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Binary consensus-based cooperative spectrum sensing in cognitive radio networks

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Abstract

We propose to use binary consensus algorithms for distributed cooperative spectrum sensing in cognitive radio networks. We propose to use two binary approaches, namely diversity and fusion binary consensus spectrum sensing. The performance of these algorithms is analyzed over fading channels. The probability of networked detection and false alarm are characterized for the diversity case. We then compare the performance of our binary-based cooperative spectrum sensing framework to that of the already-existing averaged-based one. We show that binary consensus cooperative spectrum sensing is superior to quantized average consensus in terms of agility, given the same number of transmitted bits. We furthermore derive a lower bound for the performance of the average consensus-based spectrum sensing.

We then extend our diversity-based framework to propose a weighted approach in which each secondary user utilizes a set of weights to account for different local sensing qualities of its neighbors as well as different communication link qualities from them. We mathematically characterize the optimum weights.

Finally, the impact of network configuration (in terms of average distance between the secondary users) and the resulting correlated measurements (due to shadow fading) are considered on the overall networked detection performance. More specifically, we consider the impact of the average distance on both the correlation of the sensing measurements of the secondary users and the connectivity of the underlying graph among them. We discuss interesting underlying tradeoffs when increasing or decreasing the average distance.

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Chapter 1

Introduction

1.1 Cognitive Radios and Cooperative Spectrum Sensing

In spite of the fact that RF spectrum is becoming a more and more scarce resource for new wireless services, it is still under-utilized in time and space [1]. The spectrum is basically assigned to licensed users or Primary Users (PU) who have higher priorities or legacy rights on having access to a specific part of the spectrum. However, this does not mean that a PU is constantly using the bandwidth assigned to it. Indeed, studies by Federal Communications Commission (FCC) show that the utilization of the current allocated spectrum is inefficient. Cognitive Radios (CR), introduced by Mitola [2], emerged as a possible solution to this deficiency. This approach tries to use *white spots* or *spectrum holes* where a PU is inactive in order to increase the spectral efficiency. The need for such a technology comes from the fact that the demand for higher data-rate wireless services has increased rapidly in recent years. In other words the current static spectrum allocation is not an efficient solution and CR technology is a dynamic spectrum allocation solution to accommodate more wireless users in the spectrum. The definition of cognitive radio adopted by FCC is as follows [3]: "A radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate interoperability, access secondary markets."

Cognitive radio is a Software-Defined-Radio (SDR) technology, i.e., the corresponding wireless users are smart objects. For instance, a wireless user in a Cognitive Radio Network (CRN) should be able to measure, sense and learn channel parameters as well as its own status. This, for instance, includes the radio characteristics of the channel, operating frequencies, and availability of the spectrum. CR users in a CRN are called Secondary Users (SUs). They try to exploit the white spots for their own communications while avoiding interference to the PUs. These unlicensed users constantly sense parts of the spectrum to detect the presence/absence of primary users, in order to use the available spectrum for their own communications. To achieve this goal, the secondary users (nodes) should sense the signal power in the corresponding channels and make decisions on the existence of primary users. This is called *spectrum sensing* in the cognitive radio terminology.

Spectrum sensing is the very first step and probably the most important step in establishing a cognitive radio network. This is due to the fact that the quality of spectrum sensing directly affects the performance of both primary (PU users) and secondary (SU users) networks. Spectrum sensing can be performed locally or cooperatively [4]. In local spectrum sensing, each secondary user makes a decision only based on its own one-time sensing. In cooperative spectrum sensing, on the other hand, a group of secondary users decide collaboratively on the existence of a PU, in order to improve the detection performance in the presence of local sensing errors and channel uncertainties. For instance, poor link quality, due to multipath fading, deteriorates local sensing and detection of an SU. Furthermore, individual/local sensing can not deal with the hidden terminal problem. Therefore, cooperative spectrum sensing has been proposed as an alternative approach [5–7].

Cooperative spectrum sensing can be classified into three main categories: centralized [7, 8], distributed [9], or relay-based [10, 11]. This classification is based on how a network of secondary users share their local information, i.e., their local sensing data. In centralized spectrum sensing, a fusion center (FC) (could be one of the SUs) collects all the sensing data of the SUs, makes a final decision on the presence of the PUs, based on all the available information, and broadcasts the decision to the cognitive radio network via a reporting channel [12]. In distributed spectrum sensing, on the other hand, a group of SUs reach an agreement on the existence of a PU, only based on local interactions and without a central fusion node. Finally, in the relay-based scheme, the SUs cooperate and relay the local sensing information in order to improve the overall performance. For instance, if some nodes have good sensing qualities but poor reporting channels to the FC, they can relay their measurements to the FC through other SUs with good reporting channels. It should be noted that the relay-assisted scheme is a centralized scheme.

Cooperative spectrum sensing can result in a considerable performance improvement over localized approaches as it exploits the spatial diversity of the SUs. For instance, it can get around the hidden-terminal problem as mentioned earlier. Distributed spectrum sensing is furthermore less vulnerable to FC failures. Moreover, distributed schemes are scalable and nodes can be easily added to or removed from the cooperative network. It should be noted, however, that although cooperation in spectrum sensing results in a better performance, it comes at the cost of a higher energy and bandwidth usage. Reducing cooperation overhead is therefore the main motivation of the work of this thesis.

Distributed average consensus algorithms [13] have been a subject of several studies in recent years. Applications include distributed and parallel computing [14], wireless sensor networks [15], and cooperative control of multi-agent systems [16]. In such problems, the goal is to achieve average consensus, on local information, over a network of agents. In [9, 17], average consensus and Kriged Kalman Filtering approaches have been proposed for distributed cooperative spectrum sensing. For instance, in [9], it is shown that the average consensus scheme results in a higher detection and a lower false alarm probability of the CR network. In [18], a general framework for binary consensus, i.e., agreement over the occurrence of an event, is proposed. In this approach, binary data is communicated over the network, with the goal of the whole network reaching the majority of the initial votes based on local interactions.

In this thesis, we show how such binary consensus approaches can be utilized for fast and distributed cooperative spectrum sensing. We consider two binary consensus approaches in this thesis: fusion and diversity [18]. In our binary consensus spectrum sensing, each SU makes a binary decision on the existence of the PU based on its one-time local sensing. It then communicates its vote with its neighbors over rapidly-changing fading channels. Thus, as opposed to sending raw measurements, the nodes exchange their binary votes in each transmission, which can save the communication overhead of the SUs. In Chapter 2, we characterize the performance of binary-based cooperative spectrum sensing and show how it results in a considerably higher agility, as compared to the average consensus spectrum sensing. The rapid convergence of spectrum sensing approaches is considerably important since the secondary users need to use the available spectrum as fast as possible. We also discuss the underlying tradeoffs between the binary and average consensus-based spectrum sensing approaches in terms of the asymptotic behavior. In Chapter 3, we extend our binary-based framework to a weighted approach to account for different sensing and link qualities. We show how each node can optimize its weights based on its knowledge of the sensing qualities of its neighbors and the probabilities of connectivity of the corresponding local links. Finally, Chapter 4 shows the impact of correlated measurements on the binary cooperative spectrum sensing and the optimum average distance between the nodes in the network. We conclude in Chapter 5.

1.2 Local Sensing techniques for Cognitive Radios

In this part, we briefly summarize some of the most common local spectrum sensing techniques.

1.2.1 Energy Detection

Energy detection is the most common method of spectrum sensing due to its low implementation complexity and the fact that it does not require a priori knowledge on the form of the signal of PU [6, 19]. An energy detector measures the power of the received signal and compares it with a pre-defined threshold. Its performance, however, degrades in low signal-to-noise ratio scenarios [20]. In such cases, noise and undesired signals may be detected as a false PU. Fig. 1.1 illustrates the block diagram of a typical energy detector. The received signal $r(t)$ is passed through a bandpass filter, with a center frequency that is adjusted to the part of the spectrum whose availability is being checked. The received signal then goes through a square-law filtering followed by an integrator. Finally, a binary decision on the presence of a PU, either 0 or 1, is made using a hard-limiter.

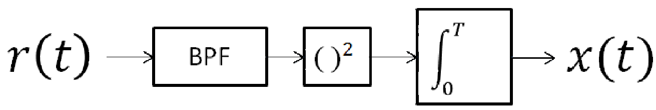

Figure 1.1: Block diagram of a typical energy detector for local spectrum sensing.

1.2.2 Cyclostationary Feature Detection

Cyclostationary feature detection technique exploits the cyclstationary features of the received signal to detect the presence of a primary user [6, 21]. A signal may have cyclostationary features due to inherent periodicity in the signal or in its statistics, e.g., mean and autocorrelation [22]. A cyclostationarity-based detector can differentiate noise from a primary user since noise has no specific redundancy in it. Furthermore, cyclostationarity can be used for distinguishing among different types of transmissions and primary users. The cyclic autocorrelation function of a signal $r(t)$ can be computed as

$$R_r^\alpha(\tau) = E[r(t+\tau)r^*(t-\tau)e^{j2\pi\alpha t}],$$

resulting in the following cyclic spectral density:

$$S(f,\alpha) = \sum_{\tau=-\infty}^{\infty} R_r^\alpha(\tau)e^{-j2\pi f\tau},$$

where α is the cyclic frequency. When the fundamental frequency of a PU signal matches the cyclic frequency, a peak occurs in cyclic spectral density function. In this method, it is assumed that cyclic features (frequencies) of the PU signal are known a priori [23].

1.2.3 Matched Filtering

Matched filtering is the optimum technique for detecting primary users when the signal of the PU is fully known to the SUs. Detection time in this scheme is shorter than the previous technique, which is a great advantage. However, it is required that the SUs demodulate and decode the signal of the primary user. This increases the complexity of the receiver of SUs considerably. In addition, each receiver needs to have perfect knowledge of the signal of the PU and other specifications such as operating frequency, type of modulation, scrambling, and coding.

1.2.4 Other Detection Techniques

Wavelet detection [24], compressed sensing [25], and waveform-based [26] techniques are other proposed methods for local spectrum sensing. Waveform-based sensing, for instance, exploits known patterns that are transmitted for different purposes. For instance, synchronization packets, preambles, pilot symbols or spreading sequences can be utilized. A preamble is a known sequence that is sent at the beginning of each burst of data. Therefore, by computing the correlation of the received signal with the known preamble, a decision on the existence of the primary users can be made. More specifically, the receiver of an SU will calculate $\mathcal{R}\big\{ \sum_{n=1}^{N} r(t)s^{*}(t)\big\}$, where $s(t)$ is a known signal, $r(t)$ is as defined earlier, and $\mathcal{R}\{\cdot\}$ denotes the real part of a complex value. In general, waveform-based sensing is more robust than energy detector and cyclostationarity-based methods because of taking advantage of specific patterns in the signal. However, this requires partial knowledge of the signal of the PU. Matched filtering is the most accurate but the most complex technique, while energy detection is the least accurate but the simplest one in terms of implementation. Compressed sensing is another approach recently proposed for spectrum sensing. This approach is specially useful for wideband spectrum sensing [27, 28].

In summary, there exist interesting tradeoffs between accuracy and complexity of implementation and/or processing of the aforementioned approaches. Choosing the right technique depends on factors such as the signal characteristics of the PU, knowledge available at CR network on PU, the required accuracy in local sensing, and the given sensing time. In this thesis we assume that SUs are equipped with simple energy detectors to focus on the cooperation and consensus among the nodes. Our framework, however, can be extended to consider other forms of local sensing.

Chapter 2

Distributed Consensus-based Cooperative Spectrum Sensing

Consider a cooperative network of M secondary users trying to reach consensus on the existence of a primary user. We assume an odd M in this thesis. Each secondary user has its own initial opinion, based on its local spectrum sensing. It will then exchange its information with other secondary users in order to improve its assessment of the existence of a primary user. The transmissions among the secondary users occur over fading channels and are furthermore corrupted by the receiver noise. As such, a communication link may not necessarily be established between a pair of secondary users due to poor link quality. Furthermore, the underlying topology of a group of secondary users that are cooperating for spectrum sensing can be time-varying. Therefore, we model the underlying network of the secondary users as an undirected random graph $\mathcal{G}(\mathcal{V}, \mathcal{E}(k))$, where $\mathcal{V} = \{1, \cdots, M\}$ represents the vertex set (the set of secondary users) and $\mathcal{E}(k)$ is the link set (the set of available communication links among the secondary users) at time k, in order to focus on the impact of network connectivity and fading channels on cooperative spectrum sensing. In a random graph (also called rapidly changing in this thesis), the underlying topology changes from one time instant to the next. Furthermore, the probability of the existence of a link, at any given time, is independent of its existence in previous times or the existence of other links. In each time step, the graph is not necessarily fully-connected and each link exists with the probability p. If a link exists, its quality is assumed perfect. Let CNR represent the ratio of the channel power to the receiver noise power. Then, there exists a link from node i to node j, at time k, if $\mathrm{CNR}_{i,j}(k) > \mathrm{CNR}_{\mathrm{TH}}$, i.e., the link quality is above a minimum acceptable threshold. We take $\mathrm{CNR}_{i,j}$s to be i.i.d. random variables with the same mean value as $\overline{\mathrm{CNR}} = \overline{\mathrm{CNR}}_{i,j}$. Thus, we assume that the secondary users operate over a small enough area such that the channels between each pair can be considered stationary and with the same average. Let p represent the probability that a link exists, from node j to node i, at a given time. Assuming exponentially-distributed multipath fading, we have $p = \mathrm{prob}(\mathrm{CNR}_{i,j}(k) > \mathrm{CNR}_{\mathrm{TH}}) = e^{-\mathrm{CNR}_{\mathrm{TH}}/\overline{\mathrm{CNR}}}$. In Chapter 3, we generalize this to the case of different average CNR for each pair of SUs and consequently different ps. We next study the spectrum-sensing model.

