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Conceptions of First-Year Secondary Mathematics Intern Teachers

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CONCEPTIONS OF FIRST-YEAR SECONDARY MATHEMATICS INTERN TEACHERS

BY

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B. S., Biological Science, Fort Lewis College, 1975

M. A., Secondary Education, University of New Mexico, 2005

DISSERTATION

Submitted in Partial Fulfillment of the
Requirements for the Degree of

Doctor of Philosophy

Multicultural Teacher and Childhood Education

The University of New Mexico
Albuquerque, New Mexico

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DEDICATION

For Alice

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ABSTRACT

Beginning mathematics teachers enter their classrooms with a network of conceptions about mathematics and how they will go about teaching it. These conceptions are based on a set of beliefs and body of mathematical knowledge. They are robust and deeply ingrained, and they influence how teachers approach the teaching and learning of mathematics. When teacher educators examine the relationship between teachers' conceptions and their practices, they are able to better understand how teachers define their practice. This understanding, then, can inform their work in preservice and inservice teacher education.

This study focused on the conceptions of three first-year secondary mathematics intern teachers and examined how these conceptions were manifested in their classroom practices. Using case study methodology, I observed the particpants in their classrooms weekly, and they participated in ongoing mentorship during the year. Their university mathematics methods course provided opportunities for them to engage in reflective discourse regarding any connections made between their experiences in the course and in their classrooms. The data collected was analyzed to document the teachers' conceptions

and how these conceptions were influenced and reshaped by their classroom, mentoring, and methods course experiences.

The participants initially entered their classrooms and taught in the traditional manner of lecture/demonstration of new problems. Then students practiced similar problems as the teacher walked around and offered help. This routine persisted for one of the participants throughout the year, but for two participants, classroom events triggered a reaction to the status quo of this classroom routine. A different set of conceptions were manifested, and with the support of mentoring and the mathematics methods course, these two participants implemented changes in their practices.

The findings of this study suggest that beginning mathematics teachers enter their classrooms with a set of implicitly acquired conceptions of mathematics from their personal experiences as learners of mathematics. The findings also suggest that when their conceptions were classroom tested and challenged during their methods course and mentorship, these teachers were able to make explicit the basis of their conceptions, expose them to critique and analysis, and reshape them or develop new conceptions.

TABLE OF CONTENTS

Page

LIST OF FIGURES

Page

LIST OF TABLES

Page

Chapter I

Introduction

> When we no longer know what to do, we have come to our real work. When we no longer know which way to go, we have begun our real journey. The mind that is not baffled is not employed. The impeded stream is the one that sings.
>
> Wendell Berry (1999, p. iv)

Emily (pseudonym), a first-year secondary mathematics intern teacher, reflected on her journey as she began her teaching career:

> In my first weeks of teaching, I followed the textbook curriculum one chapter section at a time. Homework was given from the textbook or the little workbook that came with the text. When my intern partner and I covered the first chapter, we decided to give the first test. I wanted to give a more challenging test or at least throw in some word problems just to see what the kids were capable of, but Matt (pseudonym), my intern partner, disagreed. We ended up picking problems from the workbook to make a test, at his suggestion. I hated it! Grading the test was even worse. If there were three problems from each section that required the same way of solving, then wouldn't doing two of three correctly have been sufficient to show the student understands the concept? Basically, the test was just busy work for students.

Two weeks passed with me doing the same lecture that Matt did, giving the same homework, and discarding my own intuitions. I was afraid to stray away from the "traditional" way of teaching, believing that that's how math was taught here in America. It really started to bother me when one of the teachers that teaches math in the same school showed me that there is a test that comes with the textbook that teachers use. There was a multiple choice test in three levels of difficulty; it was up to the teacher to choose which test would be more suitable. Never in my life did I take a multiple-choice test in any of my math classes. The only multiple choice tests that I did take to test out of the class I found utterly useless for showing mathematical knowledge of a person. As a student I found these multiple choice tests a joke.

Yet here I was teaching math the way I never thought I would, believing that there is no other way to teach the standards. How could I forget the way I tutored math with props and connections to the "real" world?

After talking to my mentor, the very next day I had my students sitting in groups of five or six. Together students had to solve a long three-part word problem, sort of a mathematical riddle that probably did not cover any standards. Every adult told me it was too difficult of a problem to give seventh graders, even if it didn't require anything more than having skills in adding numbers. It took me a long time to prepare for

> this group activity but once I gave it to the students they all jumped right in and worked on solving the problem together. The only thing I needed to do was hang back and check their work once they solved one of the parts of the problem. Some of the students found solutions to the riddle that I never even saw or thought possible. It was amazing. I had a smile on my face all day. (Reflective Writing, September 2007)

In this vignette, Emily was uncertain about how to incorporate her ideas into her mathematics lessons while conforming to a set of classroom norms that had been set up by her teaching partner, Matt. To resolve this inner conflict, Emily made a conscious decision to mimic his lessons containing elements of lecture, demonstration, and problem sets from the textbook. But after only a few weeks, Emily opted to break away from her reliance on her partner's lessons. While she still vacillated between using Matt's lessons and the few activities she had developed from her previous educational experiences, one of Emily's early beliefs about the teaching and learning of mathematics remained intact; students learn more effectively if they have opportunities to work on mathematical problems collaboratively and justify their ideas and thinking. But in the early weeks of her teaching, Emily tolerated worksheet-driven lessons her students were seemingly "comfortable" with and lamented that the more challenging problem-based activities that demanded teamwork, reasoning, and communication were rare. Berry's (1999) description of not knowing in the opening quote pinpointed Emily's dilemma: she was baffled and frustrated and felt her efforts were being impeded by a set of classroom norms she had little part in creating.

This short vignette describing Emily's first weeks in the classroom is not so different from other first-year mathematics teachers. Many enter their classrooms with images about the mathematics students will learn and how they will learn it (Ball, 1988; Eisner, 1992; Kagan, 1992). These images, based on a set of beliefs and a body of mathematical knowledge, manifest themselves in day-to-day practices in their classroom. This did not always happen in Emily's classroom, and it does not always go smoothly for many beginning mathematics teachers. Their initial experiences in mathematics teaching and learning do not always align with their *conceptions* regarding mathematics (Ernest, 1988). These differences can make classroom teaching difficult and frustrating and may take time to reconcile.

Statement of the Problem

Many young adults, long before they enroll in their first education course, have developed a web of interconnected ideas, or conceptions, about mathematics, about teaching and learning mathematics, and about schools (Ball, 1988). When these young adults become teachers and assume classroom responsibilities, they tend to hold primarily the same conceptions before they enter their university teacher education program as they do when they exit the program (Kagan, 1992). Their deeply embedded conceptions of mathematics and its teaching and learning align primarily with conceptions that were promoted and accepted in the United States in the early part of the 20th century (Darling-Hammond, 2006). At that time, behavioral learning theory, coupled with bureaucratic policies, sought to control teachers' behaviors in the classroom. As a result, mathematics teachers in U.S. classrooms adopted lecture and demonstration strategies with the expectation that their students would independently practice what was demonstrated.

This lecture and demonstration practice of teaching and learning mathematics conflicts with current constructivist and social learning theories advocated by the National Council of Teachers of Mathematics (1989, 2000), the National Science Foundation (1996), and the U.S. Department of Education (2000), but it is culturally embedded in U.S. mathematics classrooms; that is, people have learned the activity of teaching through informal participation over long periods of time (Stigler & Hiebert, 1997). This makes it very difficult to change these conceptions in a short-term teacher education program.

Purpose of the Study

The purpose of this study was to examine the mathematical conceptions (beliefs and knowledge) of three first-year secondary mathematics intern teachers and how these conceptions were manifested in their classrooms. Researchers and educators have long recognized that teachers' interpretations and implementation of curricula are influenced significantly by their conceptions (Peterson & Clark, 1986; Romberg & Carpenter, 1986) and their conceptions of mathematics greatly influence how they approach the teaching and learning of it (Ma, 1999; Philipp, 2007; Thompson, 1992). With this in mind, to understand teaching from teachers' perspectives, researchers need to understand the beliefs and knowledge with which teachers define their work (Nespor, 1987). Examining teachers' conceptions about the relationship between mathematics and the teaching and learning of mathematics provides the means to understand their actions, experiences, and their interpretations of the day-to-day events that occur in their classrooms. Especially during this early time in their careers, as in the cases of the participants in this study, beginning teachers' conceptions and practices can be tested and challenged, perhaps reconstructed, and old beliefs may be refined or give way to new ones (Tobin, Tippins, & Hook, 1992).

Research Questions

The research questions that guided this study are:

(1) What initial conceptions regarding the teaching and learning of mathematics do secondary mathematics intern teachers reveal in their first year of teaching?

(2) How do these conceptions change during their first year?

a) What influence does a year-long mentoring process have on the interns' conceptions?

b) What role does a university mathematics methods course have in supporting and restructuring the interns' conceptions?

The remainder of this chapter is organized to introduce the reader to the context of this study: the participants in the study and the university alternative licensure program in which they were enrolled. Also, to help provide some historical context regarding my own conceptions and the role they could potentially play in this study, I share some background on my work as a secondary mathematics teacher and my current job that entails teaching mathematics methods courses and preparing, supporting, and mentoring secondary mathematics intern teachers. Then, I briefly consider the research literature that informs the research questions. I will initially introduce what the literature says about the philosophical perspectives teachers have regarding the nature of mathematics. I continue by examining how mathematics reform efforts in the last two decades have influenced teachers' conceptions regarding the teaching and learning of mathematics. This is followed with a look at the relationship between beginning teachers' conceptions of mathematics and their classroom practices. I end with a brief look at how conceptions of beginning teachers of mathematics can be influenced, restructured, and even enriched. I conclude this chapter with a look at the importance of a study of this kind, any assumptions I have regarding it, a brief overview of the study, and a list of defined terms used throughout this chapter and those that follow.

Context for the Study

SMEST Program

Emily, Matt, and John are participants in a university alternative licensure program called SMEST (pseudonym), Science and Mathematics Education of Secondary Teachers. This 14-month program provides opportunities for its post-baccalaureate participants to earn a state level-I teaching license in mathematics. Prior to entering the classroom in the fall semester, participants complete a summer pre-service course of study and field experience in a local middle or high school. Upon successful completion, the participants become interns and are granted a provisional license by the state public education department. They are assigned as paired teacher-teams and share a full time secondary teaching position with the public schools. During both fall and spring semesters, interns receive mentoring support in the classroom and continue to take coursework related to curriculum, methodology, and content pedagogy. At the end of this

14-month experience, interns become eligible to apply for a standard teaching license and take on the role of an individual classroom teacher.

Participants

The three participants in this study, Emily, Matt, and John, are between 26 and 30 years of age. Emily is a White Anglo-European immigrant who came to the United States from Bosnia twelve years ago. As an English language learner, Emily is somewhat self-conscious about her accent, but she approaches life vivaciously and with passion. Her teaching partner, Matt, is a White Anglo-American and a product of the same public schools in which he is now employed. He is very intelligent and has a strong sense of order and discipline. Both Emily and Matt share a teaching position in a predominantly White Anglo-American middle school. John, who is also a product of the public schools in which he is employed, is of African American/Mexican decent. He is very mindful of his actions and respectful of all that is around him. He shares a mathematics/science teaching position with a SMEST science intern who is not part of this study in a high school with a predominantly Latino/a population.

My Role in this Study

This study was conceived more than fifteen years ago when I was a secondary mathematics teacher participating in a professional development project. As I progressed through the project, I continued to challenge my conceptions of mathematics and adopted a *problem solving perspective* regarding the teaching and learning of mathematics. With encouragement from my administrators, I began sharing my problem solving classroom approaches with my peers and the larger mathematical education community; as a result, I received the National Science Foundation's prestigious Presidential Award for Excellence in Mathematics Teaching. Eventually, with other concepts learned during the professional development project (i.e. mentoring), and encouragement from the mentors within the project, I began visiting classrooms of secondary mathematics teachers who were seeking change, mentoring them in ways that helped them test, challenge, and refine their own conceptions of what mathematics is and how it could be taught and learned. I became a mentor in the SMEST program, and as I learned more about mathematics teaching and learning through university coursework and in my relationships with my mentees, I decided to conduct this study. I felt it was important to teach the university secondary mathematics methods course my SMEST interns took as part of their coursework in the program; I wanted an additional data collection opportunity in my study that could encourage the interns to confront and restructure their conceptions about mathematics. When this opportunity became available, I launched the study.

I chose a qualitative, interpretive case study design (Merriam, 1998) that allowed me to observe, mentor, and examine the reflective writing of participants both in the methods course and in the context of their mathematics classrooms. Within these three case studies, I constructed narratives based on those observations, conversations, and reflections, hoping these stories would illuminate the participants' experiences and the connections they made between their classroom experiences at the university and in their own classrooms. It is these connections made explicit through conversations and reflections that helped the interns reveal their conceptions about mathematics teaching and learning.

Research that Informs this Study

I have had many conversations in the past fifteen years with other mathematics teachers about the question, what does it mean to do mathematics? Many responded in ways that described how mathematics is represented in their schools and in their own classrooms. They talked mostly about how they explained mathematical concepts to students and then gave them related problems to do in class and for homework. Many also talked about students' lack of success when they were asked to work similar problems on the test and the additional practice, or remediation, students would need before they took the make-up test. In their minds, these teachers believed this was what constituted classroom mathematics.

The conversations I had with teachers regarding their classroom were predicated by their conceptions of the nature of mathematics (Hersh, 1986). In his book, *What is Mathematics, Really?*, Reuben Hersh (1997) explores the nature of mathematics. In it, he maintains that people don't like mathematics because of the way it is *misrepresented* in schools. His view of classroom mathematics that millions of Americans experience in school is one that is an impoverished version of the subject, bearing little resemblance to the mathematics of life or work or even the mathematics in which mathematicians engage.

In order to position my study within this conceptual framework, I examined literature related to teachers' conceptions about the nature of mathematics and how those conceptions were manifested in their classroom practices. In addition, I examined literature describing the relationship between teachers' conceptions and their practices, and then considered factors, such as current reform initiatives, mentorship, and university coursework, that could not only influence beginning teachers' conceptions, but alter them in meaningful ways. What follows is a brief review of this literature.

Teachers' Conceptions of Mathematics

Mathematics teachers' conceptions are contained within a context of beliefs and knowledge about what constitutes mathematics and what they perceive their role as teachers of mathematics (Cooney, 1994; Thompson, 1992). Their conceptions might be thought of as lenses or perspectives that affect one's view of some aspect of the world or as dispositions toward action (Cochran-Smith & Lytle, 1992; Philipp, 2007), but there is one important distinction between the two; beliefs may be held with varying degrees of conviction and are not consensual whereas knowledge is held with certainty. Put another way, disputability is associated with beliefs; truth is associated with knowledge (Thompson, 1992).

Knowledge of mathematics is fundamental to being able to help someone else learn it (Ball, 1988) and having this knowledge is one of the most important influences on what is done in mathematics classrooms and ultimately what students learn (Fennema & Franke, 1992). However, the importance of possessing mathematical knowledge is not sufficient by itself to account for differences among mathematics teachers. For example, two teachers can have similar knowledge, but they can have totally different perspectives regarding the nature of mathematics and its teaching and learning; one teacher may view mathematics as the mastery of facts, skills, and procedures. The other teacher may view mathematics as an opportunity to engage in problem situations that can lead to mathematical understanding or expertise through the interplay of conceptual and procedural knowledge (Hiebert & Carpenter, 1992).

Ernest (1988) describes three perspectives teachers may have regarding the nature of mathematics: Problem-solving, Platonist, and Instrumentalist. Teachers embracing the problem solving perspective view mathematics as a dynamic process of inquiry, adding to the sum of knowledge. Mathematics is not a finished product; its results remain open to revision. It is a continually expanding field of human creation and invention in which patterns are generated and then distilled into knowledge.

Teachers embracing the Platonist perspective view mathematics as a static rather than dynamic process, as a unified body of knowledge. Here, the mathematics is discovered, not created, and consists of a realm of interconnecting structures and truths bound together by logic and meaning that are often explained by the teacher.

In the Instrumentalist perspective, teachers view mathematics as a discipline consisting of an accumulation of facts, rules, and skills to be memorized and used in pursuance of some external end. Students merely deal with these facts, rules, and procedures and demonstrate the ability to use and recall them without necessarily understanding the reasons behind them. Teachers embracing this perspective use demonstration strategies that are followed by students practicing what was demonstrated by the teacher (Romberg, 1992; TIMSS, 1995).

In his description of the three perspectives of mathematics, Ernest (1988) also views them in a hierarchical nature. With Instrumentalism at the lowest level, it involves knowledge of facts, rules, and methods as separate entities. The second level consists of the Platonist perspective, involving the understanding of mathematics as a consistent, connected, and objective structure. At the top level resides the problem solving perspective where mathematics is viewed as a dynamically organized structure situated in a social and cultural context (Ernest, 1988).

Since it is possible for teachers to have more than one perspective making up their conceptions regarding mathematics and its teaching and learning (Thompson, 1992), how do teachers evaluate and reflect on their teaching when embracing conflicting perspectives? Perhaps they can operate with conflicting perspectives because of the nature of the organization of their beliefs. Green (1971) claimed that beliefs are held in clusters, and that these clusters of beliefs are isolated and protected from other clusters. This clustering prevents “confrontation” among clusters of beliefs and makes it possible to hold conflicting sets of beliefs. As a result of clustering, it is possible for teachers to possess multiple perspectives regarding mathematics within their beliefs systems, and this may help explain why teachers sometimes profess one perspective and practice another at any given time (Thompson, 1992).

The Influence of Mathematics Reform on the Teaching and Learning of Mathematics

For more than two decades, reforms in mathematics education proposed by the National Council of Teachers of Mathematics [NCTM] (1989, 2000) and the National Science Foundation [NSF] (1996) have aligned with the problem solving perspective (Ernest, 1988). This is reflected in the vision of the *Curriculum and Evaluation Standards*, a document published by NCTM (1989):

> Students should be exposed to numerous and various interrelated experiences that encourage them to value the mathematics enterprise, to develop mathematical habits of mind, and to understand the role of mathematics in human affairs;…they should be encouraged to explore, to guess, and even to make and correct errors so they gain confidence in their ability to solve complex problems;…they should read, write, and discuss mathematics; and…they should conjecture, test, and build arguments about a conjecture's validity. (NCTM, 1989, p. 5)

This visionary statement promotes the notion of students learning within the problem solving perspective (Ernest, 1988) and forms the basis for the reform movement in mathematics education over the past two decades. The statement describes how students should become pattern seekers, experimenters, describers, inventors, conjecturers, and guessers to explore, develop, and investigate mathematical ideas. Through problem situations, teachers expect students to find solutions when none are apparent, listen to each others' ideas, reflect on and write about their mathematical thinking, and *connect* their learning to other problem situations they may have encountered (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997).

When students connect new learning to knowledge they already have, they are actively creating new knowledge by reflecting on their physical and mental actions. Known as *constructivism*, this learning theory suggests that ideas are made meaningful when children integrate them into their existing structures of knowledge (Abbot and Ryan, 1999; Grennon Brooks & Brooks, 1993). It is the learning model (Kuhs & Ball,

1986) most likely to be advocated by those teachers who subscribe to the problem solving perspective of mathematics and who view mathematics as a dynamic discipline, dealing with self-generated ideas and involving methods of inquiry (Ernest, 1988).

Several curricula were conceived and written to support the problem solving perspective to teaching and learning envisioned in the NCTM Standards (NCTM, 1989). The authors of one such curriculum advocated for critical thinking, communication, problem solving, and teamwork as important pedagogical components (Alper, Fendel, Fraser, & Resek, 1989). These authors emphasized the problem solving perspective (Ernest, 1988) throughout the curriculum, and because problem solving is not commonly embraced by teachers in their classrooms (Romberg, 1992; TIMSS, 1995), they also strongly recommended teacher professional development to enhance successful implementation. This professional development structure gave teachers opportunities to examine, restructure, and reflect on their conceptions and classroom practices; it also provided mentorship opportunities for teachers implementing the curriculum where classroom visits and lesson observations were done on a regular basis.

The curriculum briefly described above was one of several curricula written in the spirit of the *Curriculum and Evaluation Standards* document (NCTM, 1989) and was one of five curricula to receive exemplary status from an expert panel of mathematicians, educators, and policy makers who reported their findings to the U.S. Department of Education (Math Panel, 1999). But NCTM, acknowledging that mathematics education is a dynamic process, released a follow-up document in 2000. Called *Principles and*

Standards of School Mathematics [PSSM], NCTM (2000) continued to build on its vision of mathematics teaching and learning established in 1989:

> Imagine a classroom where all students have access to high-quality, engaging mathematics instruction. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures. Students draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. (NCTM, 2000, p. 3)

This vision of doing mathematics further supported the belief that teachers and students alike must engage in mathematics teaching and learning as a problem-solving process. In its *Curriculum and Evaluation Standards* document, NCTM (1989) positioned itself by stating that there is a common core of mathematical ideas that all students should have an opportunity to learn. It included thirteen standards that identified and described the mathematical content and processes of doing mathematics surrounding these ideas. NCTM (2000) consolidated and reorganized these curriculum standards by designating five content and five process standards. In his book on teaching secondary mathematics, Brahier (2005) referred to and stressed the importance of the five process standards:

problem solving, communication, connections, reasoning, and representation. He referred to these as the umbrella standards and stated that they constitute what students should be doing as they engage in problem situations, constructing understanding of mathematical ideas in areas such as algebra or probability. "These umbrella standards should permeate every mathematics lesson every day in the classroom." (p. 24)

Incorporating the five process standards into classroom practices can support teachers embracing the problem solving perspective and challenge and even re-structure their conceptions regarding mathematics teaching and learning (Brahier, 2005; NCTM, 2000). For beginning teachers, this is very important as it can become part of their own window of understanding and their process of learning to teach.

The Relationship between Teachers' Conceptions and Their Practices

Mathematics teachers' conceptions have a powerful impact on their teaching practices (Ernest, 1988). What they know and believe acts as a filter through which they interpret teaching and learning and their actions and decisions are often guided by what they believe to be true about mathematics and the teaching and learning of mathematics (Cochran-Smith & Lytle, 1992; Philipp, 2007). However, teachers' conceptions of mathematics teaching and learning are not always consistent with their classroom practices (Ball, 1988; Cooney, 1985; Ernest, 1988; Shulman, 1986; Thompson, 1992) and are subject to various constraints and contingencies:

- social context of the school
- political climate
- knowledge base.

Teachers' conceptions are influenced by the school, school district, and community in which they are situated (Ernest, 1988). Expectations from students, parents, teachers, and administrators can lead teachers to use practices in the classroom incongruent with their conceptions. For example, a teacher who embraces the problem solving perspective regarding mathematics may feel pressure to follow a different approach to teaching mathematics used by fellow mathematics department members. Despite having differing beliefs about mathematics and its teaching, teachers in the same school are often observed to adopt similar classroom practices (Ernest, 1988).

The larger societal political climate can also account for the mismatch between teachers' espoused conceptions of mathematics and what is actually enacted in their classrooms. For example, the NCTM documents (1989, 2000) advocating for mathematics education reform today may have an influence on teachers' professed beliefs but there may be little evidence of putting these beliefs into action in their classrooms. Teachers may also be responding to state mandates and testing resulting from the *No Child Left Behind* (2002) legislation in ways that impact their conceptions of mathematics and their resulting practices (Thompson, 1992).

A great deal of knowledge is essential to successfully implement certain models of mathematics teaching (Ball, 1988; Dewey, 1964; Fennema & Franke, 1992; Shulman, 1986). For example, using a constructivist model of learning that aligns with the problem solving perspective (Kuhs & Ball, 1986) requires teachers to possess a broad knowledge base in mathematics in order to recognize and capitalize on student ideas that arise naturally out of classroom discourse. If a teacher's conceptions of mathematics align with the problem solving perspective, yet he/she does not have the corresponding knowledge and skills, then that teacher may have to resort to a different, lower level perspective in Ernest's (1988) three-tiered hierarchy, such as the Instrumentalist or Platonist perspective.

Changing Intern Teachers' Conceptions of Mathematics

Teachers' conceptions of mathematics are robust; they are concretized and deep-seated (Ball, 1988) and form the foundations on which teachers build their practices. When beginning teachers are given opportunities to interact in their school community in different ways and reflect on those actions, then their conceptions of mathematics can be tested, challenged, reconstructed; perhaps old conceptions can be refined or give way to new ones (Tobin, Tippins, & Hook, 1992).

Throughout the course of this study, participants had many opportunities to interact in their school community and reflect on their experiences. A mentoring component was provided for participants as a way to engage them in discussion and reflection. In her five-year study, Thea Dunn (2005) examined the conceptions of over 400 prospective teachers regarding their conceptions about the teaching and learning of mathematics. She focused on critical reflection, allowing participants to examine how equity, justice, caring, and compassion could inform their educational goals. In doing so, she challenged her participants' thinking and used her role of mentor to guide her participants to restructure and broaden their conceptions of mathematics teaching and learning.

In another study involving the mentorship of eight student teachers, Philippou and Charalambous (2005) conducted interviews three different times over the course of the study. The analysis of their data revealed that mentors could influence student teachers' conceptions through their own teaching style, the feedback they gave and the latent messages they implicitly conveyed to their mentees.

In addition to studies involving mentorship, case studies have been designed that challenge preservice and inservice teachers' conceptions of mathematics (Feiman-Nemser, McDiarmid, Melnik, & Parker, 1987), engage them as learners of mathematics and mathematics pedagogy within a constructivist environment (Ball, 1988; Feiman-Nemser & Featherstone, 1992; Liljedahl, 2005; Tuft, 2005), and provide them with experiences in mathematical discovery (Liljedahl, 2005). The combination of these three approaches has shown to be very effective in changing preservice teachers' beliefs (Liljedahl, Rolka, & Rosken, 2007). In addition, Benken (2005) and Cobb, Wood, and Yakel (1990) have conducted case studies examining the relationship of teachers' conceptions and their practices. In these studies, the participants restructured their conceptions in light of their classroom experiences, giving them reasons to *question* and make changes in their practices.

As beginning teachers interact with students, parents, peers, and other members of the school community, they may have opportunities to evaluate and reorganize their conceptions through reflective acts and thought. In so doing, teachers can reconcile their classroom practices with their conceptions and even reconcile conflicting beliefs themselves (Ernest, 1988).

Significance of this Study

Conceptions are well-developed before individuals enter teacher education programs (Ball, 1988; Kagan, 1992). In fact, Eisner (1992) describes how young people are subject to professional socialization: "Beginning at age five or six when they begin school, children acquire images of what teachers do in classrooms and internalize what they believe being a teacher entails" (p. 610). In other words, teaching is something children and young adults learn to do more by growing up in a culture than by studying it formally in a teacher education program. Therefore, it is important to design and conduct studies within teacher education programs that challenge, and perhaps change, our students' cultural frameworks for teaching

Fieldwork is a critical part of teacher preparation programs (Pajares, 1993), and it is important to examine various components, such as mentoring and classroom observations. Changing the way we observe our teachers while teaching and then mentoring them in ways that promote a reflective practice may help to restructure beginning teachers' conceptions of mathematics. Furthermore, to help pre-service and intern teachers integrate theory into their practice, the university methods course they enroll in must be examined as another vehicle for challenging teachers' conceptions (Benken, 2005; Cobb et al., 1990; Tuft, 2005) and supporting new conceptions as they do work in the field.

There are various studies that have examined pre-service and inservice teachers' conceptions at both the elementary and secondary level (Ball, 1988; Benken (2005); Cobb, Wood, and Yakel (1990); Dunn, 2005; Feiman-Nemser, McDiarmid, Melnik, & Parker, 1987; Feiman-Nemser & Featherstone, 1992; Liljedahl, 2005; Philippou &

Charalambous, 2005; Tuft, 2005). While many have focused on the impact of university coursework on conceptions, few have focused on the impact of mentoring. Even fewer have focused on the impact of both university coursework and mentoring, and these have involved pre-service teachers as participants.

In this study, I chose to examine the impact of both university coursework and mentoring on the conceptions of first-year intern teachers enrolled in a university alternative licensure program. The structure of the university methods course was similar to other studies; the goal was to engage students as learners of mathematics and mathematics pedagogy in a constructivist environment (Ball, 1988; Feiman-Nemser & Featherstone, 1992; Liljedahl, 2005; Tuft, 2005). The structure of the mentoring in this study consisted of classroom observations followed by reflective conversations (Costa & Garmston, 1999). The goal was to foster professional growth in these intern mathematics teachers by creating a stance of exploration and experimentation regarding classroom practices and, as a result, a refinement of their cognitive maps or conceptions.

This study was significant because the participants were intern teachers. They learned the craft of teaching in the context of their own classroom while taking supporting university coursework and engaging in intensive mentoring throughout the year. Based on this and the two goals stated in the previous paragraph, this study was unlike any study found in the research literature. It contributed to a body of knowledge regarding the impact of a methods course and/or mentoring on teachers' conceptions as other studies have, but, in addition, it also examined how these first-year teachers regarded their university coursework and connected it to their classroom teaching. At the same time, this study examined how the mentoring impacted their ability to reconcile any differences between their conceptions and classroom practices.

Feiman-Namser (2001) tells us, “We still know very little about what thoughtful mentor teachers do, how they think about their work, and what new teachers learn from their interactions with them”(p. 17). With this in mind, mentoring, and its interplay with classroom observations and the university mathematics methods course, provided a unique structure for sharing my thinking about my work with these first-year intern teachers. The mentoring was a critical piece in this study because there is little research to inform us how mentors impact mathematics teachers’ conceptions in today’s reform climate.

Assumptions Regarding this Study

The following were assumed to be true and fundamental to this study:

1. Based on the current climate of reform in mathematics education today (NCTM, 1989, 2000), the problem solving perspective (Ernest, 1988) of mathematics was deemed desirable and appropriate in the classroom.
2. The information conveyed to me in the interns’ reflective writings, classroom observations, and reflective conversations was an accurate representation of the participants’ view of reality.
3. The classroom practices of the three participating intern teachers and the learning atmosphere in their classrooms reflected the typical conditions of their classrooms.

Overview of Study

This study focused on three first-year secondary mathematics teachers. Its intent was to give these teachers opportunities to confront their conceptions about mathematics and mathematics teaching and learning in the context of their university mathematics methods course and in their own classrooms. Through interactions with me, their mentor and university methods instructor, peers, and other community members, Emily, Matt, and John were given opportunities to talk about, reflect on, and reconcile differences between their espoused models of teaching and learning mathematics and the enacted models they carried out in their classrooms. Their conceptions were challenged by their classmates and me as they engaged in mathematical learning through problem solving situations in their university methods course; their conceptions were also challenged as they engaged in conversations with me each week during classroom visits. Each participant contributed words, actions, and perspectives, creating a unique context from which to view and understand their teaching and learning, albeit my understanding of these teachers' words and actions may have been influenced by my own conceptions of mathematics. Even so, the stories of each of these teachers provided a window through which we all could examine our own conceptions of what mathematics is and what it means to teach and learn it.

Definition of Terms

The following terms are used throughout this paper and should be connected with the context of teaching and learning mathematics.

