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A Numerical Solution for Weak Disturbances in a Large Scale Hydromagnetic Field

Patrick J. Blewett

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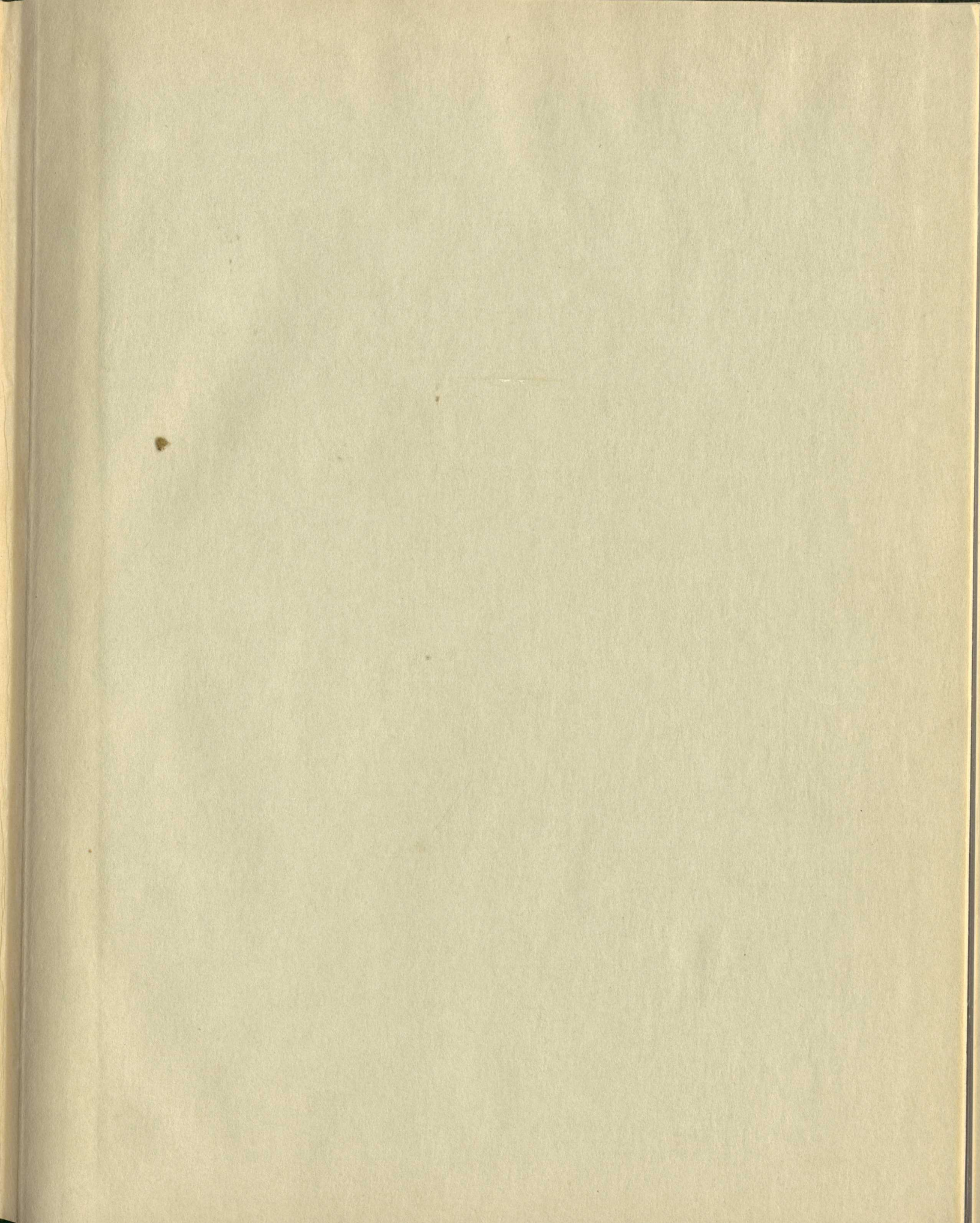


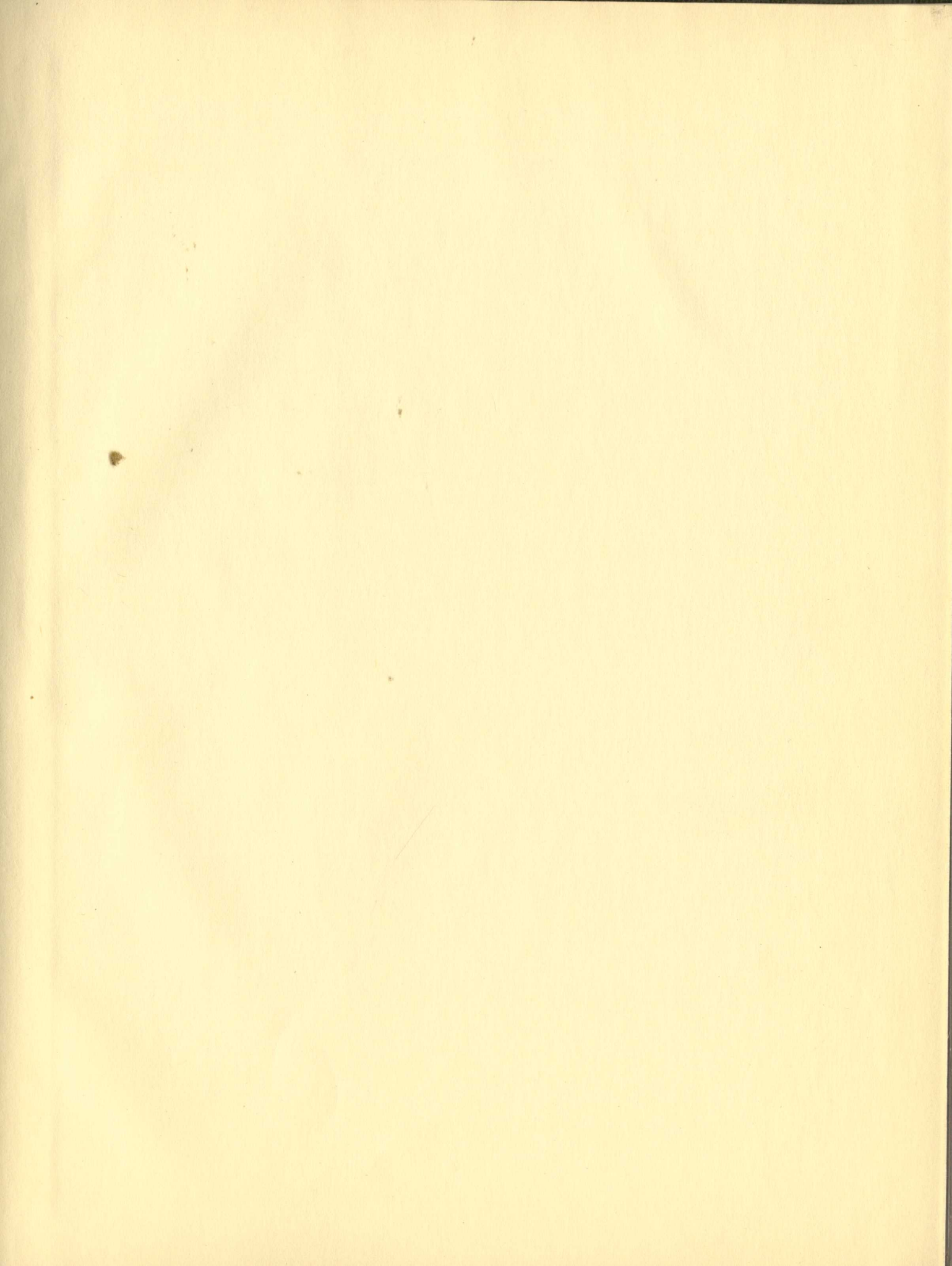
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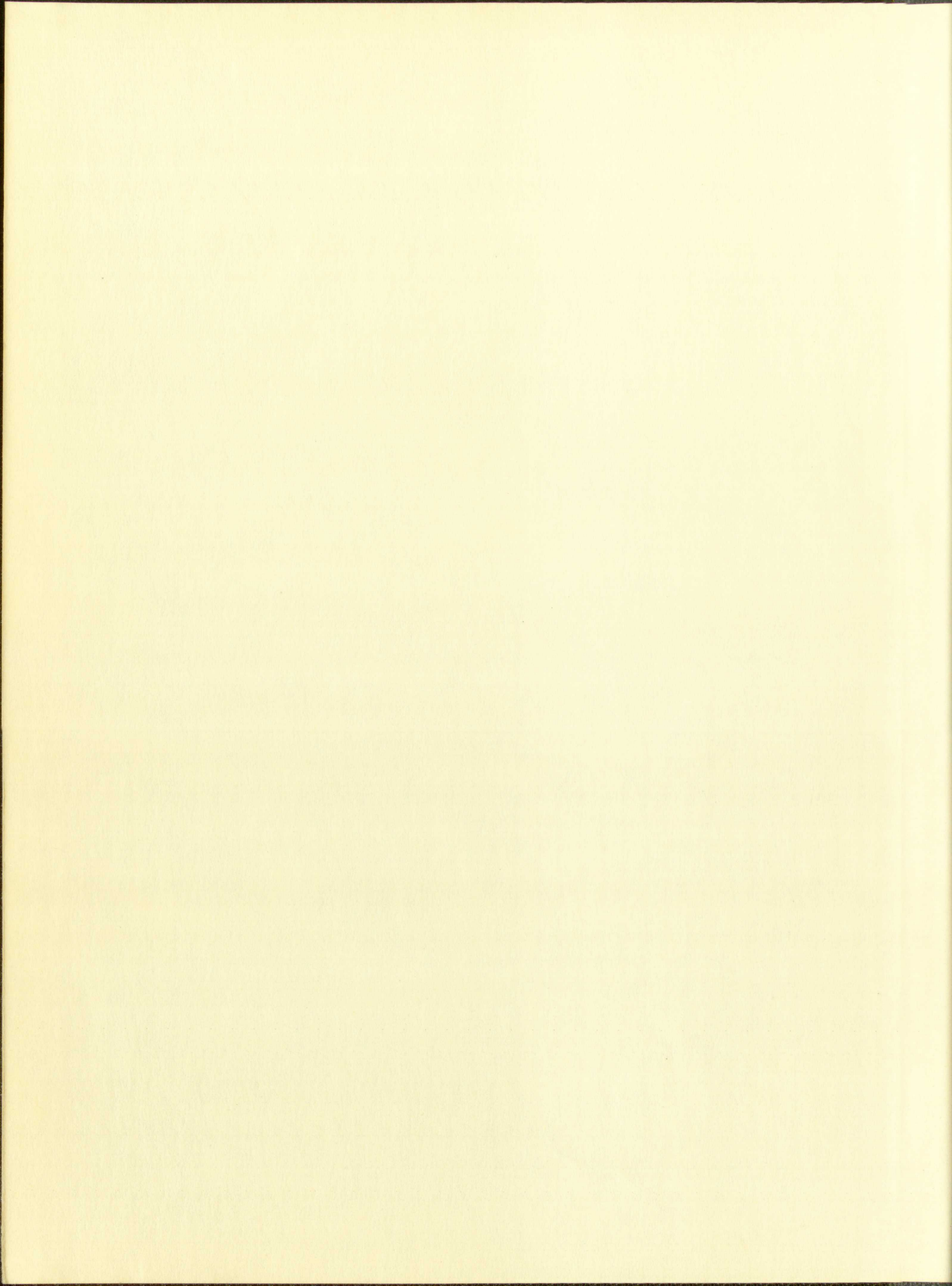
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A NUMERICAL SOLUTION FOR WEAK DISTURBANCES
IN A LARGE SCALE HYDROMAGNETIC FIELD

By

Patrick J. Blewett

A Thesis

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Physics

The University of New Mexico

1959



A NUMERICAL SOLUTION FOR THE EQUATION
IN A LARGE SCALE HYDRODYNAMIC FIELD

By
PETERSON & MANNING

A thesis
submitted in partial fulfillment of the
requirements for the degree of
Master of Science in Physics

The University of New Mexico

1959

This thesis, directed and approved by the candidate's committee, has been accepted by the Graduate Committee of the University of New Mexico in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

E. Castetter
DEAN

June 9, 1959
DATE

Thesis committee

Arnold Kukulian
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This thesis directed and approved by the candidate's committee has been accepted by the Graduate Committee of the University of New Mexico in partial fulfillment of the requirements for the degree of

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Edwin S. Redkey

June 1957
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Thesis committee

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Elsasser¹ has shown that the equations of magnetohydrodynamics for an ideal, incompressible, unbounded, perfectly conducting fluid in a magnetic field may be put in the symmetrized form

$$\begin{aligned} \frac{\partial \vec{p}}{\partial t} + (\vec{q} \cdot \nabla) \vec{p} + \nabla \psi &= 0 \\ \frac{\partial \vec{q}}{\partial t} + (\vec{p} \cdot \nabla) \vec{q} + \nabla \psi &= 0 \\ \nabla \cdot \vec{p} = \nabla \cdot \vec{q} &= 0 \end{aligned} \quad (1)$$

where

$$\begin{aligned} \vec{p} &= \vec{v} + \vec{B}^* \\ \vec{q} &= \vec{v} - \vec{B}^* \\ \psi &= \frac{p_0}{\rho_0} + \frac{(\vec{p} - \vec{q})^2}{8} \\ \vec{B}^* &= (\mu \rho)^{-\frac{1}{2}} \vec{B} \end{aligned} \quad (2)$$

\vec{v} , p , ρ , and \vec{B} are respectively the fluid velocity, the pressure, the density, and the magnetic induction vector.