As mentioned in Section 1.2.4, in this thesis we assume that all the secondary users utilize energy detectors for local sensing. An energy detector [29] consists of a square-law function, followed by an integrator. Let B and T denote the bandwidth of the bandpass filter and the integration duration of a local energy detector respectively. We assume that all the secondary users utilize energy detectors with the same parameters. Let $r_i(t)$ represent the received signal of the ith secondary user, in sensing of the primary user. We have the following two hypotheses:

$$r_i(t) = \begin{cases} n_i(t) & \mathcal{H}_0 \\ h_i s(t) + n_i(t) & \mathcal{H}_1, \end{cases} \tag{2.1}$$

where $s(t)$ is the unknown signal of the primary user, $n_i(t)$ is the zero-mean additive white Gaussian receiver noise of the ith SU, and h_i is the channel gain from the primary user to the ith user, which has a Rayleigh distribution. Let γ_i denote the Signal-to-Noise Ratio (SNR) from the primary user to the ith secondary user.

Furthermore, let $x_i(0)$ represent the output of the energy detector of the ith node at time $t = 0$, as shown in Fig. 1.1. We have the following expression for $x_i(0)$:[1] $x_i(0) = \int_0^T r_i^2(t)dt$, which results in the following distribution [29]:

$$x_i(0)|\gamma_i \sim \begin{cases} \chi^2_{2TB} & \mathcal{H}_0 \\ \chi^2_{2TB}(2\gamma_i) & \mathcal{H}_1, \end{cases} \tag{2.2}$$

where χ^2_{2TB} and $\chi^2_{2TB}(2\gamma_i)$ are the central and non-central chi-square densities respectively, with $2TB$ degrees of freedom and non-centrality parameter $2\gamma_i$. More specifically, we have the following distribution for $x_i(0)$:

$$f_{x_i(0)|\mathcal{H}_1,\gamma_i}(z) = \frac{1}{2}e^{-\frac{z+2\gamma_i}{2}}\big(\frac{z}{2\gamma_i}\big)^{\frac{TB-1}{2}} I_{TB-1}(\sqrt{2\gamma_i z}), \tag{2.3}$$

$$f_{x_i(0)|\mathcal{H}_0,\gamma_i}(z) = \frac{1}{2^{TB}\Gamma(TB)} z^{TB-1}e^{-\frac{z}{2}}, \quad z \geq 0, \tag{2.4}$$

where $I_{TB-1}(\cdot)$ is the modified Bessel function of the first kind. Furthermore, γ_is are taken as i.i.d exponentially-distributed variables.[2] We relax the identically-distributed assumption in Chapter 3 and the independent assumption in Chapter 4. The SUs then communicate among themselves in order to improve their local assessments. We define the performance metric of our cooperative network as follows:

Definition 1. *For a cooperative spectrum sensing algorithm, we define the probability of networked detection and false alarm, in the kth time step, as follows:*

$$P_{d,\text{net}}(k) = \text{prob(all the nodes vote for } 1|\mathcal{H}_1) \tag{2.5}$$

$$P_{f,\text{net}}(k) = \text{prob(at least one node votes for } 1|\mathcal{H}_0). \tag{2.6}$$

[1] It should be noted that there is only a one-time local sensing and the time progression of the next section is due to communication and consensus iterations among the secondary users.

[2] In [30], the author shows that the probability density function of $x_i(0)$ (after averaging over γ_i) can be represented as the convolution of a χ^2 PDF, with $2TB - 2$ degrees of freedom, and an exponential PDF, with parameter $2(\overline{\gamma} + 1)$.

By *vote* we refer to the binary decision of a secondary user, i.e., voting 1 means that a secondary user decides that a primary user exists while voting 0 denotes otherwise. In the following section, we propose a framework for binary consensus-based cooperative spectrum sensing over fading channels. Moreover, we characterize the probabilities of networked detection and false alarm.

2.1 Distributed Consensus Algorithms for Cooperative Spectrum Sensing

Consider a cognitive radio network with M secondary users. In our proposed binary consensus cooperative spectrum sensing framework, each secondary user makes a binary decision (vote) on the existence of a primary user, based on its local sensing. The secondary users then exchange binary votes over fading channels and update their votes based on the communicated information. This process will go on for a while. The goal of binary consensus is to achieve the majority of the initial votes. In our case, however, we are interested in cooperative spectrum sensing which may not correspond to the majority of the initial votes. For instance, due to low reception qualities from the primary user, the majority of the initial votes may not correctly reflect the existence of the primary user. Let $b_i(0) \in \{0, 1\}$ represent the binary decision or vote of the ith secondary user at time step $k = 0$. Then, $b_i(k) = 1$ indicates that the ith secondary user decides that a primary user exists at time k while $b_i(k) = 0$ indicates otherwise:

$$b_i(0) = \begin{cases} 1 & \text{if} \quad x_i(0) > \eta \\ 0 & \text{else,} \end{cases} \tag{2.7}$$

where η is the local decision threshold of the secondary users, which can be numerically optimized based on the knowledge of $\bar{\gamma}$, B and T, and through the minimization of the average probability of detection error [9]. Define $\pi_{11} \triangleq \text{prob}(b_i(0) = 1|\mathcal{H}_1)$ as the average probability (averaged over the distribution of the channel) that the ith secondary user votes for 1 initially, given $\mathcal{H}_1$ hypothesis. Under the Rayleigh fading assumption, γ_is are exponentially distributed. Furthermore, in this chapter, we assume that all the secondary users experience the same average SNR, in the reception from the primary user, denoted by $\overline{\gamma}$. Thus, we have

$$P_{d_i} = \text{prob}(b_i(0) = 1|\mathcal{H}_1, \gamma_i), \tag{2.8}$$

where P_{d_i} is the local initial probability of detection of the ith SU, and the following average initial probability of detection (π_{11}) and probability of false alarm (π_{10}) respectively.

$$\begin{aligned} \pi_{11} &= E_{\gamma_i}[P_{d_i}] = \int_0^\infty P_{d_i} \frac{1}{\overline{\gamma}} e^{-\frac{\gamma_i}{\overline{\gamma}}} d\gamma_i \\ &= e^{-\frac{\eta}{2}} \sum_{m=0}^{TB-2} \frac{1}{m!}\Big(\frac{\eta}{2}\Big)^m + \Big(\frac{\overline{\gamma}+1}{\overline{\gamma}}\Big)^{TB-1} \left(e^{-\frac{\eta}{2(\overline{\gamma}+1)}} - e^{-\frac{\eta}{2}} \sum_{m=0}^{TB-2} \frac{\eta}{m!} \frac{\overline{\gamma}}{2(\overline{\gamma}+1)} \right), \end{aligned} \tag{2.9}$$

$$\begin{aligned} \pi_{10} &= \text{prob}(b_i(0) = 1|\mathcal{H}_0) \\ &= \text{prob}(x_i(0) > \eta|\mathcal{H}_0) = 1 - \frac{\Gamma_l(TB, \frac{\eta}{2})}{\Gamma(TB)}, \end{aligned} \tag{2.10}$$

where $E_{\gamma_i}[\cdot]$ denotes the expectation operator. $\Gamma(\cdot)$ and $\Gamma_l(\cdot,\cdot)$ are Gamma and lower incomplete Gamma functions respectively.

In [18, 31], authors propose two binary consensus approaches, *fusion* and *diversity*, for a network that is trying to reach consensus over a binary value. Reaching consensus, in this context, means reaching the majority of the initial votes. In the fusion-based approach, each user fuses the received votes of other users that it can communicate with, namely neighbors, and updates its state based on the majority of the received votes. It will then send its updated vote to all its neighbors in the next time step. This process will go on until a given time for operation is reached. This strategy is suitable, in particular, when the graph connectivity is low as it creates virtual links between nodes. It is shown in [18] that fusion-based binary consensus asymptotically reaches consensus (not necessarily accurate consensus though) if the underlying graph representing the network has non-zero link existence probability. In the case of diversity, on the other hand, each node uses its transmissions to repeat its initial vote, without fusing its received information. It then fuses its received votes only at the end of the given time. This strategy, is more robust to link errors. It is also shown that diversity-based approach converges to the majority of the initial votes asymptotically if $p \neq 0$ on all the links.

In a cooperative network of SUs, each node measures the energy of the PU and makes a binary decision on the existence of a PU transmission. Therefore, a group of SUs with different binary initial votes aim to reach agreement on an event (existence of a PU). This inspires applying binary consensus-based approaches of [18] to cooperative spectrum sensing, as we explore next.

2.1.1 Diversity-Based Binary Consensus for Spectrum Sensing

In this part, we apply diversity-based binary consensus approach of [18] to cooperative spectrum sensing. In this strategy, each secondary user utilizes its communications to other SUs to repeat its initial vote. Consider the case where the SU network is given $K+1$ time steps, for $K \geq 0$, to reach an agreement (K transmissions with the last time step to finalize the decision). Each node then uses all its transmissions to repeat its initial vote and only fuses the received information at the end.[3] This

[3] As soon as the ith SU receives the vote of the jth SU, the jth SU can stop retransmissions if the ith SU sends back an ACK message. This results in a more efficient performance. While we do not consider this case, our framework can be easily extended to address it.

strategy can, in particular, be useful in reducing the impact of link failures on the exchange of information between the SUs. Let $\mathbf{b}(k) = [b_1(k), \cdots, b_M(k)]^T$ represent the vector of the votes of all the secondary users at time step k, where ^{T} denotes matrix/vector transpose. Then, the dynamics of the network evolves as follows, given K transmissions,

$$\begin{aligned} b_i(k) &= b_i(k-1), \quad k \in \{1, \cdots, K-1\} \\ b_i(K) &= \text{Dec}\left(\frac{1}{M}\Big(b_i(0) + \frac{1}{Kp}\sum_{t=0}^{K-1}\sum_{j=1}^{M} a_{ij}(t)b_j(t)\Big)\right), \quad K \geq 0, \end{aligned} \tag{2.11}$$

and in matrix form,

$$\begin{aligned} \mathbf{b}(k) &= \mathbf{b}(k-1), \quad k \in \{1, \cdots, K-1\} \\ \mathbf{b}(K) &= \text{Dec}\left(\frac{1}{M}\Big(\mathbf{b}(0) + \frac{1}{Kp}\sum_{t=0}^{K-1} \mathbf{A}(t)\mathbf{b}(t)\Big)\right), \quad K \geq 0, \end{aligned} \tag{2.12}$$

where $\mathbf{A}(k) = [a_{ij}(k)]_{1\leq i,j\leq M}$ is an $M \times M$ adjacency matrix of the SU network, at time step k, with $a_{ii}(k) = 0$. The off-diagonal elements of the adjacency matrix are Bernoulli random variables with $\text{prob}(a_{ij}(k) = 1) = p$ for $i \neq j$. We have $\text{Dec}(z) = \begin{cases} 1 & z \geq 0.5 \\ 0 & z < 0.5 \end{cases}$. Note that if $\text{Dec}(\cdot)$ is applied to a vector, it operates entry-wise. In [18], it was shown that diversity-based binary consensus algorithm over random graphs achieve asymptotic majority consensus. This is due to the fact that repeated transmissions over a link with $p \neq 0$ results in asymptotic connectivity with the probability of 1. Note that this, however, does not mean that the asymptotic probability of networked detection is 1 for spectrum sensing.

Let $S(0) = \mathbf{1}^T\mathbf{b}(0)$, the sum of the initial votes of M secondary users, represent the state of the network, where $\mathbf{1}$ is an $M \times 1$ all-one vector. In the following lemma and the corollary that follows, we characterize the probability of networked detection and false alarm of diversity-based cooperative spectrum sensing.

Lemma 1. *Assume a cognitive radio network, with M secondary users communicating over a rapidly-changing network topology, where p denotes the probability of the existence of a link between any two SUs at any time step. For a sufficiently-large odd M, the probability of networked detection, at time step K is approximated by*

$$P_{d,net}(K) \approx \sum_{i=0}^{M} \binom{M}{i} \left[(1-\pi_{11}) Q\left(\frac{(\frac{M}{2}-i)\sqrt{K}}{\sqrt{\frac{1-p}{p} i}} \right) \right]^{M-i} \times \left[\pi_{11} Q\left(\frac{(\frac{M}{2}-i)\sqrt{K}}{\sqrt{\frac{1-p}{p}|i-1|}} \right) \right]^{i}, \tag{2.13}$$

where $Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{+\infty} e^{-t^2/2} dt$ and π_{11} is defined in Eq. (2.9).