- Authority: one who determines the direction and/or correctness of the mathematics in the classroom
- AYP: Adequate Yearly Progress
- Belief: perspective often associated with disputability and subject to debate (Thompson, 1992)
- Belief system: beliefs that are organized into clusters or bundles (Green, 1971)
- Classroom focused model of teaching: teaching based on knowledge of effective classrooms; not generally associated with any mathematical perspective of learning (Kuhs & Ball, 1986)
- Cognitive Coaching: mentoring strategy that allows participants to engage in a reflective conversation
- Cognitive demand: the level and kind of student thinking required to solve a task
- Conceptions: beliefs and knowledge that affect one's view of some aspect of the world or disposition toward action (Philipp, 2007)
- Content performance model of teaching: teaching based on mathematical mastery of rules and procedures; associated with the Instrumentalist perspective of mathematics (Kuhs & Ball, 1986)
- Content understanding model of teaching: teaching that is driven by content and emphasizes conceptual understanding; associated with the Platonist perspective on mathematics (Kuhs & Ball, 1986)

- Constructivism: learning theory that suggests individuals take in new information, internalize it, and assimilate it into existing internal networks to fit with what is already known
- Disciplinary agency: disposition developed passively when strategies involving teacher demonstration and student practice are used (Boaler, 2002)
- Discourse: process of engaging in conversation or discussion
- Human agency: disposition developed actively when strategies allowing discourse and reflection through problem solving are encouraged (Boaler, 2002)
- Instrumentalist perspective: view of mathematics as consisting of facts, rules, formulas, and skills to be memorized and used (Ernest, 1988)
- Knowledge: dynamic construct of one's conceptions made up of content, pedagogy, and student cognition; usually socially constructed and indisputable (Fennema & Franke, 1992)
- Learner focused model of teaching: teaching that is based on the learner's personal construction of mathematical knowledge; associated with the Problem solving perspective of mathematics (Kuhs & Ball, 1986)
- Learning with understanding: a self-regulated process of resolving inner cognitive conflicts through concrete experience, discourse, and reflective thought (Hiebert & Carpenter, 1992)
- NCLB: No Child Left Behind
- NCTM: National Council of Teachers of Mathematics
- NSF: National Science Foundation

- Platonist perspective: view of mathematics as a precise language of logic, proof, and definitions that are explained by the teacher (Ernest, 1988)
- Problem: In Instrumentalism, a problem is considered to be part of a problem set in which all problems in the set are of the same type.
- Problem solving activity: in the problem solving perspective, a problem, or problem solving activity, is defined as any task for which there is there is no prescribed or memorized rule or method, nor is there the perception by students that there is a specific “correct” solution method.
- Problem solving perspective: view of mathematics as a generative process by which students construct and invent their own mathematical rules and formulas (Ernest, 1988)
- Resistance: a process that involves agency and the conscious choice to take action or not (Rodriguez, 2005)
- SMEST: Science and Mathematics Education of Secondary Teachers, a university alternative licensure program
- TIMSS: Third International Mathematics and Science Study

Chapter 2

Review of the Literature

> One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it…..
>
> (Ruben Hersh, 1986, p. 13))

There have been many who claim that the mathematics students learn in schools does little to represent the world of mathematics done outside of mathematics classrooms. Paul Lockhart (2002) said it very succinctly: "I'm not complaining about the presence of facts and formulas in our mathematics classes, I'm complaining about the lack of mathematics in our mathematics classes." (p. 5) Reuben Hersh (1997) talks about how the *mystery* of mathematics grows in our schools by looking at the subject as answers without questions:

> That mistake is made only by people who have had no contact with mathematical life. It's the questions that drive mathematics. Solving problems and making up new ones is the essence of mathematical life. If mathematics is conceived apart from mathematical life, of course it seems dead. (p.18)

All the mathematical methods and relationships that are now known and taught in our mathematics classrooms started out as questions, yet our students do not see these questions. Instead, they are being asked to memorize what has already been done and decided in mathematics. Perhaps this is what Lockhart means by a lack of mathematics in our classrooms. If it is, then what can be done to bring classroom mathematics to life? George Polya (1971) said that teachers of mathematics have a great opportunity:

> If they fill the allotted time with drilling their students in routine operations, they kill their interest, hamper their intellectual development, and misuse their opportunity. But if they challenge the curiosity of their students by setting them problems proportionate to their knowledge, and help them solve their problems with stimulating questions, they may give them a taste for, and some means of, independent thinking. (p. v)

It seems, then, that when students are curious enough about things to ask their own questions and take problems in new directions, then they will have opportunities to experience the 'essence of mathematical life'.

In this chapter, I will provide an overview of the literature relevant to the teaching and learning of classroom mathematics. The goal is threefold: first, I develop a conceptual framework for understanding teachers' conceptions regarding the nature of and the teaching and learning of mathematics. It is important to point out that teachers' conceptions regarding mathematics will impact their practices (Ernest, 1988; Thompson, 1992); these practices may or may not reveal what many teachers profess as an important belief: students should learn mathematics with *understanding*. For that reason, I examine the meaning behind learning with understanding. Then, I look at the relationship between teachers' conceptions of mathematics and their classroom practices. Finally, I examine ways in which conceptions regarding mathematics and its teaching and learning can be confronted, restructured, and refined.

Teachers' Conceptions of Mathematics

The National Council of Teachers of Mathematics [NCTM] and the National Science Foundation [NSF] have been instrumental in advocating for the adoption of a problem solving approach to the teaching and learning of mathematics (NCTM, 1989, 2000; NSF, 1996). NCTM first recommended almost 30 years ago that "Problem solving must be the focus of school mathematics" in *An Agenda for Action* (NCTM, 1980). Since then, NCTM has supported teachers wishing to develop a problem solving framework for their classrooms. Teachers need to realize that developing problem solving abilities in students may mean that they need to work on problems that may take hours, days, and even weeks to solve. Some problems may be relatively simple exercises to be accomplished independently; others should involve small groups or an entire class working cooperatively (NCTM, 1989). Whatever the case, when engaging in problem solving, students should be able to adapt a variety of appropriate strategies as they build new mathematical knowledge and reflect on the process (NCTM, 2000).

The mathematics reform advocated by NCTM and NSF using a problem solving approach requires teachers to make significant changes, and it cannot take place unless teachers' conceptions about mathematics and its teaching and learning change (Ernest, 1988). What conceptions regarding mathematics do teachers hold? How can they confront them and change them in order to engage learners in the process of problem solving?

Knowledge and Beliefs

Mathematics teachers' conceptions consist of *knowledge* of mathematics and *beliefs* regarding the nature of mathematics and the teaching and learning of mathematics (Ernest, 1988). Knowledge and beliefs may be thought of as *lenses* or *perspectives* that affect one's view of some aspect of the world or as dispositions toward action (Cochran-Smith & Lytle, 1992; Philipp, 2007); lenses and perspectives may also be thought of as being contained within the context of knowledge and beliefs, about what constitutes mathematics and what teachers perceive as their role as teachers of mathematics (Cooney, 1994; Thompson, 1992).

Understanding teachers' knowledge of mathematics, their beliefs about the nature of mathematics, and its teaching and learning is difficult and complex. Knowledge of mathematics is multifaceted; it consists of *content, pedagogy*, and *student cognition* (Fennema & Franke, 1992). These three components interact in complex ways, and as a result, this knowledge is continually changing, developing, and growing through these interactions. Teachers' knowledge of mathematical content (i.e. concepts, procedures, and problem-solving processes), and the manner in which it is organized, intersects with their pedagogical knowledge of classroom procedures, strategies, and routines; Shulman (1987) believes this intersection of knowledge, coupled with how students think (student cognition), characterizes a teacher's knowledge base. With this knowledge base, teachers possess the capacity to transform their content knowledge in ways that are pedagogically powerful and adaptive to the variations in ability and background presented by their students (Fennema & Franke, 1992). In other words, they transform their mathematical knowledge so their students can successfully interact with it and learn.

Carpenter, Fennema, Peterson, Chiang, and Loef (1989) conducted a study that examined teachers' content and pedagogical knowledge and their knowledge of children's thinking and problem solving strategies. Called Cognitively Guided Instruction [CGI], this problem-based framework investigated young children's thinking and learning primarily in the domains of addition and subtraction. Consider the example below:

> Diego has 4 dollars. How much more money does he need to buy a toy
> that costs 13 dollars?

Most young children in kindergarten through first grade will solve the above problem by modeling the *action* described in the problem. That is, if they use counters, they make a set of four counters, and then add more until they reach a total of 13. Then they count the nine added to find the answer. A more advanced strategy involves counting up from 4 to 13, keeping track of the number of counts, perhaps by extending a finger for each one.

Carpenter and his colleagues (1989) found the teachers in this study knew a lot about addition and subtraction, but their knowledge was not well organized when it came to the various problem types, solution strategies, and students' difficulty regarding addition and subtraction. They also found that with time, teachers could acquire knowledge about their students' thinking and solution strategies and then make appropriate instructional decisions regarding individual students.

A study of this kind illustrates the importance of possessing mathematical knowledge and having the ability to transform it in ways that become accessible to students. It also indicates that when teachers' knowledge is transformed during instruction, it becomes tied to the context in which it was developed. When it is confronted by a new situation, this knowledge can then be adapted and stored as new knowledge, thereby increasing the teachers' knowledge base (Fennema & Franke, 1992).

Content, pedagogy, and student cognition, all aspects of mathematical knowledge, form a complex interactive web. To get a good grasp of a teacher's knowledge, these components should not be separated, nor should they be separated from a teacher's beliefs. A teacher's mathematical knowledge interacts with his/her beliefs as well during the course of classroom instruction. But beliefs differ from knowledge in a fundamental way: they are associated with disputability whereas knowledge is associated with truth (Thompson, 1992). Put another way, knowledge consists of standards or criteria that are socially constructed and indisputable, whereas beliefs are seen as individual constructs and are subject to debate. Leatham (2006) argues that knowledge can also consist of individual constructs as well:

> Of all the things we believe, there are some things we 'just believe' and other things we 'more than believe—we know.' Those things we 'more than believe' we refer to knowledge and those things we 'just believe' we refer to beliefs. Thus beliefs and knowledge can profitably be viewed as complimentary subsets of the things we believe. (p.92)

Based on how Leatham (2006) articulates this division between knowledge and beliefs, discussions about teachers' knowledge of mathematics needs to include discussions about their beliefs as well (Thompson, 1992). Mathematics teachers operate for the most part on individual constructs; their actions are guided by what they believe to be true about mathematics and its teaching and learning rather than what may actually be true (Liljedahl, et al., 2007). There will be things they “just believe” and things they “more than believe” as they operate from day-to-day in their classrooms.

Conceptions Regarding the Nature of Mathematics

In mathematics education, disputability resides within teachers’ conceptions of the nature of mathematics. Even before they enroll in their teacher education programs, teachers’ conceptions regarding the nature of mathematics are well formed by years of studying mathematics in structured school classrooms (Ball, 1988; Eisner, 1992; Kagan, 1992).

Ernest (1988) describes three perspectives teachers tend to hold regarding the nature of mathematics: Instrumentalist, Platonist, and Problem-solving. Similarly, Dionne (1984) describes these three perspectives as Traditional, Formalist, and Constructivist, respectively. Teachers embracing the Instrumentalist perspective view mathematics as a set of facts, rules, formulas, and skills and procedures to be memorized and used. Rules are considered the basic building blocks of mathematical knowledge; knowledge of mathematics means being able to use these rules to do problems and get answers. This perspective is associated with the content-performance model of teaching (Kuhs & Ball 1986). For example, teachers may expect students to calculate the area of a triangle using the formula $A = 1/2\ b\ h$. Doing this calculation many times may create automaticity of this procedure, but it does not guarantee conceptual understanding. When subscribing to Instrumentalism, teachers are not concerned with conceptual understanding, but only in one's ability to demonstrate mastery of skills.

Teachers who embrace the Platonist perspective see mathematics as a precise mathematical language consisting of logic, proof, and definitions that are explained to students. Instruction makes content the focus of mathematical activity and emphasizes the understanding of ideas and processes. This perspective is associated with the content-conceptual understanding model of teaching (Kuhs and Ball, 1986). In the example of finding the area of the triangle discussed above, teachers, in addition to having students perform calculations for the area, would explain to students how the formula is derived and how it is related to other concepts and procedures.

Teachers that subscribe to the Problem solving perspective view mathematics as a *constructive* process done by students where mathematical engagement involves creating or generating their own rules or formulas, thereby inventing or re-inventing the mathematics. To illustrate this using the area of the triangle example, students would be expected to use invented methods or procedures for finding the area formula. These could be based on prior knowledge (i.e. formula for the area of a rectangle), or by some other means, concentrating on the why and not just the how or what. This *constructivist* model of mathematics learning (Clements & Battista, 1990; Fosnot, 1996; Grennon-Brooks & Brooks, 1993) centers on the students' active engagement in doing mathematics, in exploring and formalizing ideas. Teachers view their students as responsible for judging the merit and adequacy of their own ideas and having the ability to validate conjectures and defend their conclusions (NCTM, 1989, 2000).

Conflicting perspectives regarding the nature of mathematics may form and reside within teachers; as a result, there may be conflicting theories of teaching and learning mathematics (Kitchen, Roy, Lee, & Secada, 2009; Thompson, 1992). For example, a teacher's conceptions may be associated with memorizing formulas and "applying" them to a problem set (Instrumentalism); they may also be associated with students' understanding of the formulas, examining problem situations, looking for patterns, and creating and defending conjectures (problem solving). Whatever they are, these perspectives are formed from teachers' own experiences as learners of mathematics (Fosnot, 1989; Skott, 2001), and may manifest themselves as conceptions about the teaching and learning of mathematics. Because of this, it is often difficult to sort out teachers' conceptions about the nature of mathematics and its teaching and learning.

Belief Systems

Since it is possible for teachers to possess more than one perspective making up their conceptions regarding mathematics and its teaching and learning, how do they evaluate and reflect on their teaching when embracing conflicting perspectives? Green (1971) described beliefs as belief systems, structures organized into clusters or bundles that are more or less isolated from one another. This isolation, then, serves to protect these clusters from "cross-fertilization" and confrontations, thereby making it possible to hold conflicting perspectives and still operate within one perspective. Green (1971) also described belief systems in a different way; he indicated that individuals may hold beliefs with different degrees of conviction. For example, beliefs within a bundle could be *central*, those being the most strongly held, or they could be considered *peripheral*, those most susceptible to change or examination. With regard to teachers' perspectives on the nature of mathematics, one perspective might be predominantly made up of central beliefs within the system, whereas another perspective may be composed of weaker peripheral beliefs. In this case, teachers could operate within more than one perspective.

Aguirre (1995) conducted a case study that illustrates the phenomenon of operating within a system of dual beliefs. The study involved an 11-year veteran teacher, Mr. Martin, who possessed two distinct perspectives regarding mathematics. He wanted to "break away" from his dominant Platonist perspective on teaching and incorporate newly formed beliefs related to small group activities and student collaboration falling within the problem solving perspective. These peripheral beliefs, or perhaps this new cluster of beliefs (Green, 1971) regarding collaboration were emerging but fragile as they developed over the course of the study (Aguirre, 1995). During one episode, Mr. Martin suspended his desire to have his students collaborate on the situated problem when a group of students was not meeting the goals of the lesson. He reverted back to his stronger, central beliefs that mathematics is a set of rules and procedures and the teacher should explicitly tell students what to do to solve problems. Why did his newly formed beliefs regarding collaboration give way to his more didactic approach? Aguirre (1995) asserts that belief bundles may interact in complex ways; these beliefs work together with teacher cognition and goals to explain the moment-to-moment decisions and actions of teachers.

Conceptions of Teaching and Learning Mathematics

Imagine a mathematics classroom where the teacher encourages students to interact with a small piece of information, ask questions about it, and respond from their *own* perspectives without fear of being wrong or not having the right answer. For example, consider the *handshake problem* below:

> In a room of 20 people, how many total handshakes would there be if each person shook every other person's hand exactly once?

As students ponder the mathematics of this problem, they have opportunities to approach the problem using various strategies and approaches that make sense to them. For example, students could draw pictures or diagrams or act out the problem. They could reduce the problem to a simpler problem or simpler problems and look for patterns. As students solve simpler forms of the original problem, they could gain insights that could be used to solve the original, more complex problem. Additionally, solutions to simpler problems could be organized into a table or chart, and this information could be analyzed and communicated with peers. At this point, students could assess the validity of their solution to the problem and perhaps extend their thinking about the problem by finding a general rule or applying the results, procedures, or processes to other problem situations.

The classroom scenario in the preceding paragraph describes learning in a classroom environment where students, with the guidance of their teacher, determine the path taken to solve the problem. This problem solving perspective (Ernest, 1988) to teaching and learning mathematics provides opportunities for students to "do" mathematics. An integral part of this "doing" of mathematics means searching for and finding patterns. Mary Baratta-Lorton (1995) said looking for patterns trains the mind to search out and discover the similarities that bind seemingly unrelated information together in a whole. It is seen as a way of thinking that is essential for making generalizations, seeing relationships, and understanding the logic and order of mathematics. Stein, Smith, Henningsen, and Silver (2000) give a similar description of what "doing" mathematics entails:

> The category of doing mathematics includes many different types of tasks that have the shared characteristic of having no pathway for solving the task explicitly or implicitly suggested and therefore requiring non-algorithmic thinking. This category includes tasks that are non-routine in nature, are intended to explore a mathematical concept in depth, embody the complexities of real-life situations, or represent mathematical abstractions (p.23).

Perhaps the "doing" of mathematics described above is the very essence of what Lockhart (2002) and Hersh (1986) are referring to as the mathematics that is missing in our classrooms. This lack of doing mathematics is depicted in a different scenario of the handshake problem in the paragraph below.

Now imagine a mathematics classroom where students are given the answer to the *handshake problem*. In other words, the teacher may derive the formula for the students or just write the formula on the chalkboard. Then the teacher asks students to solve a set of problems involving different numbers of people using the formula written on the chalkboard. As students ponder the mathematics of this problem, they may only be concerned with putting the number of people into the formula and generating the number of handshakes that result. They do not have the same opportunities to engage in "doing" the mathematics as do the students described in the first scenario. These students are merely practicing a procedure specified by their teacher. In this scenario, students are given the answer or rule to understand and remember, rather than a problem to solve.

This second scenario describes the Instrumentalist perspective without the derivation of the formula, or the Platonist perspective when the teacher explains how the formula comes about (Ernest, 1988). The Instrumentalist approach is typical in most mathematics classrooms in the United States; the teacher usually starts the class period with the more difficult problems from the previous day's assignment. These are worked by the teacher or sometimes the students at the chalkboard or whiteboard. What follows is an explanation of new material and demonstration of how to work the new problems. A problem set is then assigned, most often from the textbook or a workbook. Students work on these paper-and-pencil exercises independently while the teacher moves around the room answering their questions. The problems that are not completed are usually regarded as homework. The following day, the teacher starts the class as before with the more difficult problems from the previous day's assignment and the routine of the previous day is repeated (National Research Council, 2001; Romberg, 1992; Stigler & Hiebert, 1997; U.S. Department of Education, 2000).

The image of teaching described above in the second scenario was well documented during 1993 in a study that compared the mathematics teaching in eighth grade classrooms in the United States, Germany, and Japan. Called the *Third*

International Mathematics and Science Study [TIMSS], researchers in one component of the study videotaped 231 eighth-grade mathematics classrooms: 81 in the United States, 100 in Germany and 50 in Japan (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). The goals of the video component of the study were simple and straightforward:

- To learn how eighth-grade mathematics is taught in the United States.
- To learn how eighth-grade mathematics is taught in two comparison countries, Germany and Japan.
- To learn something about the way teachers from the United States view reform and whether they are implementing teaching reforms in their classrooms.

After several months of watching the videotapes, researchers and educators from all three countries formulated their impressions and interpretations. One professor of mathematics education summarized the results this way:

> In Japanese lessons, there is the mathematics on one hand, and the students on the other. The students engage with the mathematics, and the teacher mediates the relationship between the two. In Germany, there is the mathematics as well, but the teacher owns the mathematics and parcels it out to students as s/he sees fit, giving facts and explanations at just the right time. In U.S. lessons, there are the students and there is the teacher. I have trouble finding the mathematics. I just see interactions between students and teachers.

The videotapes did show evidence that the content was not totally absent in mathematics lessons in the United States, as was described by the mathematics education professor above, but the content was less advanced and required less mathematical reasoning than in the other two countries. Teachers used Instrumentalist approaches (Ernest, 1988) to present mathematical definitions and demonstrate procedures for solving specific problems. Students were asked to memorize the definitions and practice the demonstrated procedures. The mathematics teaching was extremely limited, where teachers focused for the most part on a very narrow band of procedural skills, and students spent most of their time acquiring isolated skills through repeated practice (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999).

Japanese teachers in the videotapes appeared to take a less active role, allowing their students to invent their own procedures for solving problems that were quite demanding, both procedurally and conceptually. Using problem solving approaches (Ernest, 1988), they also facilitated students' learning by helping them connect and apply recently developed procedures in subsequent problem situations.

In many of the German lessons from the videotapes, teachers used Platonist approaches (Ernest, 1988) to lead their students through a development of procedures for solving general classes of problems. There was emphasis on technique that included both the rationale underlying the procedure and the precision with which the procedure was executed.

The videotaped lessons from TIMSS (Stigler et al., 1999) provided some insights for researchers and educators into the teaching and learning of mathematics. First, differences in teaching methods were observed across the three cultures (countries) in the study. Comparing these differences led the researchers to conclude that there is a distinct way of teaching in the United States, and this way of teaching differs markedly from both the German and Japanese way. Prior to the study, the researchers assumed that teachers in the United States used different approaches in their classrooms, but these differences paled when they looked across the three countries from a cross-cultural comparative perspective.

Secondly, based on the teaching variations across the three cultures, the researchers concluded that teaching is definitely a cultural activity. Teachers learn their craft through years of participation in classroom life (Ball, 1988; Eisner, 1992; Kagan, 1992), and largely take for granted some of the most widespread attributes of teaching in their own culture. That is, people within a particular culture learn the activity of teaching through the informal participation over long periods of time (Stigler & Hiebert, 1997), and the images and conceptions they acquire about teaching are deeply embedded; this makes it very difficult to change these conceptions in a short-term teacher training program. In other words, teaching is something that one learns to do more by growing up in a culture than by studying it formally.

Finally, the TIMSS video study results provided information about the effect of educational policies on classroom teaching. The videos show little evidence that teachers in the United States are changing when they try to improve their teaching by aligning with current reform recommendations (NCTM, 1989, 2000; NSF, 1996). When teachers do try to change their teaching, it is often done in superficial ways because there is clearly lacking a system for helping them improve their knowledge base. Teachers have no means of contributing to the gradual improvement of teaching methods and improving their own skills. They are commonly left alone, an action sometimes justified on grounds of freedom, independence, and professionalism (Stigler & Hiebert, 1997).

How, then, did an Instrumentalist approach to mathematics teaching and learning become so prevalent today in mathematics classrooms in the United States? Teaching in this manner has been widely accepted since the beginning of the 20^{th} century (Darling-Hammond, 2006). At that time, the confluence of Thorndike's behavioral learning theory and bureaucratic policies sought to control and simplify teaching instead of preparing teachers as *knowers* and *thinkers*. John Dewey's interest in empowering teachers with knowledge for thoughtful, responsive teaching did not win out with policymakers. Over the years, even when the various prescribed curricula were proven inadequate to the real complexities of teaching, teachers were left to their own conceptions of teaching—largely how they themselves were taught (Ball, 1988; Darling-Hammond, 2006; Eisner, 1992; Kagan, 1992).

The Instrumentalist approach to teaching revealed in the TIMSS videotapes of teaching in the United States illustrates its dominance in mathematics classrooms across the country; many teachers believe mathematics teaching and learning should be sequenced in a linear fashion, where one idea builds on another and where proficiency in one skill/concept is used to develop proficiency in the next (Reys, Lindquist, Lambdin, & Smith, 2007; Shepard, 2001). This perspective regarding the teaching and learning of mathematics is based on observable behaviors and on the idea that if a student produces a particular behavior, then learning takes place. Using an Instrumentalist approach, a teacher may demonstrate how to work a particular kind of problem, such as finding the area of a triangle, and students may learn how to produce that kind of response when given other triangles. The focus on the behavior of finding the area excludes any consideration of higher cognitive demand or thinking processes that students use to get their answers. They are able to find the area using the formula, but may not know why the area is half the base times the height. This example demonstrates how an Instrumentalist perspective to learning allows teachers to focus on student outcomes in the form of specific behaviors or skills while they ignore *learning mathematics with understanding*. Simply put, students may be able to demonstrate a desired mathematical skill without understanding what it means.

In the first scenario of the *handshake problem*, students are expected to develop methods for solving the problem and explain how the number of handshakes is related to the number of people involved. These students are likely to be learning with understanding; they take something from this problem solving task that is deep and lasting, something they might be able to use in other problem situations. Davis (1992) refers to this something as “residue”.

Teaching for and Learning with Understanding

Within many mathematics teachers’ conceptions is the belief that students should learn mathematics with understanding (Hiebert & Carpenter, 1992). But what does it mean to learn with understanding? There are certainly many different ideas about what this means. William Brownell (1946) pointed out many years ago that it is better to think of understanding as that which comes naturally while students solve mathematical problems rather than as something we should teach directly. David Perkins (1998), a noted cognitive psychologist, views this phenomenon as the ability to think and act flexibly with what one knows. Wiggins and McTighe (1998) view understanding in a similar fashion: to understand a topic or subject is to use knowledge and skill in sophisticated, flexible ways. Students need to make conscious sense and apt use of the knowledge they are learning and the principles underlying it (p. 24).

Hiebert and Carpenter (1992) describe understanding metaphorically in terms of how information is represented and structured: A mathematical idea or procedure or fact is understood if it is part of an internal network. More specifically, the mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and strength of the connections. Figure 1 illustrates how a mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections (p. 67).

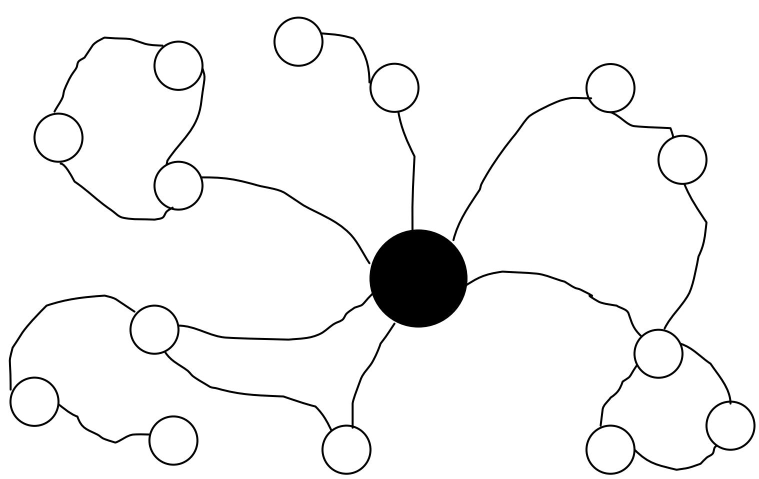


Figure 1. Internal networks.

When we use the ideas we already have (open circles) to construct a new idea (filled circle), a network of connections is developed between ideas. The more ideas used and the more connections made, the better the understanding (Van de Walle, 2007).

A Constructivist Framework

Based on the metaphorical model in Figure 1 above, learning with understanding requires that individuals create and re-create internal representations and networks. A widely accepted theory, known as constructivism, suggests that individuals take in new information, internalize it, and assimilate it into existing networks to fit with what is already known. The existing networks may need to be altered or reshaped in order to accommodate the new information to give it any meaning. Reshaping the existing networks to accommodate new ideas is accomplished through reflective or purposeful thought (Fosnot, 1996). This means sifting through existing ideas to find those that are the most useful in giving meaning to the new ideas. From the perspective of constructivism, learning with understanding is viewed as a self-regulated process of resolving inner cognitive conflicts that often become apparent through concrete experience, collaborative discourse, and reflective thought (Grennon Brooks & Brooks, 1993).

Constructivism suggests that individuals must be active participants in the development of their understanding. Students must have opportunities to “wrestle” with and “chew on” ideas and to work at fitting them into their existing internal networks. Abbot and Ryan (1999) view constructivist learning as an intensely subjective, personal process and structure that each person constantly and actively modifies in light of new experiences. According to Clements and Battista (1990) and Betts (1991), constructivism has these basic tenets:

- Knowledge is actively created or invented by the individual, not passively received from the environment. Students invent new ways of thinking about the world.
- Children create new mathematical knowledge by reflecting on their physical and mental actions. Ideas are constructed or made meaningful when children integrate them into their existing structures of knowledge.
- No one true reality exists, only individual interpretations of the world. These interpretations are shaped by experience and social interactions.
- Learning is a social process in which children grow intellectually with those around them. Students are not only involved in discovery and invention, but in social discourse involving explanation, negotiation, sharing, and evaluation.

A Socio-cultural Framework

Based on the tenets described above, a constructivist classroom should create a community of learners in which students build a positive social culture within the classroom. Four features of this social culture contribute to the overall effectiveness of the constructivist classroom (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997):

- Ideas expressed by any individual have the potential to contribute to everyone's learning and consequently warrant response and respect.
- Students must respect the need for everyone to understand their own methods, and must recognize that there are often a variety of methods that will do the job.
- Mistakes must be seen by students and the teacher as places that afford opportunities to examine errors in reasoning. They should raise everyone's level of analysis and be used constructively.
- The persuasiveness of an explanation or the correctness of a solution should depend on the mathematical sense it makes, not on the popularity of the presenter.

According to Battista (1999), when fostering a social culture supportive of constructivist pedagogy, classrooms often forgo traditional arrangements with desks in rows and opt for tables that will accommodate small groups of students. This constructivist classroom may contain an assortment of manipulatives and measuring devices for hands-on activities such as graphing calculators, which are accessible at all times (Crawford & Witte, 1999). Classroom walls are usually covered with student work, showing evidence of student collaboration. Signs, lists, and exhibits may be created by students rather than the teacher, an indication of student ownership in the classroom. Kohn (1996) describes constructivist classrooms as resembling a "working with" environment rather than a "doing to" environment with regard to student and teacher relationships.

A classroom environment that supports students' learning with understanding provides opportunities for students to communicate with each other, to participate in social interaction, and to share thoughts and ideas (Forster, 2002). Alper, Fendel, Fraser, and Resek (1995), like Battista (1999), describe a communicative and constructivist classroom this way:

> Desks are turned to face each other. Students are talking a lot in their groups, are animated, and asking questions. Two students have made a transparency and are proceeding toward the overhead projector to explain their work. They are responsible for calling on their classmates and answering their questions. There are several exchanges, and then applause erupts as the students sit down. Then the teacher asks another group of students to come to the overhead projector. (Alper et al., 1995, p. 632)

Students finding themselves in a classroom environment described above would regard their mathematics learning in active terms. Rather than memorize procedural algorithms in order to eventually pass a test, these students would see their role as learning and understanding mathematical relationships (Boaler, 2002; Forster, 2002).

When students are confronted with a new mathematics problem, the extent to which they are able to use mathematics depends on the knowledge they have, the classroom practices in which they engage in as they learn, and the relationships they have developed with the discipline of mathematics (Boaler, 2002). Students in both classroom scenarios described earlier regarding the number of handshakes may have similar procedural mathematical knowledge, but the classroom practices they engage in as they learn mathematics are very different. As a result, the relationships they build with the discipline of mathematics will be very different as well.

