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A positive-real design for robotic manipulators

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A POSITIVE-REAL DESIGN FOR ROBOTIC MANIPULATORS

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ABSTRACT

In this paper, we show that feedback-linearization in conjunction with the passivity property of rigid robots can guarantee the robustness of the closed-loop robotic system despite large uncertainties in the inertia matrix. The approach may be extended to the case of uncertain velocity-dependent terms under additional assumptions.

I. INTRODUCTION

Given the following joint-space description of a rigid robot [1]

$$D(q)\ddot{q} + h(q,\dot{q}) = f \tag{1.1}$$

Most position-control techniques for the above equation fall under one of two categories: Feedback-Linearization or Passivity designs [2]. Unfortunately, inexact cancellations in the inner-loop of the feedback-linearizability approach (also known as inverse-dynamics, computed-torque, inner/outer) may result in loss of stability which limits the applicability of these results as discussed in [3,4]. The passivity approach on the other hand, does not lend itself to the many linear control designs available from the feedback-linearizability approach.

In this paper, we show that a combination of feedback-linearization and passivity designs of the robot controller will guarantee the robust stability of the closed-loop system without the exact knowledge of the matrix $D(q)$. In section II we introduce the problem and our notation. The main results are given in section III, and our conclusions are in section IV.

II. PROBLEM STATEMENT

In this paper, we consider the class of rigid robotic systems described in joint-space by the following equations

$$D(q)\ddot{q} + h(q,\dot{q}) = f \tag{2.1}$$

with $q(t)\in R^n$, and the control $f(t)\in R^n$. D and h contain the robot's parameters, some of which may be unknown. The matrix D is a symmetric positive-definite inertia matrix, and h is the vector of centrifugal, Coriolis and gravity forces. A state-space description of equation (2.1) is given by

$$\begin{bmatrix}\dot{q}\\ \ddot{q}\end{bmatrix} = \begin{bmatrix}\dot{q}\\ D^{-1}(-h)\end{bmatrix} + \begin{bmatrix}0\\ D^{-1}\end{bmatrix} f. \tag{2.2}$$

This nonlinear system is feedback-linearizable as described in [3]. Assuming D, and h are available, the controller design based on the linearizing transformation is given by

$$f = D(\ddot{q}_d - u) + h \tag{2.3}$$

where u is designed to obtain a desired closed-loop linear system. This will lead to

$$\begin{bmatrix}\dot{q}\\ \ddot{q}\end{bmatrix} = \begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\begin{bmatrix}q\\ \dot{q}\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix} u \tag{2.4}$$

where

$$u = D^{-1}(f-h). \tag{2.5}$$

In fact, if one considers $e_1=q_d-q$, $e_2=\dot{e}_1$ and the choice of u below

$$u = -Ke = -K_1e_1-K_2e_2, \tag{2.6}$$

The error equation becomes

$$\ddot{e}_1 + K_2\dot{e}_1 + K_1e_1 = 0, \tag{2.7}$$

which, by choosing $K_1 > 0$, and $K_2 > 0$ will guarantee that e_1 and e_2 go to zero asymptotically.

III. ROBUST MOTION CONTROLLERS

Since D and h are usually unknown or too complex to be evaluated at every sampling instant, a computed version f_c of f is applied to the system (2.1) where

$$f_c = D_c(\ddot{q}_d - u) + h_c \tag{3.1}$$

where D_c, and h_c are computed versions of D, and h. One therefore obtains a calculated version u_c of the input u to the linear system (2.4). Let u_c be given by

$$u_c = \ddot{q}_d - D^{-1}(f_c-h) \tag{3.2}$$

Then, substituting (3.1) into (3.2), one gets

$$u_c = \Delta\ddot{q}_d + \delta + (\Delta-I)M^{-1}(K_2e_2+K_1e_1) \tag{3.3}$$

where

$$\Delta = I-D^{-1}D_c, \quad \delta = D^{-1}(h-h_c). \tag{3.4}$$

One is now concerned with the stability of a linear system with a nonlinear feedback described by

$$\dot{e} = (A-BK)e + B[\Delta(Ke+\ddot{q}_d)+\delta] \tag{3.5}$$

A more compact description of the error system is given by

$$\dot{e} = A_ce + Bv \tag{3.6}$$

The first step in our design is to choose the linear gain matrix $K=[K_1 \quad K_2]$, and an output matrix C, in order to guarantee that the closed-loop system (A_c,B,C) is SPR. Then, using the passivity results [5], one can show the asymptotic stability of (3.6) when h is known but D is not.

Theorem 1: Let K_1, K_2 be two diagonal matrices with

i) $K_1=diag(k_{1i})$; $k_{1i}>0; i=1,\dots,n$

ii) $K_2=diag(k_{2i})$; $k_{2i}>0; i=1,\dots,n$

iii) $(K_2)^2 > K_1$; $i=1,\dots,n$

Then the system given by

$$\dot{e} = \begin{bmatrix}0 & I\\ -K_1 & -K_2\end{bmatrix} e + \begin{bmatrix}0\\ I\end{bmatrix} u = A_ce + Bu,$$
$$y = [K_1 \quad K_2]=Ce$$

is Strictly-Positive-Real (SPR).