Proof. Let $\mathbf{Y}(K) = \mathbf{b}(0) + \frac{1}{Kp} \sum_{t=0}^{K-1} \mathbf{A}(t)\mathbf{b}(t)$ and $y_i(K)$ represent the ith entry of $\mathbf{Y}(K)$. In [18], it was shown that, for a sufficiently-large M, we can evoke the Central Limit Theorem (CLT) to approximate the distribution of $y_i(K)$ with a Gaussian density, with mean $\mu_{y_i(K)} = S(0)$ and variance $\sigma^2_{y_i(K)} = |S(0) - b_i(0)| \frac{1-p}{pK}$. Conditioning on $b_1(0), \cdots, b_M(0)$ and considering the fact that $a_{ij}(t)$s, for $i \neq j$, are mutually independent for different pairs of SUs, Gaussian random variables $y_1(K), \cdots, y_M(K)$ will also become independent. Therefore, we have

$$\begin{aligned} &\text{prob}\big(y_1(K) > \frac{M}{2}, \cdots, y_M(K) > \frac{M}{2} | b_1(0), \cdots, b_M(0)\big) \\ &\approx \prod_{i=1}^{M} Q\left(\frac{\frac{M}{2} - S(0)}{\sqrt{|S(0) - b_i(0)| \frac{1-p}{p}}} \sqrt{K} \right) \\ &= Q^{M-S(0)} \left(\frac{\big(\frac{M}{2} - S(0)\big)\sqrt{K}}{\sqrt{S(0)\frac{1-p}{p}}} \right) Q^{S(0)} \left(\frac{\big(\frac{M}{2} - S(0)\big)\sqrt{K}}{\sqrt{|S(0)-1|\frac{1-p}{p}}} \right). \end{aligned} \tag{2.14}$$

The last equality is written by noting that $b_i(0)$s are either 0 or 1. We can then derive the probability of networked detection of the secondary users as follows,

$$
\begin{aligned}
P_{d,\text{net}}(K) &= \sum_{i=0}^{M} \text{prob}\big(b_1(K) = 1, \cdots, b_M(K) = 1 | S(0) = i\big)\text{prob}(S(0) = i|\mathcal{H}_1) \\
&= \sum_{i=0}^{M} \text{prob}\Big(y_1(K) > \frac{M}{2}, \cdots, y_M(K) > \frac{M}{2} | S(0) = i\Big)\text{prob}(S(0) = i|\mathcal{H}_1),
\end{aligned}
\tag{2.15}
$$

where $\text{prob}(S(0) = i|\mathcal{H}_1) = \binom{M}{i}\pi_{11}^i(1-\pi_{11})^{M-i}$. Substituting Eq. (2.14) in Eq. (2.15) results in the Lemma. $\square$

We then have the following for the asymptotic value of $P_{d,\text{net}}(K)$

$$
\lim_{K\to\infty} P_{d,\text{net}}(K) = \sum_{i=\lceil \frac{M}{2} \rceil}^{M} \binom{M}{i} (1 - \pi_{11})^{M-i}\pi_{11}^i. \tag{2.16}
$$

Therefore, the asymptotic behavior of diversity-based binary consensus spectrum sensing, over random graphs, is independent of the network connectivity and only depends on the number of secondary users and $\overline{\gamma}$ (through π_{11}).

Corollary 1. *For the cognitive radio network of Lemma 1, the probability of false alarm is*

$$
\begin{aligned}
P_{f,net}(K) = 1 - &\sum_{i=0}^{M} \binom{M}{i} \left[(1 - \pi_{10})\left(1 - Q\left(\frac{(\frac{M}{2} - i)\sqrt{K}}{\sqrt{\frac{1-p}{p}i}}\right)\right)\right]^{M-i} \\
&\times \left[\pi_{10}\left(1 - Q\left(\frac{(\frac{M}{2} - i)\sqrt{K}}{\sqrt{\frac{1-p}{p}|i - 1|}}\right)\right)\right]^{i},
\end{aligned}
$$

where π_{10} is defined in Eq. (2.10).

Proof. The proof is similar to that of Lemma 1. $\square$

As expected, we can see from Corollary 1 and Eq. (2.10) that the probability of false alarm is independent of $\overline{\gamma}$ (it is only a function of noise parameters). Similarly,

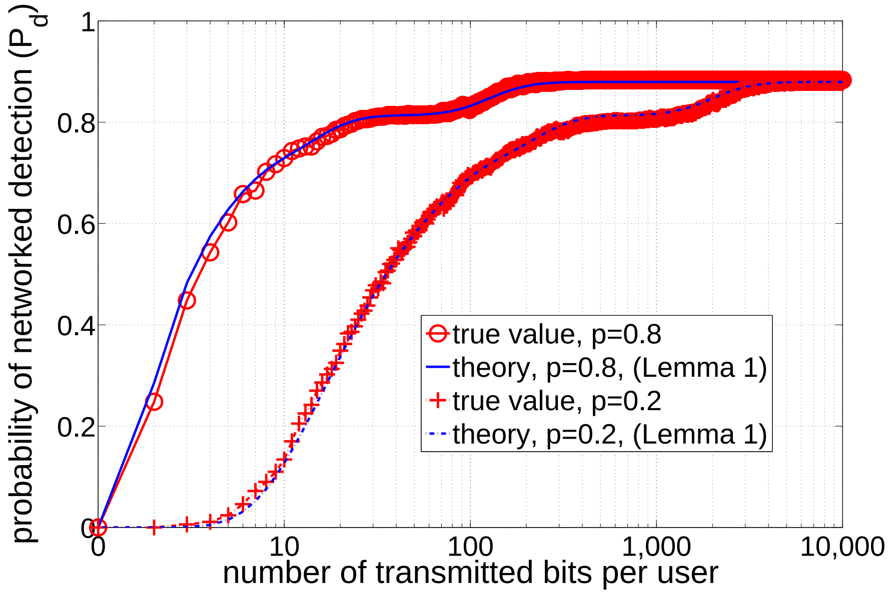

Figure 2.1: Theoretical and simulated probability of networked detection for diversity-based binary consensus cooperative spectrum sensing, with $M = 51$ and $\overline{\gamma} = 2$ dB.

as we discussed for $P_{d,\text{net}}(K)$, the asymptotic value of $P_{f,\text{net}}(K)$ is independent of the network connectivity (p). Fig. 2.1 shows the probability of networked detection, for a network of $M = 51$ secondary users. The theoretical approximation, for the probability of networked detection, is compared to the true value obtained from simulation for $\overline{\text{CNR}} = 0$ dB. It can be seen that Eq. (2.13) matches considerably well with the true probabilities. Furthermore, the probability of networked detection for both $p = 0.2$ and $p = 0.8$ converges to the same value asymptotically. This is due to the fact that the asymptotic value of diversity-based approach is independent of network connectivity as shown in Eq. (2.16). Furthermore, the asymptotic probability of networked detection is not 1 because there is always a non-zero probability that the majority of initial votes is zero under $\mathcal{H}_1$.

2.1.2 Fusion-Based Binary Consensus for Spectrum Sensing

In [18], the properties of fusion-based binary consensus were discussed in a general context. It was shown that the fusion-based approach does not necessarily reach the majority of the initial votes asymptotically, if less than $M-1$ nodes vote the same initially. On the other hand, diversity-based binary consensus achieves the majority of the initial votes almost surely for $p \neq 0$.[4] The advantage of the fusion-based strategy is that local information propagates faster through the network than the diversity-based approach. Consequently, the agility of fusion-based binary consensus is higher. Therefore, the fusion strategy is suitable, in particular, when the graph connectivity is low as it creates virtual links between the nodes. Thus, there are some tradeoffs between diversity and fusion approaches, in terms of agility and asymptotic behavior, as was shown in [18] and briefly summarized in Section 2.2.

In this part, we apply the fusion-based binary consensus approach [18] to cooperative spectrum sensing. In this case, each secondary user updates its binary decision, at every step, based on the received votes from its neighbors. In the next time step, it then transmits its updated vote to its neighbors. The dynamics of the fusion strategy can be expressed as

$$\mathbf{b}(k) = \mathrm{Dec}\left(\frac{1}{M}\Big(\mathbf{I} + \mathbf{A}(k)\Big)\mathbf{b}(k-1)\right), \quad k \in \mathbb{Z}^+,$$

or

$$b_i(k+1) = \mathrm{Dec}\left(\frac{1}{M}\Big(b_i(k) + \sum_{j=1}^{M} a_{ij}(k) b_j(k)\Big)\right) \quad \forall\, i \in \{1, \cdots, M\},\ k \in \mathbb{Z}^+.$$

In the context of cooperative spectrum sensing, however, characterization of the probability of networked detection for the fusion-based binary spectrum sensing is

[4] Note that the majority of the initial votes may not still correspond to an accurate networked detection if $\overline{\gamma}$ is too small.

challenging and an open problem. Thus, we compare its performance with diversity-based approach of section 2.1.1 through simulations later in this chapter.

2.1.3 Average Consensus for Spectrum Sensing

In this part, it is our goal to compare the performance of our binary-based cooperative spectrum sensing approaches with that of already proposed average-based approach. In [9], authors have shown through simulation that an average-based consensus scheme can result in a considerably better performance in a cognitive radio network. However, mathematical characterization of the performance is not presented in their work. Thus, similar to the analysis we did for diversity-based spectrum sensing, in this part, we first mathematically characterize a lower bound for the probability of networked detection for average-based consensus spectrum sensing. We then compare its performance and agility with our binary consensus-based algorithms.

The standard average consensus dynamics (in a general context), over random graphs, evolve as follows [13]

$$\mathbf{X}(k+1) = \mathbf{P}(k+1)\mathbf{X}(k), \tag{2.17}$$

where $\mathbf{X}(k) = [x_1(k), \cdots, x_M(k)]^T$ is an $M \times 1$ general state vector at time step k, and $\mathbf{P}(k)$ is a doubly-stochastic matrix that corresponds to the underlying graph of the SUs at time k: $\sum_{j=1}^{M} P_{ij}(k) = \sum_{i=1}^{M} P_{ij}(k) = 1$ for $i, j \in \{1, \cdots, M\}$. We then have $\mathbf{P}(k) = \mathbf{I} - \epsilon(\mathbf{D}(k) - \mathbf{A}(k))$, where $\mathbf{A}(k)$ is the adjacency matrix of the SU network and $\mathbf{D}(k)$ is a diagonal matrix whose diagonal entries are the degrees of the nodes of the graph, i.e., $\mathbf{D}(k) = \text{diag}(d_1(k), \cdots, d_M(k))$, with $d_i(k) = \sum_{j=1}^{M} a_{ij}(k)$. Let ϱ denote the maximum degree of the network, then $\epsilon \in (0, 1/\varrho)$ [13]. Since we assume that each node can potentially be connected to any other node, then

$\varrho = M - 1$. If the graph is connected at time k, then the second largest eigenvalue of matrix $\mathbf{P}(k)$ is less than 1, i.e., the largest eigenvalue (which is 1) is isolated [32].

In the context of spectrum sensing, the SUs try to reach average consensus on their original sensing. The vector $\mathbf{X}(0) = [x_1(0), \cdots, x_M(0)]^T$ contains the local measurements and $\mathbf{X}(k)$, for $k \in \mathbb{Z}^+$, is the vector of updated sensing of the nodes after k steps of fusion. The goal is for each node to reach $\bar{x}(0) = \frac{1}{M}\sum_{j=1}^{M} x_j(0)$. After the given time for consensus is reached, each node compares its current state to a predefined threshold and makes a binary decision on the existence of a primary user. Average consensus spectrum sensing is useful because it averages the noise in local measurements. However, very high noise in even only a minority of the nodes can ruin the performance considerably. On the other hand, binary consensus-based spectrum sensing performs well when the majority of SUs have good sensing quality. Let ρ denote the predefined threshold. This parameter can be optimally designed, using an ML detection rule for the desired value of the asymptotic average consensus. In other words, it is not feasible to adjust the threshold in each iteration of consensus. The is due to the fact that $x_i(0)$s get multiplied by the entries of the stochastic matrix $\mathbf{P}(k)$ and computing the distribution of the sum of these quantities is quite challenging. As such, we design the threshold for the asymptotic case where all the SUs have reached the average of the initial measurements, for which we can compute the distribution. This threshold, which is clearly sub-optimal, is then used in each iteration of average consensus spectrum sensing. Assuming that all the SUs have the same noise distribution, we can calculate the threshold ρ from 2.2.

In [9], average consensus has been utilized for spectrum sensing. However, no mathematical characterization of the networked detection performance is provided. Thus, we next find a lower bound on the probability of networked detection.

Lemma 2. *Assume a cognitive radio network, with M secondary users communicating over rapidly-changing fading channels with p as the probability of existence of* *any link at any given time. Furthermore, assume that the M secondary users use the average consensus spectrum sensing scheme of Eq. 2.17. Then the probability of networked detection can be lower bounded as follows:*

$$P_{d,net}^{avg,cons}(k) \geq prob(\mathcal{F}|\mathcal{H}_1) - \lambda_2^k\big(E[\mathbf{P}^2]\big)\int_{\mathbf{Z}\in\mathcal{F}} \frac{\|\mathbf{Z}-\frac{1}{M}\mathbf{1}^T\mathbf{Z}\mathbf{1}\|^2}{\big(\frac{1}{M}\mathbf{1}^T\mathbf{Z}-\rho\big)^2} f_{\mathbf{X}(0)|\mathcal{H}_1}(\mathbf{Z}|\mathcal{H}_1)d\mathbf{Z},$$

where $\mathcal{F} \triangleq \Big\{\mathbf{X}(0)\big|\bar{x}(0) \geq \rho\Big\}$, $f_\zeta(\cdot)$ is the probability density function (PDF) of a general random variable ζ. $P_{d,net}^{avg,cons}(k)$ denotes the probability of networked detection for average consensus spectrum sensing at time k and $\lambda_2\big(E[\mathbf{P}^2]\big)$ is the second largest eigenvalue of matrix $E[\mathbf{P}^2]$.

