $y(t) \in \mathbb{R}^p$ is the output measurements vector,
- $A(t)$, contains polytopic and/or affine uncertainties (see [9])

$$A(t) = A_0 + \sum_{i=1}^{q} \alpha_i(t) A_i,$$

where the scalar coefficients $\alpha_i(t)$ represent unknown but slowly-varying coefficients whose values belong to an uncertainty interval

$$\underline{\alpha}_i \leq \alpha_i(t) \leq \overline{\alpha}_i,\ 1 \leq i \leq q, \tag{2}$$

- $B(t)$ and $C(t)$ are constant matrices B and C respectively.

The system (1) can then be written in the form

$$\left\{ \begin{array}{lcl} \dot{x} & = & (A_0 + \sum_{i=1}^{q} \alpha_i A_i)x + Bu \quad = \quad (A_0 + \delta A)x + Bu \\ y(t) & = & Cx(t) \end{array} \right. \tag{3}$$

Note that this model is similar to a stochastic differential equation with multiplicative noise [7]. Now, given the initial state $x(0)$, the problem is to find a state-feedback compensator $u(t) = Kx(t)$ which minimizes the Linear Quadratic (LQ) performance index,

$$J = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Ru(t) \right] \, dt \, . \tag{4}$$

where $Q = C^T C$ and R is a symmetric positive definite matrix.

2.1 Non-fragile synthesis scheme

Although one finds the controller $u = Kx$, the actual controller implemented is

$$u = (K + \delta K)x = \tilde{K}x \tag{5}$$

where K is the nominal gain, and the term δK represents drifting from the nominal solution. In this case, the index (4) becomes a function of K, the uncertain term δK, and the uncertainties α_i in (3) as shown below

$$J = J(K, \delta K, \alpha_i).$$

A possible solution to the fragility problem may be as follows:

1. Letting $\delta K = 0$, we perform a "nominal" synthesis and find a bound $\tilde{J}$ on the performance index (4),

$$J(K, 0, \alpha_i) \leq \bar{J}(K).$$

We then solve a standard guaranteed-cost problem [5] for the controller gain, $\bar{K}$ which minimizes $\bar{J}(K)$;

2. We then fix the uncertainty range δK and find a new bound $\tilde{\bar{J}}$ to (4),

$$J(K, \delta K, \alpha_i) \leq \bar{\bar{J}}(K)$$

obviously, we expect that $\bar{J}(K) \leq \bar{\bar{J}}(K)$ due to the quadratic expression of the index. Now, solving the new guaranteed-cost problem we look for $\bar{\bar{K}}$ which minimizes $\bar{\bar{J}}(K)$ and we check if

$$\left| \bar{\bar{J}}\left(\bar{\bar{K}}\right) - \bar{J}\left(\bar{K}\right) \right| < M \tag{6}$$

where M is a level that can be fixed "a-priori". If (6) is not true, we try to reduce the uncertainty level δK until is satisfied.

With this scheme in mind, we next study the multiplicative uncertainty case of equation (5) in greater detail.

2.2 Multiplicative structured uncertainties

Let the nominal state-feedback matrix K be a $m \times n$, $(m < n)$ matrix. If we allow percentage drift from the nominal entries of the matrices K and represent each entry of the perturbed matrix as a multiplicative scalar uncertainty, we have

$$\begin{aligned}(K+\delta K) &= \begin{bmatrix} k_{11}(1+\delta_{11}) & \dots & k_{1n}(1+\delta_{1n}) \\ \vdots & \ddots & \vdots \\ k_{m1}(1+\delta_{m1}) & \dots & k_{mn}(1+\delta_{mn}) \end{bmatrix} \\ &= \begin{bmatrix} k_{11} & \dots & k_{1n} \\ \vdots & \ddots & \vdots \\ k_{m1} & \dots & k_{mn} \end{bmatrix} + \begin{bmatrix} k_{11}\delta_{11} & \dots & k_{1n}\delta_{1n} \\ \vdots & \ddots & \vdots \\ k_{m1}\delta_{m1} & \dots & k_{mn}\delta_{mn} \end{bmatrix}, \; -1 < \underline{\delta}_{ij} \le \delta_{ij} \le \overline{\delta}_{ij} < 1 \,. \end{aligned} \tag{7}$$