Studies Supporting Teaching for and Learning with Understanding

There has been extensive research that has examined teaching and learning within mathematics classrooms. In one study of mathematics teaching and learning, Jo Boaler (2002) found students using different mathematical practices in a problem solving environment that encouraged them to develop different relationships with the discipline of mathematics. She interviewed 48 students in six high schools in California. Students in four of the schools learned in an Instrumentalist environment by practicing problems in the textbook demonstrated by the teacher. They were generally able to do well in subsequent textbook situations, but these same students found it difficult to apply what they learned from the textbook exercises to open, group-based mathematics problem solving situations. Although they were successful in the classroom, there was an important conflict between the mathematics practices they engaged in and the *human agency* they developed with regard to mathematics. For example, many students talked about their dislike for and rejection of mathematics because it offered little opportunity for expression and interpretation. One student put it this way during an interview: “I’m just not interested in, just, you give me a formula, I’m supposed to memorize the answer, apply it and that’s it.” (Boaler, 2002, p. 115)

Some students in the same classrooms at these four schools maintained an interest in mathematics and even liked the subject because there were only right and wrong answers to think about. They were happy to be "receivers" of knowledge and liked not having to know how or why the mathematics worked. In a second interview, a student said:

> I always like subjects where there is a definite right or wrong answer. That's why I'm not a very inclined or good English student. Because I don't really think about how or why something is the way it is. I just like math because it is or it isn't. (Boaler, 2002, p. 116)

Regardless of their like or dislike of mathematics, students in the classes at these four schools were successful in developing *disciplinary agency* (Boaler, 2002). This means they were able to learn mathematics through teacher demonstrations, explanations, and subsequent textbook situations.

In contrast, the students interviewed at the other two schools learned mathematics through open-ended mathematics problem solving situations and were able to learn and use mathematics in a variety of situations. Instead of just being passive receivers of knowledge, these students were submerged in a problem solving environment and were given opportunities to discuss, conjecture, critique, reflect, ask questions, and suggest the direction of the mathematical problem solving. As a result of these constructivist classroom practices, these students not only developed disciplinary agency, but they formed a relationship with the discipline of mathematics that allowed them to spend part of their time using standard methods and procedures and part of their time modifying those standard methods to fit new problem situations (Boaler, 2002). They formed their perceptions of what mathematics is all about from different mathematical practices they engaged in, and as a result, developed human agency as well (Boaler, 2002).

In another five-year longitudinal study, Boaler and Staples (in press), documented the progress of 700 students in three high schools. In two of the schools, named Greendale and Hilltop, mathematics was taught using Instrumental approaches; students worked independently with teacher directed lectures and demonstrations and then practiced with problems similar to what the teacher demonstrated. At the third school, called Railside, students were more engaged in problem solving approaches to mathematics and were expected to share and justify their ideas and solutions with their peers. At the beginning of the study, students at Railside were achieving at lower levels than students at the other two schools. At the end of the first year, there were no significant achievement differences between Railside students and students from the comparison schools. But within two years, the Railside students were outperforming their counterparts at the other two schools; they were more positive about mathematics and took more mathematics in high school, and many more planned to pursue mathematics in college (Boaler & Staples, in press).

What happened at Railside that changed the achievement in and disposition of students toward mathematics? The findings of the study show that teachers and students at Railside used different practices in mathematics not used at Greendale or Hilltop, practices that provided opportunities for students to develop human agency toward mathematics. Table 1, adapted from Boaler (2002) summarizes instructional time spent in the classroom.

Table 1

Instructional comparisons

Railside	Greendale and Hilltop
Teacher lecture 4% of time	Teacher lecture 21% of time
Group work 72% of time	Individual work 48% of time
Student presentations 9% of time	Student presentations .2% of time
Average time/problem: 5.7 minutes	Average time/problem: 2.5 minutes
Teacher whole class questions 9% of time	Teacher whole class questions 15% of time

It is interesting to note that students at Greendale and Hilltop sat individually and did not engage in any type of structured group work. These students also spent very little time presenting their ideas. The teachers at these two schools spent a lot of class time talking to students (21%), usually demonstrating methods and procedures. Teachers' questions were classified into seven different categories. The vast majority of questions asked by the teachers at Hilltop and Greendale fell into the procedural category (97% and 99%). At Railside, teachers asked many more varied questions. Sixty-two percent were procedural, 17% conceptual, 15% probing, and 6% into the other categories (Boaler & Staples, in press). The discourse encouraged in groups and during whole-class discussions at Railside did much to contribute to students' development of human agency and its interplay with disciplinary agency.

Forster (2002) conducted a study in Western Australia in which she encouraged active participation through problem solving from 17 students in an 11th year mathematics classroom. She found, as did Boaler (2002), that mathematical practices in the classroom take on many forms, but practices that engage students in explanation, justification, negotiation, and asking and answering questions was most effective in students developing positive relationships and human agency with the discipline of mathematics.

Practices that Support Teaching for and Learning with Understanding

Webb, Romberg, Dekker, de Lange, and Abels (2004) believe teachers must critically examine their classroom practices, practices that have been developed largely to monitor student mastery of skills and procedures and development of conceptual knowledge. Part of this critical examination involves looking at two important cognitive processes that play an important role in learning mathematics with understanding: communication and reflection (Hiebert et al., 1997). Communication in mathematics classrooms allows students to challenge each others' ideas and ask for clarification and further explanation (Forster, 2002). In doing so, it can encourage students to think more deeply about their ideas, or engage in the process of reflection. It means turning those ideas over, thinking about things from different points of view, stepping back to look at things again and consciously thinking about why something has been done. Communication works together with reflection to produce new relationships and connections. Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics (Hiebert et al., 1997). In their book, Hiebert and his colleagues (1997) described multiple scenarios from actual classrooms in which students were communicating and reflecting on their own and their classmates' mathematical ideas:

> The students in Ms. Keith's class regularly are called upon to articulate their solutions, to describe in words what they have done. In order to be able to describe their strategies, they need to reflect upon them, and to decide how to report them verbally. (p. 95)

What might classrooms that support student's communication and reflective practices look like? Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Oliver, & Wearne (1996) provide glimpses into their studies and as a result, into the classrooms in which they have spent an abundance of time. They discuss some essential features of classrooms that support students' understanding of mathematics:

- The selection of mathematical tasks
- The social culture of the classroom
- The role of the teacher

Hiebert and Grouws (2007) and Hiebert et al. (1996) suggest that mathematical tasks should be problematic, allowing students to wonder why, to inquire, to search for solutions, and to resolve incongruities. In doing so, the expectation is that they should *struggle* with mathematical ideas; that is, they should expend effort to make sense of or figure something out that is not immediately apparent. The struggle will come from solving problems that are within reach and grappling with mathematical ideas that are comprehensible but not yet well-formed (Hiebert et al., 1996, 2007). In other words, tasks that promote mathematical understanding must take into consideration the current understanding of the students. Students should have the appropriate ideas to engage and solve the problem and yet still find it challenging and interesting. Dewey (1910) and Polya (1957) devoted a good deal of time and attention to the idea that students should struggle with mathematics. "The process begins with some perplexity, confusion, or doubt. It continues as students try to fit things together to make sense of them, to work out methods for resolving the dilemma" (Dewey, 1910, p.12). Polya (1957) described struggle in this way:

> A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. (p. v)

The tension that Polya speaks of above results from the doubts, confusion, and perplexity that can result from the problematic nature of the mathematics. The handshake problem, and others like it, is a good example of problem-based tasks that can create tension and struggle as students try to make sense of the mathematics. Students who engage in these tasks and focus on the solution methods are actively looking for relationships, analyzing patterns, finding out which methods work and which do not, justifying results, or evaluating and challenging the thoughts of others. They are engaging in reflective thought about the ideas involved (Van de Walle, 2007). Hence, the tasks should engage students in discourse and reflection and thereby promote understanding.

As stated earlier, learning with understanding consists of engaging in a process of creating mental representations or connections among mathematical facts, ideas, and procedures (Hiebert & Carpenter, 1992). When students are allowed to struggle with novel mathematical situations, they will need to build new representations by reconfiguring or re-forming existing relationships among the facts, ideas, and procedures in order to assimilate the new information. Without opportunities to struggle, students may merely incorporate the new information as isolated representations and have little opportunity to build stronger connections and understanding of the new facts, ideas, or procedures.

When the teacher promotes and establishes the social culture of the mathematics classroom, then students will realize the expectation is to share their ideas (Forster, 2002). At the same time, they will understand that their ideas and methods will be honored, valued, and respected. Only when every student contributes and is heard will a community of learners develop in the classroom. It is an integral part of the system of instruction that sets students' learning mathematics with understanding as the goal (Hiebert et al., 1997). Understanding, then, can be viewed as a community of students who are becoming adept at doing and making sense of mathematics. Put another way, in a "working with" environment, the focus is on students' underlying motives in order to help them develop positive values and a love of learning; the preferred methods include the creation of a caring community and genuinely engaging curriculum (Kohn, 1996).

The "working with" engaging classroom environment is often guided by socio-constructivist views of learning (Forster, 2002). This means teachers allow their students to build their own understanding of ideas and concepts by connecting them with what they have learned previously. In doing so, mathematical tasks are designed to elicit higher ordered thinking or cognitive demand (Stein, et al., 2000). Why is this so important? *Professional Standards for School Mathematics* (NCTM, 1991) states that opportunities for student learning are not created simply by putting students in groups, by placing manipulatives in front of them, or by handing them a calculator. Rather, it is the kind and level of thinking that will determine what they will learn. If students spend their time practicing paper-and-pencil skills from textbooks and worksheets, then they will likely become better and faster at executing those skills. If they spend their time watching the teacher demonstrate methods for solving problems, they will likely become better at imitating these methods on similar problems. If, on the other hand, students spend their time reflecting and communicating on the way things work, on how various ideas and procedures are the same or different, on how what they already know relates to situations they encounter, they will be likely to make new connections, build new relationships, and construct new understandings (Hiebert et al., 1997).

Shepard (2001) believes that when teachers embrace socio-constructivist aspects in their classrooms as students build new knowledge, the mathematical tasks build socialization into the classroom discourse. Therefore, in the context of discussing the mathematics, all students are expected to share their ideas about the task and challenge their thinking and the thinking of their classmates with regard to the task.

Students in a classroom environment that promotes practices of communication and reflection have opportunities to develop disciplinary agency, but because they are involved in learning and understanding mathematical relationships, these students are also able to develop human agency (Boaler, 2002). This means students can use and apply mathematics, state and test conjectures, critique each others' ideas, suggest the direction of mathematical problem solving, ask questions, and thereby develop a sense of authority about the mathematics they are learning. These students, then, can use learned procedures when encountering problems that differ from those for which the procedures were initially learned. Their conceptual knowledge may detect useful similarities and differences between problems, and subsequently, inform the procedure regarding appropriate adjustments. In this way, conceptual knowledge extends the procedure's range of applicability and the construction of internal representations of procedures become part of larger conceptual networks (Hiebert and Carpenter, 1992).

Teachers' Conceptions Vis-à-Vis Their Practices

Teachers' classroom practices are often guided by what they believe to be true about mathematics and the teaching and learning of mathematics (Cochran-Smith & Lytle, 1992; Philipp, 2007). For example, a teacher with Instrumentalist views will demonstrate to students how to work through particular problems and then expect them to practice on similar problems. Thompson (1984), while recognizing the complex relationship between conceptions and classroom practices, observed a high degree of consistency between them in her case studies of three teachers. Each one possessed a different perspective regarding the nature of mathematics (Ernest, 1988), and in each case, their practices were consistent with their perspectives. For example, one teacher named Kay held problem solving views and engaged her students in generating and justifying their own algorithms and solutions to mathematics problems. Another teacher in her study, Lynn, had Instrumentalist views regarding mathematics; she taught in a very prescriptive manner which emphasized demonstration of rules and procedures to students which were followed by drill and practice.

Grossman, Wilson, and Shulman (1989) also reported a high degree of consistency between conceptions and practices in novice teachers. In their study, they observed a strong relationship between a beginning teacher's knowledge base and his/her resulting practices. Joe engaged his students in problem solving situations, allowing them to generate their own algorithms followed by discourse regarding their validity. Possessing both a Problem solving and Platonist perspective, Joe stressed the "whys" of mathematical procedures and often pointed out how topics were inter-related and fit into the larger scheme of mathematics. Laura, another participant in their study, had a far less sophisticated knowledge base and an Instrumental perspective regarding mathematics. As a result, she emphasized drill, rarely justifying why a procedure worked, and she discouraged students from using their own algorithms that were not included in the textbook.

Benken's (2005) case studies involving beginning secondary mathematics teachers also demonstrates the consistency that can exist between a teacher's conceptions and the classroom practices that result. Laurie, one of the participants, held an Instrumentalist perspective regarding mathematics, and coupled with her limited knowledge base and perception that she could not teach high level mathematics, she engaged her students in teacher-led discussions and memorization of basic facts and procedures. In contrast, James, another participant, had a strong and flexible knowledge base and possessed a high level of confidence in his ability to learn mathematics. As a result, his problem solving perspective regarding mathematics was manifested in his classroom practices. He believed learning mathematics involved making meaningful connections and thinking through problems using multiple strategies; his students communicated their thinking about mathematical concepts through writing and during whole class and small group discussions.

While teachers' classroom practices are often guided by their conceptions regarding mathematics and its teaching and learning, mismatches between their practices and conceptions can also exist (Ball, 1988; Cooney, 1985; Ernest, 1988; Shulman, 1986; Thompson, 1992); these mismatches are predicated by a number of key elements:

- Teacher's knowledge base
- Social and political contexts
- Teacher's level of consciousness.

A great deal of knowledge is essential to successfully implement certain models of mathematics teaching (Ball, 1988; Dewey, 1964; Fennema & Franke, 1992; Shulman, 1986). For example, constructivist models of teaching and learning that align with the problem solving perspective (Kuhs & Ball, 1986) requires teachers to possess a broad knowledge base in mathematics in order to recognize and capitalize on ideas and procedures that arise naturally out of classroom discourse. If a teacher embraces the problem solving perspective, yet he/she has a limited knowledge base, then that teacher will often resort to a lower level perspective in Ernest's three-tiered hierarchy (Ernest, 1988), such as the Instrumentalist or Platonist view of mathematics. This can be illustrated by Aguirre's (1995) case study involving Mr. Martin. He wanted to incorporate newly formed beliefs into his classroom practice, those related to small group activities and student collaboration falling within the problem solving perspective. But his knowledge base regarding pedagogy and student cognition was limited, and this caused him to abandon these new beliefs and replace them with his stronger Instrumental beliefs.

Heaton (1992), Putnam (1992), Prawat (1992), and Remillard (1992) also conducted case studies looking at the mismatch between teachers' conceptions and their classroom practices. They followed four teachers as they began implementation of a new curriculum embracing the problem solving approach to mathematics. These fifth-grade teachers entered this study with the belief that computational algorithms constitute the core of mathematics. Observations of the participants' lessons confirmed this belief; the teachers highlighted procedural aspects of the lessons from the new curriculum and downplayed any opportunities for students to engage in problem solving and reflect on and share mathematical ideas with each other. Why? Did they utilize Instrumental approaches because of a limited knowledge base? Perhaps, but other factors may have been involved. It was the arithmetic algorithms that defined their mathematics and these teachers' perspective of what it meant to understand those algorithms differed from the problem solving perspective in the new curriculum.

There is a powerful influence on teachers' conceptions from the social setting in the school, school district, and community (Ernest, 1988) and expectations from students, parents, teachers, and administrators can lead teachers to use practices in the classroom incongruent with their conceptions. For example, a teacher who embraces the problem solving approach regarding mathematics may feel pressure to follow a different approach used by fellow mathematics department members. Despite having differing beliefs about mathematics and its teaching, teachers in the same school are often observed to adopt similar classroom practices (Ernest, 1988).

In addition to the social context, the political climate within a school or district can also account for the mismatch between teachers' espoused conceptions of mathematics and what is actually enacted in their classrooms. For example, the NCTM documents (1980, 1989, 2000) advocating for problem solving reform approaches may have an influence on teachers' verbal statements but have little effect in their classrooms (Thompson, 1992); teachers may also be responding to state mandated tests in ways that impact their conceptions of mathematics and their resulting practices (i.e., teaching to the test).

Some teachers engage in self-evaluation and reflective thought; both are processes giving them opportunities to examine the gap that might exist between their conceptions and practices, and to narrow it (Ernest, 1988). This level of consciousness of their own conceptions, and the extent to which they reflect on their practices, can help teachers reconcile and integrate classroom practices with conceptions. Teachers who embrace the problem solving perspective regarding mathematics and its teaching do this naturally as they take on the role of facilitator in the classroom, help students define their roles, and determine the suitability of the mathematics in the classroom. Teachers who hold Instrumentalist and/or Platonist views of mathematics and its teaching and learning require little self-evaluation and reflective thought with respect to their roles, the roles of their students, and the mathematics they employ in their classrooms (Ernest, 1988).

Changing Teacher's Conceptions

Research has indicated that teachers' conceptions of mathematics are robust; they consist of a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools (Ball, 1988; Philipp, 2007; Thompson, 1992) and they often form the foundations on which teachers build their practices. Teachers' conceptions of mathematics develop long before they enroll in their first education course and they tend to hold on to these same conceptions after they exit their teacher education programs (Kagan, 1992). But when teachers are given opportunities to interact with their environment in different ways and reflect on those actions, their conceptions of mathematics can be tested, challenged, and reconstructed; perhaps their old conceptions can be refined or give way to new ones (Tobin, Tippins, & Hook, 1992). What do these opportunities look like?

There have been several case studies that have examined conceptions in pre-service and beginning mathematics teachers (Ball, 1988; Benken, 2005; Cobb, Wood, and Yakel,1990; Feiman-Nemser, McDiarmid, Melnik, & Parker, 1987; Feiman-Nemser & Featherstone, 1992; Liljedahl, 2005; Tuft, 2005). They were designed to challenge preservice and inservice teachers' conceptions of mathematics, engage them as learners of mathematics and mathematics pedagogy, and provide them with experiences in mathematical discovery. The combination of these three approaches has been shown to be very effective in changing preservice teacher's beliefs (Liljedahl, Rolka, & Rosken, 2007).

Tuft (2005) explored changes in pre-service elementary teachers' conceptions during the time they were enrolled in a university methods course. Tuft collected data from these 34 students using a "Mathematics Inventory" which included Likert-type questions and open-ended questions. She also collected a journal entry that focused on "What is Mathematics?" and chose four focus students to interview. In her analysis, Tuft found that these students' conceptions regarding the usefulness of mathematics and the processes used in doing mathematics were tested and shifted. The data also indicated a more positive attitude toward mathematics and the teaching of mathematics.

Peter Liljedahl (2005) conducted a study while teaching a university methods course in which the conceptions of his 35 students were challenged while being submerged in a collaborative problem solving environment. The problems were used to introduce concepts in mathematics and mathematics teaching and learning; they promoted communication and interaction within groups and whole-class discussion was frequent. In addition to the discussion that resulted from the doing of problems, Liljedahl's approach to this study was to help his students make explicit their ideas on teaching and learning mathematics by keeping a reflective journal. In it students responded to prompts such as:

1. *What is mathematics?*
2. *What does it mean to learn mathematics?*
3. *What does it mean to teach mathematics?*

These prompts were assigned three times during the course and at the end, students were given this prompt: How have your ideas changed through your participation in this course? In particular, how have your ideas about what mathematics is, and what it means to teach and learn mathematics changed?

In his analysis, Liljedahl developed several themes that resulted from students' responses regarding their beliefs about mathematics and its teaching and learning. Three relevant themes that emerged were: mathematics as a verb, humanizing mathematics, and learning through talking. Almost all students mentioned how they see mathematics as something one does as opposed to something one learns. In fact, one student commented, "I now see how important it is to allow students to work on a problem, to actually do the mathematics, to struggle, to think" (p. 4). All the participants expressed how important collaboration had been to their experience of "doing" mathematics and many explicitly indicated that it was the talking that was important. One student said it very succinctly, "My whole definition of mathematics broadened beyond problem solving and pattern finding to include communication…..I started to see talking as an integral part of learning, and that math class should be noisy at times" (p. 4).

Liljedahl's study shows that submersing students in a problem solving environment can challenge their conceptions about mathematics and what it means to teach and learn it. Many of the participants in his study entered with Instrumentalist views regarding mathematics, but these conceptions of mathematics were tested, challenged, reconstructed and even refined (Tobin, Tippins, & Hook, 1992).

University mathematics methods courses that focus on submersing students in constructivist, problem solving environments and engaging them as learners of mathematics are critical in helping students confront and perhaps restructure their conceptions, but there are individual students that may exhibit *resistance* to the goal of teaching for and learning with understanding (Rodriguez, 2005). That is, they may resist using constructivist, inquiry-based pedagogical approaches to teaching and learning in their classrooms. Why? Resistance involves agency or the conscious choice to take action or not. Teachers avoid or refuse to enact inquiry-based approaches stating that they prefer to lecture and demonstrate and then have students practice, or instead indicate that students must have the skills before they can engage in problem solving activities (Rodriguez, 2005). Lortie (1975) also contends that teachers may resist making pedagogical changes because they lack the confidence or knowledge base to move away from their Instrumentalist views to teaching they have become so accustomed to after 16 years of schooling. Regardless of their reasons, it is important for teacher educators to help prospective and beginning teachers restructure their conceptions and interact with their school communities in ways that will help lessen their resistance to pedagogical change.

Throughout the course of this study, Emily, Matt, and John had many opportunities to interact with both the university and their school community and reflect on their experiences. As beginning teachers and participants in the SMEST program, mentoring was provided as a way to support and challenge their conceptions regarding mathematics and the teaching and learning of mathematics. There have been numerous studies examining the role of mentoring and its impact on teachers' conceptions.

In her five-year study, Thea Dunn (2005) examined the conceptions of over 400 prospective teachers regarding their conceptions about the teaching and learning of mathematics. Using *Mathematics Autobiographies*, field experiences, and video clips, she focused on critical reflection, allowing participants to examine how equity, justice, caring, and compassion could inform their educational goals. In doing so, she challenged her participants' thinking and used her role of mentor to guide her participants to restructure and broaden their conceptions of mathematics teaching and learning.

In another study involving the mentorship of 89 student teachers, Philippou and Charalambous (2005) conducted interviews with eight of the student teachers three different times over the course of the twelve-week study. Each of the eight participants observed their mentors' teaching and had opportunities to exchange ideas with them regarding their teaching and class management. Participants also taught 30 mathematics lessons in their mentors' classrooms and received feedback regarding their teaching. The analysis of their data revealed that mentors could influence student teachers' conceptions through their own teaching style, the feedback they gave, and the latent messages they implicitly conveyed to their mentees. The difference between the mentors' teaching style and students' conceptions about teaching and learning mathematics impacted their beliefs about teaching and learning mathematics. However, it was important for mentors to be open to a participant's ideas and teaching style and did not impose their own ideas on the participant. During feedback, statements of support were critical. One student noted, "My mentor persuaded me that mistakes should be considered opportunities for learning rather than indications of inefficiencies" (p. 77).

Even through nonverbal channels of communication, the latent messages mentors sent to the participants affected their conceptions. One participant commented on the behavior of her mentor saying, "His whole attitude instilled doubts about my teaching competence. On seeing him observe my lesson, I often had the impression that he was ready to tell me my teaching approach is ineffective. I felt that I was the worst teacher in the world." (p. 77)

There is a growing consensus that the quality of mentoring varies (Jones, 2001) and that the mentor's role covers a wide spectrum, from mentors as teaching models and critical friends who assist new teachers with planning to simply being there to provide assistance only when requested. Feiman-Namser (2001) asserts "we still know very little about what thoughtful mentor teachers do, how they think about their work, and what new teachers learn from their interactions with them" (p. 17).

What might happen when mentorship opportunities are limited or not available to early-career teachers? Statistics show that nearly half of the new teachers in the United States leave the profession in their first five years of teaching and the attrition rate is approximately 30% for teachers in their first three years (NCTM, 2007). Darling-Hammond (2006) has documented several studies that report beginning teachers who lack professional training are about twice as likely to leave teaching in their first year as those who have had student teaching and preparation in such areas as learning theory, child development, and curriculum (p. 14). Teachers, however, who receive strong mentoring (NCTM, 2007) are more likely to stay in the profession (Luczak, 2004).

For those early career teachers that have received little or no mentoring, many effectively “close” their classrooms doors. These teachers invite their students to participate in the learning of mathematics by subscribing to their conceptions about mathematics teaching and learning. It is difficult for teachers to implement any kind of meaningful change in their conceptions and classroom practice with this closed-door policy. But when teachers open their classroom doors to their colleagues and the community, it moves them away from a view of teaching as a solitary activity, owned personally by each teacher, toward a view of teaching as a professional activity open to collective observations, study, and improvement; it invites teachers to recognize and accept the responsibility for improving not only their own practice, but the shared practice of the profession (Hiebert, Gallimore, & Stigler, 2004). Such collaboration is needed as teachers work to make changes in their conceptions and classroom practices and plays a pivotal role in their professional development (Bouck, Keusch, & Fitzgerald, 1996). Opening classroom doors provides intellectual and emotional support that teachers need as they restructure their conceptions and redefine their practice. Weisglass (1994) made this statement:

> Although we can challenge teachers with different visions of mathematics pedagogy, new curriculum, and research results, we must do more. We must give them opportunities to develop trusting collegial relationships so that they can reflect on their beliefs, construct their own understanding of

> the proposed changes, work through feelings that may inhibit their ability to change, and make their own decisions about how to respond—both to their students and to the suggestions from the reform movement.

Conceptions regarding mathematics and mathematics teaching should be founded on dialogue among teachers and among teachers and their students, all responding to each other on the basis of what has been said or done. Teacher educators that are in a position to take action have the responsibility to take action to ensure that beginning and early career mathematics teachers have opportunities and resources to engage in dialogue and thereby restructure their conceptions of mathematics and mathematics teaching and learning.

Conclusion

The predominant view of mathematics education in the United States is one of rules and procedures, memorization and practice, and exactness in procedures and answers. As a result, schools and districts expect immediate results with regard to student achievement in the classroom and on standardized tests. NCLB (2002) and AYP (2001) also demand these results; otherwise schools are placed on probation and/or corrective action. There is no doubt that there is a need to have knowledge of facts and procedures, a need for practice and exact answers. But if this is the only way mathematics learning is regarded, then there is no reason to encourage mathematics learning with understanding. There was considerable debate and conflict in the days of Thorndike and Dewey and the debate continues today. Advocates of Instrumentalist and Platonist approaches to learning see solutions to students' short term progress, school accountability, and standardized and state assessments. These learning approaches give teachers specific knowledge and directions for designing their mathematics lessons, but it does little to address how students learn. Teachers' lessons focus primarily on the development of specific skills and concepts that do not require high cognitive demand and learning with understanding. Dewey (1910) cautioned that the practice of teaching without understanding damaged students' ability to reflect and to make sense of what they were doing:

> Sheer imitation, dictation of steps to be taken, and mechanical drills may give results most quickly and yet strengthen traits likely to be fatal to reflective power. The pupil is enjoined to do this and that specific thing, with no knowledge of any reason except that by doing so, he gets his result most speedily; his mistakes are pointed out and corrected for him, he is kept at pure repetition of certain acts till they become automatic. Later, teachers wonder why the pupil reads with so little expression, and figures with so little intelligent consideration of the terms of his problem. (pp. 51-52)

Advocates of constructivist learning realize that learning may not be immediate, but will take place when students are able to connect ideas and information to existing internal networks where prior knowledge and experiences exist (Hiebert et al., 1997). The learning outcomes are not as simple as in the Instrumentalist approaches as teachers are not able to "see" the connections their students are making. However, through students' communication and reflection, teachers can assess the progress students are making by helping them connect important mathematical ideas through the tasks they choose. This is a very challenging aspect of teaching mathematics, one that many teachers may not be prepared to take on. But as learning with understanding becomes more of a focus in our schools, teachers will need to restructure their conceptions of what teaching for understanding entails.

Chapter 3

Methodology

> When it is appropriate, usually after I have developed trust with a teacher, I offer generative metaphors and stir the waters. In a true relationship, a mentor is seen as a coach stirring things up, rather than an expert pouring knowledge into that teacher.
>
> (Susan Bethanis, 1995. p. 194)

This study focused on three first-year intern teachers in secondary mathematics classrooms over an academic school year. The guiding questions of this study were:

(1) What initial conceptions regarding the teaching and learning of mathematics do secondary mathematics intern teachers reveal in their first year of teaching?

(2) How do these conceptions change during their first year?

a) What influence does a year-long mentoring process have on the interns' conceptions?

b) What role does a university mathematics methods course have in supporting and restructuring the interns' conceptions?

It was my intent to investigate these research questions through a detailed, in-depth collection of data. In order to address the first research question, I initiated weekly classroom observations and engaged in informal, reflective conversations with the participants. In addition, I collected artifacts from their classrooms and examined their reflective writing regarding their teaching and learning experiences. To address the second research question, participants responded to on-line questionnaires that I constructed. I also maintained a schedule of weekly classroom observations and conversations over the course of the year and continued to examine their reflective writing.

Qualitative Inquiry

A qualitative approach to research was selected for this study because structures such as classroom observations, conversations, and reflective writing were used to interpret how both the participants and I, a co-participant, made sense of their conceptions and their day-to-day teaching and learning. Wolcott (2001) says participant observation serves as the unifying central activity of all qualitative work. Denzin and Lincoln (1994) stated that qualitative research involves an interpretive naturalistic approach to its subject matter which means the researcher studies the subject matter in its natural settings. Creswell (1998) reinforces this with his own interpretation of qualitative research: "The researcher's inquiry process builds a complex, holistic picture, analyzes words, reports detailed views of informants, and conducts the study in a natural setting" (p. 15). To engage in this inquiry process, it was important to have conversations with teachers about their conceptions of mathematics and what they do and think about in their classroom. Patton (1990) provides a description of qualitative research in this way:

> Qualitative research is an effort to understand situations in their uniqueness as part of a particular context and the interactions there. This understanding is an end in itself, so that it is not attempting to predict what may happen in the future necessarily, but to understand the nature of that

> setting—what it means for participants to be in that setting, what their lives are like, what's going on for them, what their meanings are, what the world looks like in that particular setting—and in the analysis to be able to communicate that faithfully to others who are interested in that setting…The analysis strives for depth of understanding. (p. 6)

It is Patton's (1990) description of qualitative research above that made this study so compelling. I chose to immerse myself as researcher-participant in order to understand how these teachers confronted their conceptions, made sense of their teaching, and how they changed their conceptions and teaching during the year. It was through conversations about classroom observations and reflective writings that I attempted to understand the meaning behind their decisions. With this in mind, I wanted to examine these teachers' conceptions by entering their classrooms, observing their behaviors, and then taking time to probe their thinking about what happened in their classroom and why.

Case Study Design

Case studies are single units of study that have boundaries involving one person, such as a teacher, to those involving whole schools, to those that focus on a curriculum or national program (Glesne, 2006). They were chosen for this study because each case involved a beginning teacher within a classroom over an academic year. As with all case studies, the focus is on discovery and interpretation (Merriam, 1998), and the knowledge gained is specific and contextual. This focus on discovery and interpretation fit well with the study's purpose and the research questions that guided the study. The descriptions and stories that resulted from each of the first-year intern teacher's year-long experiences helped to reveal their initial conceptions of mathematics as well as the changes that occurred during the year.