Proof. We have

$$\begin{aligned} P_{d,\text{net}}^{\text{avg,cons}}(k) &= \int_{\mathbf{Z}\in\mathcal{F}} \text{prob}(\mathbf{X}(k)\geq\rho\mathbf{1}|\mathbf{X}(0)=\mathbf{Z})f_{\mathbf{Z}|\mathcal{H}_1}(\mathbf{Z}|\mathcal{H}_1)d\mathbf{Z} \\ &\quad + \int_{\mathbf{Z}\in\mathcal{F}^C} \text{prob}(\mathbf{X}(k)\geq\rho\mathbf{1}|\mathbf{X}(0)=\mathbf{Z})f_{\mathbf{Z}|\mathcal{H}_1}(\mathbf{Z}|\mathcal{H}_1)d\mathbf{Z} \\ &= \int_{\mathbf{Z}\in\mathcal{F}} \text{prob}(\mathbf{X}(k)\geq\rho\mathbf{1}|\mathbf{X}(0)=\mathbf{Z})f_{\mathbf{Z}|\mathcal{H}_1}(\mathbf{Z}|\mathcal{H}_1)d\mathbf{Z}. \end{aligned} \tag{2.18}$$

The second term on the right hand side of the first line of Eq. (2.18) is equal to zero. This is due to the fact that the initial average is preserved in an average consensus algorithm, i.e., $\bar{x}(k) = \frac{1}{M}\sum_{i=1}^{M} x_i(k) = \bar{x}(0)$. Since the second integration is over $\mathcal{F}^c$, where $\bar{x}(0) < \rho$, then the second integral becomes zero. Under the assumption of independent initial assessments, i.e., independent $x_i(0)$s, we have $f_{\mathbf{Z}|\mathcal{H}_1}(\mathbf{Z}|\mathcal{H}_1) = \Pi_{i=1}^M f_{z_i|\mathcal{H}_1}(z_i|\mathcal{H}_1)$. Finding a closed-from expression for $P_{d,\text{net}}^{\text{avg,cons}}(k)$ is still challenging. Thus, we derive a lower bound in order to analyze the performance of average consensus spectrum sensing. Let $\mathbf{\Delta}(k) = \mathbf{X}(k) - \bar{x}(0)\mathbf{1}$ represent the error vector. We then have

$$\begin{aligned}
&\text{prob}(\mathbf{X}(k) \geq \rho\mathbf{1}|\mathbf{X}(0) \in \mathcal{F}) \\
&\qquad\qquad = \text{prob}(\mathbf{X}(k) - \bar{x}(0)\mathbf{1} \geq (\rho - \bar{x}(0))\mathbf{1}|\mathbf{X}(0) \in \mathcal{F}) \\
&\qquad\qquad \geq \text{prob}(|\mathbf{X}(k) - \bar{x}(0)\mathbf{1}| \leq (\bar{x}(0) - \rho)\mathbf{1}|\mathbf{X}(0) \in \mathcal{F}) \\
&\qquad\qquad \geq \text{prob}\big(\|\mathbf{\Delta}(k)\|^2 \leq \big(\bar{x}(0) - \rho)^2|\mathbf{X}(0) \in \mathcal{F}\big) \\
&\qquad\qquad \geq 1 - \frac{E[\|\mathbf{\Delta}(k)\|^2|\mathbf{X}(0) \in \mathcal{F}]}{\big(\bar{x}(0) - \rho\big)^2},
\end{aligned} \tag{2.19}$$

where the expectation is taken over the graph randomness. The last line in Eq. (2.19) is derived using Chebyshev's inequality for Hilbert-space-valued random elements ([33] Theorem 2.1). Next we derive an upper bound for $E[\|\mathbf{\Delta}(k)\|^2|\mathbf{X}(0)]$.

We have $\mathbf{\Delta}(k) = \mathbf{P}(k)\mathbf{\Delta}(k-1)$ [34]. From Rayleigh-Ritz inequality, we know that $\frac{z^T B z}{\|z\|^2} \leq \lambda_2(B)$ for all vectors z such that $\mathbf{1}^T z = 0$, where B is an arbitrary matrix with the eigenvector of $\mathbf{1}$ corresponding to its first (largest) eigenvalue. Since $\mathbf{1}^T\mathbf{\Delta} = 0$, by conditioning on the previous step and using Rayleigh-Ritz inequality, we have

$$\begin{aligned}
E\big[\|\mathbf{\Delta}(k)\|^2|\mathbf{\Delta}(k-1)\big] &= \mathbf{\Delta}^T(k-1)E\big[\mathbf{P}^T(k)\mathbf{P}(k)\big]\mathbf{\Delta}(k-1) \\
&\leq \lambda_2\big(E[\mathbf{P}^T(k)\mathbf{P}(k)]\big)\|\mathbf{\Delta}(k-1)\|^2.
\end{aligned}$$

Then, through induction and by noting the stationarity of matrix $\mathbf{P}(k)$, we can write

$$E\big[\|\mathbf{\Delta}(k)\|^2|\mathbf{X}(0)\big] \leq \lambda_2^k\big(E[\mathbf{P}^2(k)]\big)\|\mathbf{\Delta}(0)\|^2. \tag{2.20}$$

Next we characterize $\lambda_2(E[\mathbf{P}^2(k)])$ which is the second largest eigenvalue of matrix $E[\mathbf{P}^2(k)]$. For simplicity we drop the time index k. We have $\mathbf{P} = \mathbf{I} - \epsilon(\mathbf{D} - \mathbf{A})$. Furthermore, the off-diagonal entries of the adjacency matrix are Bernoulli distributed random variables, with the probability $\text{prob}(a_{ij} = 1) = p$ for $i \neq j$. Diagonal entries of matrix $\mathbf{D}$ are binomial random variables $d_i \sim \mathcal{B}(M-1, p)$ for $i \in \{1, \cdots, M\}$. By noting that $\mathbf{P} = \mathbf{P}^T$, we next characterize the second largest eigenvalue of $E[\mathbf{P}^2]$. We have $E[\mathbf{A}^2] = ((M-1)p - (M-2)p^2)\mathbf{I} + (M-2)p^2\mathbf{1}\mathbf{1}^T$, and $E[\mathbf{D}^2] = \big((M-1)(M-2)p^2 + (M-1)p\big)\mathbf{I}$.

Moreover, $[\mathbf{DA}]_{ij} = \sum_{m=1}^{M} a_{im}a_{ij} = \sum_{\substack{m=1\\ m\neq i,j}}^{M} a_{im}a_{ij} + a_{ij}^2$. Therefore,

$$E\big[\mathbf{DA}\big] = ((M-2)p^2 + p)(\mathbf{1}\mathbf{1}^T - \mathbf{I}). \tag{2.21}$$

We then have

$$\begin{aligned} E[\mathbf{P}^2] &= \mathbf{I} - 2\epsilon(E[\mathbf{D}] - E[\mathbf{A}]) + \epsilon^2(E[\mathbf{A}^2] + E[\mathbf{D}^2] - 2E[\mathbf{DA}]) \\ &= \big(M(M-2)\epsilon^2p^2 + 2M\epsilon^2 p - 2M\epsilon p + 1\big)\mathbf{I} \\ &\qquad + \big(2\epsilon p - (M-2)\epsilon^2p^2 - 2\epsilon^2 p\big)\mathbf{1}\mathbf{1}^T. \end{aligned}$$

$E[\mathbf{P}^2] = \alpha\mathbf{I} + \beta\mathbf{1}\mathbf{1}^T$, for any α, $\beta \in \mathbb{R}$, is a special form of a circulant matrix. It is straightforward to see that the eigenvalues of a circulant matrix are $\lambda_1 = \alpha + M\beta$ and $\lambda_j = \alpha$ for $j \in \{2, \cdots, M\}$. Therefore,

$$\lambda_2\big(E[\mathbf{P}^2(k)]\big) = M(M-2)\epsilon^2p^2 + 2M\epsilon^2p - 2M\epsilon p + 1. \tag{2.22}$$

Combining Eq. (2.18), (2.19) and (2.20) yields

$$\begin{aligned} P_{d,\mathrm{net}}^{\mathrm{avg,cons}}(k) &= \int_{\mathbf{Z}\in\mathcal{F}} \mathrm{prob}(\mathbf{X}(k) \geq \rho\mathbf{1} | \mathbf{X}(0) = \mathbf{Z}) f_{\mathbf{X}(0)|\mathcal{H}_1}(\mathbf{Z}|\mathcal{H}_1)d\mathbf{Z} \\ &\geq \int_{\mathbf{Z}\in\mathcal{F}} \left(1 - \frac{E[\|\boldsymbol{\Delta}(k)\|^2 | \mathbf{X}(0) = \mathbf{Z}]}{\big(\bar{x}(0) - \rho\big)^2}\right) f_{\mathbf{X}(0)|\mathcal{H}_1}(\mathbf{Z}|\mathcal{H}_1)\mathbf{Z} \\ &= \mathrm{prob}(\mathcal{F}|\mathcal{H}_1) - \lambda_2^k\big(E[\mathbf{P}^2(k)]\big) \\ &\qquad \times \int_{\mathbf{Z}\in\mathcal{F}} \frac{\|\mathbf{Z} - \frac{1}{M}\mathbf{1}^T\mathbf{Z}\mathbf{1}\|^2}{\big(\frac{1}{M}\mathbf{1}^T\mathbf{Z} - \rho\big)^2} f_{\mathbf{X}(0)|\mathcal{H}_1}(\mathbf{Z}|\mathcal{H}_1)d\mathbf{Z}. \end{aligned} \tag{2.23}$$

□

Corollary 2. *The lower bound obtained in Lemma 2 for the probability of networked detection in average consensus-based cooperative spectrum sensing is asymptotically exact and independent of the network connectivity.*

Proof. For $\varrho = M - 1$, we have $\epsilon = \frac{1}{M-1}$. Therefore, from Eq. (2.22) we write

$$\lambda_2\big(E[\mathbf{P}^2(k)]\big) = \frac{M(M-2)}{(M-1)^2}p(p-2) + 1. \tag{2.24}$$

Since $-1 \leq p(p-2) < 0$ and $0 < \frac{M(M-2)}{(M-1)^2} < 1$, therefore $\lambda_2\big(E[\mathbf{P}^2(k)]\big) < 1$. Thus, our lower bound in Lemma 2 tends to $\text{prob}(\mathcal{F}|\mathcal{H}_1)$ for large enough k. Moreover, from Eq. (2.18), it can be easily confirmed that since $\lim_{k\to\infty} \text{prob}(\mathbf{X}(k) \geq \rho\mathbf{1}|\mathbf{X}(0) \in \mathcal{F}) = 1$, then $\lim_{k\to\infty} P_{d,\text{net}}^{\text{avg,cons}}(k) = \text{prob}(\mathcal{F}|\mathcal{H}_1)$, which is the exact asymptotic value and independent of network connectivity. □

2.2 Simulation Results

In this section, we compare the performance of our binary consensus-based cooperative spectrum sensing approaches with that of average consensus. All the simulations in this thesis are implemented in MATLAB. As mentioned earlier, in average consensus spectrum sensing, local measurements ($x_i(0)$s) are exchanged among the secondary users. In a realistic scenario, measurements need to be quantized before transmission [35, 36]. Therefore, in this section, we simulate quantized average consensus and compare its detection performance with the binary consensus schemes for cooperative spectrum sensing. Let $q(\cdot)$ denote an R-bit quantizer. This quantizer is a mapping $q : \mathbb{R} \to \mathbb{Z}$, converting $z \in \mathbb{R}$ to its nearest integer value $n \in \mathbb{Z}$. Let A_{max} and A_{min} indicate the expected maximum and minimum of the input to the quantizer. We take $A_{min} = 0$ because the output of the energy detectors is positive. Thus, the quantization step-size becomes $\delta = \frac{A_{max}}{2^R}$. We can write

$$q(z) = \begin{cases} 2^R - 1, & z \geq (2^R - 1.5)\delta \\ n, & (n - 0.5)\delta \leq z < (n + 0.5)\delta \\ 0, & z \leq 0.5\delta. \end{cases}$$

It is straightforward to rewrite Eq. (2.17) for the quantized case as $\mathbf{X}(k+1) = \mathbf{X}(k) + (\mathbf{P}(k+1) - \mathbf{I})q(\mathbf{X}(k))$, where $q(\cdot)$ is applied entry-wise to its vector argument. Next, we compare the performance of binary consensus schemes with quantized average consensus. In order to have a fair comparison, we quantize the transmitted data of the average consensus case to R bits per transmission and evaluate the performance of these algorithms in terms of the number of transmitted bits per SU. We further assume that all the secondary users have the same $\overline{\gamma}$ and investigate two cases of low and high network connectivity ($p = 0.2$ and $p = 0.8$). Moreover, we set $R = 5$ bits to achieve the best agility for the average consensus algorithm. Optimal R was found through simulations. Basically, small values of R are not acceptable because it degrades the performance of the average consensus algorithm, while large values of R reduces the agility. So R is set according to this tradeoff. We take $\epsilon = \frac{1}{M-1}$ and $TB = 5$.