Equation (7) then leads to the uncertain controller:

$$u(t) = \left(K + \sum_{i=1}^{m} \sum_{j=1}^{n} \delta_{ij} \Psi_i^{(m)} K \Psi_j^{(n)} \right) x(t) = K_\Delta x(t) \,. \tag{8}$$

where $\Psi_i^{(m)}$, $\Psi_j^{(n)}$ are $m \times m$ and $n \times n$ rank-one matrices with a "1" entry located at the i-th and j-th position of the main diagonal, respectively. Considering the closed-loop system, we have

$$\dot{x}(t) = \left(A_0 + \sum_{i=1}^{q} \alpha_i A_i + B \left(K + \sum_{i=1}^{m} \sum_{j=1}^{n} \delta_{ij} \Psi_i^{(m)} K \Psi_j^{(n)} \right) \right) x(t) \,. \tag{9}$$

The closed-loop system matrix has the following form:

$$A(t) = A_0 + \sum_{i=1}^{q} \alpha_i A_i + BK + \sum_{i=1}^{m} \sum_{j=1}^{n} \delta_{ij} B_i K \Psi_j^{(n)} \tag{10}$$

where $B_i = B\Psi_i^{(m)}$. If we substitute into equation (4) the expression of the controller (8), we have that

$$J = \int_0^\infty \left(x^T Q x + x^T K_\Delta^T R K_\Delta x \right) \, dt \,. \tag{11}$$

Let us consider the term

$$x^T K_\Delta^T R K_\Delta x \tag{12}$$

in equation (11): due to expression of the uncertainty term (7), it is possible to bound this term by

$$\alpha(K) x^T K^T R K x$$

where

$$\alpha(K) = \frac{\sup_{\delta_{ij}} \lambda_{\max}\left(K_\Delta^T R K_\Delta\right)}{\lambda_{\min}\left(K^T R K\right)},$$

the supremum operation is performed over the uncertainty set and $\lambda_{\max}$ and $\lambda_{\min}$ indicates respectively the maximum and the minimum eigenvalues of a matrix. It is easy to see that the bound

$$\alpha(K) x^T K^T R K x > x^T K_\Delta^T R K_\Delta x$$

works only when K is known in advance. The index (11) is then bounded by

$$\mathcal{J} = \int_0^\infty \left(x^T Q x + \alpha(K) x^T K^T R K x \right) \, dt \tag{13}$$

which gives rise to the following *non-linear* dynamic optimization problem:

Find K such that

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & \left(A + \sum_{i=1}^{q} \alpha_i A_i\right) x(t) + Bu(t) + \sum_{i=1}^{m}\sum_{j=1}^{n} \delta_{ij} B_i \hat{u}_j(t) \\ y_j(t) & = & \Psi_j^{(n)} x(t) \\ u(t) & = & Kx(t) \\ \hat{u}_j(t) & = & Ky_j(t) \end{array} \right. \tag{14}$$

is stable and minimizes the index (13).

This problem is in general difficult to solve but, in particular cases, it reduces to a convex optimization problem [10]. In the following we analyze some of these special cases.

2.2.1 Special Cases

In the single input case ($m = 1$) (7) reduces to

$$(K + \delta K) = \left[\begin{array}{ccc} k_1(1+\delta_1) & \ldots & k_n(1+\delta_n) \end{array}\right] \tag{15}$$

where δ_j are scalar coefficients ($|\delta_j| \leq \tilde{\delta}_j \ll 1$). We can then write the controller as,

$$u = K(I_n + \sum_{j=1}^{n} \delta_j \Psi_j^{(n)})x = K\Lambda(\delta)x, \tag{16}$$

where the term $\Lambda(\delta)$ indicates the diagonal matrix whose entries are $1 + \delta_i$. The index J can be easily bounded by noting that

$$\Lambda(\delta)R\Lambda(\delta) \leq (1+\theta)^2 R$$

where $\theta = \max_i(\delta_i)$. The index (4) is then bounded as follows

$$J \leq \mathcal{J} = \int_0^\infty \left(x^T Q x + (1+\theta)^2 x^T K^T R K x\right) dt\,. \tag{17}$$

Let us then examine the closed-loop system:

$$\dot{x}(t) = \left(A + \sum_{i=1}^{q} \alpha_i A_i + BK\Lambda(\delta)\right) x(t). \tag{18}$$

The system matrix $A(t)$ can be rewritten as

$$A(t) = A + BK + \sum_{i=1}^{q} \alpha_i A_i + \sum_{j=1}^{n} \delta_j BK \Psi_j^{(n)},$$

and in this case the problem is equivalent to the following static output feedback problem [11]:

Find K such that

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & (A + \sum_{i=1}^{q} \alpha_i A_i)\, x(t) + B\hat{u}(t) \\ y(t) & = & \Lambda(\delta)x(t) \\ \hat{u}(t) & = & Ky(t) \end{array} \right. , \tag{19}$$

is stable and the bound J (17) is minimized.

