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Application of Finite-Time Stability Concepts to the Control of ATM Networks

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Extended Abstract

1 Introduction

When dealing with the stability of a system, a distinction should be made between *classical Lyapunov Stability* and *Finite-Time Stability* (FTS) (or *Short-Time Stability*). The concept of Lyapunov Asymptotic Stability is largely known to the control community; on the other hand a system is said to be finite-time stable if, once we fix a time-interval, its state does not exceeds some bounds during this time-interval. Often asymptotic stability is enough for practical applications, but there are some cases where *large* values of the state are not acceptable, for instance in the presence of saturations. In these cases, we need to check that these unacceptable values are not attained by the state; for these purposes FTS could be used.

Some early results on FTS can be found in [9], [12] and [8]; more recently the concept of FTS has been revisited in the light of recent results coming from Linear Matrix Inequalities (LMIs) theory, which has allowed to find less conservative conditions guaranteeing FTS and finite time stabilization of uncertain, linear continuous-time systems (see [3]).

In this note we consider the problem of applying some sufficient conditions for finite time stabilization to design the control algorithm of an ATM network described via a discrete-time system.

The extended abstract is organized as follows: in Section 2 we provide a sufficient condition for finite time stabilization of a discrete time system; in

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Section 3 we detail the model of an ATM network; finally in Section 4 some concluding remarks and plans for the final version of the paper are given.

2 FTS of Discrete Time Linear Systems

Let us consider the following linear system

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$.

Given system (1), we consider the static state feedback controller

$$u(k) = Kx(k)\,, \tag{2}$$

where $K \in \mathbb{R}^{m\times n}$. The aim of this paper is to find a sufficient condition guaranteeing that the state of system given by the interconnection of system (1) with the controller (2) is *bounded over a finite-time interval.* The general idea of *finite-time stability* concerns the boundedness of the state of a system over a finite time interval for given initial conditions; this concept can be formalized through the following definition, which is an extension to discrete-time systems of the one given in [9].

Definition 1 (Finite-time stability) *The linear system*

$$x(k+1) = Ax(k) \qquad k \in \mathbb{N}_0$$

is said to be finite-time stable with respect to the given triplet (δ, ϵ, N)*, with* $N \in \mathbb{N}$*, if*

$$\|x(0)\| \le \delta \Rightarrow \|x(k)\| < \epsilon \qquad \forall k \in \{1, \ldots, N\}$$

$\triangle$

The following result, whose proof can be found in [3], is a sufficient condition which guarantees that the interconnection of system (1) with the controller (2)

$$x(k+1) = (A+BK)x(k) \tag{3}$$

is finite-time stable with respect to a given triplet (δ, ϵ, N).

Theorem 1 *System* (3) *is finite-time stable with respect to* (δ, ϵ, N) *if there exist a positive definite matrix* Q*, a matrix* L *and a scalar* $\gamma \ge 1$ *such that*

$$\begin{pmatrix} \gamma Q & (AQ+BL)^T \\ AQ+BL & Q \end{pmatrix} > 0 \tag{4a}$$

$$\mathrm{cond}(Q) < \frac{1}{\gamma^N}\frac{\epsilon^2}{\delta^2}\,. \tag{4b}$$

In this case the controller K *is given by* $K = LQ^{-1}$.

In some cases the system equation (1) is subject to an unknown disturbance $w(k)$. A more general version of Theorem 1, accounting for the presence of disturbances, will be provided in the final version of the paper.