Fig. 2.2 and 2.3 compare the performance of the three consensus-based spectrum sensing approaches, for the two cases of low and high network connectivity respectively. It can be seen that, although the SNR to the primary user is low ($\overline{\gamma} = 2$ dB), fusion-based binary consensus outperforms both diversity-based binary consensus and quantized average consensus spectrum sensing in terms of agility. Moreover, it can be seen that the agility of diversity-based binary consensus spectrum sensing improves tremendously by increasing network connectivity. Network connectivity, however, does not impact the performance of the fusion-based strategy significantly. The reason is that when the graph connectivity is low, fusion-based strategy creates virtual links between the secondary users. In Eq. (2.16) and (2.23), we show that the asymptotic behavior of diversity-based binary consensus and average consensus spectrum sensing is independent of network connectivity and depends only on M and $\overline{\gamma}$. Fig. 2.2 and 2.3 also confirm this.

Fig. 2.4 shows the performance of these approaches for a higher level of SNR

($\overline{\gamma} = 6$ dB) and the case of low connectivity. Comparing Fig. 2.2 and 2.4, it can be seen that increasing average SNR to the primary user, which corresponds to more correct initial votes, improves the performance of all the three approaches, with a more considerable impact on the agility of the binary-based consensus approaches.

Overall, the agility of average-based spectrum sensing is not as good as binary-based approaches due to the fact that the average-based approach transmits R bits in each iteration whereas binary-based approaches use only 1 bit per transmission. This results in a slower rate of convergence of the average consensus approach as compared to binary-based schemes, as confirmed by the figures.

It is also interesting to compare the asymptotic behavior of binary-based schemes with that of the average-based approach. From the simulation results, it is seen that the average consensus spectrum sensing performs better than binary consensus asymptotically. This is more likely as M becomes larger and not true for all cases. The reason is that average consensus approach averages the noise embedded in the measurements of the SUs. Thus, as M increases, it is more likely to have negligible asymptotic noise. In the binary consensus approaches, on the other hand, if poor link quality to the PU results in a wrong majority of the initial votes, then lower probability of correct detection can be expected asymptotically. In summary, depending on the size of the network and initial measurement noise of the SUs, average consensus may or may not have a better asymptotic performance than binary-based approaches. As an example, assume an SU network with 3 nodes. Further assume that the initial measurement of one of these nodes is very noisy, compared to the other two SUs. In such a case, average consensus spectrum sensing may converge to a value which yields a wrong decision as the high noise of this particular node can result in a high level of average noise. In binary-based spectrum sensing, however, 2 out of the 3 SUs vote for 1. In this case, the majority becomes 1 which yields the correct decision. This example shows that there are scenarios where binary-based approaches outperform average-based spectrum sensing asymptotically. In general if a large number of SUs, but not the majority of them, have poor assessments on the existence of the PU, then binary consensus-based spectrum sensing can become superior to average consensus asymptotically. Overall, the binary-based approaches improve the agility considerably, which is crucial in cooperative spectrum sensing.

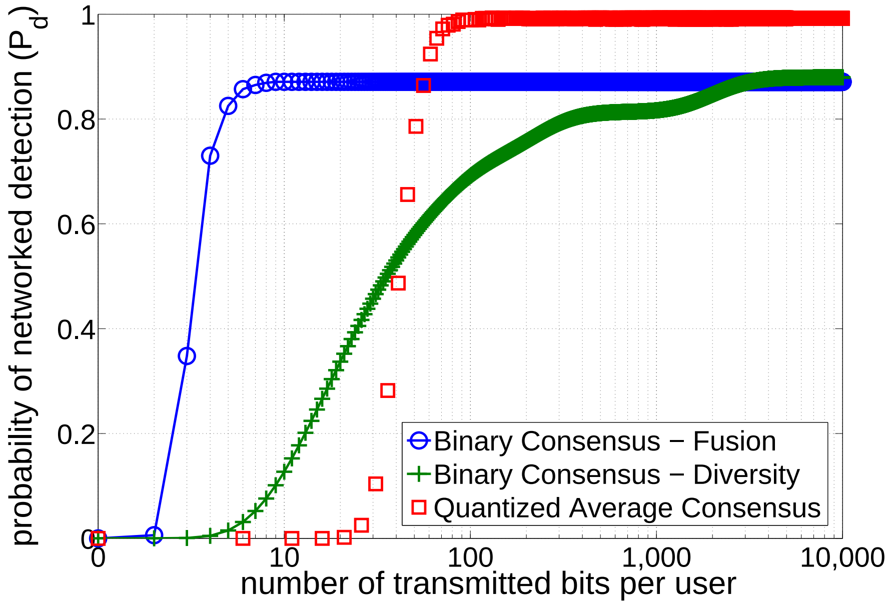

Figure 2.2: Comparison of the probability of networked detection for binary consensus and quantized average consensus schemes, with $M = 51$, $p = 0.2$, $\overline{\gamma} = 2$ dB and $R = 5$ bits.

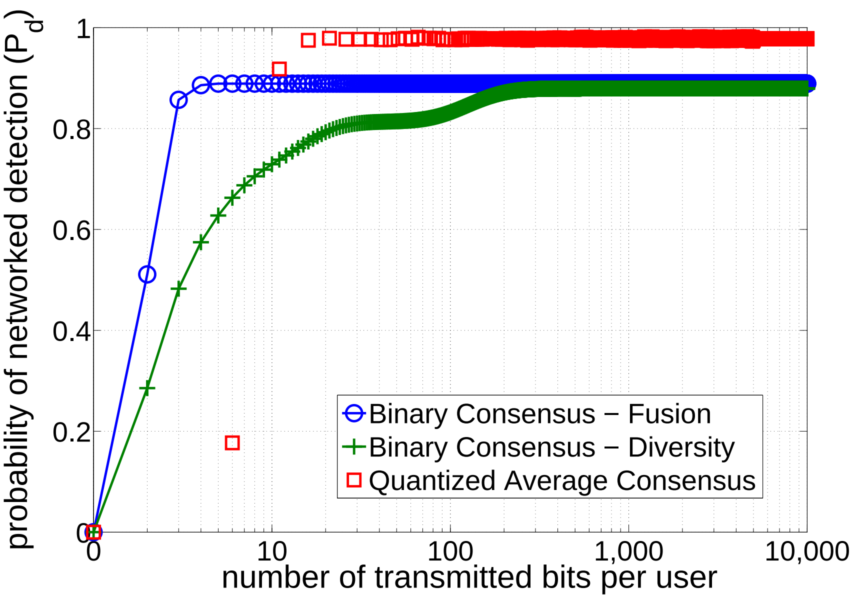

Figure 2.3: Comparison of the probability of networked detection for binary consensus and quantized average consensus schemes, with $M = 51$, $p = 0.8$, $\overline{\gamma} = 2$ dB and $R = 5$ bits.

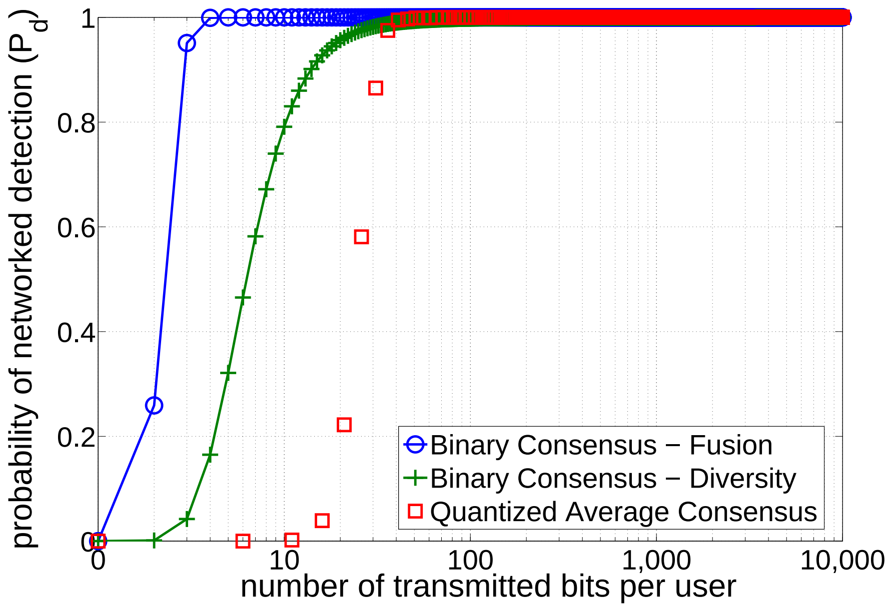

Figure 2.4: Comparison of the probability of networked detection for binary consensus and quantized average consensus schemes, with $M = 51$, $p = 0.2$, $\overline{\gamma} = 6$ dB and $R = 5$ bits.

Chapter 3

Weighted Diversity-based Cooperative Spectrum Sensing

In previous chapters, we assumed that different SUs have the same sensing quality (same $\bar{\gamma}$). Furthermore, we assumed the same probability of connectivity for each link of the network among the SUs. In a realistic scenario, the sensing quality of SUs is not identical due to many factors such as their locations. In such cases, if some information is available on the sensing qualities of the SUs, it is possible to devise better strategies that rely more on the information of SUs with good sensing qualities. On the communication side, different links among the SUs may also have different qualities. Therefore, both sensing and link qualities must be taken into account to facilitate the flow of more accurate information. In the current literature on cooperative spectrum sensing, simple weight assignment approaches are devised for the centralized case. More specifically, in [37], high sensing quality receptions are given more weights in the fusion center. In this chapter, it is our goal to find the optimum weighted strategy for our diversity-based binary distributed cooperative spectrum sensing approach, by taking into account both sensing and communication qualities. Such an optimization is, in particular, suitable for the binary approach since the SU nodes only exchange their binary votes. A binary vote does not carry all the information of the measured signal. Thus, in this chapter, we extend our framework to the case where different nodes have different sensing qualities. Furthermore, we assume that different links between the SUs also have different probability of connectivity. We then find optimum set of weights that each node should utilize to account for the differences in sensing and communication qualities.

3.1 Generalized Diversity-based Approach

We generalize our diversity-based strategy to a scheme where binary votes are given different weights. More specifically, each node designs a set of weights to apply to its incoming votes when fusing them. Our goal is to find the optimum weights and compare the performance with the non-weighted approach. We first modify the formulation of diversity-based spectrum sensing as follows, to account for different weights:

$$\begin{aligned} b_i(k) &= b_i(k-1), \quad k \in \{1, \cdots, K-1\} \\ b_i(K) &= \mathrm{Dec}\left(\left(w_{ii}(K)b_i(0) + \frac{1}{K}\sum_{t=0}^{K-1}\sum_{\substack{j=1 \\ j\neq i}}^{M} \frac{a_{ij}(t)}{p_{ij}} w_{ij}(K) b_j(t)\right)\right), \end{aligned} \tag{3.1}$$

where we assume[1] $b_j(0) \in \{-1, 1\}$ in Eq. (3.1), for all $1 \leq j \leq M$, and $\mathrm{Dec}(z) = \begin{cases} +1 & z \geq 0 \\ -1 & z < 0 \end{cases}$. Furthermore, $w_{ij}(K)$ is the weight that the ith SU assigns to the reception from the jth SU, given that the total given operation steps is K. Note that $w_{ij}(K)$ is not time-varying and does not change during the entire operation (K is an a priori given operation time). Furthermore, we have $w_{ij}(K) \geq 0$ and

[1] Note that we assume that the initial votes are $\{-1, 1\}$ rather than $\{0, 1\}$. This avoids multiplication of the designed weights by 0 votes and is a better choice when optimizing the weights.

$\sum_{j=1}^{M} w_{ij}(K) = 1$. Note that we have changed p to p_{ij} to reflect the fact that different links among the SUs have different probability of connectivity. We then have

$$y_i(K) = w_{ii}(K)b_i(0) + \frac{1}{K}\sum_{t=0}^{K-1}\sum_{\substack{j=1\\ j\neq i}}^{M} \frac{a_{ij}(t)}{p_{ij}} w_{ij}(K)b_j(t). \tag{3.2}$$

Remark 1. *It is easy to confirm that $y_i(K)$ approaches the majority of the weighted initial votes asymptotically.*

$$\lim_{K\to\infty} y_i(K) = \sum_{j=1}^{M} w_{ij}(\infty)b_j(0). \tag{3.3}$$

Lemma 3. *The probability of networked detection for weighted diversity-based cooperative spectrum sensing can be characterized as follows:*

$$P_{d,net}^{(w)}(K) = \prod_{i=1}^{M} Q\left(\frac{-\sum_{j=1}^{M} w_{ij}(K)E[b_j(0)|\mathcal{H}_1]\sqrt{K}}{\sqrt{\sum_{j=1}^{M}(1-E^2[b_j(0)|\mathcal{H}_1])w_{ij}^2(K) + \sum_{j=1}^{M}(\frac{1}{p_{ij}}-1)w_{ij}^2(K)}}\right). \tag{3.4}$$

Proof. By applying the central limit theorem, we approximate the distribution of $y_i(K)$ with a Gaussian distribution. We next characterize the mean and variance of this distribution.

$$E[y_i(K)|\mathcal{H}_1] = \sum_{j=1}^{M} w_{ij}E[b_j(0)|\mathcal{H}_1]. \tag{3.5}$$

We calculate $E[b_j(0)|\mathcal{H}_1]$ as follows,

$$E[b_j(0)|\mathcal{H}_1] = 2\overline{P}_{d_j} - 1, \tag{3.6}$$

where $\overline{P}_{d_j}$ is the initial probability of local detection of the jth SU. Note that $b_j(0) \in \{-1, 1\}$ in this chapter.

