Another Form of Correlation Coefficient between Single Valued Neutrosophic Sets and Its Multiple Attribute Decision Making Method

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Abstract. A single valued neutrosophic set (SVNS), which is the subclass of a neutrosophic set, can be considered as a powerful tool to express the indeterminate and inconsistent information in the process of decision making. Then, correlation is one of the most broadly applied indices in many fields and also an important measure in data analysis and classification, pattern recognition, decision making and so on. Therefore, we propose another form of correlation coefficient between SVNSs and establish a multiple attribute decision making method using the correlation coefficient of SVNSs under single valued neutrosophic environment. Through the weighted correlation coefficient between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. Finally, two illustrative examples are employed to illustrate the actual applications of the proposed decision-making approach.

Keywords: Correlation coefficient; Single valued neutrosophic set; Decision making.

1 Introduction

To handle the indeterminate information and inconsistent information which exist commonly in real situations, Smarandache [1] firstly presented a neutrosophic set from philosophical point of view, which is a powerful general formal framework and generalized the concept of the classic set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, paraconsistent set, dialetheist set, paradoxist set, and tautological set [1, 2]. In the neutrosophic set, a truth-membership, an indeterminacy-membership, and a falsity-membership are represented independently. Its functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x): X \to]^-0, 1^+[$, $I_A(x): X \to]^-0, 1^+[$, and $F_A(x): X \to]^-0, 1^+[$. Obviously, it will be difficult to apply in real scientific and engineering areas. Therefore, Wang et al. [3] proposed the concept of a single valued neutrosophic set (SVNS), which is the subclass of a neutrosophic set, and provided the set-theoretic operators and various properties of SVNSs. Thus, SVNSs can be applied in real scientific and engineering fields and give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. However, the correlation coefficient is one of the most frequently used tools in engineering applications. Therefore, Hanafy et al. [4] introduced the correlation of neutrosophic data. Then, Ye [5] presented the correlation coefficient of SVNSs based on the extension of the correlation coefficient of intuitionistic fuzzy sets and proved that the cosine similarity measure of SVNSs is a special case of the correlation coefficient of SVNSs, and then applied it to single valued neutrosophic multicriteria decision-making problems. Hanafy et al. [6] presented the centroid-based correlation coefficient of neutrosophic sets and investigated its properties. Recently , S. Broumi and F. Smarandache [8] Correlation coefficient of interval neutrosophic set and investigated its properties.

In this paper, we propose another form of correlation coefficient between SVNSs and investigate its properties. Then, a multiple attribute decision-making method using the correlation coefficient of SVNSs is established under single valued neutrosophic environment. To do so, the rest of the paper is organized as follows. Section 2 briefly describes some concepts of SVNSs. In Section 3, we develop another form of correlation coefficient between SVNSs and investigate its properties. Section 4 establishes a multiple attribute decision-making method using the correlation coefficient of SVNSs under single valued neutrosophic environment. In Section 5, two illustrative examples are presented to demonstrate the applications of the developed approach. Section 6 contains a conclusion and future research.

2 Some concepts of SVNSs

Smarandache [1] firstly presented the concept of a neutrosophic set from philosophical point of view and gave the following definition of a neutrosophic set.

Definition 1 [1]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x): X \to]^-0, 1^+[$, $I_A(x): X \to]^-0, 1^+[$, and $F_A(x): X \to]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Obviously, it is difficult to apply in practical problems. Therefore, Wang et al. [3] introduced the concept of a SVNS, which is an instance of a neutrosophic set, to apply in real scientific and engineering applications. In the following, we introduce the definition of a SVNS [3].

Definition 2 [3]. Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. Thus, A SVNS A can be expressed as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}.$$

Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 3 [3]. The complement of a SVNS A is denoted by A^c and is defined as

$$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \mid x \in X\}.$$

Definition 4 [3]. A SVNS A is contained in the other SVNS B, $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, and $F_A(x) \ge F_B(x)$ for every x in X.

Definition 5 [3]. Two SVNSs A and B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

3 Correlation coefficient of SVNSs

Motivated by another correlation coefficient between intuitionistic fuzzy sets [7], this section proposes another form of correlation coefficient between SVNSs as a generalization of the correlation coefficient of intuitionistic fuzzy sets [7].

Definition 6. For any two SVNSs A and B in the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, another form of correlation coefficient between two SVNSs A and B is defined by

$$N(A,B) = \frac{C(A,B)}{\max\{C(A,A), C(B,B)\}} \tag{1}$$

$$= \frac{\sum_{i=1}^{n}[T_A(x_i)\cdot T_B(x_i) + I_A(x_i)\cdot I_B(x_i) + F_A(x_i)\cdot F_B(x_i)]}{\max\left\{\sum_{i=1}^{n}[T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)], \sum_{i=1}^{n}[T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)]\right\}}$$

Theorem 1. The correlation coefficient $N(A, B)$ satisfies the following properties:

(1) $N(A, B) = N(B, A)$;

(2) $0 \le N(A, B) \le 1$;

(3) $N(A, B) = 1$, if $A = B$.