3 The Model of the ATM Network

The transmission of multimedia traffic on the broadband integrated service digital networks (B-ISDN) has created the need for new transport technologies such as Asynchronous Transfer Mode (ATM). Briefly, because of the variability of the multimedia traffic, ATM networks seek to guarantee an end-to-end quality of service (QoS) by dividing the varying types of traffic (voice, data, etc.) into short, fixed-size cells (53 bytes each) whose transmission delay may be predicted and controlled. ATM is thus a *Virtual Circuit* (VC) technology which combines advantages of circuit-switching (all intermediate switches are alerted of the transmission requirements, and a connecting circuit is established) and packet-switching (many circuits can share the network resources). In order for the various VC's to share network resources, flow and congestion control algorithms need to be designed and implemented. The congestion control problem is solved by regulating the input traffic rate. In addition, because of its inherent flexibility, ATM traffic may be served under one of the following service classes: 1) The *constant bit rate* (CBR) class: it accommodates traffic that must be received at a guaranteed bit rate, such as telephone conversations, video conferencing, and television. 2) The *variable bit rate* (VBR): it accommodates bursty traffic such as industrial control, multimedia e-mail, and interactive compressed video. 3) The *available bit rate* (ABR): it is a best-effort class for applications such as file transfer or e-mail. Thus, no service guarantees (transfer delay) are required, but the source of data packets controls its data rate, using a feedback signal provided by switches downstream which measure the congestion of the network. Due to the presence of this feedback, many classical and advanced control theory concepts have been suggested to deal with the congestion control problem in the ATM/ABR case [4, 11]. 4) The *unspecified bit rate* (UBR): it uses any leftover capacity to accomodate applications such as e-mail.

Note that the CBR and VBR service categories, a traffic contract is negotiated at the initial stage of the VC setup, and maintained for the duration of the connection. This contract will guarantee the following QoS parameters: 1) Minimum cell rate (MCR), 2) Peak cell rate (PCR), 3) cell delay variation (CDV), 4) maximum cell transfer delay (maxCTD), and 5) cell loss ratio (CLR). This then forces CBR and VBR sources to keep their rate constant regardless of the congestion status of the network. The ABR sources on the other hand, are only required to guarantee an MCR and an PCR, and thus can adjust their rates to accomodate the level available after all CBR and VBR traffic has been accommodated. In order to avoid congestion, the ATM Forum adopted a rate-based ABR control algorithm as opposed to a credit approach whereby the number of incoming cells as opposed to their rate is controlled [7]. This paper will then concentrate on the ABR service category since ABR sources are the ones to adjust their rates using explicit network feedback. In the original ATM forum specification, an ATM/ABR source is required to send one cell called a resource management (RM) cell for every 32 data cells. Switches along the path from the source to the destination then write into the RM cell their required data rate to avoid congestion. The destination switch then has information about the minimum rate required by all switches along the VC which is then relayed back to the ATM/ABR source as a feedback signal which serves to adjust its own data rate.

The earliest control algorithms for ABR consisted of setting a binary digit in the RM cell by any switch along the VC when its queue level exceeds a certain threshold [4]. This was then shown to cause oscillations in the closed-loop system. Other controllers were then suggested by various authors [6], to address this problem. Most of these controllers are either complex or did not guarantee the closed-loop stability (in a sense defined later).

In addition, one of the limiting factors of these earlier proposed controllers was that the ABR bandwidth needed to be known in the implementation of the control algorithm. This however poses a problem in multimedia applications where the ABR bandwidth is bursty and is effectively the remaining available bandwidth after the CBR and VBR traffic have been accommodated. In [11] this particular issue was dealt with using a Smith predictor which then considered the available ABR bandwidth as an unknown disturbance. While this controller had many desirable properties, it only guaranteed stability in an appropriately defined sense but had no optimality guarantees. In addition, the delays encountered along with the number of ABR sources were assumed known, although the earlier tech report [7] did not require the delays to be exactly known. In [10], robust controllers were designed when both the number of ABR sources and the delays were uncertain.

In this paper, we consider a discrete-time model for an ATM/ABR switch and source which was presented in [6] and attempt to control the system using some extensions of the results on Finite-Time Control originally presented in [3].