Letting

$$\begin{aligned} \vec{p}_0(\vec{r}, t) &= \vec{v}_0 + \vec{B}_0^* \\ \vec{q}_0(\vec{r}, t) &= \vec{v}_0 - \vec{B}_0^* \\ \psi_0 &= \frac{p_0}{\rho_0} + \frac{(\vec{p}_0 - \vec{q}_0)^2}{8} \end{aligned} \quad (3)$$

be a solution of (1) and thus the primary hydromagnetic field, Skabelund² has shown that superimposed perturbations,

$$\begin{aligned} \vec{p} &= \vec{v} + \vec{B}^* \\ \vec{q} &= \vec{v} - \vec{B}^* \end{aligned} \quad (4)$$

1. Elsasser, Phys. Rev. 79, 183 (1950)

2. Skabelund, O.N.R. Tech. Rep. 17, Contr. 1288(00), (1955)

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Blaschke has shown that the general form of the
 dynamics for a field, in particular, is
 contracting fields in a magnetic field are of the form

$$\frac{\partial^2 \psi}{\partial t^2} + \nabla^2 \psi + \psi = 0$$

$$\frac{\partial^2 \psi}{\partial t^2} + (\nabla^2 - \psi) + \psi = 0$$

$$\psi = \psi_0 e^{-i\omega t}$$

where

$$\psi = \psi_0 + \psi_1$$

$$\psi = \psi_0 - \psi_1$$

$$\psi = \frac{\psi_0 + \psi_1}{2}$$

$$\psi = (\psi_0 - \psi_1) e^{-i\omega t}$$

The general form of the dynamics for a field, in particular, is
 contracting fields in a magnetic field are of the form

Letting

$$\psi = \psi_0 e^{-i\omega t} + \psi_1 e^{i\omega t}$$

$$\psi = \psi_0 e^{-i\omega t} - \psi_1 e^{i\omega t}$$

$$\psi = \frac{\psi_0 + \psi_1}{2} + \frac{\psi_0 - \psi_1}{2} e^{-i\omega t}$$

be a solution of (1) and thus the general form of the
 field, Blaschke has shown that the general form of the

$$\psi = \frac{\psi_0 + \psi_1}{2}$$

$$\psi = \frac{\psi_0 - \psi_1}{2}$$

J. Blaschke, *Ann. Phys.*, **12**, 100 (1928)
 S. Blaschke, *Z. Phys.*, **12**, 100 (1928)

on the primary field also satisfy (1). They are related to the primary field in Lagrangian coordinates by

$$\vec{p} = (\vec{p}^0 \cdot \nabla^0) \vec{r}$$

$$\nabla \cdot \vec{p} = \nabla^0 \cdot \vec{p}^0 = 0 \quad (5)$$

$$(5)$$

where r is an integral of

$$\frac{d\vec{r}}{dt} = \vec{Q}_0(\vec{r}, t). \quad (6)$$

Note, the superscript 0's denote initial values: the quantities with tildes constitute the superimposed waves on the primary field which is denoted by the subscript 0's. Further, in the above analysis \vec{Q} was assumed small compared to \vec{P} which condition yielded the relation

$$\frac{\partial \vec{P}}{\partial t} = \nabla \times (\vec{Q}_0 \times \vec{P}) \quad (7)$$

From this equation it is seen³ that the lines of \vec{P} are frozen in a hypothetical incompressible fluid moving with velocity, \vec{Q}_0 . Hence, from equation (3b) we see that if the actual fluid's velocity is 0, the Q_0 -fluid flows with a velocity, $-\vec{B}_0^*$; whereas, when the actual fluid moves with velocity, \vec{V}_0 , the velocity of the Q_0 -fluid is increased to $\vec{V}_0 - \vec{B}_0^*$.

Strictly speaking, equations (5) and (7) hold only in the absence of reflections; however, Skabelund argues their reasonableness in the presence of reflections provided the amplitude of \vec{P} is small compared to the amplitude of \vec{Q}_0 .

3. Prandtl and Tietjens, Fundamentals of Hydro and Aero-Mechanics (McGraw-Hill, New York, 1934), p. 197

on the primary field also satisfy (1). They are related to the primary field in logarithmic coordinates by

$$\begin{aligned} (2) \quad \vec{v} &= \vec{v}_0 \cdot \vec{v} \\ (3) \quad \vec{v} &= \vec{v}_0 \cdot \vec{v} \end{aligned}$$

where x is an integral of

$$(6) \quad \frac{dx}{dt} = \dots$$

Note, the superscript 0 denotes initial values; the quantities with tildes constitute the superimposed waves on the primary field which is denoted by the superscript 0 . Further, in the above analysis \vec{v} was assumed small compared to \vec{v}_0 which

condition yielded the relation

$$(7) \quad \frac{d\vec{v}}{dt} = \vec{v}_0 \cdot \frac{d\vec{v}}{dt}$$

from this equation it is seen that the lines of \vec{v} and \vec{v}_0 are seen in a hypothetical incompressible fluid moving with velocity \vec{v}_0 . Hence, from equation (6) we see that if the actual fluid's velocity is \vec{v}_0 , the \vec{v} -fluid flows with a velocity \vec{v} ; whereas, when the actual fluid moves with velocity \vec{v}_0 , the velocity of the \vec{v} -fluid is increased to

$$\vec{v}_0 + \vec{v}$$

Similarly speaking, equations (2) and (3) hold only in the absence of reflections; however, Hasegawa argues that the absence of reflections in the presence of reflections provided the amplitude of \vec{v} is small compared to the amplitude of \vec{v}_0 .

and the scale of \vec{F} remains small compared to the scale of \vec{Q}_0 . By scale is meant the linear distance in any direction over which the relative change in the variable is unity.

Our concern here will be to investigate the form of certain solutions of equation (7); we will be content with a two dimensional analysis. Following Skabelund, \vec{Q}_0 is expanded in a Taylor's series

$$\vec{Q}_0(x, y) = \vec{Q}_0(x_0, y_0) + \left(\frac{\partial \vec{Q}_0}{\partial x}\right)_{x_0, y_0} (x - x_0) + \left(\frac{\partial \vec{Q}_0}{\partial y}\right)_{x_0, y_0} (y - y_0). \quad (8)$$

From equation (1d) \vec{Q}_0 is solenoidal, hence

$$\nabla \cdot \vec{Q}_0(x, y) = \left(\frac{\partial Q_{0x}}{\partial x}\right)_{x_0, y_0} + \left(\frac{\partial Q_{0y}}{\partial y}\right)_{x_0, y_0} = 0. \quad (9)$$

Let

$$\begin{aligned} a &= \left(\frac{\partial Q_{0x}}{\partial x}\right)_{x_0, y_0} & b &= \left(\frac{\partial Q_{0x}}{\partial y}\right)_{x_0, y_0} \\ c &= \left(\frac{\partial Q_{0y}}{\partial x}\right)_{x_0, y_0} & d &= \left(\frac{\partial Q_{0y}}{\partial y}\right)_{x_0, y_0} \end{aligned} \quad (10)$$

and from equation (9)

$$a + d = 0. \quad (11)$$

Thus, the first order approximation in component form becomes

$$\begin{aligned} Q_{0x} &= K_1 + ax + by \\ Q_{0y} &= K_2 + cx - ay \end{aligned} \quad (12)$$

where for convenience we take x_0 and y_0 as 0, and

$$\begin{aligned} K_1 &= Q_{0x}(0, 0) \\ K_2 &= Q_{0y}(0, 0). \end{aligned} \quad (13)$$

and the axis of \vec{y} remains parallel to the axis of \vec{x} . By scale we mean the linear distance in any direction over which the relative change in the variable is unity.

Our concern here will be to investigate the form of certain solutions of equation (1) which will be constant with a two dimensional analysis. The following theorem is proved in a Taylor's series

$$(9) \quad \vec{\phi}_0(x, y) = \vec{\phi}_0(x_0, y_0) + \left(\frac{\partial \vec{\phi}_0}{\partial x} \right)_{x_0, y_0} (x - x_0) + \left(\frac{\partial \vec{\phi}_0}{\partial y} \right)_{x_0, y_0} (y - y_0) + \dots$$

From equation (10) $\vec{\phi}_0$ is a solution, hence

$$(10) \quad \nabla \cdot \vec{\phi}_0(x, y) = \left(\frac{\partial \vec{\phi}_0}{\partial x} \right)_{x_0, y_0} + \left(\frac{\partial \vec{\phi}_0}{\partial y} \right)_{x_0, y_0} = 0$$

Let

$$a = \left(\frac{\partial \vec{\phi}_0}{\partial x} \right)_{x_0, y_0} \quad b = \left(\frac{\partial \vec{\phi}_0}{\partial y} \right)_{x_0, y_0}$$

$$(11) \quad a = \left(\frac{\partial \vec{\phi}_0}{\partial x} \right)_{x_0, y_0} \quad b = \left(\frac{\partial \vec{\phi}_0}{\partial y} \right)_{x_0, y_0}$$

and from equation (9)

$$(12) \quad a + b = 0$$

Thus, the first order approximation in component form becomes

$$\vec{\phi}_1 = K_1 + a_1 x + b_1 y$$

$$\vec{\phi}_2 = K_2 + a_2 x + b_2 y$$

(13)

where for convenience we take $K_1 = 0$ and $K_2 = 0$, and

$$K_1 = \vec{\phi}_1(0, 0)$$

$$K_2 = \vec{\phi}_2(0, 0)$$

(14)

The streamlines of the Q_0 -fluid are the curves satisfying

$$\frac{Q_{0y}}{Q_{0x}} = \frac{dy}{dx} = \frac{K_2 + cx - ay}{K_1 + ax + by} . \quad (14)$$

Considering this equation in the form,

$$(K_2 + cx - ay)dx - (K_1 + ax + by)dy = 0 \quad (15)$$

we see that it is exact; its solution is the general equation of a conic

$$\frac{c}{2}x^2 - \frac{a}{2}y^2 - axy + K_2x - K_1y + e = 0 \quad (16)$$

having the discriminant

$$\Delta = a^2 + bc. \quad (17)$$

Consider first the elliptical case wherein $\Delta < 0$, or

$$a^2 + bc < 0. \quad (18)$$

We will insure this condition by taking

$$a = 0, b > 0, c < 0. \quad (19)$$

There is no loss of generality here since the mixed term in (16) could always be eliminated through the proper rotation of axes. Hence, we have for the streamlines for the case of elliptical flow of our imaginary Q_0 -fluid the equation

$$\frac{c}{2}x^2 - \frac{b}{2}y^2 + K_2x - K_1y + e = 0. \quad (20)$$

For the solution of \vec{P} in equation (7), our final goal, ^{we} see from (5) that we will need a Lagrangian description of