While some case studies generate theory, others are simply descriptions of cases. Still others are more analytical in nature and display cross-case comparisons. Although there is no standard form for reporting case studies (Merriam, 1998), it is useful to at least conceptualize some kind of format for this study. Stake (1995) suggests the use of vignettes so that the reader might develop a vicarious experience and get a feel for the time and place of the study. Besides opening with a vignette, Stake (1995) has also suggested other components in a case study, those that help with the flow of ideas in the case study. These are listed below:

- The researcher identifies the issue, the purpose, and the method of the study so the reader learns how the study came to be, the background of the writer, and the issues surrounding the case.
- The researcher provides a description of the case and its context, a description the reader might make if she or he had been there.
- The researcher presents and probes a few key issues so that the reader can understand the complexity of the case.
- The researcher presents assertions and a summary of what is understood about the case.
- The researcher ends with a closing vignette, an experiential note, reminding the reader that the study is one person’s encounter with a complex case.

I started Chapter 1 with a vignette that illustrated a first-year teacher's conundrum, hoping to draw the reader into the study. I also included the importance surrounding this study, the purpose of conducting it, and a brief account of my background, again hoping to ground the study for the reader. Additionally, in Chapter 1, I provided a brief description of the study and laid out its context. In this chapter, I have added additional descriptive detail about the context of the study, its participants, and the methods used to collect and analyze the data. In later chapters, I provided a description of each case using the collected data, identified important issues, and made assertions surrounding the cases (Stake, 1995).

The format of these case studies uses the components discussed above to shape the stories of the three participants; in other words, the format shapes the written narrative (Creswell, 1998). In addition, the study's epilogue in the final chapter brings a personal experience into the narrative without disrupting the flow of the narrative in the study (Creswell, 1998). With the theme on teachers' conceptions, these case studies can be advanced because of their usefulness for teachers and teacher educators who have an interest here.

Settings

This research study took place in secondary mathematics classrooms in a large public school system in the southwestern United States. The participants were all college graduates with Bachelor of Science degrees in mathematics and met all the requirements for entry into the College of Education of a large southwestern university. They were participants in an alternative teacher licensure program, an intensive teacher education program which enables post-baccalaureate students to earn a secondary teaching license for grades 7-12. The intern teachers committed to a 14 month pre-service course of study with a concurrent year of field experience in a middle or high school classroom. In their first two months of the program during June and July, interns took two courses, a field experience course and a course called, “Teaching Reading in the Content Field”. After successfully completing this probationary pre-service coursework during the first summer, the interns were given a provisional license by the State Professional Education Department. They were hired by the local school district and then assigned in paired teacher teams to full time teaching positions. As such, they accepted full responsibility for the classroom position and functioned as full-time members of the school’s staff.

While teaching during both the fall and spring semesters, the interns completed coursework in curriculum, methodology, and content pedagogy. During the fall semester, the interns enrolled in a field experience course and a mathematics methods course entitled “Teaching of Secondary Mathematics” (See Appendix A for a description of this methods course). The methods course was of particular interest to this study because it is here that the interns’ conceptions of mathematics and mathematics teaching and learning were confronted and challenged during problem solving situations and through reflective journaling. In addition, interns also took a course in the fall entitled, “Issues in Secondary Education”. In the spring, the interns enrolled in their third field experience course and a special education class entitled, “Working with Special Needs Populations”. They also took a curriculum course entitled, “Advanced Instructional Strategies”. They completed the remainder of their coursework during the summer following the school year, taking a course in child development and a relevant elective.

The teacher preparation program described above offers an alternative to traditional teacher preparation models. The focus of the program is on learning to teach in the context of a full-time classroom experience. Throughout the 14 month program, interns receive collegial and professional support from university faculty and a mentor coach. They work in pairs with the goal of developing an immediately accessible support system, one providing opportunities to share ideas and actively reflect on their experiences. Upon successful completion of the program, the interns will have completed 30 hours of graduate coursework that can be applied toward a Master of Arts degree in secondary education. At this time, interns are able to apply for and receive a standard Level I license from the State Public Education Department. Many of the interns are re-hired by the local school district in which they complete their internship.

Case Study Participants

There were three participants in this study. John, 26 years old at the time of the study and of African American/Mexican descent, was assigned to teach two classes of Algebra I in a high school classroom. He shared a full teaching position with a female Anglo-American science intern teacher. His school, Valle Vista High School (a pseudonym), was built in the 1950's and consisted of several buildings. He and his partner were located in E building, the last building away from the administrative offices before reaching the portables and then the track and baseball field. The classroom (see Appendix E) was a former science classroom, but had become an extra classroom after the new science building had been built ten years earlier. When walking into the front entrance to the classroom, a large, prominent, fixed laboratory bench with a sink and unused aquarium occupied most of the front area. Behind it was the only chalkboard and screen in the classroom. An overhead projector sat on a table in front of the bench. On both sides of the classroom were storage cabinets and laboratory benches with sinks and gas hook-ups; they occupied most of the length of the sides of the classroom. There was also a laboratory bench set-up at the back wall of the classroom with the same standard set-up as the sides. The furniture in the classroom consisted of 15 laboratory tables that seated a pair of students; these tables had been marred, scratched, defaced, and written on during the school's 50+ years of existence. A few of the unstable ones had been kept from rocking by sliding a notebook under one of the legs. There were also two extra single desks, somewhat unstable, which were probably rejects from other classrooms, and a few loose chairs. These served to handle the overflow of students in some classes. In the back corner opposite the front entrance two teacher desks were arranged, a large one facing the front and a smaller one, with a broken drawer, butted against the back wall of the classroom. John's desk was the smaller one, and despite the salvage quality of the desk, his possessions were neatly arranged on the surface.

Valle Vista used a modified block schedule. This type of scheduling allowed John to teach his two sections of regular Algebra I, as opposed to honors Algebra I, for an extended period of 100 minutes two days a week on Tuesday and Thursday and for 50 minutes one day a week on Friday. His students were ninth-graders, some reclassified, with a sprinkling of tenth-graders as well. They were predominantly Latino/a, which reflected the school's demographics of 75% Latino/a (MuniNet Guide, 2008). See Table 2. His paired teaching partner, who was not a participant in this study, taught three 100-minute biology classes on Monday and Wednesday and 50 minutes on Friday.

Matt, age 30 at the time the study was undertaken, was a second participant in the study. A White Anglo-American, he taught seventh grade regular mathematics in a tracked mathematics program at Einstein Middle School (a pseudonym). Matt was responsible for teaching three 45-minute morning classes during the school day. He taught first period, followed by second period prep he and his partner shared. Then Matt taught third period. After an early lunch, Matt completed his teaching responsibilities with fourth period.

Emily, the third participant and 26 years old at the time the study was conducted, was Matt's paired teaching partner. She also taught seventh grade mathematics during fifth, sixth, and seventh periods, all 45-minutes and back to back. A White, Anglo-European, Emily emigrated from Bosnia to the United States with her family in 1998.

Emily's students, like Matt's, were enrolled in the regular track mathematics classes of a regular/enriched mathematics program. Their students were predominantly White, matching the school's demographics of 73% White (MuniNet Guide, 2008). Table 2 shows the demographics of the two schools where the interns taught.

Table 2

School Demographics

	Valle Vista High school	Einstein Middle School
White	19.6%	73.3%
Latino/a	75%	18%
Native American	3.2%	1.7%
Black	1.8%	3.2%
Asian	0.4%	3.8%

Matt and Emily's classroom (see Appendix F) was situated in a cluster of portable classroom buildings on the opposite side of campus from the administrative offices. The regular campus building was constructed sometime in the 1970's. When entering their classroom, a single teacher's desk used by both Matt and Emily was directly across in a corner. One side wall was completely occupied by two storage cabinets, shelves holding textbooks, and the closet for the heater. The other side wall was clear of furniture, but because the classroom was small, students' rectangular tables almost butted up against it. These 12 tables were new looking and seated two students each. There was a small white board in front with two bulletin boards on each side. An overhead projector was tucked away in a corner by a window near the heater closet. The back wall, where students entered the classroom, had windows across the rest of its length. A portable cart holding reform instructional materials also occupied the back wall of the classroom.

When they were not teaching a class, the interns spent time during the day looking at student work, planning their lessons, and observing their teacher partner. As they performed these duties, it was expected that the interns would connect and relate their teaching experiences with theory learned in courses they were enrolled at the university.

The three participants in this study all earned a B.S. in mathematics and transitioned from other jobs or careers into the teacher education program. John had been affiliated primarily with the local university and had served as tutor, grader, and teaching assistant in the mathematics department. Prior to that, he was an intern at a government scientific laboratory and worked at a non-profit organization for youth.

Matt held many jobs prior to becoming a mathematics intern teacher. Besides working as a substitute teacher, he worked as an insurance claims adjuster, a trainer and camp counselor at a sport and wellness center, a real estate appraiser, and a loan processor. It was his work at the kids' camp that influenced Matt's decision to become a teacher. Emily worked primarily as a mathematics tutor before becoming a mathematics intern. In addition to tutoring, Emily worked as a merchandiser at two department stores, developed film for a camera company, and repaired books at the university library.

Data Collection

Collection of data for this study included classroom observations, reflective conversations, reflective writings, and on-line questionnaires.

Classroom Observations

Classroom observations took place weekly beginning in August, 2007 and ending in May, 2008. I took notes and wrote comments and questions about the interns' lessons on *observational feedback* forms. I gave a copy of these comments and questions to the interns as a way to promote a reflective conversation about their lesson and continued reflective thought until the next visit. In other words, leaving the observational feedback form with the interns provided an opportunity for them to develop a reflective practice. For example, a few questions from an observational feedback form are listed below:

- What might happen when students have opportunities to share their solutions to the problem?
- What assessment opportunities were available to you during the lesson?

On the observational feedback forms, I attempted to write open-ended, nonjudgmental questions framed from a spirit of inquiry, allowing the interns to respond from multiple perspectives. In addition, it was my intent to embed presuppositions in my questions in an attempt to positively influence the thinking and feelings of the interns. For example, the first question above was aimed at getting the intern to predict possible outcomes of student discourse. The second question was intended to get the intern to examine lesson components, decide when learning may have taken place, and document that learning. An example of an observational feedback form is found in Appendix B.

Reflective Conversations

Reflective conversations took place after classroom observations, when convenient for the teacher. During the course of the study, I had approximately 30 conversations with each participant. The opportunity to have multiple conversations allowed me to capture the essence of the teachers' thinking about their teaching, and this helped them make explicit their conceptions about the teaching and learning of mathematics. Using *Cognitive Coaching* (Costa & Garmston, 1999) as a strategy, I mediated conversations with the interns by having them 1) summarize their impressions of the lesson, 2) recall supporting information, 3) analyze, compare, and infer, and 4) construct new learning. Since the goal of the conversations was to engage the teachers in reflective thought, it was necessary to allocate ample time for this to happen. Typically, these conversations took place immediately after the lesson, during the teacher's preparation period, during lunch, or after the school day. Each conversation lasted about half an hour, but there were times when some conversations exceeded an hour.

Specifically during conversations with interns, I would ask the interns how the lesson went and what they may have been thinking during the lesson. I wanted to give the interns opportunities to break down their experience into parts, to talk about their decisions regarding lesson goals, instructional strategies, assessment opportunities, interactions with students, and/or what was learned about the mathematical thinking of individual students. I intentionally paused at times during conversations to allow interns time to think about what was being said, wrote brief notes, and *paraphrased* from time to time to summarize their thoughts and validate their thinking. Also, to support the interns in constructing new learning, I asked additional inquiry-based questions that served to probe their thinking about lesson components and insights for future lessons. These questions were sometimes the same as those I wrote on the observation feedback forms, but they also stemmed from the interns' prior thinking and experiences underlying their perceptions, beliefs, and feelings.

Costa and Garmston (1999) discuss the benefits of having these reflective conversations:

- Holding conversations about one's work is essential to professional growth and development.
- Insights result from reflection on one's experiences.
- Reflecting on experience is amplified when done with others.
- Conversations are enriched when both teacher and mentor use tools of inquiry and nonjudgmental response behaviors.

Costa and Garmston (1999) maintain that professional growth is fostered when teachers (and mentors) take an active role in the conversations, a stance of exploration and experimentation, and control of their learning. Furthermore, when teachers talk about their reasons for their instructional actions and respond to questions about their perceptions and teaching decisions, they often experience a sense of professional excitement and renewed joy and energy related to their work (Peterson & Clark, 1986). This causes them to refine their cognitive maps (conceptions) and hence their instructional choices and behaviors.

After each conversation, I left the intern and intentionally found a quiet location where I could reflect on and record what was said. The observational feedback form, the brief notes I took during the conversation, and the paraphrasing I did during the conversation helped me reconstruct the intern's impressions of the lesson and any insights they or I may have gained regarding their thinking and learning.

Reflective Writing

During their mathematics methods course at the university, intern teachers were asked to reflect on and write about their experiences and to make connections between what they read, discussed, experienced mathematically, and what they experienced in their school settings (See Appendix A). These open reflections were shared with me, the instructor of the course, and the interns received written feedback at the next class meeting. This was done six times during the semester, twice each in September, October, and November of 2007. The goal of the reflections was to allow the interns' to tap into their conceptions regarding mathematics at various points during the semester and reconcile what they may have been thinking with what they were actually doing in their classrooms.

On-line Questionnaires

At the end of the semester, in December 2007, interns responded to an on-line questionnaire consisting of eight questions (See Appendix C). When I generated this questionnaire, my goal was to give interns an opportunity to reflect on their experiences in the methods course and how these experiences might have impacted their conceptions. For example, one question read:

> How did this class and your reflections about the class influence your beliefs about the teaching and learning of mathematics? In other words, are your beliefs about mathematics teaching and learning the same or different than they were at the beginning of the semester?

At the end of the spring semester, toward the end of the school year in May, the intern teachers were again asked to complete an on-line questionnaire consisting of five questions (See Appendix D). Again, as I generated these questions, I wanted to allow the interns to examine how the sum of their classroom experiences impacted their conceptions and practices in mathematics teaching and learning. What follows is an example of a question from the questionnaire:

> What has been the greatest influence(s) affecting your growth as a teacher? (i.e. mathematics methods course, other university course work, the SMEST mentor teacher, the SMEST program manager, collegial support, etc)

The goal of the question above in this questionnaire was to give the interns an opportunity to reflect on and write about the various components of their intern experience and give me additional insights regarding how the methods course and mentoring may have been a factor.

Responses to questionnaires, along with classroom observations, reflections, and conversations were used to triangulate and substantiate emerging findings (Merriam, 1998). For example, did what I learn about the teachers' thinking using one method help support my findings about their thinking using other methods? Would my classroom observations during the study support what the teachers indicated in a conversation, questionnaire, or reflective writing?

Data analysis

For qualitative studies, there is little consensus on any one specific format that should be used to analyze data (Creswell, 1998). However, for case study analysis, researchers write detailed descriptions of the cases and corresponding settings. In addition to these descriptions, Bogdan and Biklen (1992), Miles and Huberman (1994), and Wolcott (1994) advocate analysis strategies such as the following:

1. Review all data.
2. Write findings in the form of memos, reflective notes, and summaries.
3. Obtain feedback from participants on the memos, notes, and summaries.
4. Reduce the data by creating codes or categories.
5. Relate categories and develop analytic frameworks.

I read and reviewed the data for each case several times, attempting to get a sense of the overall data (Tesch, 1990). I then began to write key words or phrases in the margins of observational feedback, notes, participant reflections, and questionnaires in an effort to identify categories. The goal here was to facilitate the reduction of data, break it up and segment it into simpler general categories or codes (Coffey & Atkinson, 1996). Called *thematic analysis* (Glesne, 2006; Shank, 2002), this process involved coding and then segregating the data by codes into data clumps for further analysis and description. After segments had been coded for each participant, segments of data coded with the same words or phrases (i.e., perspectives on mathematics) were organized across the three cases.

Memos, notes, and summaries were written both before and after the reduction of data into segments, and these formed the basis of the initial and subsequent narratives written about each participant's conceptions of mathematics during the course of the study. For example, some data segments regarding my observations and conversations with the participants prompted me to write analytic notes. These analytic notes are a type of data analysis conducted throughout the research process and range from problem identification, to question development, to understanding patterns and themes (Glesne, 2006). Below is an example of an episode of analytic note-taking and the subsequent writing that ensued:

> During my observation of Matt's teaching, I have noticed that while students are working on their assignments, many raise their hands to receive confirmation of their answers with him. In my feedback to Matt, I asked him the following question:
>
> > How does one create an environment where students see themselves as the authority in evaluating their work instead of looking to the teacher as the sole authority?
>
> In our conversation that followed the lesson, Matt responded to this question by asking how he could make this happen in his classroom. He asked for suggestions about how to assert students' mathematical authority that could be included in his next day's lesson that related to the review of a quiz. I did not give him any specific suggestions, but instead asked him how he could create an atmosphere that would provide opportunities for students to exchange mathematical ideas with each other. He decided that those students that finished a quiz review sheet early could assist their classmates who continued to work on the review sheet. In this way, Matt would take himself out of the routine of assisting individual students, instead allowing his students to take on this role.

The example above and others like it played a role in searching for patterns and themes in the data and developed as I continually reviewed the words and actions of the participants during the study.

In addition to analytic note-taking and thematic analysis, Shank (2002) and Wolcott (2001) look at data analysis from a more humanistic view. Called *interpretation*, this type of analysis invites the examination, or pondering of data in terms of what the researcher makes of it. It is derived from the researcher's efforts at sense making, a human activity that includes intuition, past experience, and emotion—personal attributes that can be argued endlessly but neither proven nor disproven to the satisfaction of all (Wolcott, 2001). Because of my own intuition, experience, and connection to this study, my analysis of the data consisted of finding a balance between the themes that emerged from the data and my interpretations of those themes that came about during the study. For example, as I chunked the data systematically into different categories, I also had to perceive, or interpret that there really were differences in these categories.

Trustworthiness

Creswell (1998) and Glesne (2006) have stated that long-term engagement with participants increases the trustworthiness, or validity, of the study. Prior to the study, I began to interact with the participants in June 2007 during their summer coursework. I sat in on their field experience class two days a week, and it was there that the instructor introduced me as their SMEST mentor and coach and mathematics methods instructor. I mostly observed them during this time, but I also talked with them informally as they participated in class. I did teach a mathematics lesson during one of the classes, demonstrating a problem solving approach; thus, I had an opportunity to engage with them in a different manner. Toward the end of the course, I asked them to become participants in this study. At this time, I explained to the interns the roles they would have in the study (according to the approved IRB protocol). First, the interns were informed that there would be weekly classroom observations followed by mentoring conversations as per the SMEST program requirements. Second, the interns were informed that components of the methods course (reflective writing) would be used to document the impact on their teaching and learning. After the interns agreed to their role in the study, they signed their teacher consent form and the study commenced later in August, 2007.

The relationships I created with the interns during the summer helped with the transition to weekly classroom visits and reflective conversations once the school year began. As the school year progressed, I believe it became easier for interns to accept my presence in their classroom. I felt I was greeted warmly with each subsequent visit and the interns became more comfortable with engaging in a reflective conversation with me about their lessons. I began to feel them embrace my role as mentor, understanding that I was nonjudgmental as I supported and guided their efforts in the classroom and challenged them to engage in reflective thought.

My dual role as mentor and mathematics methods course instructor did not seem to create any problems for the interns. I believe they embraced my role as instructor and accepted the course not only as something that was required for the SMEST program, but they knew and accepted that their reflective writing requirement for the course also served as data used in the study.

Secondly, I used member checking (Creswell, 1998; Glesne, 2006) as a way to validate my understanding of the research findings. In order to member check, I solicited participants' views of the credibility of my findings and interpretations, making sure that I represented them and their ideas accurately. Three times during the study, I attached relevant pieces to e-mails regarding each participant, asking for and receiving feedback. With only one exception, each participant responded to my e-mails regarding the findings of the study and the feedback was positive in every case.

Finally, to address the issue of research validity, I have attempted to address my own bias with regard to the question of conceptions of mathematics and the teaching and learning of it. Glesne (2006) has pointed out the importance of researcher reflection upon one's subjectivity and how to use it and monitor it during the study. As a former secondary high school mathematics teacher, I have had opportunities to use various teaching and learning models with my own students. Further, I chose to engage in professional development projects in which I was able to share ideas with colleagues, learn from my own mentors, and engage in reflective thought. These experiences have certainly impacted my own conceptions of mathematics and mathematics teaching and learning and served as a foundation for creating and organizing this study.

Today, as I work with and mentor early career mathematics teachers, my goal is to support and guide them so that they can make sense of their own teaching. I believe I must give the interns opportunities to engage in reflective thought about their classroom teaching experiences rather than impose my own biases, and then allow them to make instructional decisions based on that thought. As Susan Bethanis (1995) so eloquently put it, it means that I must develop a trusting relationship while stirring things up as these teachers of mathematics envision their classroom teaching in ways that makes sense to them.

Is it possible that my position as methods instructor and classroom mentor in this study may have caused the participants to hold back during the study? Both positions can indicate ‘power’ in a relationship with students, as in the case of the methods course and giving a grade, and teachers, as in the case of the mentor/mentee relationship. The interns didn’t necessarily view me as a methods instructor who would give them a grade at the end of the semester, but because the methods course classroom was structured to immerse students in constructivist environment, they became an integral part of the assessment process, creating rubrics for assignments and their final grade. The climate of the class resembled a ‘working with’ environment rather than a ‘doing to’ environment regarding student and teacher relationships (Kohn, 1996). As a result, the participants in this study, as they engaged in the methods course, played a significant role in determining their course grade at the end of the semester. The reflective writing assignments the interns did in the methods course were graded using student-generated rubrics, but I believe the interns also completed these assigned reflections as a way to professionally grow in their teaching.

As a mentor, the interns and I had conversations about what best practices might look like in mathematics classrooms. We also talked about how there was not just one ‘right way’ to teach mathematics (NCTM, 2000). As a result, I believe the interns never felt pressure to try any instructional strategy because of my position as mentor in the SMEST program. However, if they did try something new to them, perhaps something they experienced in the methods course (i.e., group learning), we always had a conversation about their experience after.

Based on my experiences with the interns in the methods course and the relationships developed during mentoring, I believe the participants felt comfortable and at ease during the study. I also believe they regarded their experiences during the study as a way to naturally confront and challenge their own conceptions regarding mathematics and mathematics teaching and learning.

Chapter 4

Case Study Findings

> One friend, one person who is truly understanding, who takes the trouble to listen to us as we consider our problem, can change our whole outlook on the world.
>
> Elton Mayo (1999, p. 89)

Introduction

The purpose of this study was to examine the mathematical conceptions (beliefs and knowledge) of three first-year secondary mathematics intern teachers and observe how these conceptions were manifested in their classrooms. These teachers enrolled in a university alternative licensure program and launched a teaching career in secondary mathematics. They entered the teaching profession at a time when No Child Left Behind (2001) legislation had forced school districts to focus on testing and teacher accountability. Despite these pressures, each teacher willingly shared classroom experiences and their thinking about the teaching and learning of mathematics during their first year, and from the data collected, stories of each intern emerged that helped bring relevance to the questions I originally asked regarding this study: 1) What initial conceptions regarding mathematics and the teaching and learning of mathematics did secondary mathematics intern teachers reveal in their first year of teaching? 2) How did these conceptions change during their first year? What influence did a year-long mentoring process have on the interns' conceptions? What role did a university mathematics methods course have in supporting and restructuring the interns' conceptions?

This chapter was organized around the conceptions of the participants. For each intern teacher, I examined the conceptions with which they initially entered the classroom. Each one developed over time a set of implicitly acquired conceptions of mathematics from their personal experiences as learners of mathematics, and as a result, each teacher brought a unique perspective into their respective classrooms. For example, one intern was explicit in setting up a classroom focused model of teaching, one in which classroom management had precedence over the mathematics taught and learned in the classroom. A second intern entered the classroom with conflicting perspectives regarding the teaching of mathematics and vacillated between the two most of the year. The third intern began the school year as the second intern did, possessing conflicting perspectives, vacillating between one and the other, and exploring strategies that would eventually develop positive dispositions in students toward the learning of mathematics.

The remainder of the chapter examines the mathematics teaching of the interns vis-à-vis their conceptions and the changes they made during the year. As the interns' conceptions were classroom tested and challenged during their methods course and mentorship, they were able to make explicit the basis of their conceptions. The two interns possessing a dual system of perspectives exposed their conceptions to critique and analysis, and refined their conceptions throughout the year. The third intern demonstrated resistance to act on reform mathematical ideas presented during the methods course and mentorship.

Matt

> The main idea behind my instruction will be to present information in an easy to understand way. This summer I observed a teacher who used nothing but academic language and the kids learned very little. I also observed a class where the teacher presented the information in an easy to understand way and the kids learned much more. I want to present the information in ways the students can relate to.
>
> The main idea behind my classroom management will be structure, structure, structure. Keeping the kids busy from bell to bell should alleviate a large amount of behavioral problems.
>
> (Matt's teaching philosophy, June 2007)

Matt's philosophy above was in part influenced by his summer experiences in SMEST before he entered the classroom. He had an opportunity to sit in and observe classroom teaching in a summer school setting. During the year preceding SMEST, Matt worked as a substitute teacher, and because of his experiences in various classroom settings, this may also have influenced his philosophical perspective towards mathematics teaching and learning. Matt also spent his entire career as a student in the same school system in which he was now a first-year intern. He attended a middle school that had a similar demographic as Einstein and both schools fed into the same high school he attended. For that reason, he seemed to be at "home" at Einstein, connecting with many of his students in his seventh grade mathematics classes. The sum of his experiences in mathematics during this 12 year span was significant. Matt acquired images of what teaching entails and incorporated these images as part of his philosophical perspective regarding mathematics teaching and learning. Now, as Matt was about to embark on his teaching career, he was thinking about how he would present information to his students and create a classroom environment structured in ways that would minimize student behavior problems.

A Need for Structure

When he began teaching in August, Matt was true to his words. He set up a classroom-focused environment that was highly structured with the expectation that his students would demonstrate proper behavior in the classroom. That is, he would expect students to listen attentively and cooperate by following directions, answering questions, and completing the assignment. To support his desire to maintain this classroom-focused model, Matt also adopted a content-performance model of teaching. This allowed him to focus on didactic interactions in which the role of the students would be to respond to his questions and do problems demonstrated by him or the textbook. These two teaching models aligned very well with his Instrumentalist perspective on mathematics.

Matt had three different curricula from which to choose, one of which was a reform-based curriculum being advocated by the school district for middle schools. The remaining two curricula were more traditional, having sample problems in each section followed by problem sets consisting of many of the same kind of problem. Matt opted to use one of the traditional textbooks. The textbook had an accompanying workbook consisting of two parts: Activity Lab and Practice. Both components of the workbook consisted of problem sets having at least 20 practice-oriented problems.

In his first weeks of the school year, Matt used the curriculum materials faithfully and developed a strong sense of agency regarding the teaching of mathematics. His lessons consisted of bell work, lecture or demonstration, and practice. The five-minute bell work was used to focus students when the opening bell rang. It was usually a prelude to the kinds of problems students would be practicing after Matt's lecture. As he conducted his lessons, Matt demonstrated a problem, asked students questions about the mathematics, and elicited responses in a manner that engaged him one-on-one with individual students. His requests were often phrased, "Tell me what you did," and when getting a correct response from a student, Matt would often reply, "That works for me" (Classroom observation, August, 2007). After the exchange of ideas between him and the student, Matt would often summarize the outcome of their exchange for the rest of the class. During one classroom observation in August, Matt was implementing a lesson regarding order of operations; he assigned a set of problems that expected students to properly use the rules to evaluate expressions. When the class was struggling with a problem, Matt would interrupt the class as they worked on the assigned problems and ask a student who got the correct answer to explain to him what s/he did as the rest of the class listened. Then Matt summarized the student's explanation to the rest of the class. I asked Matt about his teacher-student interactions on an observation feedback form and in a later conversation:

> During the lesson, you said, "Aaron, tell *me* what you did." How can you set up your lesson so that students are more engaged with each

> other rather than just you? Why is this important? (Observation Feedback form, August, 2007)

Matt and I talked about his tendency to focus on one-on-one conversations with a student while ignoring other students. He acknowledged his one-on-one tendencies, and as we talked about the benefits of engaging more students in discussions, I presented this scenario to Matt:

> Imagine you sitting where I am in the back observing selected students from groups presenting their ideas and solutions to one another. What if you simply facilitated the discussion by asking questions when needed to keep the discourse among students going?" (Conversation, September, 2007)

Matt acknowledged the importance of engaging more students and indicated, "This is something I need to work on, but at this point in time, I need to make sure that my students can do these (the problems) when they take the A_2L" (Conversation, September, 2007). In the first few weeks of the school year, Matt was definitely concerned with establishing a structured learning environment that minimized behavior problems. His teaching was based on his previously acquired knowledge base, primarily on what he knew about managing effective classrooms, but at the same time, Matt was concerned about his students' performance on the district mandated A_2L short cycle assessments. These were taken in September, November, and March. Matt spent much of his time teaching to these assessments and filling in skill gaps demonstrated by his students. He became more relaxed in the second semester when the assessments were completed. "Now I can expand on fewer ideas, like probability, instead of following the district Scope and Sequence and other departmental guidelines." (Conversation, March

2008). In these first weeks, Matt was less concerned with incorporating pedagogical strategies that promoted student-to-student discourse, the same strategies that were being modeled in the methods class at that time. In other words, Matt was doing little to confront his conceptions that mathematics could be taught and learned in ways apart from teacher demonstration followed by independent practice by students.

Teaching Mathematics

As students worked on the routine of assigned problem sets in class, Matt walked around the classroom, helping students who had questions. It seemed that there were always an abundance of hands in the air, and Matt often scurried from student to student trying to alleviate the issues his students were having. During a subsequent visit in September, I left Matt a question on his observation feedback form (September, 2007):

> Students regard you as the classroom authority when it comes to their mathematics learning. What can you do to hand over more authority to them?

In the conversation (September, 2007) that took place after the lesson, Matt asked, What can I do to shift the authority of learning to my students?" He asked for suggestions about how to hand over more authority in tomorrow's lesson that related to a test review. I did not give him any specific suggestions, but asked him how he could create an atmosphere that would provide opportunities for students to exchange mathematical ideas with each other. After some deliberation, he decided, "Students who finish the test review sheet early the next day could assist their classmates who are still working on the review sheet." In this way, Matt hoped he could change his routine of being the only person going from student to student and answering questions and instead allow his students to take on this role as well.

Matt created the test review sheet that students received the next day. It was in the same format as the test his students would be taking the following day, consisting of a set of problems identical to the test but with different numbers. Students spent the entire period working on the review, and those that finished early had permission to walk around and help other students who were having difficulty, as he indicated they would in our conversation the day before. Matt wrote about this in a reflection:

> My class time has been filled with me running around trying to help my students. This can be overwhelming when there are a large number of students waiting. Aside from the tremendous strain this puts on me, the students that are waiting have stopped working and will not resume working until I answer their question. The students consider me to be the sole authority in the classroom.
>
> Today I tried something new. I started class by explaining that I will pick class "helpers" for the day from students who have finished their review and shown me that it was done correctly. This idea did not go as smoothly as I had hoped, but I did notice immediate changes which tell me I am on the right track. I did not have a continuous line of students around me like I do on most days. It's almost like the students just needed to know that it was alright for them to get help from another student. This extra time allowed me to spend more time

> with the students who need extra attention. This new method also kept the class more engaged. More students were working more of the time.
>
> There were some roadblocks I noticed. Some students are reluctant to ask their peers for help. I'm guessing that these students were too embarrassed to ask, but this was only the case for a couple of students who I did not mind helping. Another problem was that sometimes the helpers were a little timid when it came to helping the other students. All the helpers were volunteers, so I'm hoping this apprehension is purely because the whole concept is a little new. (Reflective Writing, September, 2007).