Proof. (1) It is straightforward.

(2) The inequality $N(A, B) \ge 0$ is obvious. Thus, we only prove the inequality $N(A, B) \le 1$.

$$\begin{aligned} N(A,B) &= \\ &\sum_{i=1}^{n}[T_A(x_i)\cdot T_B(x_i) + I_A(x_i)\cdot I_B(x_i) + F_A(x_i)\cdot F_B(x_i)] \\ &= T_A(x_1)\cdot T_B(x_1) + T_A(x_2)\cdot T_B(x_2) + \dots + T_A(x_n)\cdot T_B(x_n) \\ &+ I_A(x_1)\cdot I_B(x_1) + I_A(x_2)\cdot I_B(x_2) + \dots + I_A(x_n)\cdot I_B(x_n) \\ &+ F_A(x_1)\cdot F_B(x_1) + F_A(x_2)\cdot F_B(x_2) + \dots + F_A(x_n)\cdot F_B(x_n) \end{aligned}$$

According to the Cauchy–Schwarz inequality:

$$\begin{aligned} &(x_1\cdot y_1 + x_2\cdot y_2 + \dots + x_n\cdot y_n)^2 \\ &\le (x_1^2 + x_2^2 + \dots + x_n^2)\cdot(y_1^2 + y_2^2 + \dots + y_n^2) \end{aligned},$$

where $(x_1, x_2, \dots, x_n) \in R^n$ and $(y_1, y_2, \dots, y_n) \in R^n$, we can obtain

$$\begin{aligned} &[(N(A,B)]^2 \le \sum_{i=1}^{n}[T_A^2(x_i) + I_A^2(x_n) + F_A^2(x_n)] \\ &\cdot \sum_{i=1}^{n}[T_B^2(x_i) + +I_B^2(x_i) + F_B^2(x_i)] \\ &= N(A,A)\cdot N(B,B). \end{aligned}$$

Thus, $N(A,B) \le [N(A,A)]^{1/2}\cdot[N(B,B)]^{1/2}$.

Then, $N(A,B) \le \max\{N(A,A), N(B,B)\}$.

Therefore, $N(A, B) \le 1$.

(3) If $A = B$, there are $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$ for any $x_i \in X$ and $i = 1, 2, \dots, n$. Thus, there are $N(A, B) = 1$.

In practical applications, the differences of importance are considered in the elements in the universe. Therefore, we need to take the weights of the elements x_i ($i = 1, 2,\dots, n$) into account. Let w_i be the weight for each element x_i ($i = 1, 2,\dots, n$), $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$, then we have the following weighted correlation coefficient between the SVNSs A and B:

$$W(A,B) = \frac{\sum_{i=1}^{n} w_i[T_A(x_i)\cdot T_B(x_i) + I_A(x_i)\cdot I_B(x_i) + F_A(x_i)\cdot F_B(x_i)]}{\max\left\{\sum_{i=1}^{n} w_i[T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)], \sum_{i=1}^{n} w_i[T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)]\right\}} \tag{2}$$

If $w = (1/n, 1/n,\dots, 1/n)^T$, then Eq. (2) reduce to Eq. (1). Note that $W(A, B)$ also satisfy the three properties of Theorem 1.

Theorem 2. Let w_i be the weight for each element x_i (i = 1, 2,…, n), $w_i \in$ [0, 1], and $\sum_{i=1}^{n} w_i = 1$, then the weighted correlation coefficient $W(A, B)$ defined in Eq. (2) also satisfies the following properties:

(1) $W(A, B) = W(B, A)$;

(2) $0 \le W(A, B) \le 1$;

(3) $W(A, B) = 1$, if $A = B$.

Since the process to prove these properties is similar to that in Theorem 1, we do not repeat it here.

4 Decision-making method using the correlation coefficient of SVNSs

This section proposes a single valued neutrosophic multiple attribute decision-making method using the proposed correlation coefficient of SVNSs.

Let $A = \{A_1, A_2,\ldots, A_m\}$ be a set of alternatives and $C = \{C_1, C_2,\ldots, C_n\}$ be a set of attributes. Assume that the weight of an attribute C_j (j = 1, 2,…, n), entered by the decision-maker, is w_j, $w_j \in$ [0, 1] and $\sum_{j=1}^{n} x_j = 1$. In this case, the characteristic of an alternative A_i (i = 1, 2,…, m) with respect to an attribute C_j (j = 1, 2,…, n) is represented by a SVNS form:

$$A_i = \{\langle C_j, T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j)\rangle \mid C_j \in C, j = 1,2,\ldots,n\},$$

where $T_{A_i}(C_j)$, $I_{A_i}(C_j)$, $F_{A_i}(C_j) \in$ [0, 1] and $0 \le T_{A_i}(C_j) + I_{A_i}(C_j) + F_{A_i}(C_j) \le 3$ for $C_j \in C$, j = 1, 2, …, n, and i = 1, 2, …, m.

