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## On The Control of a High Power Backward-wave Oscillator Using Quantifier Elimination Methods

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### Abstract

This paper presents an experimental/theoretical study of methods to identify and control a repetitively-pulsed high power microwave source. A neural network was used to model the system and Quantifier Elimination (QE) theory is used to search for suitable operating conditions.

### 1. Introduction

Although many physicists are aware of control theoretical results, most of their experiments are still being controlled with classical methods. Due to the complexity of obtaining a Physics-based model of high power BWOs, researchers

utilize fully electromagnetic particle-in-cell (PIC) codes like MAGIC [1] in order to simulate certain aspects of the operation of these devices. In this paper, we choose instead to build a model based on the input/output data with the physics providing guidance but little influence. In this paper, we report on the progress of a project which combines a physics experiment along with identification methods and modern control approaches. The experiment is known as the Sinus-6 electron beam accelerator-driven backward wave oscillator (BWO). The ultimate objective of the project is to design a controller that will maximize both the power and the efficiency, or to keep a constant power across large frequency variations.

In this paper we use QE software in order to search for the maximum power and efficiency. The paper is organized as follows. In section 2. we present our experiment and our data collection. In section 3. we present an overview of QE, and QEPCAD, the software used in solving our problem along with our results. Our conclusions are given in 4.

### 2. Modeling and Data Collection

A block diagram of the experimental set-up is shown in Fig. 1. The model of the high power BWO consists of an A-K gap (electron gun) delivering an intense electron beam current  $I$  that is guided through a slow wave structure by a strong axial magnetic field. Initial experimentation with this problem has been reported in [2], and

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has yielded input/output data which is used in this research. The experimental data was collected in four separate experiments, where the A-K gap was adjusted to four different values. The A-K gap determines the electron beam diode impedance. We shall denote these four experiments as  $E_1, E_2, E_3$ , and  $E_4$ . The four intervals were divided into 95 sampling points for the first experiment, 102 sampling point for the second experiment, 78 sampling points for the third experiment, and 43 sampling points for the fourth experiment. The experimental data consists of two inputs, the cathode voltage  $u_1 = V$ , and the current  $u_2 = I$ , and the two outputs, total peak power  $y_1 = P$ , frequency  $y_2 = F$ . The RF generation efficiency  $z$  was calculated from :  $z = \frac{y_1}{u_2 \times u_1} = \frac{P}{V \times I}$ . A

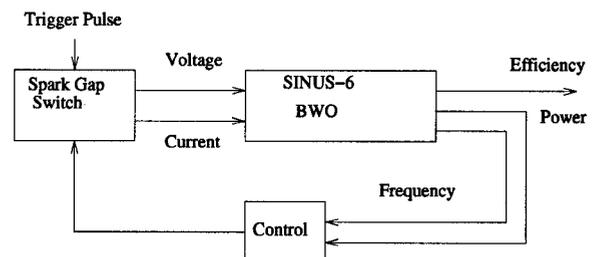


Figure 1: Block Diagram Description.

neural network approach has been used to fit the experimental input/output data for the Sinus-6 BWO. Since the Sinus-6 is extremely fast to warrant the inclusion of dynamical effects, and since the sampling interval in the experiment data is not fixed, a static, continuous neural network model is used to fit the experimental data. The nonlinear model we obtained in [3] is affine, static, and given by

$$\begin{bmatrix} F \\ P \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad (1)$$

where coefficients  $a_{ij}$  and  $b_i$  are different for different experiments. The control objective in this study is to simultaneously make both power and efficiency as large as possible. For similar static systems and control issues that arise with rapid thermal processing see [4]. This model was then used in conjunction with quantifier elimination (QE) in order to design a controller which will optimize the performance of the system.

### 3. Solution With QEPCAD

Algorithms for solving general QE problems were first given by Tarski [5] and Seidenberg [6], and are commonly called Seidenberg-Tarski decision procedures. Tarski showed that QE is solvable, but his algorithm and later modifications are exponential in the size of the problem. Researchers in control theory have been aware of Tarski's results and their applicability to control problems since the 1970's but the tedious operations made the technique very limited [7]. More recently Hong [8] has introduced a significantly more efficient partial CAD QE algorithm. In our work we use the Hong's implementation of the CAD algorithm called QEPCAD. The CAD algorithm always completely solves any QE problem. However, the computational cost is extremely high. Our experience indicates that QEPCAD can always solve, in a few seconds on a large workstation, most textbook examples. It can also solve some significantly harder problems and a few non-trivial problems. It is therefore important to simplify the QE problem as much as possible before using QEPCAD.

In order to use QEPCAD we then proceed to convert the performance objectives to a constrained optimization problem as follows.

$$V_{\min} \leq V \leq V_{\max}, I_{\min} \leq I \leq I_{\max} \left\| \begin{array}{l} w_1 P \\ w_2 \frac{P}{V I} \end{array} \right\|_{\infty} \quad (2)$$

where  $I_{\min} \leq I \leq I_{\max}$  and  $V_{\min} \leq V \leq V_{\max}$ , and equation (1) holds. It turns out to be more efficient to reformulate our performance objectives as,

$$J = \min_{V_{\min} \leq V \leq V_{\max}, I_{\min} \leq I \leq I_{\max}} \left( \left\| \frac{v_1}{P} \right\|_2 + \left\| v_2 \frac{VI}{P} \right\|_2 \right) \quad (3)$$

and let  $v_1 = w_1^2$ ,  $v_2 = w_2^2$ , and  $v_1 + v_2 = 1$ . The existence question of input variables ( $V$  and  $I$ ) is the truth of quantified statement  $\exists(V, I)[F_1(V, I) \wedge F_2(V, I) \wedge F_3(V, I) \wedge F_4(V, I)]$  where  $F_i$ ;  $i = 1, 4$  are polynomial equalities and inequalities which correspond to the optimization problem and its constraints. When this quantified formula was entered into QEPCAD, "true" was returned for some values of  $J$  and "false" was returned for others (smaller values). To find the optimal value  $I^*$ , the following question is asked for QEPCAD for some value of  $J$  to which we know a solution exists,  $\exists(V)[F_1(V, I) \wedge F_2(V, I) \wedge F_3(V, I) \wedge F_4(V, I)]$ . QEPCAD software produced the results in Table 1 for experiment  $E_1$ , and for different  $v_1, v_2$  combinations. We can see that with  $I$  decreasing and  $V$  decreasing, the power ( $P$ ) is decreased but the efficiency ( $E$ ) is increased.

### 4. Conclusions

In our future work, we will use a feedback controller as shown in Figure 1 to keep the outputs  $P$  and  $E$  at their maximum values, by regulating the inputs to their optimal values  $I^*$  and  $V^*$ .

$v's$	$I^*$	$V^*$	$F^*$	$P^*$	$E^*$
$0 \leq v_2 \leq 3E^{-8}$	6	700	9.936	540.398	0.129
$v_2 = 4E^{-8}$	6	532	9.883	495.112	0.155
$v_2 = 5E^{-8}$	6	426	9.85	466.539	0.182
$6E^{-8} \leq v_2 \leq E^{-6}$	6	375	9.834	452.791	0.201
$v_2 = 2E^{-6}$	3.50	375	9.688	291.373	0.222
$3E^{-6} \leq v_2 \leq 1$	2.75	375	9.645	242.948	0.236

Table 1: Results of QEPCAD for experiment  $E_1$

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