However, using the following proposition we show that this particular static output feedback problem can be reduced to a full-state static feedback problem:

Proposition 1 *If the coefficients δ_j are slowly time-varying, the dynamic optimization problem (17), subject to the dynamic constraints (19), is equivalent to a guaranteed-cost full static state-feedback problem.*

Proof: If we consider the variable $y(t)$ in the equation (19), we have the following nonsingular transformation of coordinates

$$y(t) = \Lambda(\delta)x(t) \tag{20}$$

the closed-loop system (18) in the y variable becomes

$$\dot{y}(t) = \left(\sum_{j=1}^{n} \frac{1}{1+\delta_j}\hat{A}_j + \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\delta_j}{1+\delta_i}\tilde{A}_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{q} \frac{\delta_j}{1+\delta_i}\alpha_k \overline{A}_{ijk} \right) y(t) + \left(B + \sum_{j=1}^{n} \delta_j B_j \right) \hat{u}(t), \tag{21}$$

$$\hat{u}(t) = Ky(t)\ ,$$

where $\hat{A}_j = A\Psi_j^{(n)}$, $\tilde{A}_{ij} = \Psi_i^{(n)} A \Psi_j^{(n)}$, $\overline{A}_{ijk} = \Psi_i^{(n)} A_k \Psi_j^{(n)}$ and $B_j = \Psi_j^{(n)} B$. We can write the system (21) in the form

$$\dot{y}(t) = \left(\mathcal{A}_0 + \sum_{j=1}^{k} \lambda_j \mathcal{A}_k \right) y(t) + \left(B + \sum_{j=1}^{n} \delta_j B_j \right) \hat{u}(t), \tag{22}$$

where $k = n + n^2 + n^2 q$, $\mathcal{A}_0 = 0$ and $\mathcal{A}_k$ is equal to one of the terms in equation (21). The index (17) has the following form

$$\mathcal{J} = \int_0^\infty y^T \Lambda^{-1}(\delta) \left(Q + (1+\theta)^2 K^T R K \right) \Lambda^{-1}(\delta) y \ dt\ . \tag{23}$$

Equation (23) can be bounded by

$$\tilde{\mathcal{J}} = \frac{1}{(1+\eta)^2} \int_0^\infty \left(y^T Q y + (1+\theta)^2 \hat{u}^T Q \hat{u} \right)\ dt\ . \tag{24}$$

where $\eta = \min_i \delta_i$. Finally we have that the index (24) is subject to the dynamic bound (21). ■

Solution Scheme: In this case, the solution of the problem is bounded by the following value of the index

$$\frac{(1+\theta)^2}{(1+\eta)^2} x(0)^T P x(0)$$

where $P = P(\lambda_i, \delta_j)$ is the solution to the matrix Riccati equation

$$P(\lambda_i, \delta_j)\tilde{\mathcal{A}}(\lambda_i, \delta_j, K) + \tilde{\mathcal{A}}(\lambda_i, \delta_j, K)^T P(\lambda_i, \delta_j) + Q + (1+\theta)^2 K^T R K = 0. \tag{25}$$

where

$$\tilde{\mathcal{A}}(\lambda_i, \delta_j, K) = \mathcal{A}_0 + \sum_{i=1}^{k} \lambda_i \mathcal{A}_i + BK + \sum_{j=1}^{n} \delta_j B_j K,\ 1 \le i \le k,\ 1 \le j \le n\ .$$

The problem can then be converted to the following convex optimization problem

$$\begin{array}{ll} \min & \dfrac{(1+\theta)^2}{(1+\eta)^2} \mathrm{tr}\mathcal{P} \\ \textit{subject to} & \mathcal{P}\tilde{\mathcal{A}}(\omega_j, \psi_i, K) + \tilde{\mathcal{A}}(\omega_j, \psi_i, K)^T \mathcal{P} + Q + (1+\theta)^2 K^T R K < 0 \end{array} \tag{26}$$

where ω_j, ψ_i each represent the upper and lower limits of the uncertainty ranges

$$\omega_j \in \Omega = \left\{ \left\{ \underline{\lambda_j}, \overline{\lambda_j} \right\},\ 1 \le j \le k \right\}$$

$$\psi_i \in \Psi = \left\{ \left\{ \underline{\delta_i}, \overline{\delta_i} \right\},\ 1 \le i \le n \right\}.$$

(Note that the expressions are numerical sets and not intervals.)