In a conversation, I had with Matt a week after the review, he reflected on what went well:

> I definitely like the direction the class went during the review. I hope to eventually get the classes to a point where they are all helping each other in every class. In general, I feel that my little experiment worked out. It still needs a lot of work, but seeing noticeable differences on day one means something is right. My goal now is to get students to help themselves so that I am directing the learning instead of being the sole source of information (Conversation, September, 2007).

Matt was beginning to respond to our conversations and the questions I left for him on observation feedback forms, and he wrote a reflection from the methods course relating to his dilemma of giving his students more authority. He had designed a lesson that allowed students to help their peers during a test review, but Matt was still in a somewhat precarious place with respect to his teaching. He admitted he wanted his students to engage in discourse, exchanging mathematical ideas and solutions, but at the same time, he still demonstrated a need for a classroom structure that would alleviate off-task behaviors. Matt had come to a crossroad: he had confronted his conceptions about how a classroom should be managed during mathematics instruction. Out of necessity, Matt enlisted help from his students to help him with student questions, but in doing so, he had to risk off-task behaviors that could disrupt the classroom. This was a big step for Matt, but changing his classroom-focused model of teaching did not change his beliefs about how the teaching and learning of mathematics should take place in a classroom. Matt still demonstrated how to work mathematics problems and expected students to listen and then practice on problems of the same type. In other words, Matt's Instrumentalist views of mathematics were still intact.

In October, with encouragement from an instructional coach in the school district, Matt opted to use a lesson from a unit in the reform curriculum with his seventh graders, thinking this lesson might advance his idea of students helping students. The lesson, *Variability in Categorical Data*, provided opportunities for students to examine and analyze a set of data regarding the different colors of candy in various bags. Matt focused on group collaboration strategies as he prepared for the lesson:

> The problem-based assignments that are designed to be done in groups should really help get my classes where I want them to be. I am

> thinking of pushing tables together so I have groups of four students sitting together. I will have to carefully plan where the students will sit to avoid behavior problems. I will also want to sit students with other students who compliment their strengths as well as help with their weaknesses (Conversation, October, 2007).

Before the variability lesson, Matt regarded group work as a reward for good behavior, and as a result, opportunities for his students to work in groups were rare. Matt noticed several things as he and his students experienced the lesson from the unit. First, he observed a high level of engagement from his students. “The kids that were the most involved tended to be the students that usually have behavior problems. This was very pleasing from a teaching perspective because these students are much less of a distraction when they are working” (Conversation, October, 2007) Secondly, Matt appreciated how the lesson in the unit established relevance for his students:

> Another thing I liked was the use of M&M’s in the problem. This grabbed their attention right away. Any type of data could have been used for this problem, but the authors chose a topic that kids would relate to. (Conversation, October, 2007).

With his teaching and learning experiences during this lesson, Matt began to draw parallels between what he was doing in his classroom with his experiences in the methods class. In a conversation (October, 2007) with me he said, “The format used in the reform-based lesson was pretty much the same type of format we have had in the problems in methods class.” Matt was also beginning to use ‘we’ more than ‘me’ when making references to his and his students' classroom experiences. For example, he stated, "We probably could have summarized for another day if we had wanted" (Conversation, October, 2007). Although Matt reflected on some of the drawbacks of doing the variability lesson, he enjoyed teaching it.

> Its philosophy is almost identical to what we have discussed in methods class, but it maps out everything for you. It's like a cheat sheet for problem-based math lessons, providing instruction for the lesson components as well as providing questions to pose throughout the lessons. I feel like this transition to teach problem based lessons is easier for me than for other teachers at my school because of the methods class. There has been much opposition to this curriculum at my school and it comes from the teachers who have been teaching for 20+ years. I guess this means more teaching opportunities for me. I don't really like the reform curriculum, but I do like the problems. It's just that I don't like having to cram everything in. The authors say it takes one day for the lesson, but it really takes two or three days (Conversation, October, 2007).

Matt's experience with the reform curriculum lesson was pivotal. His words above demonstrated some resistance to embracing a curriculum that did not align with his perspective on mathematics teaching. However, the lesson did elicit a higher level of engagement from his students, and this appeared to motivate him to use problem solving activities both from the methods class and from other resources available to him. For example, in November, Matt embraced a problem called *Poison* that he experienced in the university methods class. *Poison* is a version of the game *Nim* and provides an opportunity for both logical and algebraic reasoning at many levels. Basically, *Poison* is a game for two people using a specified number of objects. Players take turns removing one or two objects. Play continues until all objects have been taken. The last object to be removed is considered to be the poison and whoever gets stuck taking it loses the game.

When I observed Matt's lesson involving *Poison*, he introduced the problem by engaging students' interests immediately. To start, he challenged his students to play against him at the whiteboard, and there was no shortage of volunteers. Then, as groups formed and began playing the game, Matt moved around listening to their strategies and challenging their thinking along the way. For those groups that developed winning strategies, Matt extended the problem and challenged them to think about the game in different ways. As the period drew to a close, I wrote a few questions on his observation feedback form (November, 2007):

> What benefits are there for students when groups have opportunities to share their strategies with each other? How could you engage students in a discussion about the mathematics of the problem? In other words, how could you challenge your students to develop an algebraic expression that is related to a winning strategy using any number of objects and/or taking any number on a given turn?

Teacher Knowledge

The questions from the observation feedback form led to even more questions later in a conversation about instructional strategies for selecting groups, facilitating group work, and providing opportunities for groups to share ideas while allowing the teacher to learn more about students' thinking. He talked excitedly about the activity and the engagement of his students. "When I challenged them to change the conditions of the game, they went right at it. Then they called me over to challenge me to play." (Conversation, November 2007).

The problem solving activity, *Poison*, was a positive indication that Matt was internalizing his experiences in the methods course and his mentoring experiences with me; thus his conceptions were changing as he was adding to his knowledge base. Matt selected other problem solving activities to supplement the textbook he was using, some from the methods course and some from other resources. As he implemented these activities in his classroom, he also put his students into groups larger than the pairs they were normally in. However, he still moved from group-to-group answering questions and left no time in the lesson for groups to share ideas, questions, and solutions to the problems with other groups. In other words, he still maintained the teaching models he had adopted at the beginning of the school year.

In December, toward the end of the first semester, Matt implemented a lesson that challenged students to make sense of improper fractions. Every student was using the same shortcut strategy he demonstrated to convert mixed numbers to improper fractions; that is, they multiplied the dominator and whole number and then added the result to the numerator to get the improper fraction. Students seemed to be comfortable using this procedure and as they continued, I began to wonder if they knew why that procedure worked. I left a couple of questions for Matt on his observational feedback form (December, 2007):

> Your students are good at following the shortcut procedure, but do they know why it works? Do they have the understanding and flexible thinking needed when they encounter fractions like these in other contexts? What different approaches could your students develop when looking at these same problems?

During the lesson on improper fractions, I felt that Matt's content knowledge was sufficient to move his students forward from the simple demonstrated procedure to the challenge of multiple approaches. Unfortunately, we did not have a conversation about what his students could have done to advance their thinking about mixed numbers. Matt talked excitedly in October about "grabbing their attention" with the M&M's problem, but he seemed complacent about not demanding higher levels of thinking from students when converting mixed numbers to improper fractions and was satisfied they could follow the procedure he had demonstrated early in the lesson.

Early in the second semester, I noticed that students were getting more opportunities to share and present their ideas. When Jacob was presenting his solution to a problem, another student asked a question, but directed it toward Matt instead of Jacob. In our conversation (January, 2008) later, I asked Matt about this. "How can you re-direct questions aimed at you to the student making the presentation? In other words, how can you encourage students to direct their questions and comments to each other?" He acknowledged the need to do this: "I agree, handing off questions would benefit more of my students when they talk to each other about the problems. Sometimes it's just easier to answer them and go on." (Conversation, January, 2008).

Toward the end of January, in a subsequent lesson involving percents, Matt asked his students to find percentages of certain numbers: 50% of 10, 25% of 8, and 75% of 20. During the lesson, students were expected to find the answers using a demonstrated procedure discussed in class. While they were calculating the answers to this list of problems, I wondered about the different ways students might think about finding percentages of numbers. I thought about a problem involving percents that might demand more a higher level of cognition from his students. I wrote this on his observational feedback form (January, 2008):

> How do you think students would respond to this problem? Draw a four by ten grid of squares and then shade any six of the squares. What % of the grid is shaded? Find at least two ways to solve the problem. How does a problem like this increase cognitive demand in students?

Matt did not respond to this part of the observational feedback; after the lesson, our conversation focused on the problems he gave his students: "I wanted my students to be able to find 10% and 1% of numbers and use that knowledge to find other percents of numbers, such as 8% of 40." (Conversation, January, 2008). I was satisfied since our conversation focused on increasing students' cognitive demand in a way that related to his lesson, but hoped he would keep the grid problem in mind for another day.

A few weeks later, in February, Matt's students were learning about similarity. As they were solving simple proportion problems, they were cross-multiplying and solving for the missing number. While solving these problems, I wondered if they knew why this shortcut method worked and again asked Matt about it on an observation feedback form. Later in the semester, Matt challenged his students with a problem solving activity that he brought in from the methods class. Related to proportional reasoning, Matt expressed some concern in our conversation before the lesson: "I'm not sure whether they will be able to set up ratios and the proportion. How much information will I have to give them?" (Conversation, February 2008). It was evident that Matt was still not comfortable when his students had to struggle and wrestle with problem solving situations. Although his students had worked with similarity and proportional reasoning a few weeks earlier, Matt was concerned with their ability to connect the problem solving activity with their prior experiences and learn from the activity. He was still challenged by the idea of deviating from his Instrumentalist views of demonstration and practice. This challenge is illustrated in another lesson that came a month later, briefly described in the following paragraph.

In early March, during a lesson on area of polygons, Matt drew a few triangles on the whiteboard and then wrote the area formula for triangles. After his students found the area of a few triangles using the formula, they watched as he drew a trapezoid on the whiteboard; he labeled the height and the two bases, and then wrote the area formula of a trapezoid based on the drawing. His students then used it to calculate the area of several other trapezoids. Matt's lesson only consisted of calculations to find areas, given a formula. I wondered how his seventh grade students might have responded to the challenge of deriving these formulas themselves, perhaps based on their knowledge of the area of parallelograms. I asked Matt about this on his observational feedback form (March 2008):

> How can students come up with the area of a triangle based on the area of a rectangle? How can they use what they learn to derive the area formula for a trapezoid? How does giving students opportunities to make sense of what they know help to de-mystify the mathematics?

In our conversation that followed the lesson, Matt explained, "My students will need to be able to find the area of certain polygons because they will have to do this on the upcoming short-cycle test" (Conversation, March 2008).. He, like many mathematics teachers during this time, was preparing his students to take a test that would be under the scrutiny of his school and district and used to place his students in their eighth-grade mathematics class.

Despite his concern with the testing mandates, Matt was cognizant of the feedback questions I had left him in recent weeks. In response to these questions, Matt handed his students the *Staircase Problem*. This is a classic problem that gives students opportunities to discover patterns and make predictions about how many blocks or squares would be needed for a staircase with a given number of steps in the staircase. He gave students the problem with little or no introduction and expected them to read it and begin exploring. As Matt walked around from group to group, he became excited as he learned how his students were approaching the problem. He even sent one of his students over to me so she could explain to me what she did to solve and generalize the problem. It was an elegant solution and I lamented that there was no time built into the lesson for this student or other students to share their thinking about this problem with peers. Despite the absence of having students share their ideas and solutions to the problem, it appears that Matt was responding to the questions I had asked with regard to raising student cognition in problem situations.

When the testing was completed in March, Matt stated, "Now I can expand on fewer big ideas, like probability, instead of following the Scope and Sequence and other departmental guidelines" (Conversation, March 2008). In a lesson I observed in April, Matt was true to his statement. He challenged students with the task of finding the area and perimeter of a complex geometric figure. For homework, Matt gave his students a drawing of ten concentric circles. The radius of the innermost circle was given and each circle extended past the one inside by the same radius distance. With every other ring shaded, the task was to find the area of the shaded rings. I left Matt with three questions to reflect on:

> What challenges does this mathematics pose for students? How could you promote the sharing and exchange of ideas regarding this problem? What expectations do you and your students have regarding this problem? (Observation Feedback form, April, 2008)

In our conversation after his class, I asked Matt specifically how his students approached the problem. He said, "Most of the students knew they had to use the formula to get the areas, but they had trouble understanding how to subtract out the unshaded parts. I think Pi got in the way until I let them use 3.14." (Conversation, April 2008). Then we talked about the benefits of sharing ideas about the problem.

With only a few weeks left in the year, both Matt's and his partner's students engaged in a lesson on plotting points on a two-dimensional coordinate plane. They were highly engaged and some interesting designs and pictures were created from their efforts. Afterwards, Matt, like Emily, asked students to display their work on the walls of the classroom. Students were still talking about their displayed work when I returned a week later. Matt, after seeing his partner display her students' work during the year, decided to follow her lead on this. Emily had done this throughout the year and this had elicited reactions from both her and Matt's students. His students were curious about what Emily's students were doing in her classes. Perhaps this simple but profound strategy of displaying students' work impacted both Matt and his students and eventually led Matt to do the same.

Changing Conceptions

Matt's conceptions at the beginning of the school year were narrowly focused. In the first few months, he exclusively implemented demonstration and practice strategies and supported these with a model of teaching that ensured classroom control and minimized behavioral problems. When students challenged his authority, he took action. For example, when a student became a distraction during a lesson, Matt quickly moved this student to a different location in the classroom, often to an unoccupied table. However, as the school year progressed, Matt became more relaxed and more tolerant of classroom noise. In his final reflection during the methods class, Matt wrote about his challenge to change his students' thinking at the beginning of the year that he was the sole authority regarding their mathematics learning: "The hardest part of getting my students to problem solve was eliminating the 'learned helplessness' that was filling my classroom. It took a while, but students started to think for themselves before calling me over. I still need to work on not providing solutions right away." (Reflective Writing, December, 2007)

As the school year progressed, Matt began to incorporate problem solving and other process standards in his lessons, but this did not come easily. He admitted he was surprised to find that communication was one of the process standards and said:

> I didn't really think of this as being important. Initially, I varied the way I provided information purely to avoid monotony, but now my students are providing work orally, visually, and written (written especially when we used the reform curriculum lesson) (Questionnaire, December 2007).

At the end of the year, when Matt was given an opportunity to reflect on how he had changed, he mainly talked about his relationships with students: "I believe one of the major changes I made was with the way I interacted with the students. I was able to relax more with them throughout the year, as opposed to being such a disciplinarian." (Questionnaire, May, 2008) Matt also wrote about how he changed his instructional strategies with regard to questioning, saying he learned to answer a question with a question, hoping that would help clarify students' thinking and move them forward. Matt rarely mentioned how the methods course impacted his conceptions, but he specifically mentioned at the end of the year how having a mentor in his classroom influenced the changes he made in his teaching.

> Outside of experience, which was probably the biggest influence, I would have to say the mentoring was the most helpful. The help and feedback was given on a regular basis and was done so in a way that was not forceful or condescending. It was also nice to have this mentor to turn to for ideas with specific topics or problems (Questionnaire, May 2008).

I believe that mentorship in Matt's intern year did play a role in changing his conceptions about the teaching of mathematics. The questions I left for Matt and the conversations we had during the year allowed him to enhance his knowledge base and begin to question and change some of his instructional strategies. For example, Matt began to question his role as the classroom authority and worked at getting students to take on that role. He stated early in the year, "In general, I feel my little experiment worked out. My goal now is to get students to help themselves so that I am directing the learning instead of being the sole source of information." (Conversation, September 2007).

When asked to sum up his experiences in the first year, Matt was very persistent in writing about relationships with students.

> I was challenged by my students' lack of motivation, anxiety, skill gaps, and behavior, but I believe I found a job that I am willing to do for more than two to three years. I loved being in the classroom with the students. Everything outside of the classroom (meetings, in-service, paperwork, scheduling, etc.) is enough to make somebody want to change professions, but the interaction with the students more than made up for it (at least for me). I had some trouble working with some people at the school, but I expect that to be the case wherever I go. I felt like the school was far too uptight regarding dealing with students, both from the administration as well as some of the teachers. I always used to think that when students complained about teachers they were just complaining because they don't like school. I now believe that there are some very bad teachers as well as teachers that just don't like kids. It's hard to watch other teachers yell at students that you have for extremely petty reasons. Many times I felt the need to defend students from teachers but knew that it would be inappropriate. (Questionnaire, May 2008)

This end-of-year reflection indicated that Matt was less concerned about changing any conceptions he had regarding mathematics teaching and learning than establishing relationships with students. He had implemented some problem solving activities throughout the year; perhaps Matt presented these problem solving activities because he was genuinely interested in the mathematics that was intrinsic to the problems. The bottom line, however, was that throughout the year, Matt established and refined a classroom-focused model of teaching that was supported primarily by his Instrumentalist mathematical perspectives. Figure 2 summarizes Matt's conceptual framework during the school year.

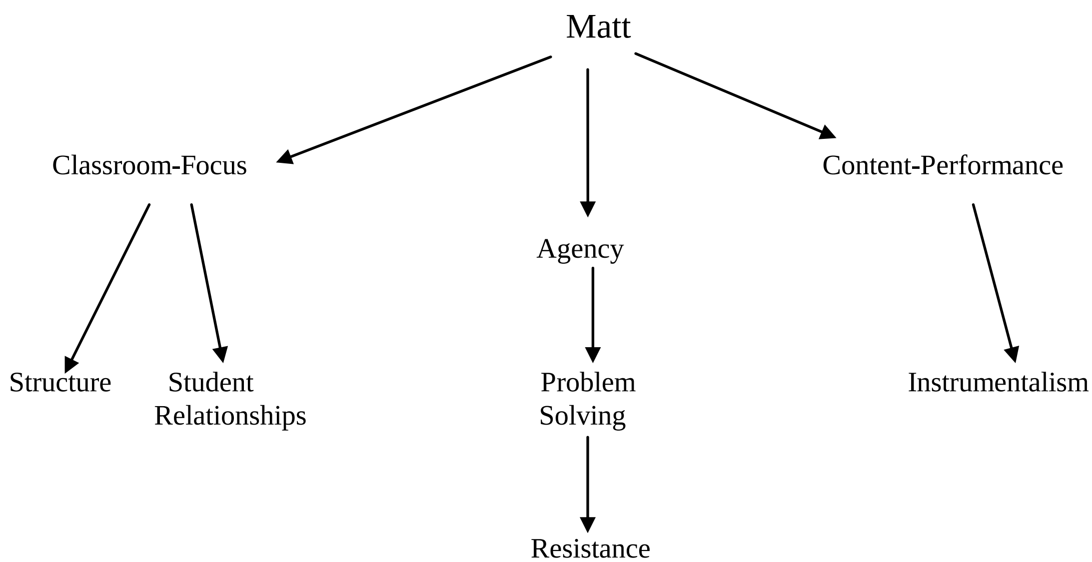


Figure 2. Matt's Classroom Perspectives.

Matt entered his classroom in August with a strong sense of agency regarding classroom structure and mathematics teaching by adopting a dualistic approach to his teaching. He established a Classroom-focused model of teaching that would provide his desired classroom management, and to address the mathematics curriculum, Matt adopted a Content-performance model of teaching. His conceptual framework was solid during the year even as he experimented with problem solving, but he demonstrated resistance to this perspective, believing his approaches to teaching were sound and firmly grounded in the school's culture.

Emily

> My primary goal as a teacher is to help young minds become successful academically, to learn some of life's lessons and morals along the way, and most of all to see that learning is fun.
>
> (Emily's teaching philosophy, June 2007)

Emily's philosophy, stated two months before she entered the classroom, says nothing of the mathematics she intended to teach, but it does reveal her desire to focus on students and making a difference in their lives. Her philosophy seems very much connected to her past experiences.. Emily was born in Bosnia, a small country in Eastern Europe. She lived there during a period when her country was engaged in a war with neighboring countries. Emily was only a child at the time, and despite the large number of civilian casualties, she and her family managed to survive. Surviving with her family may have actually helped Emily develop a passion for life and for the students she would eventually encounter in her career.

A few years after the war, Emily and her family immigrated to the United States. She enrolled in a public high school and graduated two years later. Since she studied mathematics for only two years in high school in the United States, Emily acquired much of her formal educational experiences in mathematics in her home country of Bosnia. After graduation from high school, Emily enrolled at the local university where she earned a B.S. in mathematics.

Searching for Teacher Identity

In my conversations with Emily regarding her experiences in mathematics, she often talked about the differences in the mathematics she experienced in Bosnia and the mathematics she experienced in the United States:

> In Bosnia, there was a national curriculum. Lot's of word problems. We didn't have to do the same repetitive problems over and over like here. We worked a lot with tables, came up with rules, and made predictions. All the subjects were integrated, not separated like here. The tests in Bosnia usually had five problems. Never in my life did I take a multiple-choice test in any of my math classes until I came here. In class we did paired work and there was lots of discussion (Conversation, August 2007).

Emily's first few weeks of teaching mathematics as an intern were difficult and frustrating. As an early mathematics learner, she developed the conception that mathematics was about problem solving, and only when she immigrated to the United States did she experience mathematics as an exercise in repetition, continually solving problems of the same type. In addition to her school education, Emily's father had a great influence on her conceptions of mathematics. "My dad was the one who introduced me to mathematics beyond the textbooks and curriculum. He would give me fun and challenging problems and we always used to work on them together" (Conversation, September, 2007). Now, as Emily contemplated her teaching during these first few weeks, she was unsure about how to proceed. At first she mimicked her partner, Matt, and did just what he did. Emily adopted his Content-performance model of teaching. That is, she covered the textbook, one section at a time, and gave individual seatwork and homework either from the textbook or from the accompanying workbook. Feeling pressure to conform to this way of presenting the mathematics, Emily and I talked about her dilemma. In a conversation in September, she said, "I was afraid to stray away from this 'traditional' way of teaching, believing that was how mathematics was taught in the United States."

Emily's uncertainties in the classroom were also related to classroom management. Her classroom was noisy at times with off-task behavior and Emily admitted that it was important to address this: "I need to take disciplinary measures more seriously so I can avoid trouble. It's hard to concentrate on discipline when I think children will be children and sometimes silliness is ok as long as they do their work, right?" (Conversation, September 2007)

When it was time to give the first chapter test; Emily became distraught when the mathematics department head at her school encouraged her to use a multiple choice test that was part of the textbook publisher's resource kit. She rejected this option as she had little experience with multiple choice tests in mathematics. "I wanted to give a more challenging test or at least throw in some word problems just to see what the kids were capable of" (Reflective Writing, September 2007). Matt convinced her otherwise, and in the end, problems were simply chosen from the workbook to create the first chapter test. With Emily's lack of experience and her "fear" of straying from the traditional way of teaching mathematics, she relented and succumbed to the wishes of her partner.

In those first weeks of the school year, Emily was teaching her three seventh grade classes in a way she never thought she would. "My first month of teaching was hard. I had no experience teaching math to a full classroom. I had very little idea of how to approach teaching and ended up copying my intern partner's lessons" (Reflective Writing, September 2007). Before she entered the SMEST program, Emily worked as a tutor, using hands-on strategies and helping her students make connections. Now, after helping her partner create a test she did not believe would benefit her students, Emily was anxious and worried that her students would not do well.

The day after the chapter test, I came in for a visit. We talked about the test, and the poor performance from her students. Despite the practice her students had before the test on similar problems, many failed the test. Afterwards, I asked Emily to talk about her next steps. I did not give any specific suggestions about what to do, but I must have triggered something in Emily, giving her permission to access and reactivate her conceptions regarding problem solving that had been suppressed for so many weeks. In a later reflection, Emily put it this way: "That talk with Alan made me realize that there are no rules or limitations or one way to teach mathematics; every teacher has their own technique and it was time for me to figure out my own" (Reflective Writing, September 2007). The very next day after our conversation, Emily put her students into groups and engaged them in a problem solving activity that had multiple solutions and could be solved using different approaches. The problem came from a resource she brought from Bosnia and required students to use logical reasoning to arrive at their answers. She wrote about this in a reflection:

> It took me a long time to prepare for this group activity, but once I gave it to the students, they all jumped right in and worked on solving the problem together. The only thing I needed to do was hang back and check on them once in a while. Some of the students found solutions to the problem that I never even saw or thought of. It was amazing. I had a smile on my face all day (Reflective Writing, September 2007).

Teaching Mathematics

In October, Emily still accepted the routine of textbook and workbook driven lessons, but after witnessing the high level of student engagement in her initial problem solving activity, she also searched for and adapted problem solving activities for her students. She wanted her students to experience problem solving tasks in groups and realized the Content-performance model of teaching would have to give way to a more Learner focused model of teaching. In other words, Emily wanted her students engage in a process where they were sharing ideas about problem solving activities and making connections to other mathematical experiences.

Emily chose the *Stacking Cups Problem* from her university methods class thinking this would be a good problem solving activity for getting her students to measure, find patterns, and make connections. Basically, student groups were given six drinking Styrofoam cups and a ruler. Base on the measurement of one cup and subsequent measurements when the cups were nested, the students had to determine the height of a stack of 50 cups by finding a rule or extending the data table or graph. Then they had to design and build a box that would accommodate the 50 cups without any waste of material. Emily was impressed with the level of engagement her students demonstrated during this problem solving task, but in a conversation with her later, she indicated her students were still comfortable with the routine of worksheets and problem sets from the textbook. She stated, "This is still their idea of 'doing' mathematics, and when I give them an opportunity to do more problem solving activities, they often show little evidence of critical thinking about the problems. Instead, they just want me to lead them through the problem from start to finish." (Conversation, October 2007). This posed a dilemma for Emily:

> These kids don't see math as something connected to the real world. The issue with the *Stacking Cups Problem* was that they were so concerned with building a box that they forgot what they were building the box for. The activities I am asking them to do prompt them to make connections, and most students don't see them. They only understand how to write down the answer, such as a number, but they can't explain where that answer or number comes from. How can I help these kids see the connections? Did I not see them when I was their age? It seems I am trying more to reprogram these students than teaching them math. I know how to make the connection and I know the students can, but I just need to get them to articulate their thoughts into sentences that make sense (Conversation, October 2007).

What did Emily mean by “reprogramming” her students? It was clear by this time that she was unhappy with her students’ expectations of “doing” mathematics; she wanted them to experience mathematics as something fun that they could connect with. At this point in time, Emily was certainly in touch with her problem solving perspective regarding mathematics. Her teacher identity was developing, but as a first-year teacher, it was difficult and time consuming for Emily to create and adapt problem solving tasks for her students. By November, she was feeling overwhelmed:

> Coming up with the activities takes so much time and sometimes I find myself being pulled to teach math the easy way by following the textbook. My intern partner pointed out that I am using four different sources to teach and cover the topics, and I must admit I get overwhelmed, and mistakes are bound to happen. Even if I can come up with successful activities, I am not always sure how to connect them to the standards. The important thing, however, is that with these activities, I may be planting a seed thirsty for math knowledge in a few kids. I just want them to see their own endless possibilities.
>
> (Conversation, November 2007).

Despite her mixed feelings during the month of November, Emily still felt the need to give her students opportunities to engage in problem solving.

Although she was still “pulled” toward the easier textbook lessons her partner Matt used, Emily’s early problem solving perspectives were well intact and at the forefront of her thinking. The need to "plant seeds" and help students make connections in their mathematics learning was paramount for Emily.

In December, as the end of first semester was approaching, Emily continued her search for problem solving activities where she could promote collaboration, critical thinking, and making connections. Many of the problems, like the *Stacking Cups Problem*, were adapted from problem solving activities she brought into the classroom from her university methods course. As the instructor of this course, I gave students opportunities to experience problem solving activities that were adaptable and accessible to a wide range of students. Emily used almost every problem she experienced in the methods course with her students. She later reflected on her experiences with these problems:

> I really enjoyed the activities we did in methods. They demonstrated that there may be more than one solution to a problem and that there are different ways of approaching the problems. The most fun was the *Locker Problem* and the *Probability Menu*. It also showed how much preparation is needed on the teacher's part for such activities; it definitely made me think about teaching and preparing lessons (Questionnaire, December 2007).

Emily did not have to use the problem solving tasks from the methods class, but adapting them for her students was a way for her to deviate from the routine of the textbook problem sets her students were so used to. Sharing these problems with her students and experiencing them in the context of her own classroom provided an opportunity for Emily to continue to establish her identity as a mathematics teacher and develop a sense of agency with regard to teaching.

As the second semester began in January, Emily adapted problem solving lessons her classmates shared as part of the university methods course requirements. For example, John shared a lesson in the class called the *Orange Juice Problem*. The goal of the lesson was for students to develop proportional reasoning by creating different ratios of concentrated juice with water and then answering the question, “Which is orangeier?” After Emily had a student read the introduction to the *Orange Juice Problem*, one student asked, “How are we supposed to know this?” Emily’s response was simple: “Get into your groups and start sharing ideas. Then I’ll come around and answer any questions” (Classroom observation, January 2008). I asked Emily the following questions on her feedback form:

> At what point are you willing to answer questions from groups? What kinds of questions can you ask groups to help them move forward? At what point during the lesson can you provide opportunities for student groups to share ideas and strategies with other groups? (Observation Feedback form, January 2008).

At this point, which was early in the second semester, groups still had not had opportunities to engage in discourse with one another. In other words, groups still did not share, challenge, and present their ideas about the mathematics. They relied on Emily for validation of their thinking as she moved from group to group; thus, they demonstrated little authority for their own learning. After school, we talked about questioning strategies and building time into lessons for idea sharing. Emily talked about her lesson:

> It seems every group had the same question: how do we do this? I had to help them set up the proportions, and some of them still had trouble. By the time they could even talk about it, the class was almost over. (Conversation, January 2008).

This was a frustrating lesson for Emily. She was still in a place where she believed she had to 'rescue' her students by rushing from group to group and felt uncomfortable when they struggled with the problems. Her students were all too happy when she did their thinking for them.

Besides adapting problems from her classmates in the methods class, Emily accessed the resources she had in the classroom. There were three different classroom sets of textbooks. Two of the resources were more "traditional" in the sense that each chapter section had problem examples followed by problem sets for students to practice. For each textbook, there was an accompanying workbook that Emily scoured to find problem solving activities to adapt for her students. Emily occasionally selected word problems at the end of problem sets from the textbooks, but she often felt she needed to assign all or part of the problem sets as well so her students could more easily transition to the word problems.

The third resource consisted of reform mathematics materials that Emily mostly disregarded. She experienced difficulty choosing problems from this resource because the four-week long units were thematic. Selecting problems meant taking them out of context, and she felt the problems would not always make sense to her students. Emily did, however, choose *The Locker Problem* from one of the units. *The Locker Problem* was introduced to her in the university methods course, so Emily may have chosen it on the basis of her experience with the problem in class.