The discriminant of the quadratic equation is

$$\Delta = b^2 - 4ac$$

Considering this equation in the form

$$(a+x^2)y^2 + (b+2xy)x + (c+x^2) = 0$$

we see that it is exact for the solution of the equation in the form of a circle

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

having the discriminant

$$\Delta = b^2 + 4ac$$

Let us consider first the elliptical case

$$a > 0, b > 0$$

We will assume this condition is satisfied

$$a > 0, b > 0, c > 0$$

There is no issue of reality in this case. In fact, the ellipse is always real and exists. Hence, we have for the ellipse the case of elliptical form of our equation.

tion

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

For the solution of the equation in the form of a circle see from (2) that we will have the same result

the Q_0 -fluid flow. For this we return to equation (6) from which

$$\begin{aligned}\dot{x} &= Q_{0x}(\vec{r}, t) \\ \dot{y} &= Q_{0y}(\vec{r}, t)\end{aligned}\quad (21)$$

and from the equations of expansion, (12), we have

$$\begin{aligned}\dot{x} &= K_1 + by \\ \dot{y} &= K_2 + cx\end{aligned}\quad (22)$$

from which

$$\begin{aligned}\ddot{x} &= b[K_2 + cx] \\ \ddot{y} &= c[K_1 + by]\end{aligned}\quad (23)$$

This pair of equations then yields the solutions

$$\begin{aligned}x &= Ce^{\sqrt{bc}t} + De^{-\sqrt{bc}t} - \frac{K_2}{c} \\ y &= Ae^{\sqrt{bc}t} + Be^{-\sqrt{bc}t} - \frac{K_1}{b}.\end{aligned}\quad (24)$$

Equations (24) are subject to the initial conditions

$$\begin{aligned}t=0: \quad x &= x^0 & \dot{x} &= K_1 + by^0 \\ y &= y^0 & \dot{y} &= K_2 + cx^0.\end{aligned}\quad (25)$$

These conditions are sufficient for the determination of the integration constants in equations (24). The result is

$$\begin{aligned}x &= \sqrt{\frac{b}{c}} \left[y^0 + \frac{K_1}{b} \right] \sin \sqrt{bc_1} t - \sqrt{\frac{b}{c}} \left[\frac{K_2}{\sqrt{bc_1}} - \sqrt{\frac{c_1}{b}} x^0 \right] \cos \sqrt{bc_1} t + \frac{K_2}{c_1} \\ y &= \left[y^0 + \frac{K_1}{b} \right] \cos \sqrt{bc_1} t + \left[\frac{K_2}{\sqrt{bc_1}} - \sqrt{\frac{c_1}{b}} x^0 \right] \sin \sqrt{bc_1} t - \frac{K_1}{b}\end{aligned}\quad (26)$$

where $c_1 = -c$ and thus, $c_1 > 0$. Equations (26) are the equations of motion of the Q_0 -fluid in Lagrangian coordinates.

the general form, the value of c is determined (2) from which

$$x = x_1 + y_1$$

(31)

$$y = y_1 + y_2$$

and from the equation of asymptotes, (17) we have

$$x = k_1 + y_1$$

(32)

$$y = k_2 + y_2$$

from which

$$x = k_1 + y_1$$

(33)

$$y = k_2 + y_2$$

This pair of equations then gives the solutions

$$x = C_1 e^{at} + C_2 e^{-at} - \frac{b_1}{a}$$

(34)

$$y = C_1 e^{at} + C_2 e^{-at} - \frac{b_2}{a}$$

Equations (31) are subject to the initial conditions

$$x = x_0, \quad y = y_0$$

(35)

$$x = x_0, \quad y = y_0$$

These conditions are satisfied for the determination of

the integration constants in equations (34). The result is

$$x = \frac{1}{\sqrt{c^2 - 4}} \left[\frac{c + \sqrt{c^2 - 4}}{2} e^{at} + \frac{c - \sqrt{c^2 - 4}}{2} e^{-at} \right] - \frac{b_1}{a} + \frac{b_1}{\sqrt{c^2 - 4}}$$

(36)

$$y = \frac{1}{\sqrt{c^2 - 4}} \left[\frac{c + \sqrt{c^2 - 4}}{2} e^{at} + \frac{c - \sqrt{c^2 - 4}}{2} e^{-at} \right] - \frac{b_2}{a} + \frac{b_2}{\sqrt{c^2 - 4}}$$

where $c = -c$ and $\sqrt{c^2 - 4} > 0$. The case $c = 0$ and $\sqrt{c^2 - 4} = 2i$ is

the case of a pair of complex conjugate roots of the characteristic equation.

That they constitute the parametric equations for an elliptic trajectory of a particular Q_0 -fluid particle can be seen by noting

$$\begin{aligned}x &= \sqrt{\frac{E}{c_1}} [E \sin \theta - F \cos \theta] + \frac{K_2}{c_1} \\y &= E \cos \theta + F \sin \theta - \frac{K_1}{b}\end{aligned}\quad (27)$$

where the substitutions are easily recognized. From (27) then

$$\frac{\left[y + \frac{K_1}{b}\right]^2}{E^2 + F^2} + \frac{\left[x - \frac{K_2}{c_1}\right]^2}{\frac{E}{c_1} [E^2 + F^2]} = 1. \quad (28)$$

Thus, we have for the paths of the various Q_0 -fluid particles ellipses centered about $\left(\frac{K_1}{c_1 b}, \frac{K_2}{c_1}\right)$ with an eccentricity

$$e = \sqrt{1 - \frac{c_1}{b}}. \quad (29)$$

The following graph, figure 1, will help to visualize the flow of particles which constitute the positive x axis when $t = 0$. For calculation purposes it was assumed that

$$K_1 = K_2 = 0, \quad \frac{b}{c_1} = -2. \quad (30)$$

Now with equations (26) and (5) \vec{P} can be determined for an arbitrary small-amplitude initial distribution. Rewriting equation (5a) in component form

$$\begin{aligned}\tilde{P}_x &= \tilde{P}_x^0 \frac{\partial x}{\partial x^0} + \tilde{P}_y^0 \frac{\partial x}{\partial y^0} \\ \tilde{P}_y &= \tilde{P}_x^0 \frac{\partial y}{\partial x^0} + \tilde{P}_y^0 \frac{\partial y}{\partial y^0}\end{aligned}\quad (31)$$

and substituting equations (26) into the above yields

$$\begin{aligned}\tilde{P}_x &= \tilde{P}_x^0 \cos \sqrt{6} c_1 t + \tilde{P}_y^0 \sqrt{\frac{E}{c_1}} \sin \sqrt{6} c_1 t \\ \tilde{P}_y &= -\tilde{P}_x^0 \sqrt{\frac{E}{c_1}} \sin \sqrt{6} c_1 t + \tilde{P}_y^0 \cos \sqrt{6} c_1 t\end{aligned}\quad (32)$$

That they constitute the asymptotes of the hyperbola
 the trajectory of a particle is a hyperbola as can be
 seen by noting

$$x = \sqrt{\frac{a}{2}} [E \cos \theta - F \sin \theta] + \frac{a}{2}$$

$$y = E \sin \theta + F \cos \theta - \frac{a}{2}$$

where the substitution $\theta = \theta_0 + \omega t$ is used.
 then

$$\frac{\sqrt{\frac{a}{2}} [E \cos \theta - F \sin \theta] + \frac{a}{2}}{E \cos \theta - F \sin \theta} = 1$$

Thus, we have for the path of the particle a hyperbola
 also ellipse centered about $(\frac{a}{2}, \frac{a}{2})$ with semi-axes

$$c = \sqrt{\frac{a}{2}}$$

The following graph shows the path of a particle in
 flow of particles which originate at the origin and
 $c = 0$. For calculation purposes $a = 1$ is assumed.

$$K = K_0 = 0, \quad \frac{a}{2} = 0.5$$

For other equations (2) and (3) the same procedure
 for an arbitrary semi-major axis a and eccentricity e
 writing equation (2) in standard form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{a^2 e^2} = 1$$

and substituting $x = a \cosh t$ and $y = a e \sinh t$
 $\frac{a^2 \cosh^2 t}{a^2} - \frac{a^2 e^2 \sinh^2 t}{a^2 e^2} = 1$
 $\cosh^2 t - \sinh^2 t = 1$

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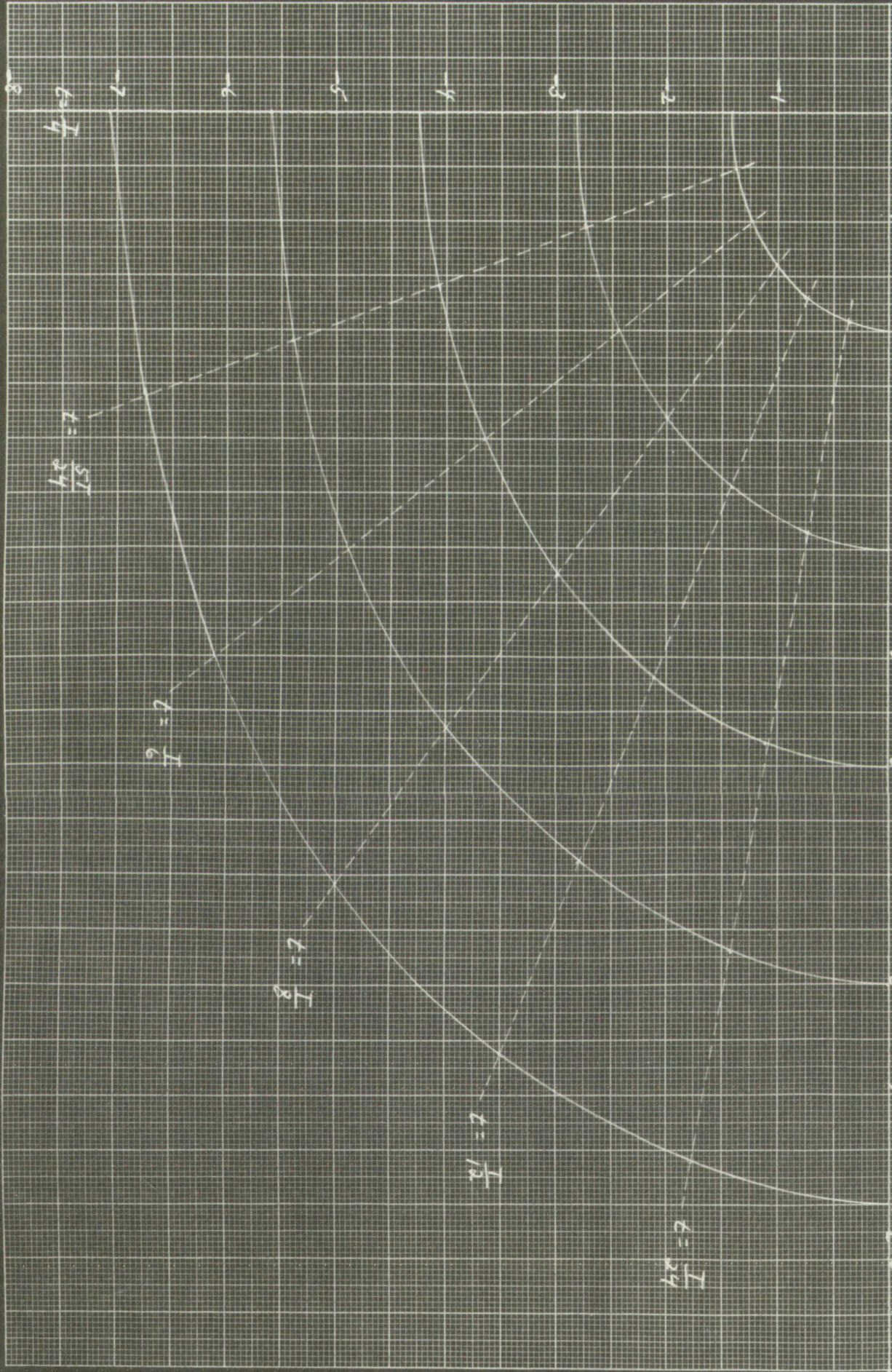
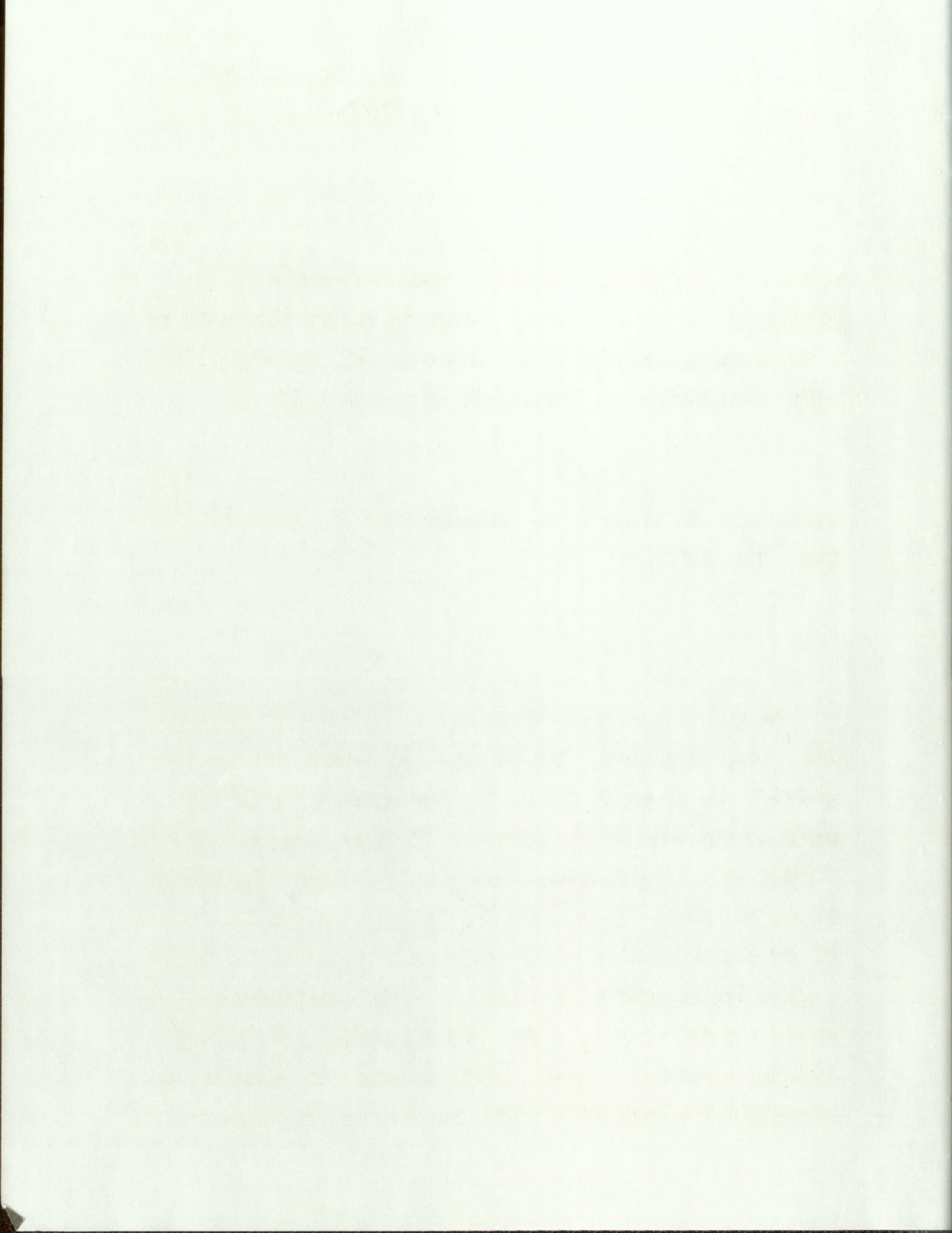