Next we derive an expression for the variance of $y_i(K)$. We have,

$$E[y_i^2(K)|\mathcal{H}_1] = E_{\mathbf{b}(0)|\mathcal{H}_1}[E[y_i^2(K)|\mathbf{b}(0)]]$$

$$= E_{\mathbf{b}(0)|\mathcal{H}_1}\Big[w_{ii}^2(K)b_i^2(0) + \frac{2}{K}w_{ii}(K)b_i(0)\sum_{t=0}^{K-1}\sum_{j\neq i}^{M} w_{ij}(K)b_j(0)$$

$$+ \frac{1}{K^2}\Big(\sum_{t_1=0}^{K-1}\sum_{m\neq i}^{M}\frac{a_{im}(t_1)}{p_{im}}w_{im}(K)b_m(0)\Big) \times \Big(\sum_{t_2=0}^{K-1}\sum_{n\neq i}^{M}\frac{a_{in}(t_2)}{p_{in}}w_{in}(K)b_n(0)\Big)\Big]$$

$$= E_{\mathbf{b}(0)|\mathcal{H}_1}\Bigg[w_{ii}^2(K) + \frac{2}{K}w_{ii}(K)b_i(0)\sum_{t=0}^{K-1}\sum_{j\neq i}^{M} w_{ij}(K)b_j(0)$$

$$+ \frac{1}{K^2}\Bigg(\sum_{t=0}^{K-1}\sum_{\substack{m\neq i\\ n\neq i}}^{M}\frac{E[a_{im}(t)a_{in}(t)]}{p_{im}p_{in}}w_{im}(K)w_{in}(K)b_m(0)b_n(0)$$

$$+ K(K-1)\sum_{\substack{m\neq i\\ n\neq i}}^{M} w_{im}(K)w_{in}(K)b_m(0)b_n(0)\Bigg)\Bigg]$$

$$= E_{\mathbf{b}(0)|\mathcal{H}_1}\Bigg[w_{ii}^2(K) + \frac{2}{K}w_{ii}(K)b_i(0)\sum_{t=0}^{K-1}\sum_{j\neq i}^{M} w_{ij}(K)b_j(0)$$

$$+ \frac{1}{K}\Bigg(\sum_{\substack{m,n\neq i\\ m\neq n}}^{M} w_{im}(K)w_{in}(K)b_m(0)b_n(0) + \sum_{n\neq i}^{M}\frac{1}{p_{in}}w_{in}^2(K)$$

$$+ (K-1)\sum_{\substack{m,n\neq i\\ m\neq n}}^{M} w_{im}(K)w_{in}(K)b_m(0)b_n(0) + (K-1)\sum_{n\neq i}^{M} w_{in}^2(K)\Bigg)\Bigg]$$

$$= w_{ii}^2(K) + 2w_{ii}(K)E[b_i(0)|\mathcal{H}_1]\sum_{j\neq i}^{M} w_{ij}(K)E[b_j(0)|\mathcal{H}_1]$$

$$+ \sum_{\substack{m\neq i\\ n\neq i}}^{M} w_{im}(K)w_{in}(K)E[b_m(0)b_n(0)|\mathcal{H}_1] + \frac{1}{K}\sum_{n\neq i}^{M}(\frac{1}{p_{in}} - 1)w_{in}^2(K).$$

We also know that $E[y_i(K)|\mathcal{H}_1] = \sum_{j=1}^{M} w_{ij} E[b_j(0)|\mathcal{H}_1]$. Therefore, we have the following expression for the variance:

$$\begin{aligned}
e_i(K) &= w_{ii}^2(K) + 2w_{ii}(K)E[b_i(0)|\mathcal{H}_1]\sum_{j\neq i}^{M} w_{ij}(K)E[b_j(0)|\mathcal{H}_1] \\
&+ \sum_{\substack{m\neq i\\ n\neq i}}^{M} w_{im}(K)w_{in}(K)E[b_m(0)b_n(0)|\mathcal{H}_1] + \frac{1}{K}\sum_{n\neq i}^{M}(\frac{1}{p_{in}} - 1)w_{in}^2(K) \\
&- \sum_{m,n\neq i}^{M} w_{im}(K)w_{in}(K)E[b_m(0)|\mathcal{H}_1]E[b_n(0)|\mathcal{H}_1] \\
&- 2w_{ii}(K)E[b_i(0)|\mathcal{H}_1]\sum_{j\neq i}^{M} w_{ij}(K)E[b_j(0)|\mathcal{H}_1] - w_{ii}^2(K)E^2[b_i(0)|\mathcal{H}_1].
\end{aligned}$$

It is straightforward to see that

$$e_i(K) = \sum_{j=1}^{M}(1 - E^2[b_j(0)|\mathcal{H}_1])w_{ij}^2(K) + \frac{1}{K}\sum_{j=1}^{M}(\frac{1}{p_{ij}} - 1)w_{ij}^2(K). \tag{3.7}$$

which completes the proof. □

Lemma 3 shows how each SU can optimize its weights in a distributed manner that only requires its local knowledge. More specifically, the ith SU should solve the following optimization problem by noting that $\overline{P}_{d_j} > 0.5$:

$$\begin{aligned}
\max_{w_{i1}(K),\cdots,w_{iM}(K)} \quad & \frac{\sum_{j=1}^{M} w_{ij}(K)(2\overline{P}_{d_j} - 1)}{\sqrt{\sum_{j=1}^{M}(1 - E^2[b_j(0)|\mathcal{H}_1])w_{ij}^2(K) + \sum_{j=1}^{M}(\frac{1}{p_{ij}} - 1)w_{ij}^2(K)}}, \\
\text{s.t.} \quad & \sum_{j=1}^{M} w_{ij}(K) = 1, \qquad \forall\, 1 \le i \le M, \\
& w_{ij}(K) \ge 0, \qquad \forall\, 1 \le i, j \le M.
\end{aligned} \tag{3.8}$$

While the optimization problem of (3.8) can be solved numerically, we can not find a closed-form expression for the weights. We next show how to find a closed-form expression through a sub-optimum strategy. The derived expression would bring more insight on the impact of sensing and communication qualities, as well as other system parameters, on the optimum weights. Based on Eq. (3.4), we have the following lower bound,

$$P_{d,\text{net}}^{(w)}(K) = \prod_{i=1}^{M} Q\left(\frac{-\sum_{j=1}^{M} w_{ij}(K)E[b_j(0)|\mathcal{H}_1]\sqrt{K}}{\sqrt{\sum_{j=1}^{M}(1-E^2[b_j(0)|\mathcal{H}_1])w_{ij}^2(K)+\sum_{j=1}^{M}(\frac{1}{p_{ij}}-1)w_{ij}^2(K)}}\right)$$
$$\geq \prod_{i=1}^{M} Q\left(\frac{-(2\min(\overline{P}_{d_j})-1)\sqrt{K}}{\sqrt{\sum_{j=1}^{M}(1-E^2[b_j(0)|\mathcal{H}_1])w_{ij}^2(K)+\sum_{j=1}^{M}(\frac{1}{p_{ij}}-1)w_{ij}^2(K)}}\right). \tag{3.9}$$

Thus, the minimization of $e_i(K)$ variable will maximize the lower bound (it will minimize the denominator of the Q function). We then have the following optimization problem for the ith SU:

$$\min_{w_{i1}(K),\cdots,w_{iM}(K)} e_i(K), \tag{3.10}$$
$$\text{s.t.} \quad \sum_{j=1}^{M} w_{ij}(K) = 1, \qquad \forall\, 1 \leq i \leq M,$$
$$w_{ij}(K) \geq 0, \qquad \forall\, 1 \leq i, j \leq M.$$

It can be easily confirmed that this optimization problem is convex. We can then confirm the following optimum weights by writing the KKT conditions:

$$w_{ij}(K) = \frac{l(K)/2}{(1-E^2[b_j(0)|\mathcal{H}_1])+\frac{1}{K}(\frac{1}{p_{ij}}-1)}, \quad \forall\ 1 \leq i, j \leq M, \tag{3.11}$$

where $l(K) = \frac{2}{\sum_{j=1}^{M}\frac{1}{(1-E^2[b_j(0)|\mathcal{H}_1])+\frac{1}{K}(\frac{1}{p_{ij}}-1)}}$. From Eq. (3.11) it can be seen that the optimum weights are a function of K (given operation time), link existence probabilities and the qualities of sensing, i.e., local detection probability, as expected. It should be noted that the ith SU needs to assess the local sensing qualities of its neighbors ($\overline{P}_{d_j}$ for $j \neq i$) as well as the probability of connectivity of the corresponding links (p_{ij}) in order to find the weights. It can estimate p_{ij}s based on its reception qualities from the neighboring nodes. As for the local sensing qualities, the neighbors can send an assessment of their local sensing qualities ($\overline{P}_{d_j}$ for $j \neq i$) to the ith node at the beginning of the operation. Since this is a one-time communication, it should not increase the number of transmitted bits considerably if the operation time is large enough.

It is interesting to see the asymptotic behavior of the weights as $K \to \infty$. We have the following set of weights for the ith SU asymptotically:

$$w_{ij}(\infty) = \frac{1/\sum_{j=1}^{M}(1 - E^2[b_j(0)|\mathcal{H}_1])^{-1}}{1 - E^2[b_j(0)|\mathcal{H}_1]}, \qquad \forall \ 1 \leq i \leq M. \tag{3.12}$$

As expected, the weights are only a function of the quality of sensing asymptotically.

Fig. 3.1 illustrates the performance of the weighted diversity-based cooperative spectrum sensing as compared with the non-weighted approach of Chapter 2. It can be seen that the probability of networked detection increases by optimizing the weights. In general, the weighted approach can improve the performance more considerably if the difference of local sensing qualities is more drastic. On the other hand, the link existence probability among the SUs only affects the convergence rate to the asymptotic value.

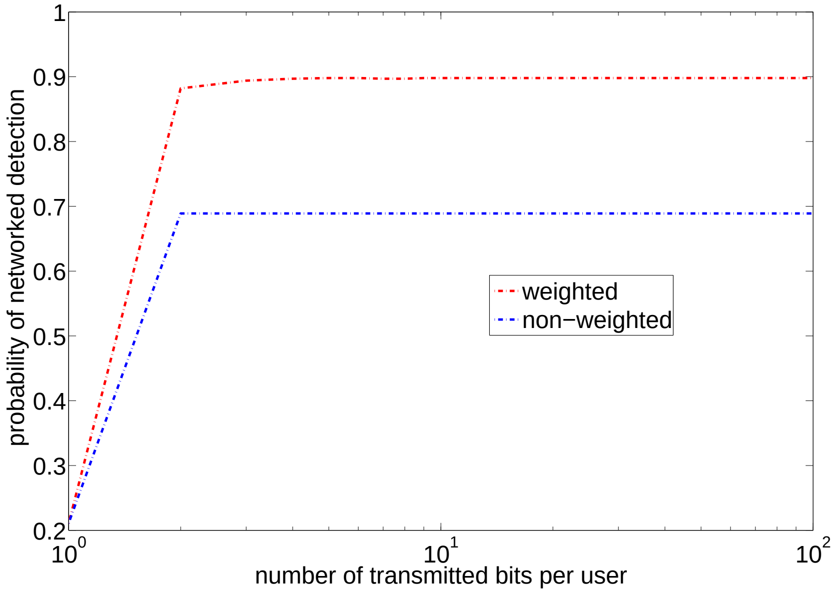

Figure 3.1: probability of networked detection for both weighted and non-weighted diversity strategies. In this case, $M = 3$, and the local probabilities of detection are $\{0.9, 0.5, 0.5\}$.

Chapter 4

Impact of Network configuration on Correlated Measurements and Connectivity

In the previous chapters, we assumed that the local sensing of the SUs are uncorrelated. Depending on the distance among the SU nodes, this may or may not be the case. More specifically, the measurements of SUs can become correlated due to the spatial correlation of the shadow fading components of the channels from the PU to the SUs, which can not be ignored at smaller distances.

In [38], basic limits on the performance of spectrum sensing with energy detectors is studied. In [39–42], node selection and node clustering approaches are proposed to get around the degraded performance due to shadowing and correlated measurements. In this chapter, we explore the impact of shadowing correlation on our distributed diversity-based cooperative spectrum sensing. It has been established in the wireless communication literature that an exponential distribution can best characterize the spatial correlation of shadow fading in the dB domain [43]. In this chapter, we explore the impact of this spatial correlation on our cooperative spectrum sensing.

Consider a group of SUs cooperating, based on the diversity-based cooperative spectrum sensing algorithm of Chapter 2. In this part, we assume the unweighted approach, which is as follows:

$$\mathbf{b}(k) = \mathbf{b}(k-1), \quad k \in \{1, \cdots, K-1\}$$
$$\mathbf{b}(K) = \mathrm{Dec}\left(\frac{1}{M}\underbrace{\left(\mathbf{b}(0) + \frac{1}{Kp}\sum_{t=0}^{K-1}\mathbf{A}(t)\mathbf{b}(t)\right)}_{\mathbf{Y}(K)}\right), \quad K \geq 0. \tag{4.1}$$

We also assume that $b_i(0) \in \{0, 1\}$. Furthermore, the channels among the SUs are taken to be independent and identically distributed, with the probability of connectivity of p for each link. This assumption assists us to focus on the impact of correlated sensing. We further assume that the channels from the PU to the cooperative network experience path loss and shadowing as we further explain later in this chapter. We use Eq. (2.2) to generate the output of the energy detectors in the cooperative network.