Note: When all entries of the state-feedback matrix K are perturbed by the same amount ($\delta_1 = \delta_2 = \ldots = \delta_n = \delta$), as was done in [4], the problem is reduced to:

Find K such that

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & \left(A + \sum_{i=1}^{q} \alpha_i A_i\right) x(t) + (B + \delta B)\,\hat{u}(t) \\ \hat{u}(t) & = & Kx(t) \end{array} \right. \qquad (27)$$

and the index (17) is minimized.

It is interesting to see, using a numerical experiment, how these simple cases can be formulated as convex optimization problems and what is the quantitative effect of a Non-fragile synthesis over the performance of the system.

3 A Numerical Example (LQ/$\mathcal{H}_2$ design)

Let us consider the following mechanical system [9], known as the "Benchmark Problem" where

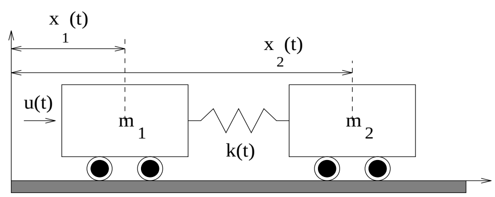

Figure 3: Benchmark Problem

1. $u(t)$ is the control input;
2. x_1, x_2 are the positions, with respect to a reference system, of the masses;
3. the masses m_1, m_2 are equal to 1 in the appropriate units;
4. the stiffness $k(t)$ is a slowly-varying parameter in the interval [0.5, 2].

The linear time-varying model which describes the behavior of the system is

$$\left\{ \begin{array}{rcl} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} & = & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k(t) & k(t) & 0 & 0 \\ k(t) & -k(t) & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u(t) \\ y(t) & = & \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + u(t) \end{array} \right. \qquad (28)$$

It is easy to see that we can represent (28) as an affine uncertain model where the matrix $A(t)$ is given by

$$A(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + k(t) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} = A_0 + k(t)A_1$$

and the matrices B, C, D are constant.

3.1 Non-fragile synthesis using a uniform perturbation scheme

Using the MATLAB^{TM} LMI toolbox and the function `msfsyn` a nominal LQ/$\mathcal{H}_2$ static state-feedback synthesis was performed. The guaranteed LQ/$\mathcal{H}_2$ performance was found to be 1.54 and the controller gain vector given by

$$K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix} = \begin{bmatrix} -2.7917 & 1.7912 & -2.3651 & -0.1045 \end{bmatrix}. \tag{29}$$

An affine family of controllers according to the following rule was generated

$$\tilde{K} = (1+\delta)K \tag{30}$$

where δ is a parameter which corresponds to a drifting of the nominal value k_i: in this case each component of K was considered to have same relative uncertainty range [4]. The fragility of the controller was tested by varying δ and, using MATLAB^{TM} LMI Toolbox standard routines `quadstab` and `pdlstab`, the values of δ for which the closed-loop system is no longer quadratically stable [10, 9] or, less conservatively, does not admit a parameter-dependent Lyapunov function [10, 12, 9] were checked. For this particular system the nominal controller (29) was taken into consideration and, examining the closed-loop system

$$\dot{x} = (A_0 + kA_1 + (1+\delta)BK)\,x(t),$$

was obtained that if δ is greater than 0.1 quadratic stability is lost, and if δ is greater than 0.78 the system does not admit a parameter-dependent Lyapunov function.
Letting δ now to be an uncertain parameter in the interval $-0.1 \le \delta \le 0.1$ a new synthesis was performed having in mind the optimization scheme expressed by the equation (27). Using Convex Optimization methods, the problem which generates the new solution is the following

$$\begin{aligned} &\min && \mathrm{tr}\mathcal{P} \\ &\text{subject to} && \mathcal{P}(A_0 + \bar{k}A_1 + (1+\bar{\delta})BK) + (A_0 + \bar{k}A_1 + (1+\bar{\delta})BK)^T\mathcal{P} + Q + (1+\theta)^2\,K^TRK < 0 \end{aligned} \tag{31}$$

where

1. $\bar{k} \in \{0.5, 2\}$, $\bar{\delta} \in \{-0.1, 0.1\}$ and $\theta = 0.1$,
2. $Q = C^TC$ and $R = 1$.