Figure 1



Equations (32) then constitute the Lagrangian description of \vec{P} . For purposes of calculation a P-whirlring centered initially about the point (10,0) was chosen with the functional form

$$|\vec{P}^0| = r e^{-\frac{r^2}{\delta^2}} \quad (33)$$

where r is the radial coordinate from the center of the whirlring and δ is a constant chosen to insure the scale of \vec{P} to be small compared to the scale of \vec{Q}_0 . For this specific calculation the components of \vec{P} were taken as

$$\tilde{P}_x^0 = -y^0 e^{-\frac{2[(x^0-10)^2 + y^{02}]}{L^2}}; \tilde{P}_y^0 = (x^0-10) e^{-\frac{2[(x^0-10)^2 + y^{02}]}{L^2}} \quad (34)$$

where L is the scale of \vec{P} . Substitution of equations (34) into (32) yields

$$\begin{aligned} \tilde{P}_x &= \left[-y^0 \cos \sqrt{\delta c} t + (x^0-10) \sqrt{\frac{L}{\delta}} \sin \sqrt{\delta c} t \right] e^{-2[(x^0-10)^2 + y^{02}]} \\ \tilde{P}_y &= \left[y^0 \sqrt{\frac{L}{\delta}} \sin \sqrt{\delta c} t + (x^0-10) \cos \sqrt{\delta c} t \right] e^{-2[(x^0-10)^2 + y^{02}]} \end{aligned} \quad (35)$$

Of particular interest is the change of the shape of the \vec{P} -ring with time. The results for motion through one quadrant are shown in figure 2. The \vec{Q}_0 -flow though not shown is the same as in figure 1. Further, all concentric \vec{P} -rings at $t=0$ will change with time like that depicted in figure 2. Note, the magnitude of \vec{P} for a particular \vec{P} -ring is not preserved with time. This is illustrated in figure 3 where the magnitude of \vec{P} for a few representative points on the initial ring is plotted against time. It is seen that the values of $|\vec{P}|$ remain small compared to those of $|\vec{Q}_0|$ throughout the motion; the same is true of its scale length.

Equation (1) then satisfies the Lagrangian description of \vec{r} . For purposes of calculation a 2-whirling contour initially about the axis $(0,0)$ was chosen with the initial form

$$\left| \frac{\vec{r}}{r_0} \right| = r_0^{-\frac{1}{2}}$$

(23)

where r is the radial coordinate from the center of the whirling and λ is a constant chosen so that the scale of \vec{r} to be small compared to the scale of \vec{r}_0 . For this case of the calculation the coordinate of \vec{r} was taken as

$$\vec{r} = \frac{r_0 \cos(\lambda t + \phi_0)}{\sqrt{1 + \lambda^2 r_0^2}} \hat{e}_1 + \frac{r_0 \sin(\lambda t + \phi_0)}{\sqrt{1 + \lambda^2 r_0^2}} \hat{e}_2$$

(24)

where λ is the scale of \vec{r} . Substitution of equation (24) into (23) yields

$$\vec{r} = \frac{r_0 \cos(\lambda t + \phi_0)}{\sqrt{1 + \lambda^2 r_0^2}} \hat{e}_1 + \frac{r_0 \sin(\lambda t + \phi_0)}{\sqrt{1 + \lambda^2 r_0^2}} \hat{e}_2$$

(25)

Of particular interest is the change of the shape of the \vec{r} -ring with time. The results for motion through one quadrant are shown in Figure 2. The \vec{r} -ring starts out as shown in the case of Figure 1. Further, all concentric \vec{r} -rings at $t=0$ will change with time like that depicted in Figure 2. Note, the magnitude of \vec{r} for a particular \vec{r} -ring is not preserved with time. This is illustrated in Figure 2 where the magnitude of \vec{r} for a few representative points on the initial ring is plotted against time. It is seen that the value of $|\vec{r}|$ remains small compared to r_0 throughout the motion; the same is true of the total length.