As in [6] and [1], we consider the closed-loop discrete-time system:

$$Q(k+1) = \mathrm{sat}_{\{0,B\}}(Q(k) + \lambda(k-d^f) - \mu(k)) \tag{5}$$

$$R(k+1) = \mathrm{sat}_{\{0,C\}}(R(k) - \sum_{j=0}^{J}\alpha_j(Q(k-j) - Q_0) - \sum_{l=0}^{L}\beta_k R(k-l)) \tag{6}$$

where R denotes the explicit rate (ER) computed by a switch for a given VC and Q denotes the buffer occupancy of this VC at the switch. Furthermore, $\lambda(k)$ and $\mu(k)$ are the rate at the ABR source and the service rate at the switch during the interval $[k, k+1]$. d^f is the forward delay from the source to the switch and the saturation level B represents the buffer size. The saturation level C is the maximum ER. Q_0 is the desired buffer occupancy. The ABR source is greedy if the source's rate $\lambda(k-d^f)$ is equal to $R(k+1-d)$, where $d = d^f + d^b$ is the round trip delay (d^b is the feedback delay from the switch back to the source). By $\mathrm{sat}_{\{A_1,A_2\}}(\nu)$ we denote the scalar saturation function defined as:

$$\mathrm{sat}_{\{A_1,A_2\}}(\nu) = \begin{cases} A_1 & \text{if} \quad \nu < A_1 \\ \nu & \text{if} \quad A_1 \leq \nu \leq A_2 \\ A_2 & \text{if} \quad \nu > A_2 \end{cases} \tag{7}$$

In (5)-(6), the numbers J, L and the parameters α_j, β_k have to be found such that closed-loop stability and some performances are attained [5].

In [5], the author showed that when considering the linearized model of system (5)-(6) it is sufficient to consider $J = 1$ and $L = d$ in order to completely place the closed-loop poles. Hence, we consider (5)-(6) with $J = 1, L = d$ but without removing the saturation functions as done in [6]. Moreover we suppose that $\lambda(k - d^f)$ is equal to $R(k + 1 - d)$.

Let us now define both extended state and disturbance vectors

$$X(k) = \begin{bmatrix} Q(k) \\ Q(k-1) \\ R(k) \\ R(k-1) \\ \vdots \\ R(k+1-d) \\ R(k-d) \end{bmatrix} \in \mathbb{R}^{d+3} \ ; \ W(k) = \begin{bmatrix} \mu(k) \\ Q_0 \end{bmatrix} \in \mathbb{R}^2 \tag{8}$$

and define the following matrices

$$\mathbb{A} = \begin{bmatrix} 0 & 0 & ... & ... & 0 & 0 & 0 \\ 1 & 0 & ... & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & ... & ... & 0 \\ 0 & 0 & 1 & 0 & ... & ... & 0 \\ \vdots & ... & ... & \ddots & ... & ... & \vdots \\ \vdots & ... & ... & ... & \ddots & ... & \vdots \\ 0 & ... & ... & ... & ... & 1 & 0 \end{bmatrix} \ ; \ \mathbb{B}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ \vdots & \vdots \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \ ; \ \mathbb{B}_2 = \begin{bmatrix} -1 & 0 \\ 0 & \alpha_0 + \alpha_1 \end{bmatrix}$$
$$\mathbb{G} = \begin{bmatrix} 1 & 0 & ... & ... & 0 & 1 & 0 \\ -\alpha_0 & -\alpha_1 & 1 - \beta_0 & -\beta_1 & ... & ... & -\beta_d \end{bmatrix} = \begin{bmatrix} \mathbb{H} \\ \mathbb{K} \end{bmatrix} \tag{9}$$

Hence, from (8) and (9), the system under consideration reads:

$$X(k+1) = (\mathbb{A} + \mathbb{B}_1\mathbb{G})X(k) + \mathbb{B}_1\mathbb{B}_2W(k) + \mathbb{B}_1\Psi(X(k), W(k)) \tag{10}$$

with

$$\Psi(X(k), W(k)) = \mathrm{sat}_{\{0,B\},\{0,C\}}(\mathbb{G}X(k) + \mathbb{B}_2W(k)) - (\mathbb{G}X(k) + \mathbb{B}_2W(k)) \tag{11}$$

where

$$\mathrm{sat}_{\{0,B\},\{0,C\}}(\nu) = \begin{bmatrix} \mathrm{sat}_{\{0,B\}}(\nu_{(1)}) \\ \mathrm{sat}_{\{0,C\}}(\nu_{(2)}) \end{bmatrix} \tag{12}$$

Considering the nonlinear system (10), the problem we aim to solve throughout this paper may be summarized as follows.