In this chapter, we are interested in the impact of correlated sensing on the overall cooperative network performance. The local measurements of the SUs (of the PU channel) become correlated as they get spatially closer to each other. Thus, we are interested in exploring the impact of the distance among the SU nodes on the overall performance. If the network was fully connected, then it would have been best for the SUs to spread out in their given area and get as far as possible (as long as they can maintain the same individual sensing quality to the PU), in order to benefit from the spatial diversity resulted by independent uncorrelated measurements of the PU channel. In the context of a centralized cooperative spectrum sensing, [44] also confirms that the probability of networked detection, under correlated measurement assumption, decreases as compared to the case where the measurements are uncorrelated.

As the SU nodes get farther from each other, however, the probability that a link between two SUs gets disconnected becomes higher. Thus, there are interesting underlying tradeoffs regarding how far the SU nodes should get from each other. If the SU nodes are too close, they can not benefit from the cooperation as their sensor measurements would become correlated. On the other hand, as they get farther from each other, the chance of experiencing disconnected links between SU nodes increases, which degrades the overall cooperation performance. Therefore, there should exist an optimum average distance between the SUs in order to benefit from the spatial diversity for local sensing while maintaining proper connectivity.

Remark 2. *As mentioned earlier, we assume that as the SU nodes get farther from each other, the local individual sensing of each node is not impacted. This, however, can only be the case if we consider the expansion of the nodes over a small enough area. Thus, such an underlying implicit assumption should be kept in mind in our discussions in this chapter. For instance, in the hypothetical case that the distance between the nodes goes to infinity, while the local measurements will become uncorrelated, the individual sensing will most likely degrade drastically for most geometric configurations. Thus, our discussions assume that the expansion of the SU network occurs over a small enough area such that the local individual sensing qualities are not impacted.*

Remark 3. *As mentioned earlier, we assume that a link between any pair of SUs has a probability of connectivity of p. While we do not carry the explicit variable dependency in the rest of this chapter, we take this probability to be a function of the distance between the corresponding nodes. Thus, as the network expands, this probability is impacted. Furthermore, we assume uncorrelated probability of connectivity for different links between the SUs. In practice, this will not be the case. As the SU network shrinks and the distance between the SU nodes become smaller, the* *probability that the corresponding pair-wise links become correlated increases. While this impacts our mathematical derivations, it should not impact the conclusions of this chapter. Alternatively, the i.i.d assumption can be justified by assuming that the SUs communicate over different frequency bands while trying to reach consensus.*

As we discuss the impact of the expansion of the network, we need a proper metric to characterize the impact on the distance between the nodes. We define this metric as the average distance between pairs of SUs, i.e., $\bar{d} = \frac{1}{M(M-1)} \sum_{i,j=1}^{M} d_{ij}$. As the distance between two SUs changes, this impacts the path loss component of that corresponding channel, which is the average of the distribution of the variable that characterizes that link. We can take all the three dynamics of path loss, shadowing and multipath to characterize the links between the SUs. Alternatively, we can only consider shadowing and path loss or multipath and path loss to follow the discussions of this chapter.

In general, it is challenging to mathematically investigate the problem of optimum positioning and the corresponding tradeoffs for a general placement of the SUs. In this chapter, we start with simple mathematical analysis and proceed to discuss the underlying tradeoffs through simulations. Our discussions of this chapter can then serve as a good starting point for future work in this area. The correlation of the shadowing component of the channels from the PU to the ith and jth SUs is $\xi^2 e^{-\frac{d_{ij}}{d_0}}$ where d_{ij} is the distance between the ith and jth SUs and d_0, ξ^2 are the correlation distance and power of shadowing process respectively.

Next, we characterize the links from the PU to each SU to better highlight the impact of correlated measurements. Let $\Upsilon_{m,n} = [\gamma_{\mathrm{dB},m}, \gamma_{\mathrm{dB},n}]^T$, where $\gamma_{\mathrm{dB},i} = 10\log(\gamma_i)$ represents the received SNR of the ith SU in sensing of the PU signal, when located at position q_i. Furthermore, q_b is the position of the primary user. We can characterize $\gamma_{\mathrm{dB},i}$ by a 2D non-stationary random field with the following form [45]: $\gamma_{\mathrm{dB},i} = O_{\mathrm{dB}} - 10\tau 10\log_{10}(\|q_i - q_b\|) + \gamma_{\mathrm{dB,SH},i} + \gamma_{\mathrm{dB,MP},i}$. The distance-dependent path loss has a linear decay in the dB domain. Then, O_{dB} and -10τ represent its offset and slope respectively. Furthermore, $\gamma_{\mathrm{dB,SH},i}$ and $\gamma_{\mathrm{dB,MP},i}$ are independent random variables, representing the effects of shadowing and multipath in the dB domain respectively. Since multipath fading decorrelates very fast, we take $\gamma_{\mathrm{dB,MP},i}$ to be spatially uncorrelated. We furthermore take each $\gamma_{\mathrm{dB,MP},i}$ to be a zero-mean Gaussian random variable with the variance of $\sigma^2_{\gamma_{\mathrm{dB,MP}}}$.[1] Thus, we assume a Gaussian distribution for $\gamma_{\mathrm{dB,MP},i}$ to facilitate mathematical derivations. As for the shadowing variables, a Gaussian distribution with an exponential spatial correlation has been shown to best characterize the distribution of $\gamma_{\mathrm{dB,SH},i}$ [46]. Thus, that is the distribution we will utilize in this chapter. Define the following variables:

$$H_{m,n} \triangleq \begin{bmatrix} 1 & -10\log_{10}(\|q_m - q_b\|) \\ 1 & -10\log_{10}(\|q_n - q_b\|) \end{bmatrix}, \theta \triangleq [O_{\mathrm{dB}}, \tau]^T, \text{ and} \tag{4.2}$$

$$R_{m,n} \triangleq \begin{bmatrix} \xi^2 & \xi^2 e^{-\frac{\|q_m - q_n\|}{d_0}} \\ \xi^2 e^{-\frac{\|q_m - q_n\|}{d_0}} & \xi^2 \end{bmatrix} \tag{4.3}$$

The distribution of the SNR (in the dB domain) of a pair of SU measurements is then best characterized by a bivariate Gaussian distribution. Let $f_{\Upsilon_{m,n}}(\Upsilon_{m,n})$ represent the joint PDF of $\gamma_{\mathrm{dB},m}$ and $\gamma_{\mathrm{dB},n}$ for the mth and nth SUs. We then have

$$f_{\Upsilon_{m,n}}(\Upsilon_{m,n}) = \frac{1}{2\pi\Big(\det\big[R_{m,n} + \sigma^2_{\gamma_{\mathrm{dB,MP}}} I_2\big]\Big)^{1/2}} \times e^{-\frac{1}{2}\left(\Upsilon_{m,n} - H_{m,n}\theta\right)^T \left(R_{m,n} + \sigma^2_{\gamma_{\mathrm{dB,MP}}} I_2\right)^{-1} \left(\Upsilon_{m,n} - H_{m,n}\theta\right)}. \tag{4.4}$$

As mentioned in Chapter 2, the distribution of the output of an energy detector is best characterized by a non-central chi square. From Eq. (2.8), we have

$$P_{d_i} = \mathrm{prob}(x_i(0) > \eta | \mathcal{H}_1, \gamma_i) = Q_{TB}(\sqrt{2\gamma_i}, \sqrt{\eta}), \tag{4.5}$$

[1]Note that while Nakagami or exponential are shown to better match the distribution of $\gamma_{\mathrm{MP},i}$ in the non-dB domain, log-normal has been shown to provide a reasonable fit [46].

where $Q_{TB}(\cdot,\cdot)$ is the generalized Marcum Q-function. We then have

$$\pi_{11} = \text{prob}(x_i(0) > \eta|\mathcal{H}_1) = \int_{-\infty}^{\infty} Q_{TB}(\sqrt{2 \times 10^{0.1\gamma_{\text{dB},i}}}, \sqrt{\eta}) f_{\gamma_{\text{dB},i}}(\gamma_{\text{dB},i}) d\gamma_{\text{dB},i}, \tag{4.6}$$

where $f_{\gamma_{\text{dB},i}}(\gamma_{\text{dB},i})$ is the marginal distribution of $\gamma_{\text{dB},i}$ (SNR at the ith SU in the dB domain). Let $r_{mn} = E[b_m(0)b_n(0)|\mathcal{H}_1]$ denote the correlation between the mth and nth SUs, we can write

$$\begin{aligned} r_{mn} &= E[b_m(0)b_n(0)|\mathcal{H}_1] = \text{prob}(x_m(0) > \eta, x_m(0) > \eta|\mathcal{H}_1) \\ &= \int_{-\infty}^{\infty} Q_{TB}(\sqrt{2 \times 10^{0.1\gamma_{\text{dB},m}}}, \sqrt{\eta}) Q_{TB}(\sqrt{2 \times 10^{0.1\gamma_{\text{dB},n}}}, \sqrt{\eta}) f_{\Upsilon_{m,n}}(\Upsilon_{m,n}) d\gamma_{\text{dB},m} d\gamma_{\text{dB},n}. \end{aligned}$$

As can be seen, as the SU nodes m and n get closer to each other, their measurements become more correlated, characterized by the exponential correlation term $\xi^2 e^{-\frac{d_{ij}}{d_0}}$, which impacts the correlation of their corresponding binary votes and the resulting networked detection process.

Lemma 4. *Assume a cooperative network of M (odd) secondary users communicating using diversity-based consensus cooperative spectrum sensing. Then, we can characterize the mean and variance of $\mathbf{Y}(K)$ as follows, under $\mathcal{H}_1$: $E[\mathbf{Y}(K)|\mathcal{H}_1] = M\pi\mathbf{1}$ and $C(K) = E\Big[\big(\mathbf{Y}(K) - E[\mathbf{Y}(K)]\big)\big(\mathbf{Y}(K) - E[\mathbf{Y}(K)]\big)^T|\mathcal{H}_1\Big] = \big(\frac{(M-1)\pi}{K}\frac{1-p}{p}\big)\mathbf{I} + \big(M\pi - M^2\pi^2 + \sum_{\substack{m,n=1 \\ m\neq n}}^{M} r_{mn}\big)\mathbf{1}\mathbf{1}^T$, where $\pi_{11} = \pi$ of Eq. (4.6) and $\mathbf{Y}(K)$ is as marked in Eq. (4.1).*

Proof. Consider $\mathbf{Y}(K) = \mathbf{b}(0) + \frac{1}{Kp}\sum_{t=0}^{K-1}\mathbf{A}(t)\mathbf{b}(t)$ of Eq. (4.1). It can be easily confirmed that

$$E[\mathbf{Y}(K)|\mathcal{H}_1] = ME[b_i(0)|\mathcal{H}_1]\mathbf{1} = M\text{prob}(x_i(0) > \eta|\mathcal{H}_1)\mathbf{1} = M\pi\mathbf{1}, \tag{4.7}$$

where $S(0)$ is the sum of the initial votes. Note that averaging is done over the distribution of the $a_{ij}(t)$s. We next show that $E[y_i^2(K)|\mathbf{b}(0)] = S^2(0) + \frac{S(0)-b_i(0)}{K}\frac{1-p}{p}$:

$$\begin{aligned}
E[y_i^2(K)|\mathbf{b}(0)] &= b_i(0) + 2b_i(0)(S(0) - b_i(0)) \\
&\quad + \frac{1}{K^2p^2}\Big[\sum_{t=0}^{K-1}\sum_{n,m\neq i}^{M} E[a_{in}(t)a_{im}(t)]b_n(0)b_m(0) \\
&\quad + \sum_{\substack{t1,t_2=0\\ t_1\neq t_2}}^{K-1}\sum_{n,m\neq i}^{M} E[a_{in}(t_1)a_{im}(t_2)]b_n(0)b_m(0)\Big] \\
&= b_i(0) + 2b_i(0)(S(0) - b_i(0)) \\
&\quad + \frac{1}{Kp^2}\Big[\sum_{n,m\neq i}^{M} p^2 b_n(0)b_m(0) + \sum_{n\neq i}^{M} p(1-p)b_n(0) \\
&\quad + (K-1)p^2\sum_{n,m\neq i}^{M} b_n(0)b_m(0)\Big] \\
&= b_i(0) + 2b_i(0)(S(0) - b_i(0)) + \frac{1}{Kp^2}[p(1-p)(S(0) - b_i(0)) \\
&\quad + Kp^2(S(0) - b_i(0))^2] \\
&= S^2(0) + \frac{S(0) - b_i(0)}{K}\frac{1-p}{p}. \qquad (4.8)
\end{aligned}$$

We also calculate $E[y_i(K)y_j(K)|\mathbf{b}(0)]$ for $i \neq j$. Note that conditioning on $\mathbf{b}(0)$, $y_i(K)$ and $y_j(K)$ are independent, for $i \neq j$, due to the i.i.d link assumption among the SUs. We then have

$$E[y_i(K)y_j(K)|\mathbf{b}(0)] = E[y_i(K)|\mathbf{b}(0)]E[y_j(K)|\mathbf{b}(0)] = S^2(0). \qquad (4.9)$$