-Y

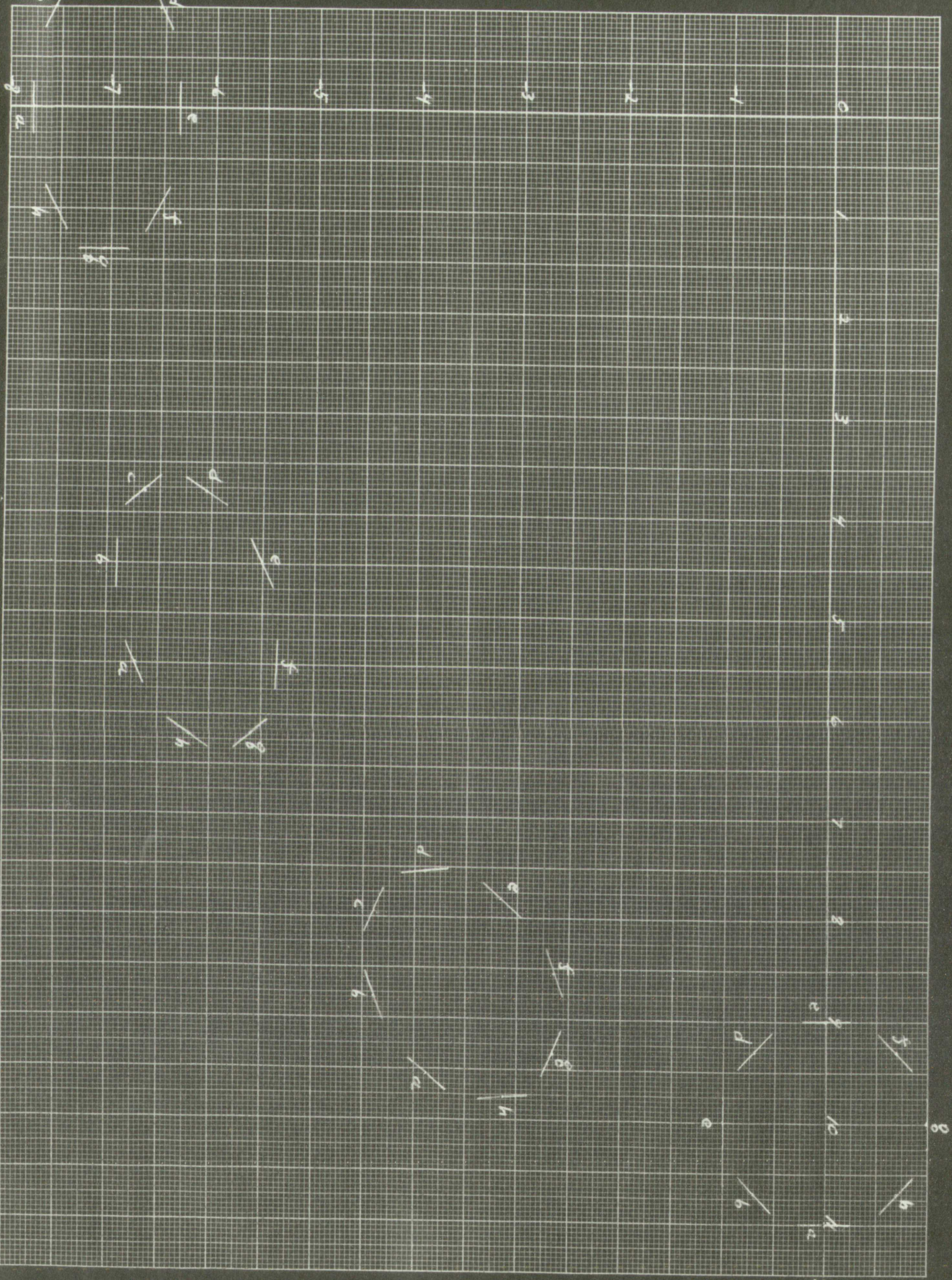


Figure 2



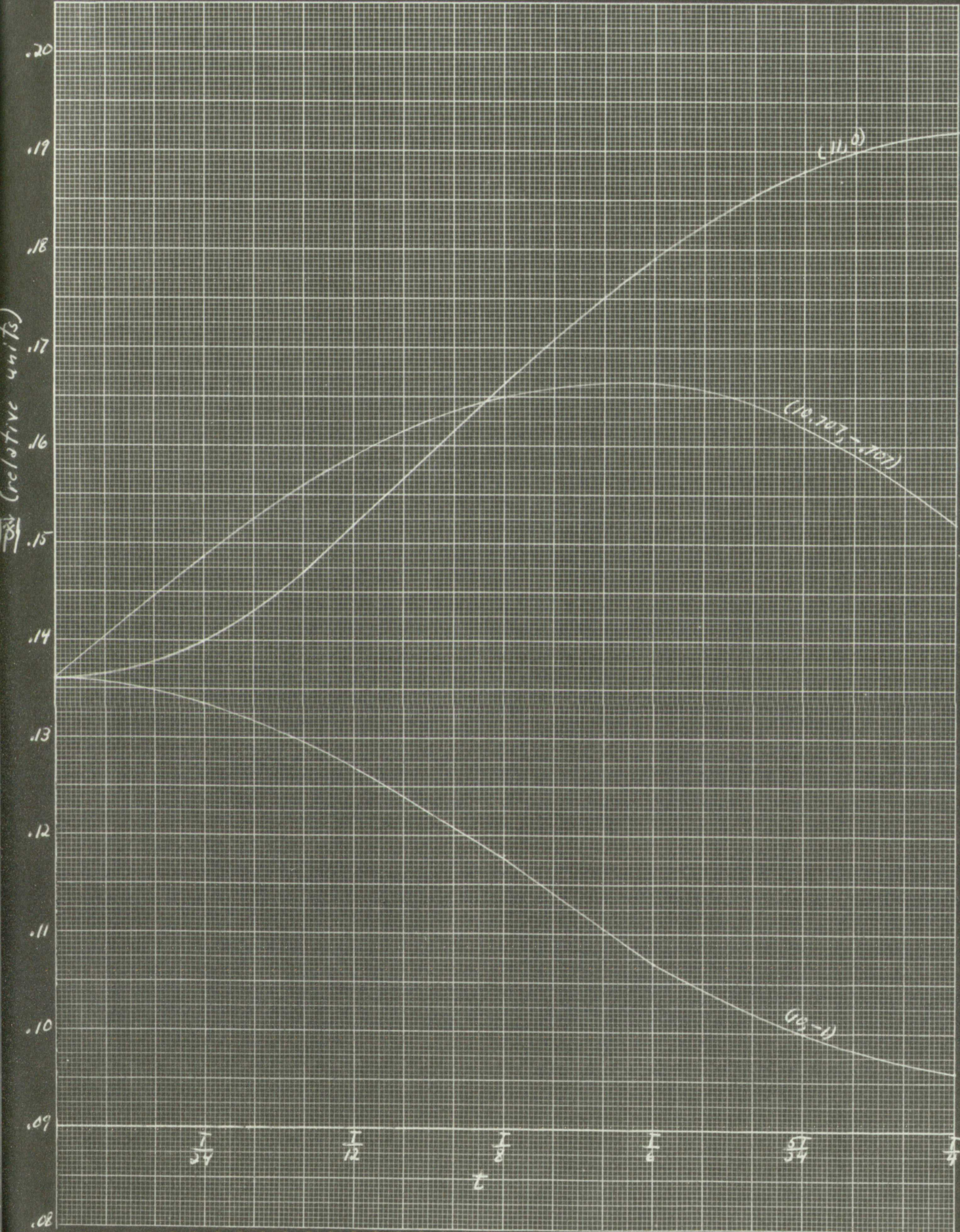
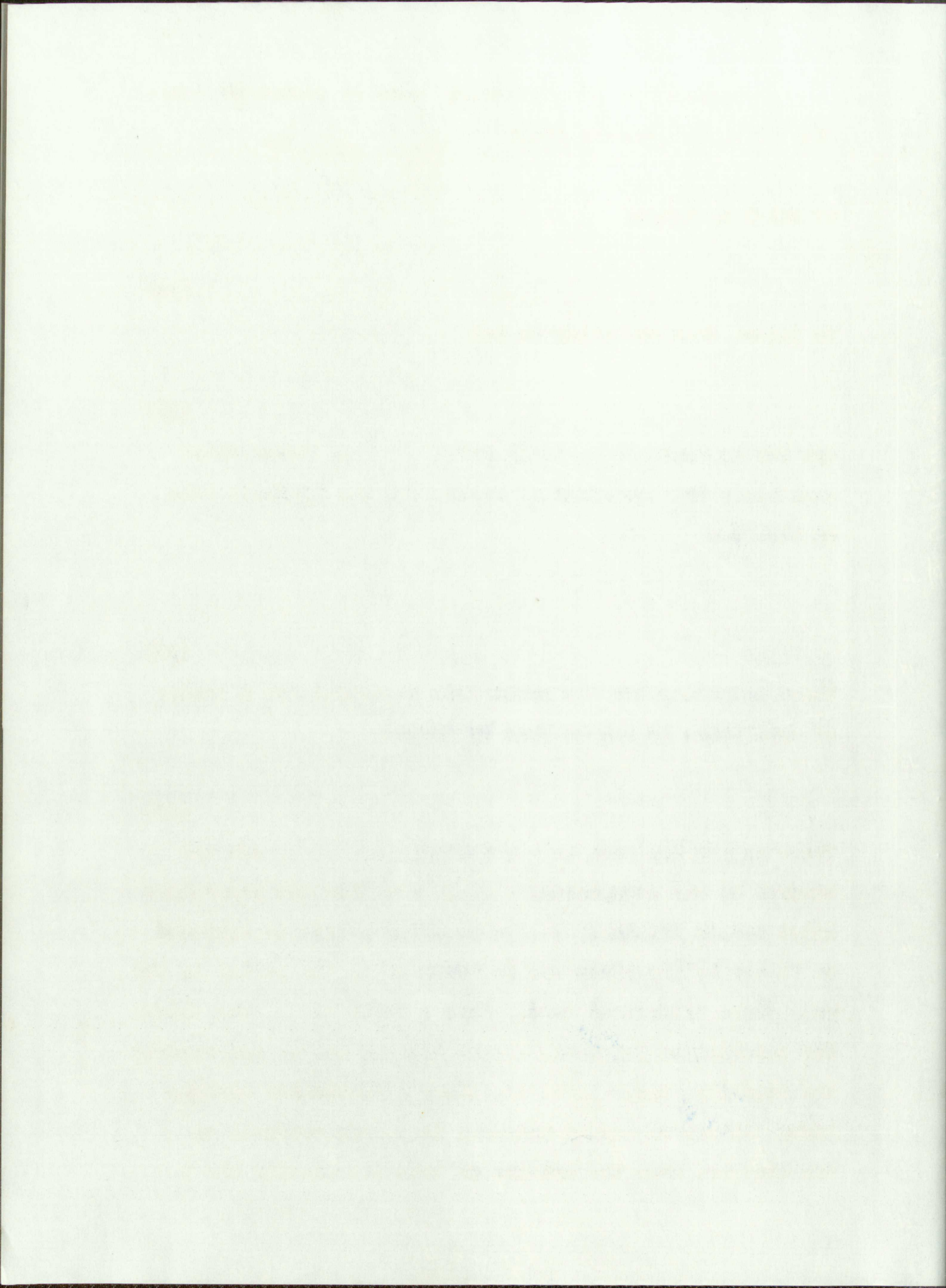


Figure 3



To investigate the hyperbolic case we return to equation (16), the general conic

$$\frac{c}{2}x^2 - \frac{b}{2}y^2 - axy + k_1x - k_2y + c = 0$$

in which we demand

$$a^2 + bc > 0. \quad (36)$$

To insure this condition we let

$$a = 0, b > 0, c > 0 \quad (37)$$

and for convenience we again let $k_1 = k_2 = 0$. Under these conditions the equations of motion for the Q_0 -fluid take on the form

$$\begin{aligned} x &= \sqrt{\frac{c}{b}} y^0 \sinh \sqrt{bc} t + x^0 \cosh \sqrt{bc} t \\ y &= y^0 \cosh \sqrt{bc} t + \sqrt{\frac{c}{b}} x^0 \sinh \sqrt{bc} t. \end{aligned} \quad (38)$$

These equations are the parametric equations for a family of hyperbolae as can be seen by forming

$$\frac{y^2}{y^0^2 - \frac{c}{b} x^0^2} - \frac{x^2}{\frac{c}{b} [y^0^2 - \frac{c}{b} x^0^2]} = 1. \quad (39)$$

These hyperbolae have an eccentricity, $e = \sqrt{1 + \frac{b}{c}}$, and are bounded by the asymptotes, $y = \pm \sqrt{\frac{c}{b}} x$. The flow for representative points initially on the positive x axis is depicted in figure 4; the times are in terms of T , the period in the comparable elliptical case. Here a ratio of $\frac{b}{c} = \frac{1}{2}$ was chosen for calculation purposes. From figure 4 we see how rapidly the velocity, i.e. Q_0 , varies along a particular streamline. As our expansion demands, it is proportional to the distance from the origin; in this calculation the y

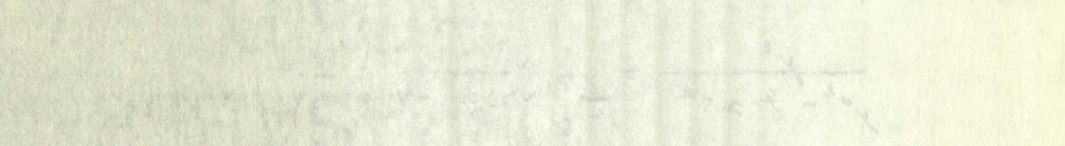
To investigate the effect of the distance between the electrodes on the rate of electrolysis, the following experiment was carried out.

$$2H_2O \rightarrow 2H_2 + O_2$$

To measure the current, a voltmeter was connected in series with the electrodes. The voltmeter was set at 2.0 V and the current was measured. The distance between the electrodes was varied and the current was recorded. The results are shown in the table below.

Distance between electrodes (cm)	Current (A)
1.0	0.15
2.0	0.25
3.0	0.35
4.0	0.45
5.0	0.55

From the table, it can be seen that the current increases as the distance between the electrodes increases. This is because the resistance of the electrolyte decreases as the distance between the electrodes increases.



These results are in agreement with the theory that the resistance of the electrolyte is inversely proportional to the distance between the electrodes. The current is directly proportional to the resistance of the electrolyte. Therefore, as the distance between the electrodes increases, the resistance of the electrolyte decreases and the current increases.