It is important to remark that, in this case, 4 matrix inequalities must be considered because the two parameters (k, δ) are involved in the inequality and all the possible combinations between the maximum and the minimum values of the uncertainty intervals have to be chosen. Using again the function `msfsyn` a new "center" value for the K vector was obtained

$$K = \begin{bmatrix} -3.0930 & 2.0916 & -2.6365 & -0.0396 \end{bmatrix}$$

and the guaranteed LQ/$\mathcal{H}_2$ performance in this case was equal to 1.7.
The difference between the two guaranteed costs is equal to

$$|1.7 - 1.54| = 0.16$$

which corresponds to a 10.36% worsening of the LQ/$\mathcal{H}_2$ cost as a price paid to guarantee Non-fragility.

3.2 Non-fragile synthesis using a general perturbation scheme

A more realistic experiment was carried out by perturbing each entry of the K vector independently from each other. We allow for example that,

1. k_1 has a 15% drift around its nominal value,
2. k_2 has a 20% drift around its nominal value,

3. k_3 has a 10% drift around its nominal value,

4. k_4 has a 30% drift around its nominal value.

In this case the Convex Optimization problem is given by (26) and, using the state space transformation (20), we have an affine uncertain linear model (27) which has 36 parameters (2^{36} inequalities!) but in this case most of them are ineffective because most all the matrices in equation (27) are zero. The new entries of the K vector are now found to be

$$K = \begin{bmatrix} -238.4746 & 266.7781 & -21.8273 & -670.6591 \end{bmatrix}$$

and the guaranteed LQ/$\mathcal{H}_2$ performance was 5.0198, which is worse than the previous case when each entry of the K is equally perturbed by 10% of its nominal value.
The effect of this type of synthesis in the two cases is now clear: A trade-off exists between the minimization of a performance indices and the fragility of the compensator.

4 Conclusions

In this paper, the effect of the LQ robust synthesis of uncertain static state feedback controller for linear systems with structured uncertainties in the dynamic matrix was observed. Simple theoretical results and upper bounds on the performance index were obtained when multiplicative structured uncertainties are allowed in the controller. A guaranteed-cost approach, using Linear Matrix Inequalities was the computational tool used in the numerical experiment (benchmark problem) and the main result was that a price had to be paid in terms of performance in order to guarantee non-fragility. Future directions of research include the synthesis of dynamic Non-fragile controllers, and the relation between the order of the controller and its fragility characteristics.

References

[1] S. Bhattacharyya, H. Chapellat, and L. Keel, *Robust Control: The Parametric Approach.* Upper Saddle River, NJ: Prentice-Hall PTR, 1st ed., 1995.

[2] J. Rosenthal and X. Wang, "Output feedback pole placement with dynamic compensators," *IEEE Trans. Automat. Control*, vol. AC-41, no. 6, pp. 830–843, 1996.

[3] L. Keel and S. Bhattacharyya, "Robust, fragile, or optimal?," in *Proceedings IEEE Amer. Contr. Conf.*, (Albuquerque, NM), pp. 1307–1313, 1997.

[4] W. Haddad and J. Corrado, "Non-fragile controller design via quadratic Lyapunov bounds," in *Proceedings of the 35th IEEE Conf. on Decis. and Control*, (San Diego, CA), 1997.

[5] D. Bernstein and W. Haddad, "Robust stability and performance via fixed-order dynamic compensation with guaranteed cost bounds," *Math. Control Signals Systems*, vol. 3, pp. 139–163, 1990.

[6] J. Ackermann, *Robust Control - System with Uncertain Physical Parameters.* Berlin: Springer-Verlag, 1st ed., 1993.

[7] P. Dorato, C. Abdallah, and V. Cerone, *Linear-Quadratic Control, An Introduction.* Englewood Cliffs, N.J.: Prentice Hall, 1995.

[8] B. Barmish, C. Hollot, F. Kraus, and R. Tempo, "Extreme point results for robust stabilization of interval plants with first order compensators," *IEEE Trans Auto. Control*, vol. AC-37, pp. 707–714, 1992.

[9] P. Gahinet, A. Nemirovski, A. Laub, and M. Chilali, *LMI Control Toolbox.* The Mathworks, 1995.

[10] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control theory.* Philadelphia, PA: SIAM, 1994.

[11] V. Syrmos, C. Abdallah, P. Dorato, and K. Grigoriadis, "Static output feedback–a survey," *Automatica*, vol. 33, no. 2, pp. 125–137, 1997.

[12] P. Gahinet, P. Apkarian, and M. Chilali, "Affine parameter-dependent Lyapunov functions for real parametric uncertainty," in *Proceedings 1994 Conf. Dec. Contr.*, (Lake Buena Vista, FL), pp. 2026–2031, December 1994.