Teacher Knowledge

One of Emily's creations, the *Landscaping Project*, consisted of a scaled drawing of her backyard. She challenged students to help her determine the amount of grass seed and number of flowers needed based on certain parameters including the perimeter and area of her yard. As I sat in on this class, I observed a high level of engagement on the part of Emily's students. Although engaged, each student group repeatedly required Emily's assistance and approached the problem in ways suggested by Emily. I left Emily with three questions on her observational feedback form that day:

> What did you learn about students' thinking as you walked from group to group? Based on what you learned, what opportunities could you give student groups to share their thinking and ideas with other groups? How did you allow your students to become the authority with regard to their learning today? (February, 2008)

Our conversation after the lesson focused on the questions above. She said, "I know I rush from group to group answering many procedural kinds of questions, but it's hard to think of questions on the spot that might help them solve the problem (Conversation, February 2008).. Emily's students were used to getting answers in this way and seemed anxious about getting them. The questions I asked Emily were intended to help her focus on student cognition as a way to help her develop her knowledge base regarding pedagogical strategies and student cognition.

As Emily experimented with ways to group her students, she continued to ask questions, and during our conversations, we talked about group size, group norms, and roles within groups, to mention a few things. When I came to visit her classroom and students were engaged in group work, there were always opportunities to ask Emily additional group-related questions. For example, I asked Emily these questions:

> When allowing students to select their own groups, what parameters or constraints would you discuss with them first? What kinds of questions can you ask groups to help them move forward with their mathematical thinking? How can you facilitate discourse among groups so that each group is sharing important ideas and strategies about the problem? How will you assess the work that groups are doing? (Observation Feedback, March 2008)

The questions I asked Emily were some of the same I had asked earlier in the semester. She was determined to have her students work on problem solving activities together, so this became the primary topic of our conversations. Emily admitted, “Managing students during group work is challenging, and I need to break the habit of rushing from group to group answering questions. Sometimes, I am only talking to one student, not the whole group. I need to make sure the whole group is engaged, not just one or two students. Instead, I need to have questions ready so groups can make progress.” (Conversation, March 2008).

A few weeks later, with Emily's prompt, I observed a student from one group summarize another group's strategies for solving a problem. It was evident that they were beginning to share ideas with each other and reflect on them. This observation led me to believe that Emily had reflected on the recent feedback questions and mentoring conversations. She was consciously making an effort to make instructional changes with regard to group work, student discourse, and helping students develop more authority. In other words, Emily was allowing students and student groups to share their mathematical thinking, encouraging more autonomy, and was facilitating the process.

Emily began to realize that when her students displayed their own and their groups' work on the classroom walls, the social culture of the classroom became more positive. Students often looked at and talked about their displayed work during class. During one of my visits in April, a student from one of Matt's classes asked Emily if she would ask him to do the kinds of hands-on activities that she was doing with her classes. Emily asked this student how she knew about the hands-on activities, assuming it was a friend who told her. The student simply replied she knew because she sees the posters and students' work on the walls and wonders what her classes are doing. Emily lamented not having done enough of this:

> I feel like I should work more on activities so students can display their work around the classroom and maybe then, when they walk in, they will have a sense of pride and realization that the classroom is a place for learning. It's nice to have a classroom that looks like a math learning wonderland. (Conversation, March 2008)

Emily's words above demonstrate that she was beginning to understand the importance of developing a social culture in which students' work and the discourse that follows was an integral part of the whole. The experience of listening to students' comments about their displayed work was beginning to positively shift her disposition toward teaching mathematics and her students' dispositions toward learning it.

In April, I walked into the classroom and sat next to Emily at one of the empty student tables. She was in the back of the classroom observing Matt teach his class and looking at some of her students' work. After a short while, she whispered to me, "I need to talk to you." We went outside and sat on a couple of makeshift stone benches near the classroom and began a conversation (April, 2008), one that I will never forget. Emily began by proclaiming, "I was never twelve years old." I did not understand what she meant by her statement until she continued.

> I was 12 years old during the third year of the Bosnian war. As you can imagine, I did not grow up the same way as my seventh-graders here. Instead of going out and playing and doing what 12 year old girls normally do with their friends, I was only concerned about helping my family survive. I don't know if I am making connections with my students and I'm worried I don't understand how they are thinking about the mathematics. Am I providing them with the learning environment they deserve?" (Conversation, April 2008).

I did not understand fully what Emily and her family had to do to survive, but I assured her she was developing a strong positive rapport with her seventh-graders and that other teachers were challenged as well with their students' mathematical thinking. This conversation was important for Emily; it demonstrated that she recognized the need to develop her teacher knowledge base with regard to student cognition.

Throughout the year, I asked Emily many questions about what might be happening with regard to how her students were thinking. In one of her favorite problem solving activities, the *Locker Problem*, some of the student groups in her class were having a difficult time getting started while other groups 'ran with' the problem. Emily had initially read and acted out the problem with cardboard 'lockers', but after twenty minutes, she was still finding it difficult to help a few groups move forward with their thinking. In our conversation that followed the lesson, Emily summarized her impressions, talking primarily about one group: "They sat on the floor in the back, cut out paper lockers, and reenacted the problem over and over." (Conversation, May 2008). I then asked her what this group might have been able to share with other struggling groups regarding their thinking about the problem. This led us to have a conversation about the importance of implementing strategies during the lesson for allowing groups to share ideas.

During another lesson in May, with only a few weeks left in the school year, Emily adapted an activity she had experienced in the methods course related to the probability of rolling two dice and examining two-dice sums. Students appeared very engaged during this activity, and based on results obtained after repeating this activity a few times, they built a nice representation of the data they had collected on the white board. However, during the analysis, both Emily and her students erroneously made statements regarding the theoretical probability of two-dice sums.

The classroom lessons described above show a first-year teacher's struggle to build a solid knowledge base in the context of her own classroom. In the conversation following the *Locker Problem*, Emily felt compelled to share with me her dilemma with student cognition, and as a result of our conversation, she continued to work on pedagogical strategies that allowed students and student groups to share mathematical ideas. In addition, during the *Two Dice Sums* problem, Emily was unable to move students' thinking forward in the problem situation. Questions that might have been used to probe students' thinking about theoretical probability did not surface at a critical time during the lesson and the lesson ended abruptly.

Changing Conceptions

Over the course of the school year, Emily's classroom lessons consisted of problem solving activities taken from the mathematics methods course, her peers, and from her own creative efforts, but her teaching also reflected traditional textbook approaches as well. Reflecting over the year, she said, "I didn't really have much of an idea in the beginning about what it would really take to teach math, but the methods class helped me to think about teaching differently" (Questionnaire, May 2008). Shortly before this reflection during a visit in April, Emily put three problems on the whiteboard. As students worked in pairs, Emily was in a relaxed mood, casually walking around the classroom, engaging with students. Emily built time into her lesson for presentations, and she had no problem getting her students to volunteer. After the first student presentation, there was a hearty round of applause. During the presentation of another problem, a second student asked Emily if the answer was correct, but Emily simply asked her to share her thinking with her peers, and then when the student was finished, commented, “You did a great job!” When discussion about the three problems came to a close, Emily paused and then said, “You guys are pros at this” (Classroom Observation, April 2008). Early in the school year, she would have gone from student to student, frantically checking every student’s answers. Now, eight months later, Emily had created a classroom environment where her students shared ideas and solutions and engaged in discourse with one another while she facilitated the process. She attributed much of her development as a teacher to the modeling of lessons in the methods course, but I believe she also refined her conceptions as a result of responding to the feedback questions and the conversations we had during the mentoring process.

Even though there was observable change in Emily’s conceptions, her lessons during the second semester still consisted of instrumentalist teaching approaches in which lecture and demonstration preceded practice. For example, late in March, Emily became uncertain about her students’ abilities to find the length of the hypotenuse or a leg of a right triangle. She made the decision to give students more practice by giving them additional problems from a worksheet. A month earlier, Emily demonstrated how to calculate the area of several geometric figures with little thought of why or how the formulas worked. Then she gave students a set of problems to practice calculating area. These lessons, and others like them, were consistent with the lessons her partner Matt implemented, but Emily lamented doing this kind of classroom instruction. In her end-of-the-year reflection in May, Emily wrote:

> I never liked giving students repetitive problems that could escape their memory as soon as a new topic came along. I plan to minimize or hopefully eliminate such lessons and assignments and will do my utmost next year to create a problem solving community of learners. I want to create assignments that have open-ended problems and classroom discussions that will allow my students to make connections, share ideas, and use prior and new knowledge to solve problems. (Questionnaire, May 2008).

Emily can not undo the lessons she implemented during the year that involved giving students repetitive problems for practice. She admits that many of her lessons fell short of her goals: "I have come to realize that any lesson that wasn't a great success can be improved upon and now I don't think of it as a failure but something to learn from and make better next time around." (Conversation, May 2008). Did the sum of all her experiences during this first year help Emily refine her philosophical perspectives toward teaching mathematics? A year earlier in June, Emily's goal was to help her students "become successful academically and to see that learning is fun." In her end-of-the-year reflection above, she expanded on her perspective and wrote about creating a problem-solving community of learners by incorporating process standards into her future lessons: problem solving, communication, making connections. It was her reflexive nature that allowed Emily to build on and refine her conceptions throughout the year. Figure 3 summarizes Emily's conceptual framework during the school year.

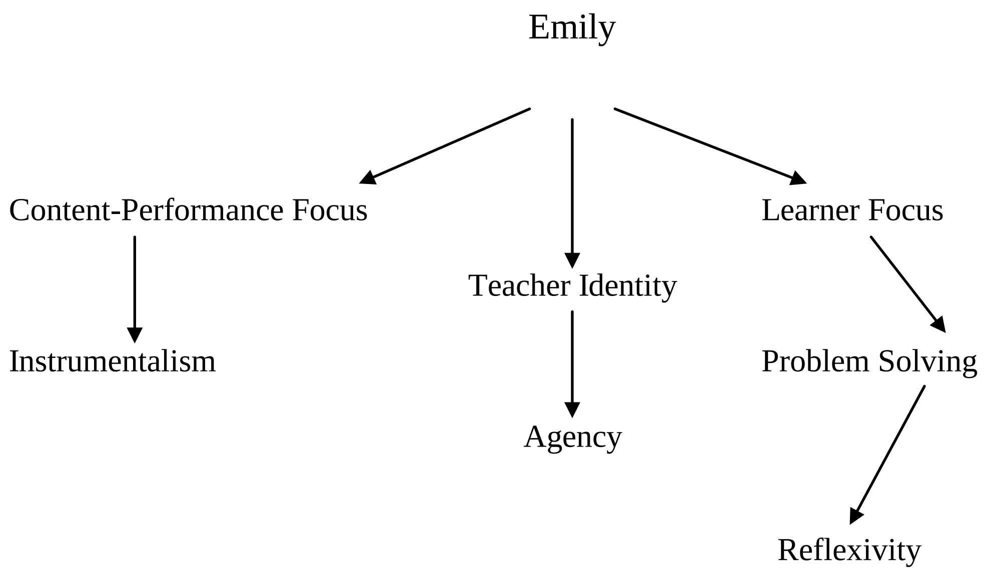


Figure 3. Emily's Classroom Perspectives.

Emily entered the classroom in August with dualistic perspectives with regard to mathematics: Problem solving and Instrumentalism. The two perspectives were both at the forefront of her thinking, and as a result, created conflict as Emily began her teaching. On the one hand, she felt she should conform to the Instrumentalist perspective since that was the norm within the school culture. On the other hand, Emily felt that implementation of her problem solving perspective would benefit her students by helping them make connections between ideas and through the sharing of ideas. This perspective eventually won over, and as Emily worked to develop pedagogical strategies for problem solving, she developed her identity and sense of agency as she reflected on the process.

John

> To describe why I teach the way I teach I would pose the questions, "What is mathematics education?" and "what is mathematics?" For the sake of time and space, and my incomplete understanding of math education and mathematics, I'll answer both at the same time to convey my philosophy of teaching.
>
> Mathematics is more than an assortment of facts, theorems, and proofs students need to understand and memorize. Math is observing and making sense of the world, our lives, through logical reasoning and numbers. I would teach math the way I do because I see math as a tool to live life thoughtfully. With that said, I see teaching as a means to guide students in constructing knowledge of the world around, helping them learn how to use that knowledge to make good decisions and develop a sense of curiosity about the life and the world around them (the bigger picture).
>
> (John's teaching philosophy, June 2007)

In his teaching philosophy above, John makes explicit his problem solving perspective regarding mathematics and its teaching and learning. He sees the mathematics teacher as someone that works with his students and helps them construct and reflect on their knowledge. I observed this during my first visit to John's classroom in August. He had a class meeting with his Algebra I students. During the meeting, John displayed four questions on the overhead projector for his students to respond to regarding their experiences in the class so far:

> What do you like and dislike about the assignments so far? How do you think you've been doing with the content so far? Do you think you've been keeping class rules and policies? What would you change? (Classroom observation, August 2007)

It was apparent that John was trying to elicit a reflective classroom atmosphere from the beginning. After giving his students time to write their responses to these questions, John managed to engage only a few of his students in a discussion, and many of their responses were quiet and short. Most students were disengaged and some had their heads down on the tables. It was frustrating for John to face the apathy of his ninth grade Algebra I students at the beginning of the school year. On the one hand, he was faced with the mathematics department's expectation that all Algebra I teachers would align their classroom instruction, proceeding section by section in the textbook; on the other, John was faced with 50 seemingly unmotivated students that had already demonstrated a poor disposition toward learning mathematics using practices they were disconnected from. John had asked his students what they would change but received little response. He had to figure out what to do to bring about this needed change in his students.

Reflecting on the Process

As frustrated as he was, John was determined to probe his students' thinking in order to learn about their disposition toward learning mathematics. He chose to write about this in his first reflection:

Early in the school year, I was giving the answers to the homework problems from section 1.4 in the algebra textbook. Not all students had done the problems and many students who did them worked them incorrectly. Afterwards, when I asked if there were questions, no student asked about any problem from the homework set. Why were there no questions? It was obvious that students did not have a complete understanding of how to work the problems, so why were they reluctant to ask questions about these problems?

As the weeks passed by, I tried to challenge my students to begin looking at the concepts in the curriculum from their own perspective and not just from that of the "five inch thick" textbook. On many occasions, I pleaded with them to start their work or turn in assignments. When I asked them to share an idea with their classmates, many thought they had the wrong answer and assumed that their "take on the problem" didn't account for anything. Based on these experiences and on comments students made about the assignments, I have reason to believe my students have determined that they are incapable of doing and learning mathematics.

Trading their insecurities for a confident approach to math will not be easy for my students. My focus has not been so much on my questioning in place of lecture because I still have to get my students to a place where they are craving to answer questions and not some

> repetitious worksheet. It is awkward saying that, but my students would much rather me lecture and give them worksheets. Asking questions is hard, not only because students care about their image in front of peers, but also because it is hard for them to articulate their thoughts. Instead of thinking outside the box when they learn, my students would rather have me "doing the learning" while they just listen. Perhaps a balance is necessary for some short intermittent periods of lecture to counter the more reformed style of teaching so that students can observe and sincerely subscribe to a more authentic way of thinking. For me the trick has been to find what that balance looks like. (Reflective Writing, September 2007)

John's reflection revealed rather succinctly his view of where his ninth grade algebra students were with respect to learning mathematics. As he implemented his lessons from chapter I in the textbook, he found that few students were responding to the department- expected strategies of lecture and demonstration followed by practice and homework. This approach to teaching was not new for John, for he had experienced mathematics classrooms in his own education where Instrumentalism was predominant. However, John struggled with this approach. He often gave students the answers to the homework problems the next day, asked if there were questions, and then worked them out at the overhead when necessary. During one lesson, John was showing students how to work a particular problem at the overhead projector. There was little student input, and when John asked his students about the procedure, no one responded. John asked, "Doesn't anybody care?" and then walked away from the overhead (Classroom observation, September 2007). John was frustrated with his students' preferences to have him do their thinking and learning. His lessons did not resemble the 'more reformed style' that he spoke of that would give students a learning experience different from what they were used to.

A few weeks later, John generated and administered a test over Chapter I. He chose problem types that were similar to the problems students experienced in their homework assignments. The classroom was very quiet during this time and some students finished the test much earlier than John had expected. Many students that turned their test in early were not able to complete all the problems, so taking more time with the test would have been an exercise in futility. John asked me after the methods class that week to come to his school so we could have a conversation about the results of the test he had given. Many of the students, about 50%, failed the test and there was some evidence of cheating on the test. John expressed his frustration with the situation and did not know what direction to go next in terms of addressing students' lack of understanding of the concepts on the test.

When I arrived the next day, John and I went to a quiet place to have a conversation. He brought a copy of the test and another test a student had cheated on. This student's test had some correct answers, but from a different version of the test. John did not tell students that more than one version of the test was being used.

The test consisted primarily of problems like the homework problems his students did not do or did not ask questions about. We began to have a conversation that focused on why the dismal results were not so surprising and on providing different, more concrete ways of helping students learn algebra. He expressed his concerns about which direction to go a few days later:

> What really gets me at this point is the disposition most of my students have about math or the class. Because of this, I find myself in limbo with the school curriculum pace and at odds with various philosophies of teaching. Inquiry based lessons are now beginning to occupy my attention and efforts. The idea of students working through activities to invent their own mathematical rules not only seems appealing, but it provides opportunities for students to have meaningful learning experiences. However, developing tasks that enables this is not easy as the battle that rages in me and in my students to embrace this style of learning is foreign—we are not used to looking at math as observed patterns. (Reflective Writing, September 2007).

Teaching Mathematics

John's reflection early in the year is very telling as he began to reveal his dissatisfaction with the status quo of the school's mathematics department expectations and the need and difficulty to make changes. Soon thereafter, he began to develop and adapt lessons that provided concrete models to help students develop understanding of concepts. For example, John used an interactive lesson called *The Chef's Hot and Cold Cubes* to give students a tangible model for understanding the addition, subtraction, and multiplication of positive and negative integers. Following an introductory story about a chef and his ability to change the temperature of foods during cooking, students added or removed hot cubes (red) and/or cold cubes (blue) to the “cauldron” and considered the outcome or change in temperature. Consider the example below:

> If the temperature within the “cauldron” is 42 degrees and you add seven blue cubes, then the temperature decreases seven degrees and becomes 35 degrees (42 + -7 = 35). If instead seven cold cubes are removed, the temperature of the “cauldron” increases seven degrees and becomes 49 degrees (42 - -7 = 49).

In a conversation I had with John, he recalled:

> Using the cubes had its benefits. Some of the students expressed interest in the activity and understood the concepts, but far more students found the cubes a distraction rather than useful. I should have provided guidelines for appropriate use of the blocks; instead of engaging in the mathematics, many of the students regarded the blocks as toys. If students would have had some exploration (play) time first, lesson results might have been more positive, my explanation of the lesson may not have been very clear. (Conversation, October 2008).

Speaking about the use of manipulatives, John said, “It just makes sense that being able to do math hands-on would be refreshing from the typical lecture and workbook math.” John could not dismiss his concern about the apathy exhibited by his students:

> My students in general have an unmotivated disposition about them that leaves me questioning whether or not the task I present is arousing enough to really help them probe patterns and discover mathematical

> concepts successfully. Even more frustrating may be my lack of experience communicating these types of problems. (Conversation, October 2008).

The *Chef's Hot and Cold Cubes* activity prompted John to pay more attention to group collaboration strategies; for example he experimented with group size and roles within groups, to name a few things. In another lesson in October, John assigned problems involving inequalities; students were confused about why the inequality sign was flipped when dividing or multiplying by a negative number. Students were working in groups as they worked on these problems; they were in groups of four, some worked in pairs, but some worked independently. In one group of four, I heard a student ask, "Okay, what did we do wrong here?" On his observation feedback form, I asked John about the multiple grouping scenarios and which one might have allowed the students to more readily accomplish the task at hand. Then I asked him how he could best facilitate student learning while in groups and how he would know what they were learning.

During October, John used a variety of resources to find meaningful learning experiences for his students. He adapted an algebraic model called *Mystery Bags* to help his students understand how to manipulate and solve equations in one variable. He allowed and even encouraged them to explore different ways to solve these problems. Consider the example below:

> If three mystery bags and two coins are on one side of the scale and two mystery bags and 12 coins are on the other side of the scale (and

> the scale balances), how many coins are in each bag if the same number is in each bag?

This was very engaging for some students and many were willing to come to the chalkboard to share their solutions. As I observed this lesson, I could overhear student groups talking about the mathematics with each other and each presenter seemed to feel "safe" while at the chalkboard. John facilitated the process by saying, "Tell us what you did" and asking, "Anyone have questions for __________?" when the presentation was over.(Classroom observation, October 2007).

A few weeks later, John implemented an activity called *Temperature Scales* in which students examined the relationship between the Celsius and Fahrenheit temperature scales. He had placed six thermometers in six different liquids of different temperatures at the front bench before class; then he called on different students to come up and read the temperatures in both Fahrenheit and Celsius scales. The students were actually measuring the temperatures and collecting data, rather than just having the data given to them. Students plotted the temperatures on a graph and examined the relationship between the two. Once they found the rule, or derived the formula, John asked students to apply it to a few problems. During this period, a student asked, "Then all I have to do is put 80 in for C?" John quickly gave her a "thumbs up". This prompted me to ask John these questions on his observation feedback form (October 2007):

> How many of your students still regard you as the authority, the one to go to? How do we as teachers begin to instill a sense of agency in our students that they can do this? How do we give that authority to them?

In our conversation that followed, John recognized what might have happened if he had "thrown" the student's question back to the class. "I missed an opportunity for this student and the class to think more about why substituting 80 for C made sense." (Conversation, October 2007).

The next week, John's students were engaged in a problem solving activity he brought in from the methods class; the *Stacking Cups Problem*. Some groups did not appear to be groups at all as chairs were not moved so that students could easily engage in the problem, communicate better, and feel part of the group and process. But there were other groups talking about and negotiating their measurements, units, and what to do next. As John moved from group to group, he listened as students talked about the errors that might have been made and what to do about them, but he did not intervene. John was making a conscious effort to allow his students to determine the correctness of their solutions as they interacted with one another and the mathematics. I believe he had given great thought to my feedback questions and our conversation regarding authority and agency from the week before. After the class, we talked more about developing student agency towards mathematics. I also challenged John to think about how he could set up and facilitate group work so that collaboration was considered critical by students to the overall success of the group and the members within.

Teacher Knowledge

The positive changes that emerged in October in some of John's students became a catalyst for generating more questions about how he could evaluate their learning. He wrote about this in a reflection:

> Assessing students has to be one of the most challenging aspects of teaching. One has to be thoroughly involved with a student's thinking to truly evaluate their comprehensions of concepts. It is my belief that very little can be inquired about a student's thought processes by only having them regurgitate facts and fill out worksheets that only have right or wrong answers. So one question I have had for a while is "how do teachers go about observing student cognition and then describe what it reflects?" I am not convinced that a percentage of correct answers are a holistic indication of student comprehension. (Reflective Writing, November 2007).

John's reflection regarding assessment prompted him to design and experiment with rubrics as a way to incorporate student thinking into his assessment strategies; early success with these rubrics was marginal, largely due to the fact that his students did not want to use them.

> My rubric may be too complicated for the students. Perhaps instead of stating every little detail it should include, I was thinking of posing a question for each category of the rubric that students can answer as they complete their work. Their grade will be determined by whether they can answer all the questions. (Conversation, November 2007).

As he continued to reflect on student thinking and learning, John began to connect his own learning experiences with the kinds of learning he wanted his students to experience:

> Most of us who have learned mathematics over so many years of our lives haven't really learned it meaningfully. Well, at least I haven't learned it meaningfully—thus, I lack enormous amounts of content knowledge. It's one thing to regurgitate formulas and manipulate equations, but it's more to know where they come from and why. It really appears that I'm at a disadvantage in creating meaningful lessons because my experiences lead me to construct formulas that have no connections with the world around me. This makes it difficult, not only to say I know my content, but then to guide students into making meaningful connections with new concepts. (Reflective Writing, November 2007).

John's testimony above regarding his lack of content knowledge prompted more questions and more conversations about how he might assess his students' thinking and how his students might begin to monitor their own learning. His reflective writings from the methods class focused on chapter readings, articles, and discussions related to assessment and teaching and learning strategies, demonstrating his effort to connect the ideas from the methods class to his classroom. These readings and discussions prompted John to write this reflection about designing rubrics that would allow students to re-submit their work:

> Learning takes shape as students engage in concepts on several occasions and in a variety of contexts. I don't think it's a fair assessment when students are still making connections and forming

> understanding of concepts and are asked to produce solutions that indicate very little about those connections. If students are working hard and need more time to put the fragments together to understand the material, then why not let them improve to that level? I don't see the reason in penalizing students for not having the ability to form a solid understanding the first twenty or thirty minutes they encounter a concept. Penalties for late work or meeting deadlines can be worked out somewhere, but the end goal should be that they learn the mathematics. (Reflective Writing, November 2007).

Students' dispositions toward mathematics learning seemed to become more positive as John continued to give students opportunities to engage in problem solving activities. They felt comfortable and safe in sharing ideas as they worked on problem solving activities, and I no longer observed heads resting on tables as I had earlier in the year. John continued to search for and adapt problems that would allow him to promote student collaboration and discourse. In late November, John created a version of the *Orange Juice Problem* that he also shared with his peers in the methods class. In this problem, his students had to examine four different recipes for mixing orange juice concentrate with water and then determine which one provided the orangeiest taste. During the lesson, John used an adapted assessment tool for evaluating students while they worked in groups on the problem. Called PQRS, John explained to his students that he would be making notations as he walked around the room about the **Q**uestions group members were asking, **R**esponses to those questions, and **S**upport group members were demonstrating for each other. In addition, John made notes about the **P**resentations students were making with regard to their and their group's thinking about the problem. John believed this informal assessment served to engage his students in the mathematics and help them value student collaboration. Now, at the end of each week, John's students were given an opportunity to reflect on their mathematical practices during class and within their groups by answering four questions:

How did you contribute to your group's efforts?

Describe how you presented your ideas or solutions to the class.

How did you support your classmates during class?

What questions did you ask during class?

The four questions above demonstrate John's effort to promote communication and reflection as part of the learning process. These questions also show evidence of John's commitment to engage students in a learner-focused classroom supported in a constructivist environment.

In December, toward the end of the semester, John checked out a set of graphing calculators from the school's mathematics department. It was John's goal to incorporate this technology into his mathematics lessons to give students opportunities to develop proficiency with this tool while exploring and learning mathematical concepts. The use of the graphing calculator in John's lesson that day allowed his students to compare and contrast linear functions in a single class period. For example, students began graphing the line $y = x + 2$ on their calculators. Then John asked them to examine and explain the behavior of the lines as they changed the equation to $y = 3x + 2$ and $y = x – 4$, among others. His classroom exhibited a distinctive quality that day. Every student was engaged and when there were questions, group members excitedly worked with each other to resolve them. Unfortunately, the graphing calculators were being shared and had to be returned a few days later. To help build continuity and connections with this technology, I offered to lend John a set of graphing calculators he could keep in his room for the remainder of the year. When he returned them in June, I asked him, "What did you accomplish by incorporating the graphing calculators into your curriculum?" He simply replied,

> It isn't what I accomplished, but what my students were able to accomplish. They had opportunities to learn algebra in another way and opportunities to explore and expand their thinking. I even got a letter of gratitude from one of my students where she thanked me for letting them use the calculators. (Conversation, June 2008).

During the second semester, John chose to use two stand-alone mathematics units from a reform-based curriculum to complement the use of the graphing calculators. In the first unit, students were asked to maximize the profit of a bakery that made certain kinds of cookies. In the second unit, students were asked to find out when rockets in a fireworks display would reach the top of their trajectory so they could set the timing mechanism for their explosion. In both units, solving the essential questions was complex, and the students needed to work together. By solving simpler problems along the way, students were better able to address the essential questions and were more successful in the end. Each unit was designed to last four to five weeks, but John extended each one to seven or eight weeks. He adapted each unit to make them more accessible to his students and conducted a “review” of concepts before launching into each unit. For example, the bakery unit focused on the intersections of systems of linear inequalities as a way to examine profit. John reviewed linear inequalities and their graphs beforehand and eliminated some of the nonessential activities in the unit. During one of my observations during the review, John instructed students to get into their groups to work on the problem, but there was little response from his students. I left John this question on his observation feedback form:

> If you want your students to work in groups and this is what you value, how can you communicate this to your students?

Later we talked about how important it is to revisit the classroom and group norms John had posted on the classroom walls.

Toward the end of the semester, a student responded to a problem John had written on the chalkboard. “Mister, I have no idea how to do this. Could you refresh me?” John re-directed the question to other students and one girl responded. After an exchange of ideas between the two students, the first student asked, “Why are you talking to me like Mr. Adams (pseudonym)?” She responded that she just was not going to hand him the answer, but he had to work for it and think about it. (Classroom observation, April 2008). This student-to-student episode is another indication of John’s response to my feedback questions and our conversations regarding student authority over their learning. John could have saved time by answering the question but recognized the value in allowing students to resolve the issue instead.

Changing Conceptions

John began the school year in a traditional way using strategies the mathematics department expected: lecture, demonstration, and practice. The dismal results of the first chapter test motivated him to question and make changes in his classroom routines and practices; as a result, John began to use problem solving activities that were more engaging to students. Using these activities prompted John to examine and question his assessment strategies. In his final reflection from the methods class, John wrote:

> Developing rubrics and assessing students is still one of my weaker areas of teaching right now. I want to improve in this area, and I think our discussions in methods class were helpful. I also really valued the *Probability Menu* and the algebra related problem solving activities. It was useful to see someone model these lessons in a constructivist environment. The articles, the text readings, and the activities and other assignments all supported each other, and making connections was easy the way it was set up. (Questionnaire, December 2007).

When asked how the methods class influenced his beliefs about the teaching and learning of mathematics, John responded, “I think I now have a bent towards problem-based classrooms and the methods class really gave me some formal support for how and where to start in making this a reality in my classroom” (Questionnaire, December 2007).

At the end of the school year in May, John was given the opportunity to reflect on the sum of his classroom experiences, the changes he made along the way, and the reasons those changes came about. He quickly acknowledged that he began by following the textbook used at the school detail by detail using demonstration and practice strategies in teaching the concept, but at some point, he began to approach the curriculum with a "less is more" attitude.

> I wanted to see my students discover and uncover fewer mathematical concepts and make meaningful connections between the concepts in great depth and at several different levels, as opposed to covering an abundance of ideas without giving them time to make meaningful connections. (Questionnaire, May 2008).

John wrote about his professional growth as being influenced mainly by the mentoring process he experienced throughout the year, the pedagogical support and reflection opportunities the mentor provided. He also cited university coursework in teaching mathematics methods and special education courses that provided a variety of support for how he approached students and teaching.

> The mentor provided feedback in the form of reflection questions, lesson ideas, and manipulatives/tools that guided me in the teaching process. His support enabled me to experiment and discover what "works" and what doesn't in the classroom. Our conversations also helped me to consider a variety of perspectives on my approach to classroom management, the manner in which I taught, and the content of what I taught. The courses influenced my teaching by more formally discussing theories of learning and providing differentiation of instructional strategies. (Questionnaire, May 2008).