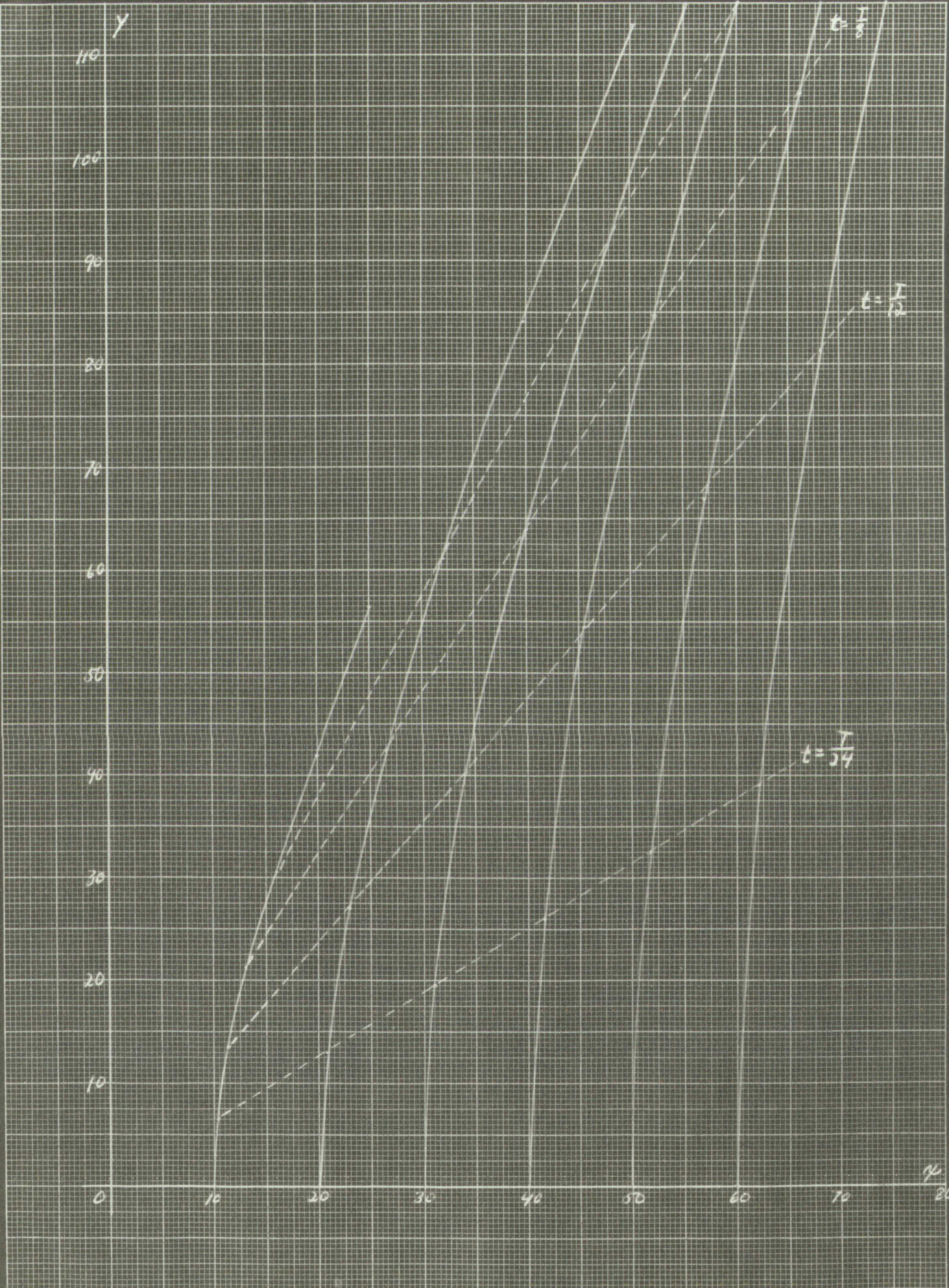
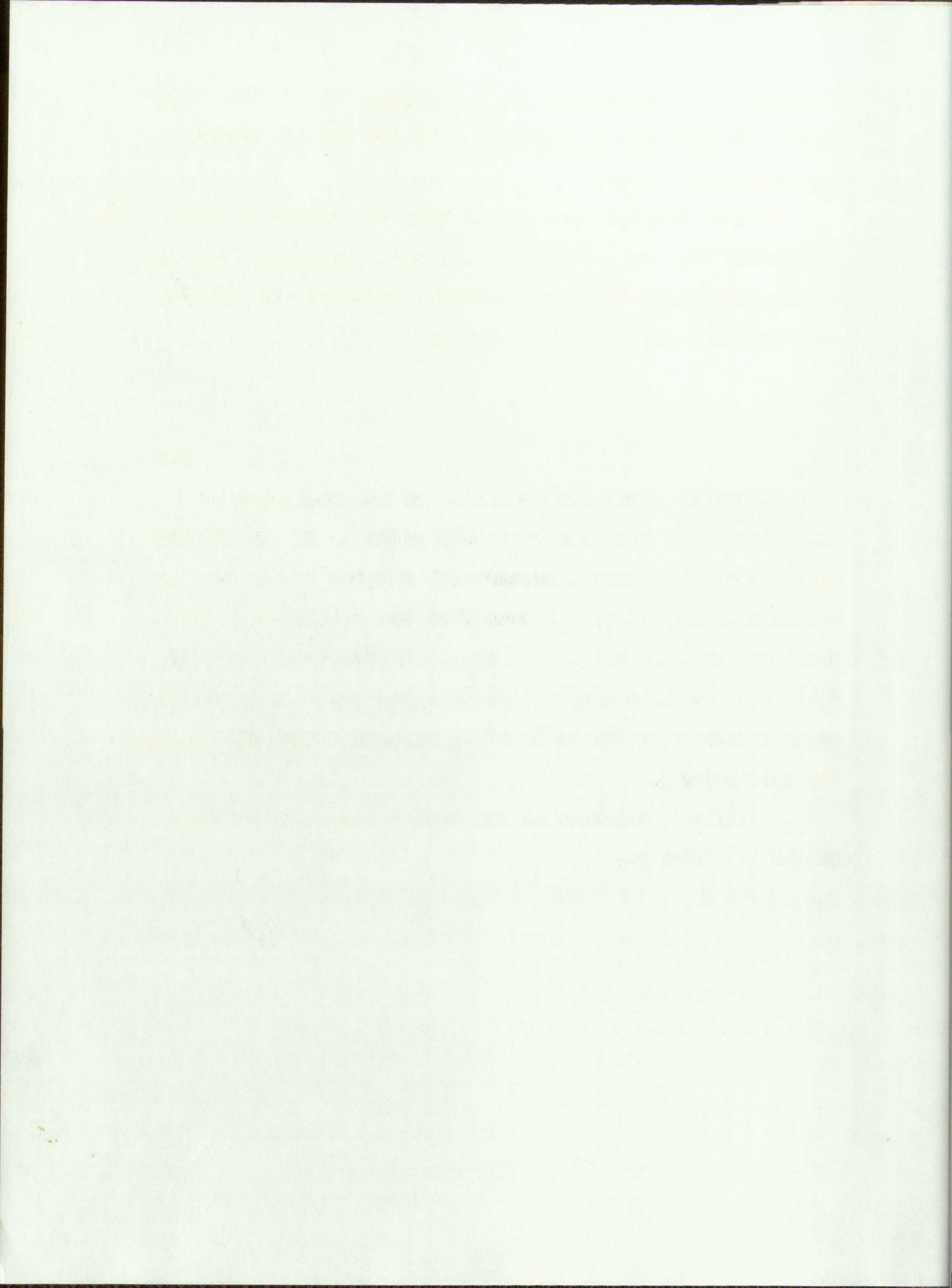


Figure 4



component of \vec{Q}_0 dominates the x component due to the ratio chosen and also due to the fact that our initial points are along the x axis.

Figures 5,6,7,8, and 9 show then the effect of this hyperbolic flow on a \vec{P} -ring of the same functional form as in the elliptical case, but centered initially at (25,0). The equations for P in this case are

$$\begin{aligned}\tilde{P}_x &= \left[-\gamma^0 \cosh \sqrt{6}ct + \sqrt{\frac{1}{6}} (x^0 - 25) \sinh \sqrt{6}ct \right] e^{-2[(x^0 - 25)^2 + y^{02}]} \\ \tilde{P}_y &= \left[-\gamma^0 \sqrt{6} \sinh \sqrt{6}ct + (x^0 - 25) \cosh \sqrt{6}ct \right] e^{-2[(x^0 - 25)^2 + y^{02}]} \quad (40)\end{aligned}$$

Figure 10 shows the variation of the magnitude of \vec{P} with time for a few representative points. In calculating these curves the common exponential damping factor was ignored; however, it can be seen that the variation in the amplitude of \vec{P} is small compared to the amplitude of \vec{Q}_0 . Also, the previous graphs show that the scale of \vec{P} remains small compared to the scale of \vec{Q}_0 in accord with our previous assumptions.

Lastly a tabulation of the \vec{P} -ring tangents for these graphs is included.

component of \vec{F} ...
 shown ...
 are along the x-axis.

Figures 1(a), (b), and (c) show the effect of ...
 hypothesis that ...
 in the elliptical case, but ...
 the equation for \vec{F} in this case are

$$\vec{F} = \left[\gamma \frac{m \vec{v}}{r^2} + (x^2 - y^2) \frac{m \vec{v}}{r^2} \right] e^{-\alpha r}$$

$$\vec{F} = \left[\gamma \frac{m \vec{v}}{r^2} - 2(x^2 - y^2) \frac{m \vec{v}}{r^2} \right] e^{-\alpha r}$$

Figure 10 shows the variation of the magnitude of \vec{F} ...
 which then for a few representative points, is calculated ...
 these curves the constant exponential damping factor was ...
 noted; however, it can be seen that the variation in the ...
 magnitude of \vec{F} is well approximated by the equation ...
 Also, the present curves show that the curve of \vec{F} ...
 small compared to the value of \vec{F} in accord with our previ- ...
 one assumption.

Lastly a tabulation of the \vec{F} -value tangents for these ...
 graphs is included.