Problem 1 *Determine a matrix $\mathbb{K}$, a set of admissible initial conditions S_0 and a set of admissible disturbances D_0 such that:*

1. *The closed-loop matrix $\mathbb{A} + \mathbb{B}_1\mathbb{G}$ is asymptotically stable.*

2. *The closed-loop trajectories remain bounded for any $X(0) \in S_0$ and any admissible disturbance $W(k) \in D_0$, $\forall n$.*

3. *The steady state buffer occupancy is equal to the desired threshold Q_0.*

It is important to note that the satisfaction of point 3 in Problem 1 forces us to study the existence of possible equilibrium points corresponding to the case $W(k) = W_e$ where W_e is a constant value. It is easy to check that an equilibrium point X_e for system (10) satisfies

$$R_e(k-l) = R_e(k-l+1) = R_e = \mu_e(k) = \mu_e, \; l = 0, 1, ..., d \tag{13}$$

$$Q_e(k) = Q_e(k-1) = Q_e = Q_0 \tag{14}$$

$$\sum_{l=0}^{d} \beta_l R_e = \sum_{l=0}^{d} \beta_l \mu_e = 0\,. \tag{15}$$

Since, in general μ_e is not equal to 0, condition (15) implies that

$$\sum_{l=0}^{d} \beta_l = 0 \tag{16}$$

The satisfaction of this equality implies that β_d is computed from the last d entries. In other words, one can write:

$$\beta_d = -\sum_{l=0}^{d-1} \beta_l \tag{17}$$

As a consequence of (17), we have that the vector $\mathbb{K}$ in (9) can be written as

$$\mathbb{K} = \begin{bmatrix} -\alpha_0 & -\alpha_1 & 1-\beta_0 & -\beta_1 & ... & ... & \sum_{k=0}^{d-1} \beta_k \end{bmatrix} \tag{18}$$

The above observations lead us to an alternative representation of model (5)-(6) and therefore of model (10). At this aim, consider the following vectors

$$Y(k) = \begin{bmatrix} Q(k) - Q_0 \\ Q(k-1) - Q_0 \\ R(k) - \mu_e \\ R(k-1) - \mu_e \\ \vdots \\ R(k-d) - \mu_e \end{bmatrix} \in \mathbb{R}^{d+3} \text{ and } \nu(k) = \mu(k) - \mu_e \in \mathbb{R} \tag{19}$$

which correspond to a change of variables around the equilibrium point X_e, and define the matrix

$$\mathbb{B}_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \mathbb{R}^{d+3} \tag{20}$$

¿From (19) and (20), the closed-loop system under consideration reads:

$$Y(k+1) = (\mathbb{A} + \mathbb{B}_1\mathbb{G})Y(k) + \mathbb{B}_3\nu(k) + \mathbb{B}_1\Psi(Y(k), \nu(k)) \tag{21}$$

with

$$\Psi(Y(k), \nu(k)) = \mathrm{sat}_{\{-Q_0, B-Q_0\},\{-\mu_e, C-\mu_e\}}\left(\mathbb{G}Y(k) + \begin{bmatrix} -1 \\ 0 \end{bmatrix}\mu(k)\right) - \left(\mathbb{G}Y(k) + \begin{bmatrix} -1 \\ 0 \end{bmatrix}\mu(k)\right)$$

where the saturation is defined similarly to (12).

4 Control Design via FTS Theory

In order to solve Problem 1 we will make use of the change of variables that leads the system in the form (21), "centered" around the equilibrium point. Then we will guarantee that the system operates in linear regime, i.e. the saturation never occurs, by imposing that the arguments of the saturation functions never exceed the limits. This is accomplished using the concept of Finite-Time Stability defined in Section 2.

In the final version of the paper we will show how Theorem 1 can be applied to our case, giving a solution to Problem 1. Some simulation results will be also included.

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