For convenience, the values of the three functions $T_{A_i}(C_j)$, $I_{A_i}(C_j)$, $F_{A_i}(C_j)$ are denoted by a single valued neutrosophic value (SVNV) $a_{ij} = \langle t_{ij}, i_{ij}, f_{ij}\rangle$ (i = 1, 2, …, m; j = 1, 2,…, n), which is usually derived from the evaluation of an alternative A_i with respect to an attribute C_j by the expert or decision maker. Thus, we can establish a single valued neutrosophic decision matrix $D = (a_{ij})_{m\times n}$:

$$D = (a_{ij})_{m\times n} = \begin{bmatrix} \langle t_{11}, i_{11}, f_{11}\rangle & \langle t_{12}, i_{12}, f_{12}\rangle & \cdots & \langle t_{1n}, i_{1n}, f_{1n}\rangle \\ \langle t_{21}, i_{21}, f_{21}\rangle & \langle t_{22}, i_{22}, f_{22}\rangle & \cdots & \langle t_{2n}, i_{2n}, f_{2n}\rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle t_{m1}, i_{m1}, f_{m1}\rangle & \langle t_{m2}, i_{m2}, f_{m2}\rangle & \cdots & \langle t_{mn}, i_{mn}, f_{mn}\rangle \end{bmatrix}.$$

In the decision-making method, the concept of ideal point has been used to help identify the best alternative in the decision set. The ideal alternative provides a useful theoretical construct against which to evaluate alternatives. Generally, the evaluation attributes can be categorized into two kinds, benefit attributes and cost attributes. Let H be a collection of benefit attributes and L be a collection of cost attributes. An ideal SVNV can be defined by an ideal element for a benefit attribute in the ideal alternative A^* as

$$a_j^* = \langle t_j^*, i_j^*, f_j^*\rangle = \left\langle \max_i(t_{ij}), \min_i(i_{ij}), \min_i(f_{ij})\right\rangle \text{ for } j \in H,$$

while an ideal SVNV can be defined by an ideal element for a cost attribute in the ideal alternative A^* as

$$a_j^* = \langle t_j^*, i_j^*, f_j^*\rangle = \left\langle \min_i(t_{ij}), \max_i(i_{ij}), \max_i(f_{ij})\right\rangle \text{ for } j \in L.$$

Then, by applying Eq. (2) the weighted correlation coefficient between an alternative A_i (i = 1, 2, …, m) and the ideal alternative A^* is given by

$$W(A_i, A^*) = \frac{\sum_{j=1}^{n} w_j\left[t_{ij}\cdot t_j^* + i_{ij}\cdot i_j^* + f_{ij}\cdot f_j^*\right]}{\max\left\{\sum_{j=1}^{n} w_j\left[t_{ij}^2 + i_{ij}^2 + f_{ij}^2\right], \sum_{j=1}^{n} w_j\left[(t_j^*)^2 + (i_j^*)^2 + (f_j^*)^2\right]\right\}} \tag{3}$$

Then, the bigger the measure value $W(A_i, A^*)$ (i = 1, 2, . . . , m) is, the better the alternative A_i is, because the alternative A_i is close to the ideal alternative A^*. Through the weighted correlation coefficient between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best one can be easily identified as well.

5 Illustrative examples

In this section, two illustrative examples for the multiple attribute decision-making problems are provided to demonstrate the application of the proposed decision-making method.

5.1 Example 1

Now, we discuss the decision-making problem adapted from the literature [5]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investment company must take a decision according to the three attributes: (1) C_1 is the risk; (2) C_2 is the growth; (3) C_3 is the environmental impact, where C_1 and C_2 are benefit attributes and C_3 is a cost attribute. The weight vector of the three attributes is given by $w = (0.35, 0.25, 0.4)^T$. The four possible alternatives are to be evaluated under the above three attributes by the form of SVNVs.

For the evaluation of an alternative A_i with respect to an attribute C_j (i =1, 2, 3, 4; j =1, 2, 3), it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to an attribute C_1, he or she may say that the possibility in which the statement is good is 0.4 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.2. For the neutrosophic notation, it can be expressed as $a_{11} = \langle 0.4, 0.2, 0.3\rangle$. Thus, when the four possible alternatives with respect to the above three attributes are evaluated by the expert, we can obtain the following single valued neutrosophic decision matrix D:

$$D=\begin{bmatrix}\langle 0.4,0.2,0.3\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.8,0.2,0.5\rangle \\ \langle 0.6,0.1,0.2\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.5,0.2,0.8\rangle \\ \langle 0.3,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0.5,0.3,0.8\rangle \\ \langle 0.7,0.0,0.1\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.6,0.3,0.8\rangle \end{bmatrix}.$$

Then, we utilize the developed approach to obtain the most desirable alternative(s).