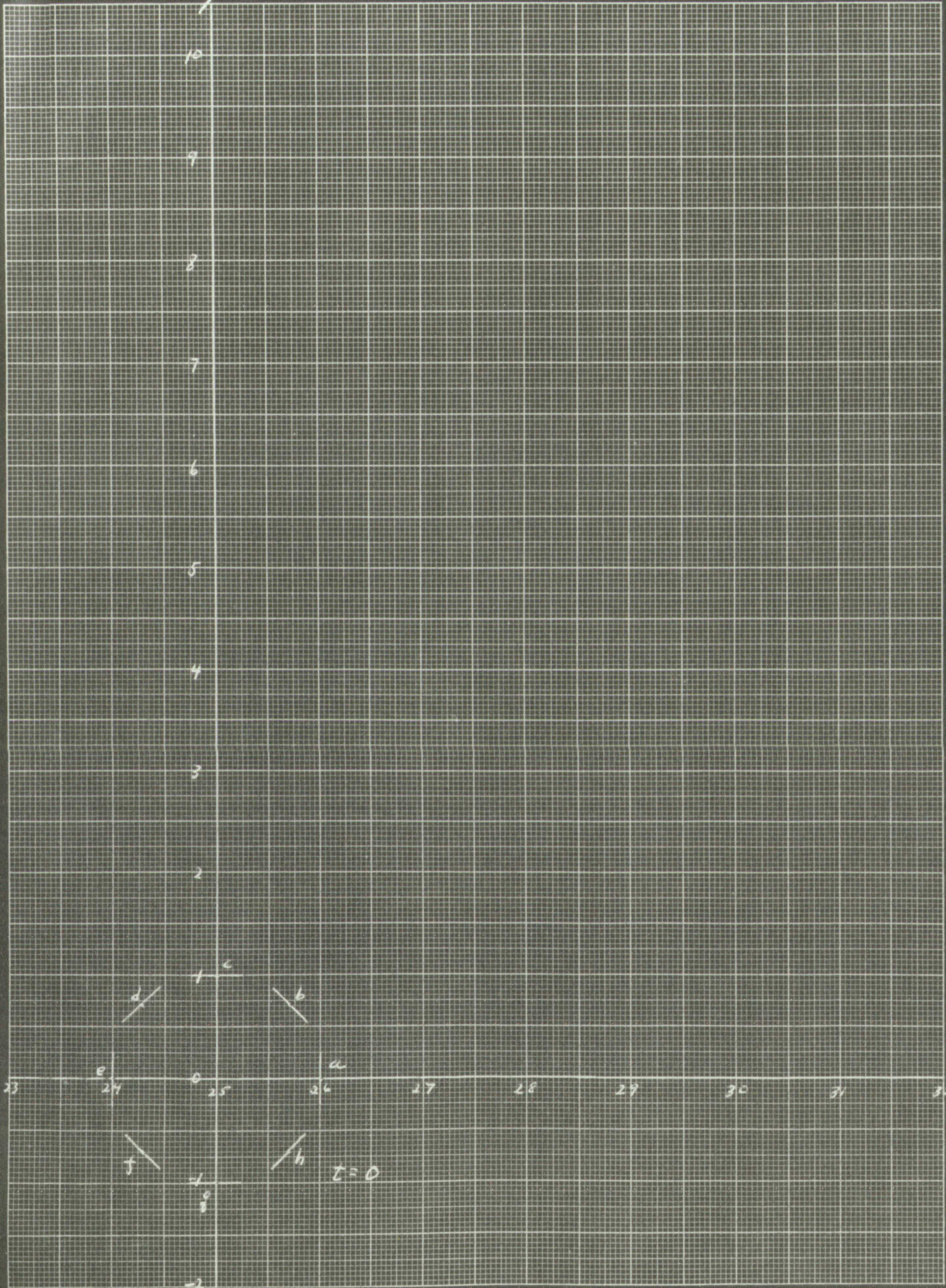


Figure 5



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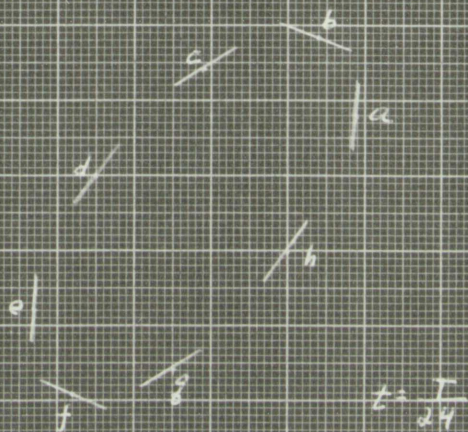
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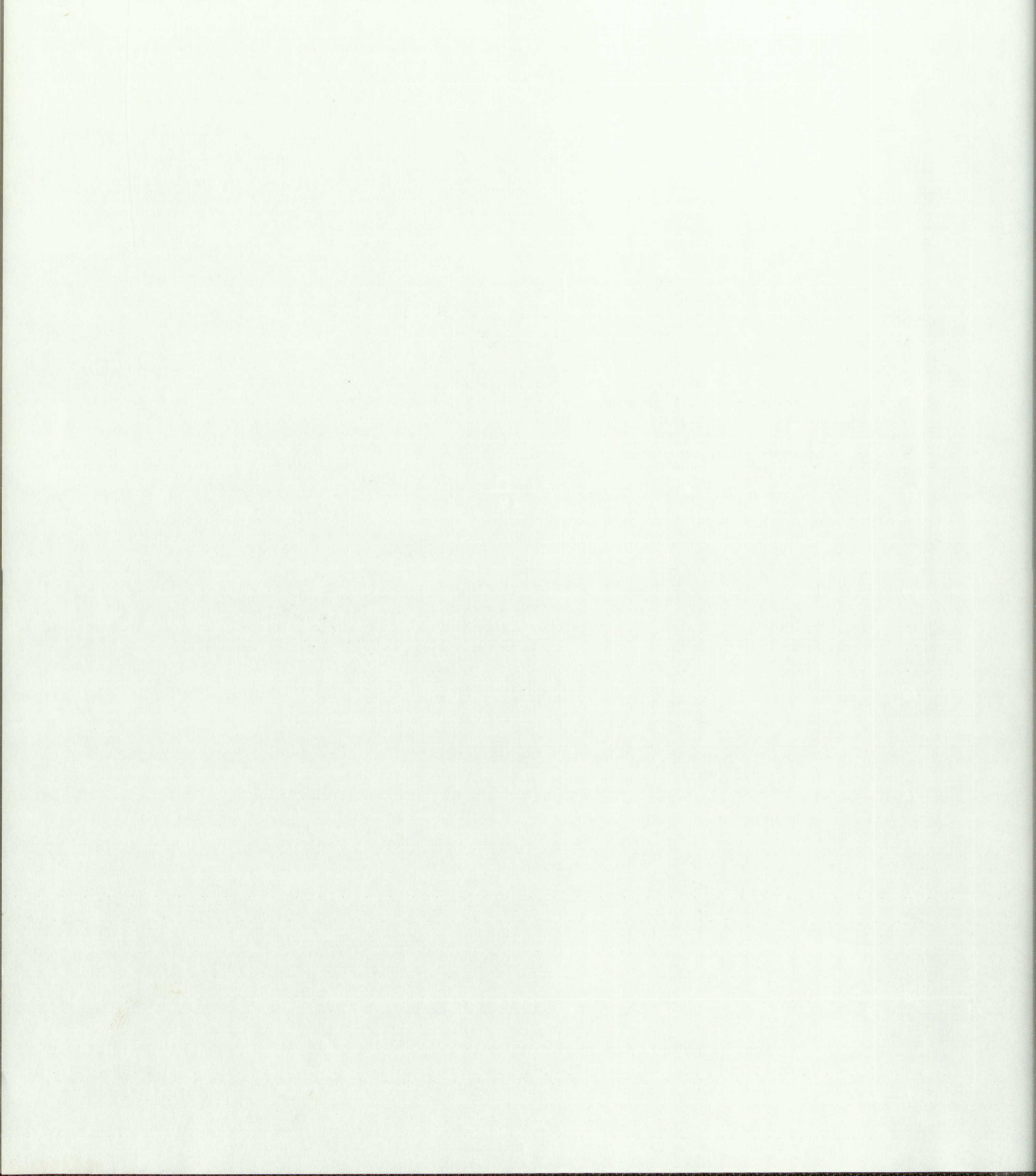
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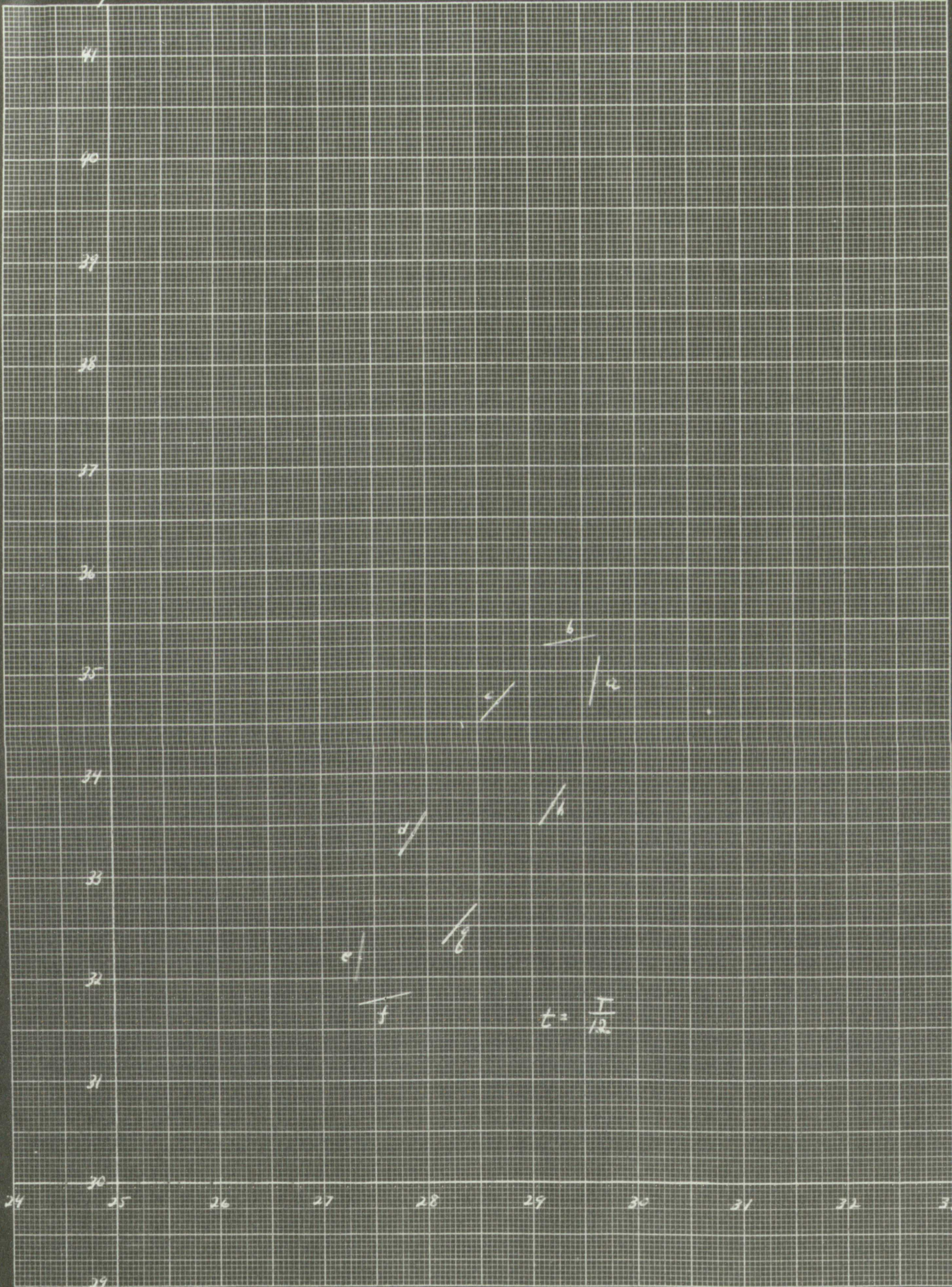