Let $C(K) = E\Big[\big(\mathbf{Y}(K) - E[\mathbf{Y}(K)]\big)\big(\mathbf{Y}(K) - E[\mathbf{Y}(K)]\big)^T|\mathcal{H}_1\Big]$ denote the covariance matrix of vector $\mathbf{Y}(K)$ under $\mathcal{H}_1$. Therefore, we have

$$\begin{aligned}
E[y_i^2(K)|\mathcal{H}_1] &= E_{\mathbf{b}(0)|\mathcal{H}_1}\left[E[y_i^2(K)|\mathbf{b}(0)]\right] \\
&= E[S^2(0)|\mathcal{H}_1] + \frac{E[S(0)|\mathcal{H}_1] - E[b_i(0)|\mathcal{H}_1]}{K}\frac{1-p}{p} \\
&= \sum_{m=1}^{M} E[b_m(0)|\mathcal{H}_1] + \sum_{\substack{m,n=1\\m\neq n}}^{M} E[b_m(0)b_n(0)|\mathcal{H}_1] + \frac{(M-1)\pi}{K}\frac{1-p}{p},
\end{aligned} \tag{4.10}$$

and the following for $i \neq j$,

$$\begin{aligned}
E[y_i(K)y_j(K)|\mathcal{H}_1] &= E_{\mathbf{b}(0)|\mathcal{H}_1}\left[E[y_i(K)y_j(K)|\mathbf{b}(0)]\right] = E[S^2(0)|\mathcal{H}_1] \\
&= \sum_{m=1}^{M} E[b_m(0)|\mathcal{H}_1] + \sum_{\substack{m,n=1\\m\neq n}}^{M} E[b_m(0)b_n(0)|\mathcal{H}_1].
\end{aligned} \tag{4.11}$$

Therefore,

$$\begin{aligned}
[C(K)]_{ii} &= M\pi - M^2\pi^2 + \sum_{\substack{m,n=1\\m\neq n}}^{M} r_{mn} + \frac{(M-1)\pi}{K}\frac{1-p}{p}, \\
[C(K)]_{ij} &= M\pi - M^2\pi^2 + \sum_{\substack{m,n=1\\m\neq n}}^{M} r_{mn} \quad \text{for } i \neq j.
\end{aligned} \tag{4.12}$$

□

Remark 4. *Consider* $\mathbf{Y}(K) = \mathbf{b}(0) + \frac{1}{Kp}\sum_{t=0}^{K-1}\mathbf{A}(t)\mathbf{b}(t)$. *If we evoke the central limit theorem, then the distribution of* $\mathbf{Y}(K)$ *can be approximated by the following Gaussian distribution:* $\mathbf{Y}(K)|\mathcal{H}_1 \sim \mathcal{N}(M\pi\mathbf{1}, C(K))$ *based on the mean and covariance that are found in Lemma 4. Note that the accuracy of this CLT approximation would depend on the level of the correlation between the local measurements of the SUs.*

We next show how a more simplified expression for the probability of networked detection can be derived for this case, by separating the correlated and uncorrelated parts of $\mathbf{Y}(K)$ and utilizing the distribution of Remark 4.

Lemma 5. *Consider* $\mathbf{Y}(K) = \mathbf{b}(0) + \frac{1}{Kp}\sum_{t=0}^{K-1}\mathbf{A}(t)\mathbf{b}(t)$. $\mathbf{Y}(K)$ *can be decomposed as* $\mathbf{Y}(K) = U(K) + v\mathbf{1}$, *where* $v = S(0)$ *and* $[U(K)]_i = \sum_{\substack{j=1\\j\neq i}}^{M}\Big(\frac{1}{Kp}\sum_{t=0}^{K-1} a_{ij}(t)b_j(t) - b_j(t)\Big)$, *for* $1 \leq i \leq M$. *For sufficiently large* M *and by evoking the CLT (see Remark 4), we have* $U(K)|\mathcal{H}_1 \sim \mathcal{N}(\mathbf{0}_M, \sigma^2_{U(K)}\mathbf{I}_M)$ *and* $v|\mathcal{H}_1 \sim \mathcal{N}(M\pi, \sigma_v^2)$, *where* $\sigma^2_{U(K)} = \frac{(M-1)\pi}{K}\frac{1-p}{p}$, $\sigma_v^2 = M\pi - M^2\pi^2 + \sum_{\substack{m,n=1\\m\neq n}}^{M} r_{mn}$ *and* $\mathbf{0}_M$ *is a zero vector of length* M.

Proof. We can write

$$y_i(K) = S(0) + \sum_{\substack{j=1\\j\neq i}}^{M}\Big(\frac{1}{Kp}\sum_{t=0}^{K-1} a_{ij}(t)b_j(t) - b_j(t)\Big). \tag{4.13}$$

It can be easily confirmed that $\sigma_v^2 = M\pi - M^2\pi^2 + \sum_{\substack{m,n=1\\m\neq n}}^{M} r_{mn}$. Furthermore, $E\big[[U(K)]_i|\mathcal{H}_1\big] = 0$ and $[U(K)]_i$ and $[U(K)]_j$ are uncorrelated for $i \neq j$. We also have

$$\begin{aligned}
E\Big[\big([U(K)]_i\big)^2|\mathcal{H}_1\Big] &= E\Big[\Big(\sum_{\substack{j=1\\j\neq i}}^{M}\Big(\frac{1}{Kp}\sum_{t=0}^{K-1} a_{ij}(t)b_j(t) - b_j(t)\Big)\Big)^2|\mathcal{H}_1\Big]\\
&= \sum_{\substack{j=1\\j\neq i}}^{M} E\Big[\Big(\frac{1}{Kp}\sum_{t=0}^{K-1} a_{ij}(t)b_j(t) - b_j(t)\Big)^2|\mathcal{H}_1\Big]\\
&= \sum_{\substack{j=1\\j\neq i}}^{M} E\Big[\Big(\frac{1}{Kp}\sum_{t=0}^{K-1} a_{ij}(t) - 1\Big)^2 b_j(0)|\mathcal{H}_1\Big]\\
&= \frac{(M-1)\pi}{K}\frac{1-p}{p}.
\end{aligned} \tag{4.14}$$

This completes the proof. □

The probability of networked detection for the diversity strategy with correlated measurements can be then written as

$$\begin{aligned} P_{d,\text{net}}(K) &= \text{prob}(y_1(K) > \frac{M}{2}, \cdots, y_M(K) > \frac{M}{2}|\mathcal{H}_1) \\ &= \int_{-\infty}^{\infty} \text{prob}(y_1(K) > \frac{M}{2}, \cdots, y_M(K) > \frac{M}{2}|v) f_{v|\mathcal{H}_1}(v|\mathcal{H}_1) dv \\ &= \int_{-\infty}^{\infty} Q^M\left(\frac{\frac{M}{2} - M\pi - v}{\sigma_{U(K)}}\right) f_{v|\mathcal{H}_1}(v|\mathcal{H}_1) dv, \end{aligned} \tag{4.15}$$

where $f_{v|\mathcal{H}_1}(v|\mathcal{H}_1)$ is the PDF of v given $\mathcal{H}_1$. Note that the probability of networked detection for correlated measurements is a function of $\bar{d}$ and the correlation comes into the picture from this dependency. The expression of Eq. (4.15) is more simplified as the impact of correlation is represented by one scalar variable v.

As the SU nodes get farther from each other ($\bar{d}$ increases), the local measurements become more uncorrelated, resulting in a better cooperative performance if the connectivity of the graph of SUs would remain the same (given that the movement of SUs is limited to a small enough area such that individual sensing is not affected as explained in Remark 2). However, as $\bar{d}$ increases, the connectivity of SUs is affected as p becomes smaller with a high probability. Thus, one would expect an optimum $\bar{d}$ that would result in the best tradeoff between sensing and communication.

Fig. 4.1 and 4.2 show the impact of the average network distance ($\bar{d}$) on the probability of networked detection for two different local sensing qualities (different π_{11}). For both figures, we take the distance between any two SU nodes to be the same as $\bar{d}$. The sensing channel from the PU to an SU is taken to be log-normally distributed with an exponential spatial correlation and a mean that is dictated by the path loss component (only shadowing and path loss are considered for these figures). We assume that the movements of SUs are limited to a small enough area such that the path loss component of the channels to the PU does not change (see Remark 2). The channel between any two SUs experiences exponential multipath fading, with an average that is characterized by the path loss term, which becomes the function of the distance between the two nodes. Furthermore, the channel from the PU to each SU experiences only path loss and log-normal shadowing with exponential spatial correlation. A hypothetical curve for the case of uncorrelated measurements is also plotted. In this case, as $\bar{d}$ changes, the local sensing of the nodes are kept uncorrelated. Clearly, in such a case, it is the best for the SU nodes to be as close as possible, which results in the optimum $\bar{d} = 0$. However, this is not realistic since the local measurements become more and more correlated as $\bar{d}$ decreases. The figure shows that the performance is degraded initially due to the correlation of the shadow fading components of the individual local sensings. As the SUs get farther from each other, the measurements become more uncorrelated and the probability of networked detection increases up to a certain point. At high $\bar{d}$, the connectivity among the SU nodes is impacted to the point that it is not beneficial to increase $\bar{d}$ any further. This suggests an optimum average network distance as can be seen from the figures.

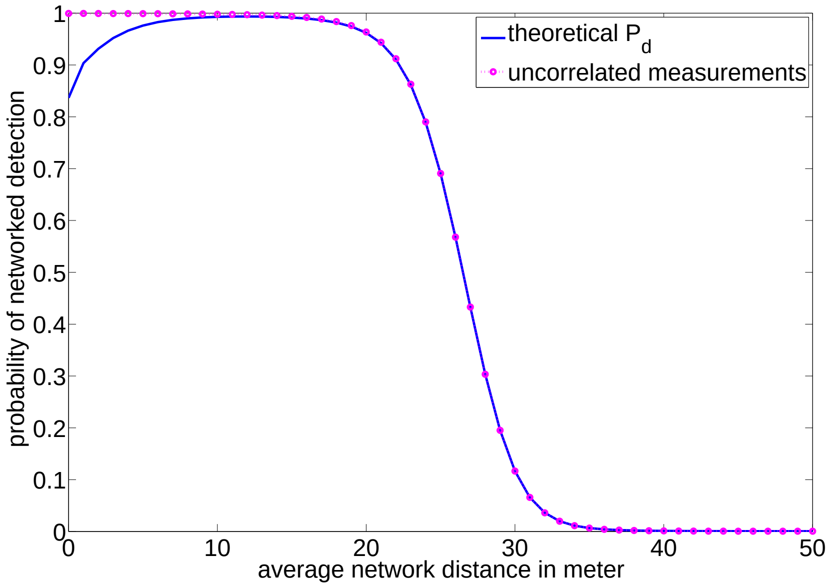

Figure 4.1: probability of networked detection for diversity strategy, $\pi = 0.85$, $M = 11$, $K = 100$.

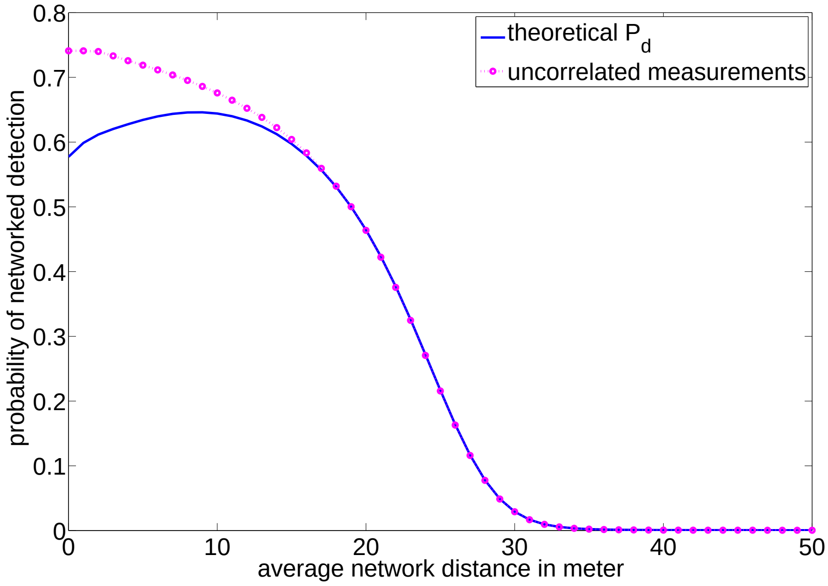

Figure 4.2: probability of networked detection for diversity strategy, $\pi = 0.6$, $M = 11$, $K = 100$.

Chapter 5

Conclusion

We proposed to use binary consensus algorithms for distributed cooperative spectrum sensing in cognitive radio networks. We proposed to use two binary approaches, namely diversity and fusion binary consensus spectrum sensing. The performance of these algorithms was analyzed over fading channels. The probability of networked detection and false alarm were characterized for the diversity case. We then compared the performance of our binary-based cooperative spectrum sensing framework to that of the already-existing averaged-based one. We showed that binary consensus cooperative spectrum sensing is superior to quantized average consensus in terms of agility, given the same number of transmitted bits. We furthermore derived a lower bound for the performance of the average consensus-based spectrum sensing.

We then extended our diversity-based framework to propose a weighted approach in which each secondary user utilizes a set of weights to account for different local sensing qualities of its neighbors as well as different communication link qualities from them. We mathematically characterized the optimum weights.

Finally, the impact of network configuration (in terms of average distance between the secondary users) and the resulting correlated measurements (due to shadow fading) were considered on the overall networked detection performance. More specifically, we considered the impact of the average distance on both the correlation of the sensing measurements of the secondary users and the connectivity of the underlying graph among them. We discussed interesting underlying tradeoffs when increasing or decreasing the average distance.

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