In a final conversation with John, he talked about how things would change drastically next year. Although he never complained about having an intern partner, John mentioned that since he would not be sharing classroom space, he intended to have students working in groups with seating charts made for groups of three or four. Since wall space was limited for displaying students' work in his first year, John indicated that student work would be posted frequently and throughout the classroom in his next classroom. He also talked about his vision of what roles his students would take on:

> Students will participate in more frequent weekly self-assessments. They will engage each other as authorities for learning mathematics, while being encouraged to dialogue about math as I take more of a back seat in the discussion. My goal is to have students take ownership of their learning. When they ask questions, I will try to answer with a question. If they say they don't get it, I will ask them for specifics and if they are still stuck, that is okay. It's okay to have misconceptions and make mistakes. When there are multiple approaches to the problems, that will enhance the discourse in the groups, and when students present, those misconceptions may disappear. (Conversation, May 2008).

When I asked John about how he could be supported the following school year, he instead talked about the mentoring support he had had as an intern: "You answered and asked questions about class management and other uncertainties, and this helped me develop a framework, guide my thinking, and reflect on my experiences. This had an impact on how things unfolded the way they did" (Conversation, May 2008). Based on his response, I could only assume John would seek out mentoring support in his new school. Figure 4 summarizes John's conceptual framework during the school year

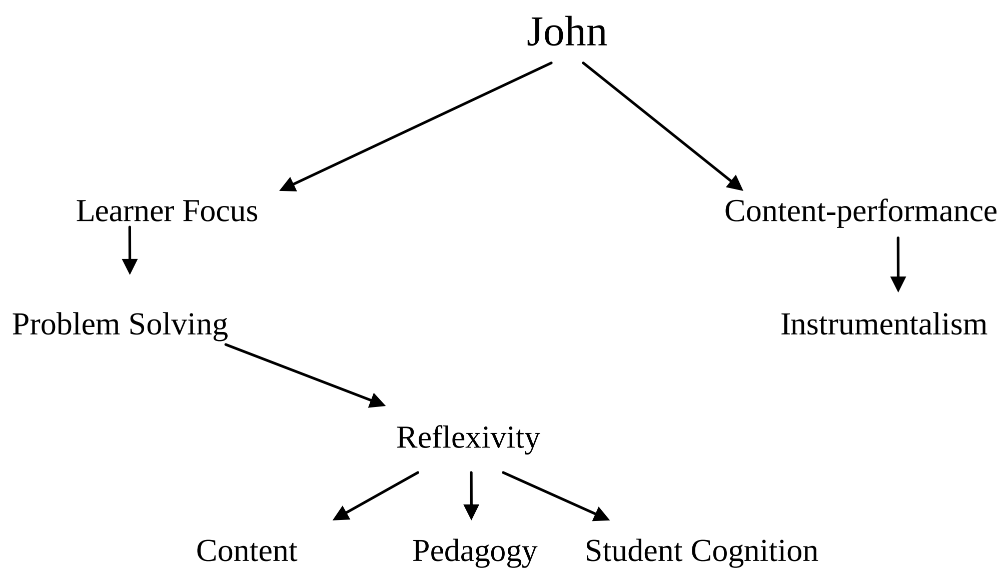


Figure 4. John's classroom perspectives.

John began his teaching believing students should construct their knowledge as he facilitated the process. This Problem solving perspective was definitely at the forefront of his thinking as he initially aligned his teaching approaches to the Instrumentalist values of his colleagues in the mathematics department. This was short-lived, however, as his students demonstrated a reluctance to engage in the lessons. John's students eventually embraced his problem solving approach as he built and refined his knowledge base while he and his students became reflexive in their learning of mathematics.

Themes That Emerged From This Study

The stories depicted in this chapter revealed many similarities, yet their differences were profound. One theme that emerged from these stories was that although all three interns began the school year with Instrumentalist views of teaching, two of the interns possessed a dual system of teaching perspectives. With that said, each intern had his/her own reasons for adopting the Instrumentalist perspective in their classroom at the beginning of the year. A second theme worthy of discussion is related to the mentoring process and university methods course the interns experienced. All three seemed to embrace the mentoring and completed the university course, but each intern responded to each in different ways. A third and equally important theme was connected to how the interns' conceptions changed with regard to the teaching and learning of mathematics. Each intern demonstrated some changes in their perspectives and disposition toward mathematics teaching and learning, but these changes unfolded profoundly different for each intern. These themes will serve to organize the discussion that follows in Chapter five.

Chapter 5

Discussion

> It is the practice of teaching, the growing sense of self as teacher, and the continual inquisitiveness about new and better ways to teach and learn that serve teachers in their quest to understand and change the practice of teaching.
>
> NCTM (1991)

During the month of October, when he had only been teaching a few months, John was given an opportunity to substitute in another teacher's classroom. It was one of many experiences that year that impacted his conceptions about the teaching and learning of mathematics. He reflects on his experience below:

> A couple of weeks ago, I was asked to substitute for a calculus class across campus. As I was discussing logarithms with students, the first thing that came to my mind was, why can't this topic be taught by having the students research logarithms in a context that they may find some interest in? The teacher's lesson plan called for me to explain, or rather state, the facts the students should commit to memory and then have them practice problems that made no connection to some prior knowledge, at least for most students. I so badly wanted to suggest an assignment similar to one I am using in my Algebra I classes. It is to have my students brainstorm and/or research areas of life they have interests in and develop a math problem, including how to solve it. This would be a great form of

assessment and an opportunity for students to answer their own open ended questions.

The teacher of the calculus class that John substituted for was an experienced veteran teacher who had left John the kind of lesson he briefly described in the vignette above. This was not atypical, for most mathematics classrooms in the United States use a similar approach: start the class by going over questions from the previous night's homework, introduce a new topic by lecturing and demonstrating how to work certain problems, give similar problems to practice on, and then assign a problem set for homework (Romberg, 1992; Stigler et al., 1999).

Why did John "so badly want to suggest" a different approach to this teacher's lesson on logarithms? After all, John's teaching career in mathematics was just beginning, and he too had used lessons in his Algebra I classes that used a similar approach as the calculus lesson. The answer to this question was indeed connected to John's "few" classroom experiences, but it also extended beyond his classroom and resided within a complex network of conceptions that he possessed (Thompson, 1992). The foundation of his conceptions of mathematics and the teaching and learning of mathematics were formed before he ever entered his classroom in August (Ball, 1988; Eisner, 1992; Kagan, 1992).

Although well-formed, John's conceptions regarding mathematics teaching and learning were being tested and challenged. By the time John had been a substitute in the calculus class, he had accumulated many classroom experiences. His conceptions were influenced by these and past experiences, weekly participation in mentorship (Dunn, 2005;

Philippou and Charalambous, 2005) and university coursework (Ball, 1988; Benken (2005); Cobb, Wood, and Yakel (1990); Feiman-Nemser, McDiarmid, Melnik, & Parker, 1987; Feiman-Nemser & Featherstone, 1992; Liljedahl, 2005; Tuft, 2005). An important step in refining John's conceptions was to provide opportunities for him to recognize the nature of and reasons for his conceptions and to recognize important moments in his teaching. Reflective conversations, using Cognitive Coaching strategies (Costa and Garmston, 1994) provided an important step for John to accomplish this.

The questions guiding this study were:

1. What initial conceptions regarding mathematics and the teaching and learning of mathematics did secondary mathematics intern teachers reveal in their first year of teaching?
2. How did these conceptions change during their first year?

a) What influence did a year-long mentoring process have on the interns' conceptions?

b) What role did a university mathematics methods course have in supporting and restructuring the interns' conceptions?

The questions above served to focus this study on the conceptions of three secondary mathematics intern teachers. From the stories that resulted, three themes emerged:

1. Although the interns began the school year with Instrumentalist views of teaching, two interns developed their problem solving perspective during the year.

2. The interns underwent weekly mentoring and completed the university methods course, but each intern responded to the mentoring and methods course in different ways.
3. The interns demonstrated some change in their perspectives and dispositions toward mathematics teaching and learning, but these changes were profoundly different for each intern.

Each theme will be discussed in this chapter and connected to the research questions and literature reviewed in Chapter 2 on teachers' conceptions of mathematics, the teaching and learning of mathematics, and the relationship between teachers' conceptions and their practices. This is followed by an epilogue, a short concluding section that elucidates my personal experience and how it relates to the study and a short discussion on the future of the participants. The chapter ends with a look at the study's limitations and recommendations that stem from this study.

The Interns' Conceptions at the Beginning

Matt, Emily, and John all began the school year using classroom practices that aligned with Instrumentalist views of teaching and learning mathematics. However, they did so for very different reasons. Matt was very comfortable with Instrumentalism; he had been successful as a student in Instrumentalist classrooms since he had begun learning mathematics. As a result, he had never considered any other approach to teaching mathematics when he became an intern teacher. Emily, Matt's classroom partner, was not as resolute about Instrumentalism as him. She only acquired Instrumentalist views beginning at age 16, when she immigrated to the United States.

Now, 10 years later, Emily did not have the same sense of agency regarding her ability to teach mathematics like her partner, so she deferred to Matt's perspective of teaching and learning. When John entered his classroom, he felt pressure to conform to his mathematics department's expectation that all Algebra I teachers would teach in the same manner. He did conform for a while, but he became frustrated with the manner in which his students responded to this teaching style and opted to find ways to change their mathematical experiences and ultimately their disposition toward mathematics.

Matt

Matt entered his classroom in August using classroom practices that aligned with the Instrumentalist perspective of teaching and learning mathematics: lecture, demonstration, practice, and homework (Romberg, 1992; Stigler, et al., 1999). He was comfortable with this approach; after all, he spent his entire academic career in a school district and university where most all of his instructors subscribed to Instrumentalist views. I say this because like Matt, I too was a product of the same school district and took the bulk of my mathematics coursework at the same university. I never experienced anything outside of the Instrumentalist view of mathematics from any instructor. As a result, Matt's identity as a teacher regarding mathematics had developed from the sum of his 16 years of mathematics classroom experiences (Ball, 1988; Eisner, 1992; Kagan, 1992). Now, as he began teaching in his own classroom, Matt had little reason to change his teaching and learning perspectives, one that had been deeply ingrained in him (Stigler, et al., 1999), and one that he now largely took for granted.

When he began his internship at Einstein middle school, Matt was comfortable using classroom practices that were consistent with his Instrumentalist perspective and his content-performance model (Kuhs & Ball, 1986) of teaching. That is, he was comfortable with teaching that was driven by students' mastery of mathematical rules and procedures. Since Matt embraced this teaching style, he also embraced the Scope and Sequence for the district's seventh grade curriculum, a document he could easily align with his Instrumentalist views. Matt's choice to adopt the district's Scope and Sequence may have also been determined by his desire to maintain a classroom-focused teaching model. That is, as Matt's teaching philosophy depicted, he was adamant about presenting information and maintaining structure in his classroom: "The main idea behind my classroom management will be structure, structure, structure. Keeping the kids busy from bell to bell should alleviate a large amount of behavioral problems.(June, 2007)" A classroom-focused model would align with Matt's thinking that classroom activity must be well-structured and efficiently organized, but unlike other teaching models, it would not address questions about mathematical content (Kuhs & Ball, 1986). The district's Scope and Sequence provided to the school would take care of the mathematical content for Matt. Matt could then focus on his conception that students would learn best when his lessons were clearly structured and followed principles of effective instruction, principles he had acquired during his academic career as a student (Ball, 1988; Eisner, 1992; Kagan, 1992).

Many of the lessons I observed started with *Bell Work*; that is, before students came to class, Matt would have a few problem examples written on the white board for them to focus on as they sat at their tables. After five to ten minutes, sometimes longer, Matt would ask for volunteers to share their answers. They sometimes did this from their table and sometimes at the whiteboard, but when they did present their ideas, they faced and talked to their teacher. After each problem, Matt often said "That works for me" if the answer was correct, and then he summarized this one-on-one exchange with the whole class. (classroom observation, August, 2007). If the answer was incorrect, Matt would explain how to get the correct answer. After all the problem solutions were explained, he launched into the rest of the lesson. This usually consisted of a few more examples of problems at the whiteboard. Then Matt provided an opportunity for students to start a problem set from the textbook or workbook that consisted of the same types of problems explained earlier.

Grounded in Matt's conceptions was the assumption that his students would learn best when his lessons were clearly structured. Students were expected to listen and pay attention to his explanations, and then practice problems afterwards. To maintain this structure, Matt would sometimes use strategies to keep students working during problem sets, and when asked if he was going to collect the assignment, he would say, "I may or may not collect this at the end of the period." (Classroom observation, September, 2007) This classroom-focused model (Kuhs & Ball, 1986) of teaching served Matt well in minimizing classroom disruptions and behavior problems while running an effective classroom. When confronted with disruptions and behavior problems, Matt quickly eliminated them by relocating the offending student to another location in the classroom and sometimes talked to the student privately during class in order to correct the behavior.

Matt subscribed to the classroom-focused model of teaching to control his students' behavior, and his Instrumentalist perspective prompted him to use a content-performance teaching model (Kuhs & Ball, 1986) as well. He believed his role as teacher was to demonstrate and explain, presenting information in an expository style. “The main idea behind my instruction will be to present information in an easy to understand way.” (Matt’s Philosophy, June 2007) By using these teaching models, Matt also believed the roles of his students would be to listen, respond to his questions, and do exercises using procedures he had modeled. Unfortunately, because of his decision to use these models, it may have been difficult for many of his students to develop and use other mathematical practices in class that would develop human agency (Boaler, 2002) toward the mathematics; that is, because there was little opportunity for student sharing and reflection on different ideas and procedures about the problems, there was little opportunity for learning with understanding (Hiebert, et al, 1996).

Emily

Emily started the school year with Matt as her SMEST partner in the same classroom, using the same classroom approaches to teaching as Matt. Emily, like Matt, also subscribed to Instrumentalist perspectives regarding mathematics when she began teaching, but she possessed a dual system of perspectives (Kitchen, Roy, Lee, & Secada, 2009; Thompson, 1992). Born in Bosnia, Emily lived there until age 16, in a country with a national mathematics curriculum that stressed the problem solving perspective. It was there that Emily developed a problem solving perspective, but her problem solving perspective may have been rudimentary considering the disruptive nature of her education during the war. Regardless of her development, Emily did talk about her mathematical experiences. “In Bosnia, we did lots of word problems; we didn’t have to do the same repetitive problems over and over like here." (Conversation, August 2007) When she moved to the United States with her family, Emily entered one of the local high schools in the same school district as her internship. It was here in the United States that she initially experienced mathematics teaching by instructors with an Instrumentalist perspective regarding the teaching and learning of mathematics.

As Emily entered the classroom with Matt in August, she possessed a dual system of conceptions regarding mathematics (Kitchen, Roy, Lee, & Secada, 2009; Thompson, 1992), one belief system acquired in Bosnia and the other acquired in the United States. Was one belief system more dominant than the other? How did the war in Bosnia disrupt her education and the development of her problem solving perspective? How did the acquisition of a second mathematics perspective later in her life influence her beliefs regarding the first? She reflected, "My first month of teaching was hard. I had very little idea of how to approach teaching in a classroom situation." (Reflective Writing, September 2007). Emily's statement here is very telling. Why did her partner Matt have such a strong teacher identity while she was struggling to find her own identity as a teacher? Were her conflicting perspectives opposing each other, and did Emily struggle to reconcile this conflict? Matt developed a strong Instrumentalist perspective over 16 years; Emily's problem solving experiences as a child in Bosnia were probably fragmented due to the war. When she immigrated to the United States at 16, Emily was indoctrinated into an educational system filled mostly with mathematics instructors with Instrumentalist views. She only had a few years to develop a knowledge base for this perspective and may have even resisted this perspective initially. Emily faced other challenges as well; she had to learn a new language and adjust to living in a new country. The result of the sum of her experiences consisted of an interrupted problem solving education as a young child and a different Instrumentalist perspective later in her youth that was limited to six years in a language she did not speak. Perhaps Emily's experiences in mathematics resulted in a knowledge base with two opposing and conflicting perspectives. This may have resulted in a lack of agency regarding teaching that led Emily to choose classroom practices that aligned with Instrumentalist perspectives (Ball, 1988; Dewey, 1964; Fennema & Franke, 1992; Shulman, 1986) in the first weeks of school, imitating what her partner did with his students. Emily appeared to be content initially to have a partner who had taken the lead in planning lessons and just followed along for a while. She rationalized her actions, or inactions, by saying, "I was afraid to stray away from this 'traditional' way of teaching, believing that was how mathematics was taught in the United States." (Conversation, September 2007). Did Emily really believe this, or did she follow Matt's lessons in the beginning because she was a woman four years younger than her male partner, one who had a strong desire to maintain a classroom-focused teaching model (Kuhs & Ball, 1986) that was highly structured for the purpose of minimizing classroom disruptions? This is unclear, but it may be that with her dual perspectives, Emily would have subscribed to a content-performance teaching model that aligned with Instrumentalist views (Ball, 1988; Dewey, 1964; Fennema & Franke, 1992; Shulman, 1986) without Matt as her partner. There was an important difference, however, between Emily and Matt. She was not as resolute about following the classroom-focused teaching model. Emily tolerated much more classroom noise than Matt, but she also recognized at times that she needed to exercise more classroom authority: “I need to take disciplinary measures more seriously so I can avoid trouble. It’s hard to concentrate on discipline when I think children will be children and sometimes silliness is ok as long as they do their work, right?” (Conversation, September 2007).

John

John, like Matt, was a product of the same school system in which he became an intern teacher. He too spent his entire academic career in a school system and a university where instruction was primarily aligned with Instrumentalist perspectives regarding mathematics teaching and learning. However, there was at least one important exception; John had a problem solving experience in tenth grade in high school. I was John’s Algebra II instructor and during the year, great emphasis was given to problem solving practices that gave students opportunities to engage in and explore problem solving activities that incorporated the process standards (NCTM, 1989, 2000). As a result, students in the class were expected to communicate and share ideas, approach and represent the problems in multiple ways, justify their reasoning about the problems, and make connections to other problems and mathematical ideas. Students initially resisted these expectations, but eventually embraced them and learned the importance of reflecting on their and their classmates’ learning (Hiebert, et al., 1996). How did this experience impact John’s conceptions of mathematics? I observed that the class was far different from his other mathematics classes in high school where Instrumentalism ‘ruled the day’, and it was unlikely that he would have had these problem solving experiences earlier than high school or after high school (Carpenter, 1992; Stigler et al., 1999). As a result of his Algebra II class, John developed a problem solving perspective, albeit limited, and began his internship with a dual system of perspectives (Kitchen, Roy, Lee, & Secada, 2009; Thompson, 1992) with regard to mathematics teaching and learning: Instrumentalist and problem solving.

John began the year in a quiet manner, using the same classroom approaches to teaching mathematics as the other mathematics department members. During mathematics department meetings, John engaged with the other Algebra I teachers in conversations about pacing and maintaining the goal of teaching the course in the same manner as them. He often returned to his classroom after the meeting thinking differently about his goals for the students. His students were not responding to the teaching methods that the teachers discussed in these meetings, so he felt different goals were needed. It was clear that John initially possessed a dual system of conceptions as he began the year. He lectured and demonstrated how to work problems from the beginning sections of the textbook and gave his students problem sets for homework. In addition to these Instrumentalist views, there existed another cluster of conceptions regarding mathematics. This was depicted in John's teaching philosophy he wrote about before he entered the classroom and through other early reflections as he demonstrated frustration with his students' dispositions toward mathematics. "Mathematics is more than an assortment of facts, theorems, and proofs students need to understand and memorize. Math is observing and making sense of the world, our lives, through logical reasoning and numbers." (John's philosophy, June 2007) John's statement about what mathematics is clearly demonstrates the existence of a different network of conceptions related to the problem solving perspective.

The social context (Ernest, 1988) of his school constrained John to enact a learner-focused model (Kuhs & Ball, 1986) of teaching mathematics. In other words, he hesitated to enact a style of teaching that focused on the students' personal construction of mathematical knowledge because of the established teaching and learning culture of the school where he was an intern. John lamented his predicament early in the year.

> Developing rich tasks that enables students to have meaningful learning experiences has not been easy and often discouraging. This is mostly because I find myself in limbo with the school's curriculum pace and at odds with various philosophies of teaching at the school. (Reflective Writing, September 2007).

John's frustration with this content-performance teaching model associated with Instrumentalist perspectives was evident during a lesson I observed one day. He was showing students how to work a particular problem at the overhead projector. Students were not paying attention, disconnected, and when he asked a question about the procedure, there was no response from any student. John asked, "Doesn't anybody care?" and then walked away from the overhead. (Classroom observation, September 2007)

Responding to Mentoring and the Methods Course

Matt, Emily and John were all interns and participants in SMEST, an alternative licensure program. As a result, each committed to university coursework, including the secondary mathematics methods course, and a weekly mentoring process that was designed to engage them in reflective conversations regarding their classroom practices. One of the goals of this study was to examine how the methods course and the mentoring influenced their thinking and conceptions regarding the teaching and learning of mathematics. The interns, because each had a unique network of conceptions regarding mathematics teaching and learning, responded differently to the methods course and the mentoring. It was their conceptions that influenced their responses, and this impacted their ability to refine and/or re-structure their conceptions.

Matt

Matt's conceptions of mathematics were never really tested and challenged until he entered the SMEST program. It was then that he became aware of problem solving perspectives regarding mathematics teaching and the constructivist theory of learning. The methods class and the mentoring each week left him with ideas and opportunities to reflect on his classroom practices and perspectives toward mathematics teaching and learning, but Matt rarely took advantage of those opportunities. Instead, he was steadfast in using demonstration/practice strategies and teaching models that supported those strategies. Why was Matt so resistant? There are a few reasons why he may have demonstrated little response. First, with resistance comes agency, the conscious decision to take action or not (Rodriguez, 2005). Matt developed early a sense of self as a teacher, believing he knew what it took to teach mathematics. He was successful as a mathematics student, spending 16 formative years building his conceptions in traditional classrooms in which he experienced much success. This may have influenced his decision not to act. Second, Matt often talked about his students' skill gaps. He felt filling in those gaps or to 'get the content' was a prerequisite to doing any kind of problem solving activity (Rodriguez, 2005). Even when his students did poorly on the first test, Matt dismissed their performance and chose to present or re-teach the material again. He openly expressed his need to follow the school district's Scope and Sequence for the seventh grade mathematics curriculum and posted this Scope and Sequence on one of the bulletin boards in the front of the classroom for students and classroom visitors to see. The mathematical topics and the order in which they were written on the Scope and Sequence provided the structure Matt desired and aligned well with his Instrumentalist views on teaching and learning. Third, Matt's resistance to respond to the methods course and mentoring may have been related to his lack of confidence or his knowledge base (Lortie, 1975) to move from his Instrumentalist views of teaching to higher level problem solving views (Ernest, 1988). When he did use problem solving activities from the methods course, they were implemented using the same approaches he used when enacting the content-performance model of teaching that dominated his practice.

During the methods course, Matt only submitted half of the reflective writings, and at the end of the first semester and school year, his responses to the questionnaires were brief, some of which consisted of only one line. To Matt, the methods course was a course requirement in the SMEST program, something he had to get through, and not much more. He acknowledged the goals of the methods course, enjoyed the problem solving activities, and participated in the class discussions, but Matt adapted little from the class into his own lessons. He demonstrated a high comfort level with his Instrumental perspectives regarding mathematics teaching and learning and did not see the necessity to justify them. As a result, Matt rarely referenced the methods course in his reflections as being connected to his own classroom experiences, and our conversations rarely focused on his experiences in the methods class. What Matt preferred instead was to write about his developing relationship with his students.

> I loved being in the classroom with the students. Everything outside of the classroom (meetings, in-service, paperwork, scheduling, etc.) is enough to make somebody want to change professions, but the interaction with the students more than made up for it (at least for me). (Questionnaire, May 2008).

Why did Matt reflect so little on his experiences during his first year of teaching? When he did reflect, Matt's focus was more on his relationships with students rather than the mathematics he and his students were experiencing. His Instrumentalist perspectives on teaching and learning mathematics and associated teaching models did not require a high level of consciousness (Ernest, 1988) or thinking about mathematics and the practice of teaching mathematics. Since there did not seem to be any conflict among his conceptions (Thompson, 1992), Matt had little to reconcile. Therefore reflexivity never became an integral part of his practice.

Emily

Teaching was an overwhelming endeavor for Emily, but her conceptions were open to refinement during her first year. The methods class provided an opportunity for her to reengage with her childhood problem solving experiences and begin developing a learner-focused model of teaching. However, if the methods class helped to motivate Emily to access and reshape her problem solving perspective, it may have also helped Emily to recognize the challenges associated with implementing pedagogical strategies needed in problem solving approaches (Ball, 1988; Dewey, 1964; Fennema & Franke, 1992; Shulman, 1986). As a result, Emily often reverted back to Instrumentalist views of teaching mathematics. She stated, “Coming up with the activities takes so much time and sometimes I find myself being pulled to teach math the easy way: following the textbook.” (Conversation, November 2007).

Despite her struggle to build a knowledge base, Emily began drawing more heavily from the problem solving activities she experienced in the methods class and implemented almost every problem in her own classroom. Emily’s goal was to have her students sitting in groups, working on challenging problems together, making connections, and having fun. Her primary concern was her ability to facilitate students’ learning in groups and being able to create and prepare problem solving activities on her own.

When Emily did create or adapt problem solving activities, her students did not always respond in positive ways. She was developing strategies from the methods class for facilitating group work and assessing what her students were thinking and learning. She said, “Even if I can come up with successful activities, I have no idea how to connect them to the standards.” (Conversation, November 2007). Still, Emily implemented the problem solving activities from the methods class, but, like any new teacher, she struggled to internalize the pedagogy and associated assessment strategies aligned with such activities that had been modeled in the methods class and described by her mentor. Emily was merely moving from group to group, keeping her students on task. She rarely gave them opportunities to share ideas with each other and reflect on each other’s thinking about the mathematics. Emily's students were having fun with the problems, and this was one of her goals, but she found it difficult to assess what her students were thinking and learning.

Emily embraced the mentorship process as it provided important opportunities to have conversations with someone about her practice of teaching and her growing sense of self as a teacher. Although Emily rarely mentioned her mentorship experience in her reflective writings, the support she received strengthened her resolve to make changes in her classroom practices. I found Emily often responding to the feedback questions and conversations we had one week by making changes in and experimenting with her classroom strategies the next week.

John

John used both the methods class and his mentorship opportunities to build on and reinforce his problem solving perspective and subscribed to a teaching model that would create a learning environment that his students would accept, and eventually embrace. He did use some of the problem solving activities from the methods class, but he focused more on developing his knowledge base regarding the pedagogical strategies discussed in the methods class that would be effective for the learning environment he had envisioned. "Developing rubrics and assessing students is still one of my weaker areas of teaching right now. I want to improve in this area, and I think our discussions in methods class were helpful." (Questionnaire, December 2007). This statement is just one example of how John's reflective writing revealed the impact that both the methods class and the mentoring process had on his conceptions. He wrote at length on creating students as problem solvers in his classroom, finding ways to assess students' thinking during the problem solving process, and giving his students ownership of their learning. John's reflexivity became an increasingly important part of his practice as he continued to build, refine and integrate his problem solving conceptions of mathematics. This transformative process was vital to John, for without it, changing his students' dispositions regarding their learning of mathematics would not have been possible. By the end of the year, John was talking about what his students were able to accomplish with graphing calculator technology. "They had opportunities to learn algebra in another way and opportunities to explore and expand their thinking." (Conversation, June 2008).

Over time, John was able to create a more constructivist learning environment while using problem solving activities from the methods course. He began to create assessment rubrics and self-assessments for students that aligned with the problem solving activities and had discussions with his students about their roles in the process. John also discussed and posted group norms and presentation expectations for both the presenter and audience. He asked many questions as he examined and internalized the constructivist strategies that were modeled in the methods class. John brought in various problem-solving activities and used, experimented with, and refined his newly learned strategies for facilitating student learning in groups. His school-expected Instrumentalist perspective to teaching began to give way as John adopted a learner-focused model (Kuhs& Ball, 1986) of teaching that aligned with his conceptions of problem solving as an approach to teaching. In his writings for the methods class and during the second semester, John spent many hours reflecting on the process:

> Inquiry lessons have occupied my attention and efforts lately. The idea of students working through activities to invent their own mathematical rules suits my personality and really provides an opportunity for them to have meaningful learning experiences. Assessing students, though, has to be one of the most challenging aspects of teaching. One has to be thoroughly involved with a student's thinking to truly evaluate their comprehension of concepts, and I *believe* that very little can be inquired about a student's thought processes by only having them regurgitate facts and fill out worksheets that have right or wrong answers. The use of rubrics makes room for a more complete evaluation of student mathematical cognition as they meet the standards for implementing problem solving activities. (Reflective Writing, November 2007).

Through his reflections, John was able to advance his thinking about assessment and other pedagogical structures within a learner-focused teaching model. Besides learning important pedagogical approaches to support this model, John also worked to develop strategies to enhance his knowledge base about how his students were thinking about the problem solving activities he presented to them (Fennema & Franke, 1992).

Changes in Conceptions

Throughout the school year, the interns had an abundance of classroom experiences and received ample opportunities to talk about and reflect on these experiences. Furthermore, they were immersed in a constructivist learning environment during the first semester while they were enrolled in the mathematics methods course. Each week, they were asked to examine and challenge their conceptions, engage as learners of mathematics and mathematics pedagogy, and invent/re-invent mathematics during the process. As a result, each week they had opportunities to refine or re-construct their conceptions of mathematics teaching and learning (Liljedahl, Rolka, & Rosken, 2007).

Matt

There is no doubt that Matt developed an awareness of the problem solving perspective of mathematics teaching and learning throughout the school year. After all, he attended the methods class every week for 15 weeks and had weekly mentoring sessions for the entire school year. With the encouragement from the district middle school coach, Matt implemented a problem solving activity in his class. He also brought in a few problem solving activities from the methods course to share with his students. However, for the most part, Matt resisted the problem solving perspective to teaching. Why? Why would Matt be willing to change his approach to teaching? Why would Matt abandon his Instrumentalist views and his adopted classroom-focused and content-performance models of teaching (Kuhs & Ball, 1986)? He had spent many years in classrooms being successful where Instrumentalism was predominant and his conceptions were robust and deeply ingrained. Matt had built a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools (Ball, 1988; Philipp, 2007; Thompson, 1992); this web formed the foundation on which he built his practice.

Matt openly admitted that he did not change his classroom teaching and learning models for fear of losing control and having to deal with behavior problems. However, once he developed more rapport with his students, he relaxed more, allowing students to work together while doing textbook or workbook pages, but he did not hesitate to split a group of students at a table to minimize talking and disruptive behavior.

Matt's Instrumentalist views at the beginning of the year were strong and solid, and they were still very much intact at the end of the year. If he did fit a few problem solving experiences into existing conceptual networks (Green, 1971), it is unclear whether he re-constructed any existing conceptual networks to accommodate these new ideas or if he just created a new cluster of beliefs apart from his existing ones. It would take more than just an awareness of the problem solving perspective and a few classroom experiences over the span of a year to appreciably change Matt's conceptions. He would need more time to begin reflecting on his role as a teacher of mathematics and his students as learners of mathematics.