Figure 7



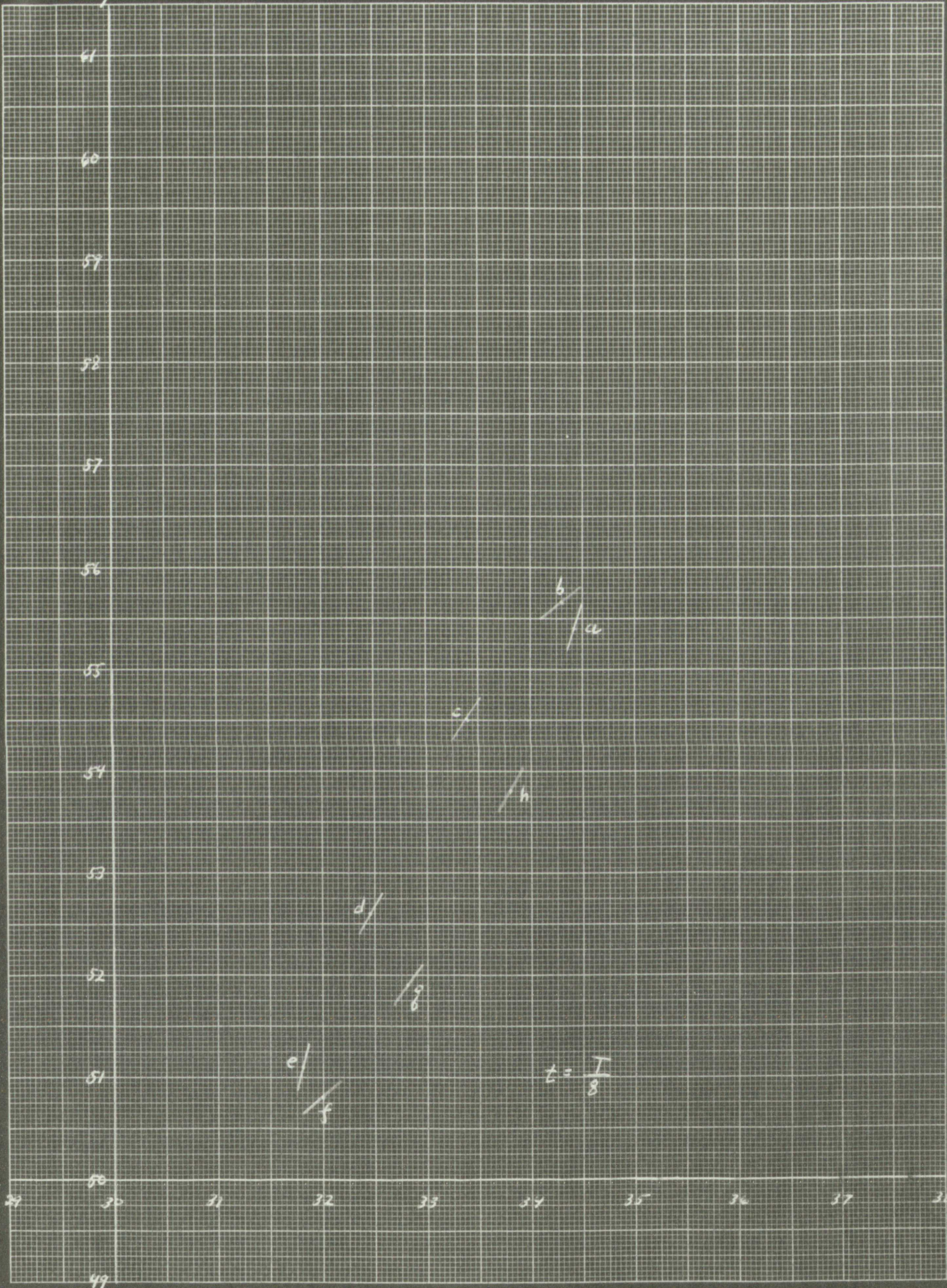


Figure 8



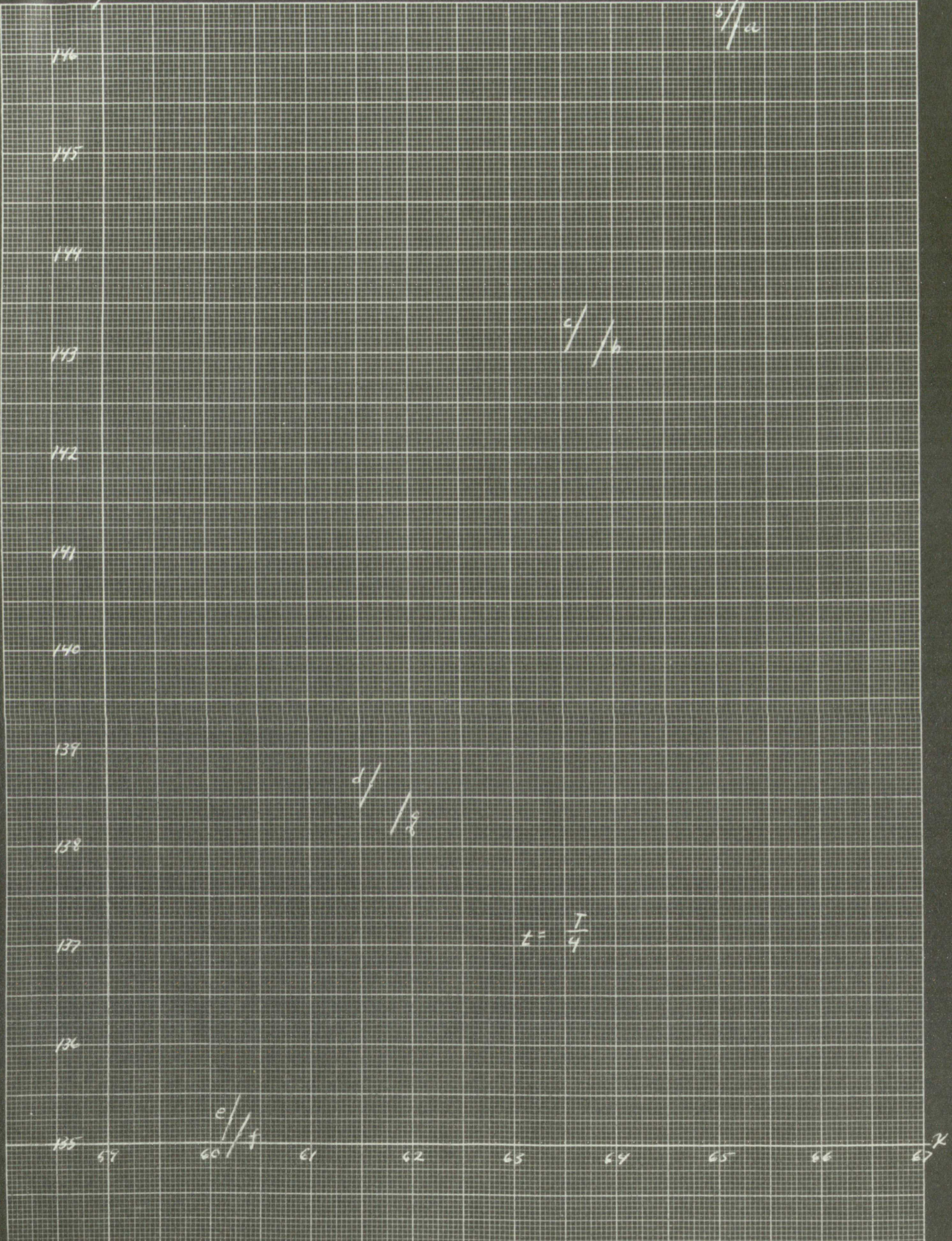


Figure 9



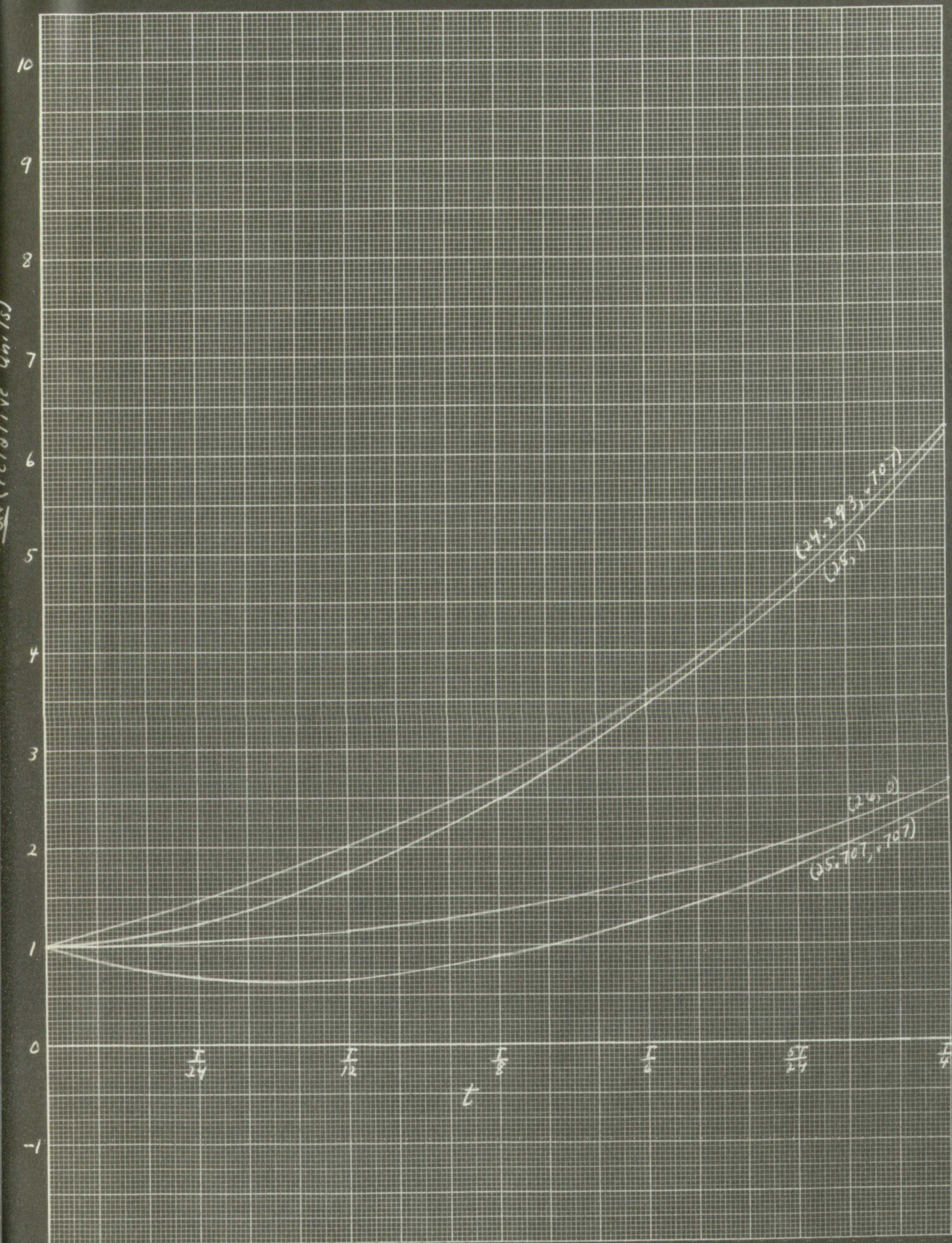


Figure 10

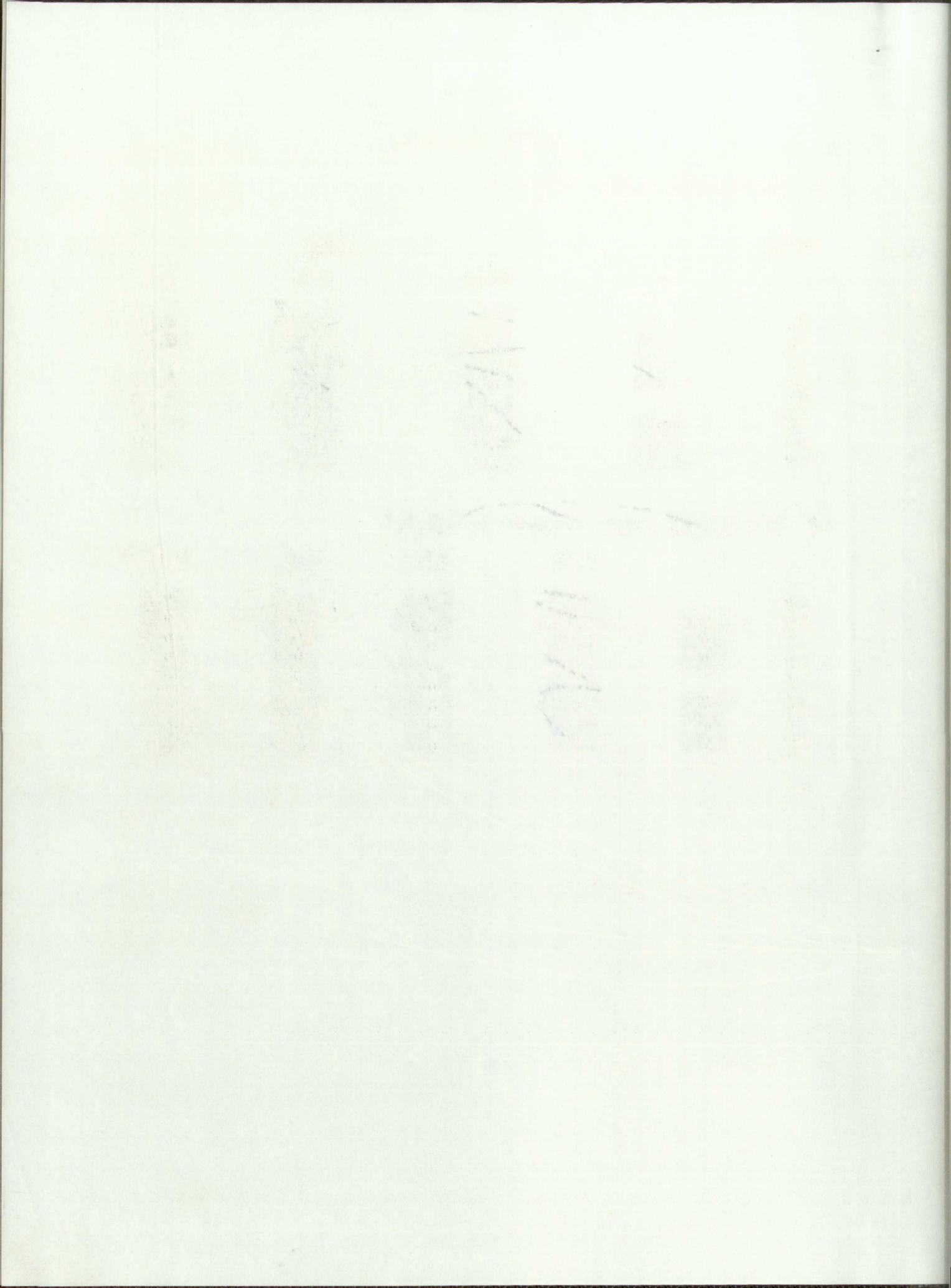


Table 1

P-ring Tangents

1. elliptical case, figure 4

<u>point</u>	<u>time</u>			
	0	T/12	T/6	T/4
a.	∞	1.23	0.410	0.00
b.	1.00	0.326	-0.066	-0.50
c.	0.00	-0.408	-1.23	∞
d.	-1.00	-7.60	1.54	0.50
e.	∞	1.23	0.410	0.00
f.	1.00	0.326	-0.066	-0.50
g.	0.00	-0.408	-1.23	∞
h.	-1.00	-7.60	1.54	0.50

2. hyperbolic case, figures 5,6,7,8,9

	0	T/24	T/12	T/8	T/4
a.	∞	9.58	5.09	3.73	2.68
b.	-1.00	-0.411	0.216	0.830	1.98
c.	0.00	0.63	1.18	1.61	2.25
d.	1.00	1.48	1.82	2.06	2.37
e.	∞	9.58	5.09	3.73	2.68
f.	-1.00	-0.411	0.216	0.83	1.98
g.	0.00	0.63	1.18	1.61	2.25
h.	1.00	1.48	1.82	2.06	2.37



1. General

total

1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00
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2. Special

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