From the single valued neutrosophic decision matrix, we can obtain the following ideal alternative:

$$A^*=\{\langle C_1,0.7,0.1,0.1\rangle,\langle C_2,0.6,0.1,0.2\rangle,\langle C_3,0.5,0.3,0.8\rangle\}.$$

By using Eq. (3), we can obtain the values of the weighted correlation coefficient $W(A_i, A^*)$ (i =1, 2, 3, 4):

$W(A_1, A^*) = 0.8016$, $W(A_2, A^*) = 0.9510$, $W(A_3, A^*) = 0.8588$, and $W(A_4, A^*) = 0.9664$.

Thus, the ranking order of the four alternatives is $A_4 \succ A_2 \succ A_3 \succ A_1$. Therefore, the alternative A_4 is the best choice among the four alternatives.

5.2 Example 2

A multi-criteria decision making problem is concerned with a manufacturing company which wants to select the best global supplier according to the core competencies of suppliers. Now suppose that there are a set of four suppliers $A = \{A_1, A_2, A_3, A_4\}$ whose core competencies are evaluated by means of the four attributes: (1) C_1 is the level of technology innovation; (2) C_2 is the control ability of flow; (3) C_3 is the ability of management; (4) C_4 is the level of service, where C_1, C_2 and C_2 are all benefit attributes. Assume that the weight vector for the four attributes is $w = (0.3, 0.25, 0.25, 0.2)^T$.

The proposed decision making method is applied to solve this problem for selecting suppliers.

For the evaluation of an alternative A_i (i =1, 2, 3, 4) with respect to a criterion C_j (j =1, 2, 3, 4), it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to a criterion C_1, he or she may say that the possibility in which the statement is good is 0.5 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.1. For the neutrosophic notation, it can be expressed as $a_{11} = \langle 0.5, 0.1, 0.3\rangle$. Thus, when the four possible alternatives with respect to the above four attributes are evaluated by the similar method from the expert, we can obtain the following single valued neutrosophic decision matrix D:

$$D=\begin{pmatrix}\langle 0.5,0.1,0.3\rangle & \langle 0.5,0.1,0.4\rangle & \langle 0.7,0.1,0.2\rangle & \langle 0.3,0.2,0.1\rangle \\ \langle 0.4,0.2,0.3\rangle & \langle 0.3,0.2,0.4\rangle & \langle 0.9,0.0,0.1\rangle & \langle 0.5,0.3,0.2\rangle \\ \langle 0.4,0.3,0.1\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.0,0.4\rangle & \langle 0.6,0.2,0.2\rangle \\ \langle 0.6,0.1,0.2\rangle & \langle 0.2,0.2,0.5\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.7,0.2,0.1\rangle \end{pmatrix}$$

Then, we employ the developed approach to obtain the most desirable alternative(s).

From the single valued neutrosophic decision matrix, we can obtain the following ideal alternative:

$$A^*=\{\langle C_1,0.6,0.1,0.1\rangle,\langle C_2,0.5,0.1,0.3\rangle,\langle C_3,0.9,0.0,0.1\rangle,\langle C_4,0.7,0.2,0.1\rangle\}.$$

By applying Eq. (3), we can obtain the values of the weighted correlation coefficient $W(A_i, A^*)$ (i =1, 2, 3, 4):

$W(A_1, A^*) = 0.7998$, $W(A_2, A^*) = 0.8756$, $W(A_3, A^*) = 0.7580$, and $W(A_4, A^*) = 0.7532$.

Thus, the ranking order of the four alternatives is $A_2 \succ A_1 \succ A_3 \succ A_4$. Therefore, the alternative A_2 is the best choice among the four alternatives.

From the two examples, we can see that the proposed single valued neutrosophic multiple attribute decision-making method is more suitable for real scientific and engineering applications because it can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations.

6 Conclusion

In this paper, we proposed another form of the correlation coefficient between SVNSs. Then a multiple attribute decision-making method has been established in single valued neutrosophic setting by means of the weighted correlation coefficient between each alternative and the ideal alternative. Through the correlation coefficient, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. Finally, two illustrative examples illustrated the applications of the developed approach. Then the technique proposed in this paper is suitable for handling decision-making problems with single value neutrosophic information and can provide a useful way for decision-makers. In the future, we shall continue working in the applications of the correlation coefficient between SVNSs to other domains, such as data analysis and classification, pattern recognition, and medical diagnosis.

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