Proof: Consider the Lyapunov equation

$$A_c^TP + PA_c = -Q$$

where

$$Q = \begin{bmatrix}2K_1^2 & 0\\ 0 & 2K_2^2-2K_1\end{bmatrix}$$

Note that $Q>0$ by the conditions of the theorem and solve for P as given below

$$P = \begin{bmatrix}2K_1K_2 & K_1\\ K_1 & K_2\end{bmatrix}$$

It can be shown that $P>0$. On the other hand, note that

$$C = B^TP.$$

Using the MKY lemma [6], the system is then SPR.

Note that the relationship $K_2^2>K_1$ is easy to satisfy. Next, we show that in the event that h is known, and using the passivity theorem [5,6], the asymptotic stability of the closed-loop system is guaranteed with a particular choice of M_c.

Theorem 2: Suppose h is known. Then, the origin of (3.6) is an asymptotically-stable equilibrium point if f_c is given by (3.1) where

$$D_c = aI; \quad h_c = h;$$

where

$$aI > D;$$

Proof: Given the choice of $h_c = h$, one gets from (3.5,3.6)

$$\delta = 0$$

and

$$v(t) = -\Delta(Ke + \ddot{q}_d) = -\Delta w.$$

Consider then the block diagram of Figure 1. The output of the nonlinear block is given by

$$z(t) = -\Delta w(t).$$

To determine if the nonlinear block is passive, check that there exists some finite μ such that for all finite T

$$\int_0^T -w^T \Delta w dt \geq \mu,$$

If one chooses $\mu = 0$, one needs to show that

$$\int_0^T w^T [D^{-1} D_c - I] w dt \geq 0,$$

for all T finite. It is then sufficient to choose

$$D_c = aI > D$$

as specified in the theorem. Using Popov's hyperstability criterion [6], one deduces that the the signals $z(t)$ and $e(t)$ in Figure 1 are bounded. Then noting that the linear block is SPR, one deduces that $e(t)$ goes to zero asymptotically.

Note that the $aI > D$ can be satisfied since D is bounded above [2] and that the choices made in the theorem result in

$$f_c = a(Ke + \ddot{q}_d) + h. \tag{3.7}$$

The above discussion then shows that the nonlinear feedback (from the linear system's output to the robot) due to the uncertainties in the inertia matrix D can be made passive.

In the more general case where both D and h are unknown, one can divide the nonlinear feedback (due to the uncertainties) into two parts: one due to D and one due to h. The later contribution may tend to make the closed-loop system unstable. As shown in the following theorem however, a linear bound on the uncertainties in h is sufficient to maintain the stability of the closed-loop system.

Theorem 3: Let the following hold

$$\|h - h_c\|_T \leq c \,||w||_T + d$$

Then, the closed-loop system is asymptotically stable if f_c is given by (3.1) and

$$D_c = aI$$

where

$$a \geq \frac{c+1}{r}$$

and

$$M > \frac{1}{r} I.$$

Proof: Consider the output of the nonlinear feedback

$$-(\Delta w + \delta) = \ddot{q} - w$$

Then one should choose a to satisfy

$$\int_0^T w^T (\ddot{q} - w) dt \geq b$$

for some finite b and all finite T. Noting that

$$\ddot{q} = aD^{-1} w + D^{-1}(h - h_c)$$

the inequality is satisfied if

$$\int_0^T w^T (aD^{-1} - I) w dt - \int_0^T w^T D^{-1}(h_c - h) dt \geq b.$$

Using the bounds in the theorem, the following sufficient condition is obtained

$$\int_0^T w^T (aD^{-1} - I - cI) w dt \geq d \,||w||_T + b,$$

or

$$(ar - c - 1)(||w||_T)^2 - d \,||w||_T - b \geq 0$$

where $M > rI$ as specified in the theorem. Since the last inequality should be verified for all $||w||_T$, a sufficient condition is obtained from

$$a > \frac{c+1}{r}; \quad -b > \frac{d^2}{4(1+c-ar)}.$$

The condition on a is that stated in the Theorem, while b is arbitrary. Therefore, one is assured that the nonlinear block stays passive. Repeating the arguments made in Theorem 2, the asymptotic stability of the origin of the closed-loop system is guaranteed.

Recently, and using network theory [7], the loss of passivity due to inexact cancellations in the computed-torque algorithm was illustrated. It was also shown that $D_c = aI$ will maintain the passivity of the closed-loop if no contact forces exist between the manipulator and its environment.

V. CONCLUSION

It was shown that a feedback-linearization approach to the control of robotic systems may be used in conjunction with passivity theory in order to guarantee the asymptotic-stability of the origin of the closed-loop system despite large uncertainties in the inertia matrix D. If the velocity-dependent terms h are also uncertain but a linear bound on the uncertainty is known, one may still design the controller for asymptotic stability. This then shows that the Lagrange-Euler equations are robust to modeling uncertainties in D and h. In particular, if one is designing an adaptive controller for these equations [7], the matrix D need not be updated since one can choose the gains to keep the contribution of $D_c - D$ passive.

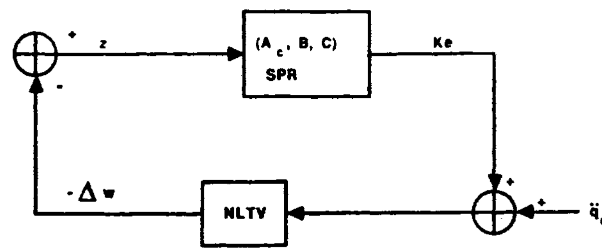

Figure 1: Hyperstability Block Diagram

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