Emily

Emily's dual conceptions of mathematics (Kitchen, Roy, Lee, & Secada, 2009; Thompson, 1992) developed in ways that would create struggle and tension during her first year in the classroom. Emily's initial problem solving conceptions were acquired in her native country of Bosnia, and part of this process was influenced and interrupted as she experienced war in her country; she spent her middle school years surviving and her high school years recovering from a traumatic period in her life. This most likely influenced the development of her conceptions of mathematics and mathematics teaching.

Emily's problem solving perspective were not reinforced or supported in her mathematics classrooms in the United States (Romberg, 1992). She was indoctrinated to a new belief system, Instrumentalism, regarding the teaching and learning of mathematics.

As an intern, Emily may have felt that it was her obligation to teach in the manner of her new country. To add to her dilemma, Emily was an English language learner at the age of 16 years, and this may have played a role in her struggle to develop a sound mathematics knowledge base in her new language.

During her time as an intern, Emily accessed her problem solving perspectives, but she did not always feel successful in using problem solving activities; "Even if I can come up with successful activities, I am not always sure how to connect them to the standards." (Conversation, November 2007). As a first year teacher, Emily also struggled to implement the pedagogical strategies needed to assess students' thinking and understanding. She openly admitted that she feared she did not relate to her seventh graders because of her war experiences as an adolescent. This lack of confidence and experience (Lortie, 1975) may have contributed to Emily's decision to use classroom approaches throughout the year that aligned with Instrumentalist perspectives where demonstration and practice are the norm (National Research Council, 2001; Romberg, 1992; Stigler & Hiebert, 1997; U.S. Department of Education, 2000). She vacillated between Instrumentalism and problem solving throughout the school year, and because of this,, Emily struggled in establishing her identity as a mathematics teacher.

When the time came to give students the first test, Emily became very uncomfortable with the multiple choice format recommended by a colleague in the math department. She was even uneasy about giving students too many of the same type of problem on the test, something her partner wanted to do. She reacted by saying, "The tests in Bosnia usually had five problems. Never in my life did I take a multiple choice test in any of my math classes until I came to the United States." (Conversation, August 2007). Contemplating this first test for her students may have been the "trigger" in helping Emily access her Bosnian-acquired conceptions.

Did Emily's conceptions regarding mathematics teaching and learning change during the school year? She was heavily influenced by her experiences in the methods course, embracing and implementing every problem solving activity she encountered there. Her beliefs about using approaches aligned with the problem solving perspective were reinforced and her content knowledge grew as a result of her methods course experiences. However, even as Emily advanced her knowledge regarding pedagogy and student cognition, this was still the basis of her frustration in the classroom. Overall, Emily's conceptions did change, but to advance her knowledge base to a level where she could be successful in a problem solving environment, Emily would need continued mentoring and opportunities for reflection.

John

John, like Emily, entered his classroom with dual perspectives with regard to teaching mathematics (Aguirre, 1995; Kitchen, Roy, Lee, & Secada, 2009), Instrumentalist and problem solving. He immediately conformed to the mathematics department's expectation of following a textbook, starting in the first chapter. Contrary to his belief that his content knowledge was weak, John did not display any deficiency. However, content was not his big concern in the first weeks of school; his students' disposition toward the learning of mathematics is what bothered him:

> On many occasions I have pleaded with students to start their work, or turn in assignments, or share an idea with the class. They immediately think they have the wrong answer and assume that their take on the problem doesn't count for anything. I have reason to believe that from these experiences and comments made about or on the assignments, students have trusted that they are incapable of doing and learning mathematics. (Reflective Writing, September 2007).

This was a dilemma for John, and soon after the first test, he chose to change his approach to his teaching and his students' learning. He did so with some trepidation, however, as he acknowledged the reluctance in his students to learn mathematics in a style that was foreign to them. He said, "In my mind, it makes sense that being able to do math hands-on would be refreshing from the typical lecture and workbook math." (Conversation, September 2007).

Like Emily, John also reached a critical turning point when he gave his students their first test of the year. He gave the test on a Tuesday and after the methods class on Tuesday evening, he asked for a meeting with me. The next day, John and I looked at the test, and his students' poor results on the test, and then talked about what he would like his next teaching steps to be. Although he did not abandon the "five inch textbook" completely, John made a decision to supplement his content-performance model of teaching (Kuhs & Ball, 1986) with problem solving approaches.

John's significant changes in his conceptions occurred not in his beliefs, for he was steadfast in his beliefs regarding the problem solving perspective. John made great strides in developing components of his knowledge base that would support his beliefs. He enhanced his content knowledge during the year by virtue of his year-long classroom experience, but John worked especially hard to develop his knowledge of pedagogy and student cognition, both important in enacting a learner focused model of teaching (Kuhs& Ball, 1986). He became very reflective, embracing many of the ideas from the methods class and those that emerged from our reflective conversations. By the second semester, John had become an avid proponent of constructivism, engaging students in discourse and self-assessment. By the end of the year, he had accomplished an important goal: changing his students' disposition toward the learning of mathematics. In so doing, John had refined and re-constructed his already complex network of conceptions.

Epilogue

Before I became a mentor in the SMEST program, I was a secondary mathematics teacher in the same school district as the interns in this study. I was surrounded by other mathematics teachers with Instrumentalist perspectives to teaching and learning and I even used demonstration/practice classroom approaches at times in my own classroom. I am sure that my own classroom learning experiences in mathematics involved demonstration/practice approaches, and I acquired those images of teaching mathematics very well. However, I was never quite comfortable with this approach. Perhaps the reason for this has something to do with how my teaching career started; I began as a biology teacher and believed science was about inquiry and getting one's hands dirty. That was how I learned biology and that is the approach I took in my classroom.

After eight years of science teaching, I took a job in industry that emphasized the importance of collaboration, communication, and teamwork in every project we did. Then, after five years, I made the decision to teach mathematics. As I did in my science classes, I began teaching mathematics as a subject of inquiry and search for patterns. A year later, I began teaching at Valle Vista High School, the very same school that John would do his SMEST internship twenty years later.

After a few years of experimenting with various classroom strategies in problem solving, a colleague and I brought a reform mathematics curriculum to Valle Vista and launched a problem solving approach to mathematics. This curriculum co-existed with the other school mathematics curriculum, and initially, few students were willing to cross over to the newer reform approach. Many of my students in the beginning resisted the problem solving approaches and were more "comfortable" with the routine of worksheets and practicing procedures. We were well into the school year before I convinced them that a mathematics class was more than the routine of drill and memorization followed by the test. Instead, I attempted to challenge their thinking by providing opportunities for them to "do" mathematics. Students generated and collected data, used graphing calculator technology to display their data, and made predictions based on their data. The old traditional textbooks were stacked in the corner of the classroom and were used to build supports for bridges and miniature car ramps. Pattern blocks, stopwatches, meter sticks, and other materials became the norm. My students' dispositions toward mathematics improved as they became more engaged in their mathematical learning.

It was during the first-year phase of implementation of the reform curriculum at Valle Vista that John enrolled in my Honors Algebra II class. Because he was older, he did not enroll in the freshman only reform classes. However, John did have problem solving experiences in the algebra class that year as I used problem solving approaches in all my classes. This may have been fortuitous; John's experiences in the Algebra II class would help form the foundation of his problem solving perspective of mathematics.

The Future of the Participants

At the time of this writing, Emily, Matt, and John were in the second semester of their second year of teaching secondary mathematics. They all found positions in high schools in the same school district in which they had been interns. Matt taught in a small, alternative high school that emphasized college preparatory courses. Students, as they approached graduation, were expected to enroll concurrently in courses at the local community college. Matt indicated he used the same models of teaching as last year and that it aligned with the expectations of the mathematics department at the school. He seemed happy with this expectation and his department head's goal for him to just get through the text by the end of the year.

Matt described his current mentoring experience as a pain and that his mentor attempted to control his thinking by telling him how things should be done in his classroom. He compared his current mentoring experience with that of his mentoring experience as an intern emphasizing the differences in the quality of the two experiences. "With the weekly observations last year, your mentoring was 20 times more in depth. So far, she has only observed my class once." (Conversation, August 2008). Matt informed me that mentoring is required by the district within a teacher's first three years and this was his one opportunity to receive it.

After this year, what opportunities will Matt have to inquire about his classroom teaching? Will there be a mentoring structure in place that will challenge his conceptions?

Emily taught in one of the larger comprehensive high schools in the district and had a classroom in one of the portable classrooms on campus. She had 30+ single student desks arranged in five rows, and so when students worked in pairs or larger groups, the desks had to be moved and rearranged. I do not know how frequently Emily's students engaged in problem solving activities but since the desks were in rows, my guess was that Emily also used demonstration/practice approaches with her classes as well. This same scenario unfolded in the study when Emily vacillated back and forth between demonstration/practice and problem solving approaches.

When I asked Emily about mentoring, professional development, and other structures in place at her school that offered support, she said, "The professional development here is so lame, nobody even shows. We're supposed to meet once a week, but I don't know what happens. The math department is in shambles." (Conversation, August 2008). With no mentor and no district support for an instructional coach at her school, support for Emily's continued development of conceptions appear to be in jeopardy.

Without any support in her first year, what opportunities does Emily have to continue to challenge and refine her conceptions of mathematics teaching and learning? She seems to be teaching in isolation, especially with a math department in "shambles". What measures can Emily take to reach out to her peers, invite them into her classroom, and engage in a collaborative process regarding her teaching? Doing this is critical, especially for early career teachers, if they intend to grow in their practice (Hiebert et al., 2004), develop their identity as a teacher, and to continue to inquire about new and better ways to teach and learn mathematics (NCTM, 1991).

John, when interviewing for mathematics positions, chose a charter school within the district that used a reform problem solving approach to mathematics. It is not surprising that he would take this direction with his career. John worked hard as he developed a learner-focused model of teaching last year. It consisted of many critical components he would rely on to serve him in his quest to understand and change his practice of teaching. With the expected mentoring support of the other two mathematics teachers at his school and the professional development opportunities his school offers, I am confident John will continue to shape his conceptions of mathematics to align with mathematics education reform initiatives.

Limitations

The time I spent with Emily, Matt, and John was limited to 15 meetings in a university methods course and one classroom visit each week during the school year. Conversations with interns were not always possible because of their teaching schedules, additional school-related responsibilities, and limited time.

The duration of the study was limited to one school year. This is merely a snapshot view of changes in conceptions and practices that may have occurred, and it did not consider whether these changes would occur later or would be sustainable over the long run. A longitudinal study would be necessary to study this.

The data collected and its analysis was colored to some extent by my preexisting conceptions regarding mathematics and its teaching and learning, teachers' roles in the classroom, and my prior experience as a teacher of secondary mathematics. In addition, my role as the participants' university methods course instructor may have influenced their responses to questionnaires, during conversations, and course assignments (reflections) integral to the study.

It was not possible for me to record, or to take into account all the influences that might have impacted Emily, Matt, and John, and the resulting changes in their conceptions and practices. Undoubtedly, there were factors (i.e. in the school or school district setting) that exerted influence on the interns' thinking and practice. Investigating these factors was outside the scope of this study.

Recommendations

In this study, I examined the conceptions of three beginning secondary mathematics intern teachers and investigated how these conceptions manifested themselves in their classroom teaching. Because conceptions are represented by a dynamic, malleable cognitive network (Hiebert and Carpenter, 1992), each participant in the study painted a unique and complex picture of what it means to teach and learn mathematics at the secondary level. Each participant's story was different, as was expected, and if each only revealed a small window into the understanding of the teacher's conceptions vis-à-vis their practices and words, their stories still contribute to an important body of research. I hope this will serve to inform teacher educators in their work to reshape preservice and inservice mathematics teachers' conceptions that align with today's reform initiatives. This study has certainly informed my continued work with first-year mathematics intern teachers. Based on the findings of this study, I make the following recommendations:

1. Mathematics courses within teacher education programs should be designed to immerse students in a problem solving environment. Conceptions are deeply engrained as students enter these programs (Ball, 1988; Eisner, 1992; Kagan, 1992), and total immersion will give these students opportunities to challenge and restructure their conceptions. Matt had been involved in an “apprenticeship of teaching” (Lortie, 1975) by having watched other teachers teach for 16 or more years of schooling. It was necessary for him to experience learning as a socially constructed process in a problem solving environment.
2. Preservice and beginning teachers should be provided with meaningful mentorship opportunities. They should be placed with mentors that have knowledge of and experience with reform mathematics initiatives so that important reflective conversations can take place on a regular basis. John, as he depicted in his reflections, demonstrated that he benefited from this opportunity. Without the mentoring, he may not have advanced his thinking to the degree that he did and enact his problem solving perspective in his classroom.
3. Beginning and early career mathematics teachers should be provided with ongoing classroom-based peer mentorship. Day-to-day conversations about classroom successes and challenges are important if teachers are going to reshape their conceptions of mathematics and the teaching and learning of mathematics. Becoming reflexive about one’s teaching is tied to agency (Rodriguez, 2005) and

teacher identity and helps one to act on new knowledge and restructure and/or refine conceptions Emily and Matt did not have this opportunity early during their first year of teaching following their internship, and as a result, had little opportunity to challenge their conceptions and grow in their practice.

4. Future studies that examine the impact of mentoring and coursework in teacher education programs on teachers' conceptions should be considered and designed for longevity. A 15-week methods course designed to illustrate teaching for understanding (Hiebert & Carpenter, 1992) may do little to undo deeply entrenched conceptions some prospective teachers have (Kagan, 1992), but it may "plant the seeds" necessary to trigger change. Teacher change should be regarded as a long-term process resulting from teacher-tested classroom strategies, reflecting on the merits of those strategies with regard to student learning, and taking the initiative to make appropriate changes, if necessary. A long term meaningful mentoring process (see recommendation 2) could be instrumental in using those "seeds" to help the mentee develop a set of conceptions and related classroom practices that could impact students' dispositions toward mathematics and promote their mathematical human agency (Boaler, 2002).

Implications

When I was involved in a professional development project in the 1990's, I made a commitment to make changes in how I regarded the teaching and learning of mathematics by adopting a reform-based mathematics curriculum. In the beginning, I did not know what to expect from the experience other than learning to implement the curriculum. I learned it was much more than implementation. An important goal of the professional development was to critically examine one's thinking about the teaching and learning of mathematics. As a participant, I did not do this in isolation. Mentoring was an integral component of the project and my mentor, Anne, flew in from Denver bi-monthly to observe my teaching and have conversations with me about my classroom experiences.

During my lessons, Anne wrote notes and questions and she sometimes left me with questions to ponder. One question she left still resonates with me today: "What are you doing that students could be doing?" After the lessons, Anne mediated our conversations using Cognitive Coaching (Costa & Garmston, 1999); she allowed me to reflect on the lesson, probed my thinking, and sometimes paraphrased my words to let me know she valued what was being said. Anne was never judgmental and she never forced her ideas on me. There were times during conversations when Anne would pause for a moment, and then redirected my thinking by asking another question. This strategy allowed me to continue to analyze and draw inferences about my lesson and my teaching.

I learned to value my relationship with my mentor and looked forward to her visits. We had reflective conversations twice a month for two years, and as a result, I was able to re-define my teaching as I created a learner-focused classroom. In short, the mentoring process was pivotal in changing and refining my conceptions regarding the teaching and learning of mathematics.

As I look back on this study and the experiences the interns and I had throughout, I have to single out the mentoring as the most important aspect of the study. The methods course was important as well as it provided a means to examine and reflect on recommended problem solving methods (NCTM, 1989, 2000; NSF, 1996), but its importance paled compared to the mentoring. Mentoring provided a means for establishing trusting relationships with the participants and provided opportunities for the interns to develop a reflective practice. Each intern, either in a conversation or through reflective writing, expressed the importance of having the mentoring experience. Early in the study, Emily said, “That talk with my mentor made me realize that there are no rules or limitations or one way to teach mathematics; every teacher has their own technique and it was time for me to figure out my own” (Reflective Writing, September 2007). Matt wrote about his mentoring experience near the end of the study:

> Outside of experience, which was probably the biggest influence, I would have to say the mentoring was the most helpful. The help and feedback was given on a regular basis and was done so in a way that was not forceful or condescending. It was also nice to have this mentor to turn to for ideas with specific topics or problems (Questionnaire, May 2008).

John communicated his mentoring experience in this way:

> The mentor provided feedback in the form of reflection questions, lesson ideas, and manipulatives/tools that guided me in the teaching process. His support enabled me to experiment and discover what works and what doesn’t in the classroom. Our conversations also helped me to consider a variety of perspectives on my approach to

> classroom management, the manner in which I taught, and the content of what I taught (Questionnaire, May 2008).

The mentoring I provided the interns was modeled after the Cognitive Coaching (Costa & Garmston, 1999) strategy I had experienced a dozen years earlier when I was the mentee and the Cognitive Coaching workshops I attended a few years later. When this study was launched in 2007, I felt it was important to use this strategy with the participants. As a result, the teachers were free to take a stance of exploration and experimentation with regard to their teaching in ways that made sense to them while I 'stirred the waters' along the way.

Mentoring new and early career mathematics teachers, as in this study, is paramount as we challenge them with a different vision of mathematics teaching and learning. To become successful, these teachers must develop trusting relationships with their mentors so they can begin to reflect on their beliefs about mathematics teaching and learning and work through feelings that may inhibit their ability to change.

Appendix A

Teaching of Secondary Mathematics

Fall 2007

Course Overview:
The goal of this course is for new and prospective secondary mathematics school teachers to learn methods of teaching and experiencing mathematics in 6-12 classrooms. The course is also designed to provide prospective teachers an opportunity to reflect on their beliefs about teaching, learning, and their expectations for students.

Required Materials:

Brahier, D. (2005). *Teaching Secondary and Middle School Mathematics* , 2nd Ed
TI-84 plus graphing calculator Selected Readings (TBD)

Web-based References:
National Council of Teachers of Mathematics: http://nctm.org
NCTM standards for middle and high school mathematics: http://standards.nctm.org

Detailed Course Description and Objectives:

Part One: What is Mathematics? What is Mathematics Reform?
We will reflect upon our own experiences, feelings, and beliefs about mathematics. We will then look at mathematics as a discipline, and compare more traditional ideas about what it means to 'know' and 'do' mathematics to the vision of mathematics advocated by the mathematics reform movement.

- What does it mean to "understand" a concept in mathematics?
- What does it mean to "do" mathematics?
- What is the role of problem solving in the mathematics classroom?
- What are current learning theories in mathematics education?

Part Two: What instructional strategies are effective in the teaching of mathematics?
Here we will take a closer look at strategies and tools that can engage students in the mathematics classroom.

- What is a good mathematical task?
- How do students make sense of the mathematics?
- What problem-solving strategies do students often use? How do those strategies progress over time?
- How can we integrate assessment of student's thinking into instruction?
- How can tools (including manipulatives, calculators and other technology) assist students in their thinking and problem solving?
- How can we interact with students as they are working on tasks in ways that support their thinking and extend their understanding?

Part Three: **How do we plan and implement lessons that promote understanding?**
In part three, we will focus on issues related to planning and implementing lessons in a linguistically and culturally diverse classroom setting. We will discuss the roles of students and teachers in the classroom, and ways to foster a classroom environment that encourages rich discussion of mathematical ideas. We will learn about a variety of lesson planning / assessment formats. You will also design and implement a problem-based mathematics lesson with your students. We will consider questions such as:

- What are the roles of the teacher and students in a mathematics classroom?
- How can we promote discussion in the classroom?
- How do we plan and implement a mathematics lesson?
- How can we adjust our instruction to meet the needs of a diverse group of learners?

Part Four: **What curriculum models are available and how do teachers make prudent decisions about which materials to use with their students?**

- What are the NCTM standards and how/why were they developed?
- What are advantages and disadvantages of various curriculum models?

Course Assignments

General notes for all assignments:
Each assignment should be a learning experience for you. Some of the things you need to do include: identify major themes in teaching and learning mathematics, apply new ideas to working with students and organizing instruction, provide examples that illustrate a point, raise questions that you don't necessarily have answers to, consider alternate points of view (especially ones you may not agree with), and critically examine your beliefs and knowledge about teaching mathematics in light of our readings and discussions. I believe that almost all meaningful learning is a result of struggling to integrate new ideas and understandings into your old ways of thinking and doing. In your assignments, I value the challenges you take on, and I am more impressed by individuals that take on a challenge and struggle with it.

I. Reflections

You will complete weekly reflections and will submit them six times during the semester. The purpose of the reflections are to allow you to examine issues that arise during the semester, to ask questions and to make connections between what you read, what we discuss, and what you experience mathematically, and what you experience in the school setting. While the reflections are informal, I do expect you to think carefully about what you are writing and to express your thoughts clearly. Each reflection should be about 2 pages, typed, and double spaced. ***Due: September 4th, September 18th, October 2nd, October 16th, October 30th, November 13th***

II. Journal Article Summary

Twice during the semester you will choose an article from a mathematics journal (i.e. *Mathematics Teacher*) that is relevant to your teaching or your thinking about mathematics learning and teaching. The article should be no longer than 4-5 pages. Your task will be to **summarize** the article and **describe** how it connects to your teaching experiences and your thinking. Each summary should be about two pages, typed and double spaced. ***Due: September 25th, November 6th***

III. Problem of the Week

During the semester, you will be given the opportunity to work on mathematics problems outside of class. These problems will allow you to explore mathematical ideas without the constraints of time. These may be open-ended problems, often mathematical classics that cannot be solved easily in a short period of time. Given at the end of one class, you should be prepared in the next to share your ideas and thinking about the problem. Your **write-up** should describe how you worked on the problem and give an explanation of your reasoning about the problem and its solution.

Due: August 28th, September 11th, October 9th, October 23rd, November 20th

IV. Lesson Development Project

Towards the end of the semester, you will work individually on a final lesson development project. Each of you will choose/design a **problem-based task** that may contain one or more content standards, one or more process standards, and one grade level, and complete the following:

- **Develop one Lesson Plan**: We will discuss a few frameworks for lesson planning in class, and you will use one of these frameworks to support you in developing the lesson. The lessons must be linked to New Mexico Math Content and Process Standards at your grade level.
- **Teach the lesson to a small group of students or the whole class.** Implement the lesson in your classroom with a small group of students or the whole class.
- **Reflect on the Lesson:** After teaching the lesson, you will reflect upon how things went, how the students responded, and what adjustments you want to make before teaching the lesson again. You will need to submit a written reflection based on your experience.
- **Class Presentation:** Each of you will prepare a brief presentation that summarizes the lesson taught, including examples of the problems/activities in the lesson, and examples of student work. Presentations should be professional, and if possible include components such as digital photographs and/or scanned copies of students' work.

Additional details about this assignment, including grading rubrics for the lesson plan, reflection on the lesson, and presentations, will be provided.

Draft of Lesson Plan Due: (optional, for feedback) November 13th, Final Lesson Plan Due: November 27th, Reflection on Teaching a Lesson Due: November 27th

Class Presentation: November 27th or December 4th (depending on the day you present)

Appendix B

Observational Feedback

- Bellwork: mixed to Improper
  - Every student used the shortest strategy; that is, they multiplied the denominator and whole number and then added the numerator. Why does this work? Your students are good at following a procedure, but do they have the understanding and flexible thinking and sense of agency needed when they encounter novel and complex problems involving fractions?
- $3\frac{1}{4} + 9\frac{1}{6} = \frac{149}{12} = \frac{144}{12} + \frac{5}{12} = 12\frac{5}{12}$

  It would be nice if students could develop alternative ways of looking at the same problem. That could be as valuable (or more) than looking at 10 more the same way.
- Students are comfortable with the class assignment/homework. Why do you think this is so?
- As always, your classroom management is admirable! :)

Appendix C

End of Fall Semester Questionnaire

1. How helpful and/or relevant were the readings? How will these readings help you as you begin your career?

2. Describe the topic or idea that we discussed this semester that you found to be the most helpful. Be as specific as possible.

3. Describe a few class activities that you found to be the most helpful. These could be class activities/menus, whole group discussions, reflections, POW's, etc. Again, please be as specific as possible.

4. Please describe any other aspect of the class (the way the class was run, readings, etc) that you found to be especially helpful.

5. What topic(s) or idea(s) would you like to have seen addressed in class, or like to have seen addressed in greater depth?

6. How did this class and your reflections about the class influence your beliefs about the teaching and learning of mathematics? In other words, are your beliefs about mathematics teaching and learning the same or different than they were at the beginning of the semester?

7. How do you envision your classroom with respect to teaching mathematics? How will you know you are successful when teaching mathematics to your students?

8. What challenges do you anticipate with regard to teaching mathematics? What support might you need?

Appendix D

End of Year Questionnaire

1) What changes have you made in your classroom instructional strategies over the course of the school year?

2) What has been the greatest influence(s) affecting your growth as a teacher? (i.e. mathematics methods course, other university course work, the SMEST mentor teacher, the SMEST program manager, collegial support, etc)

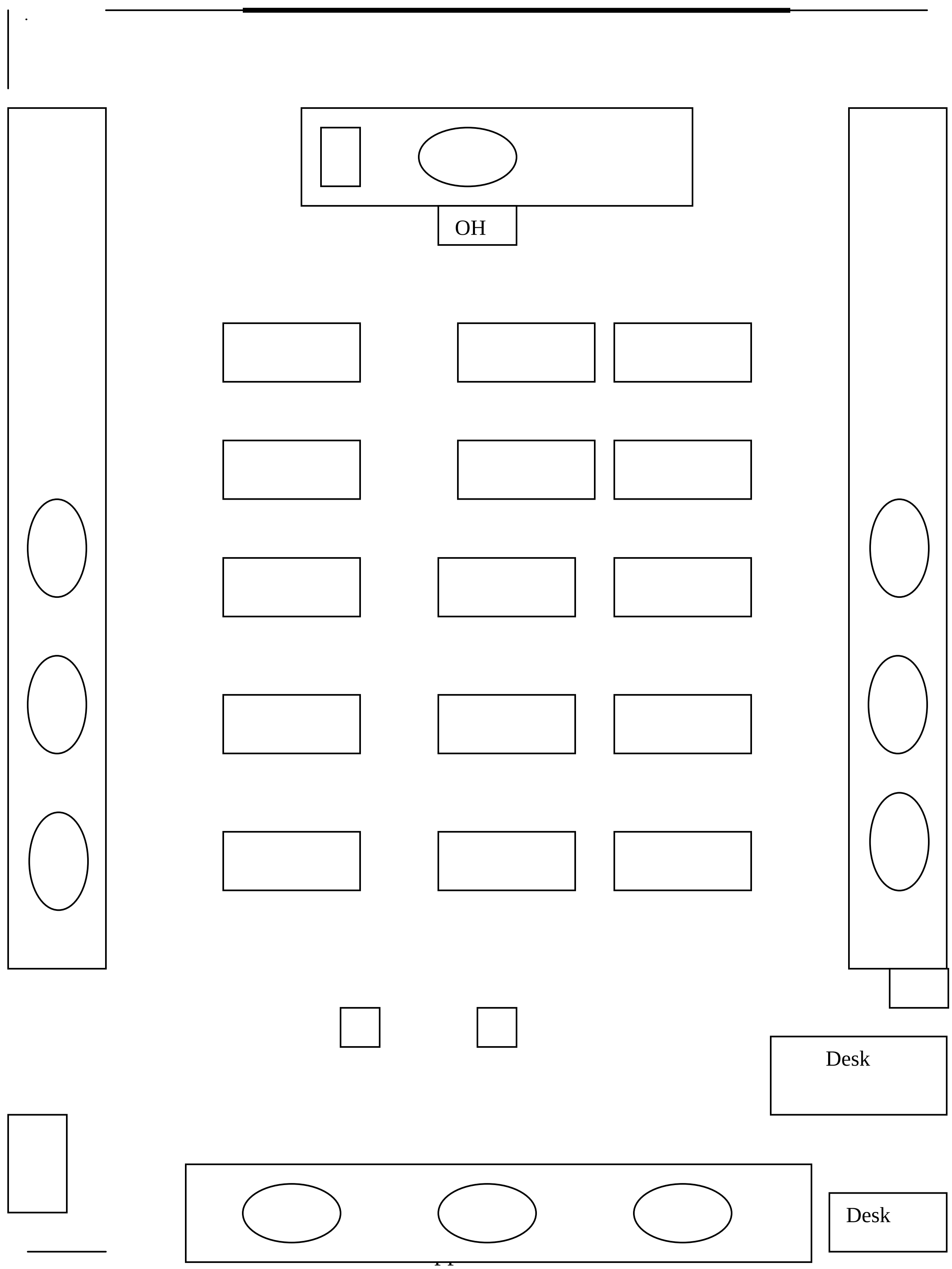
3) How did each of the influences you mention in question #2 contribute to your growth?

4) How do you envision your mathematics classroom next year? How will you know you are successful when teaching mathematics to your students?

5) What challenges do you anticipate with regard to teaching mathematics? What support might you need?

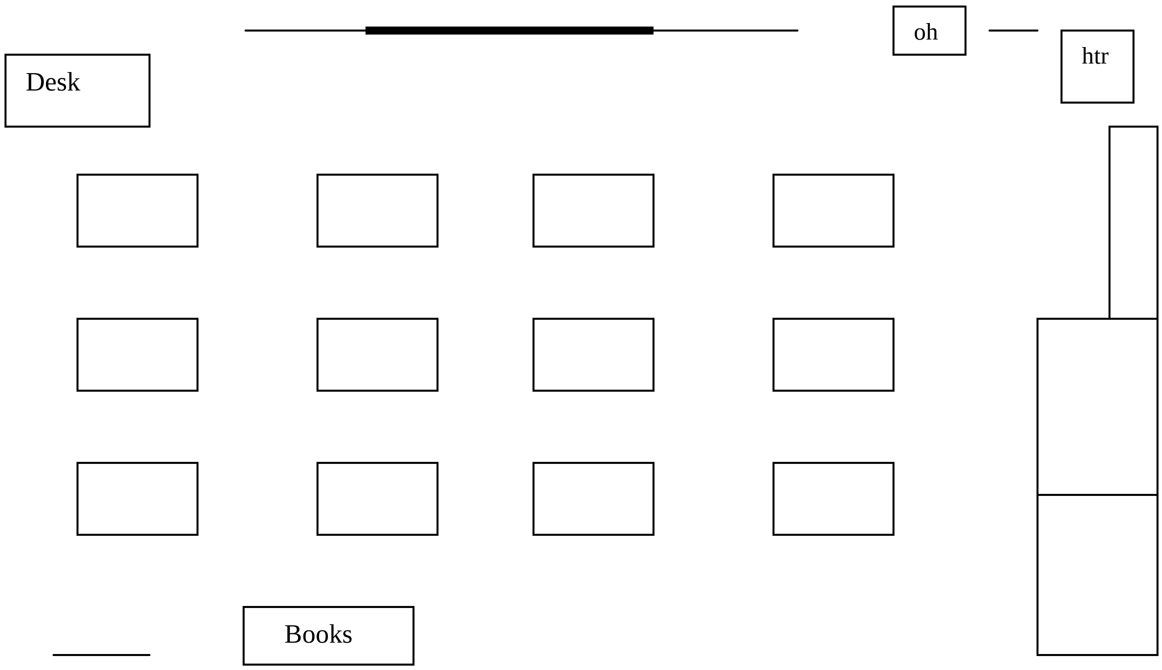
Appendix E

John's Classroom



Appendix F

Emily's and Matt's Classroom



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