

2-13-2014

Add+VantageMR® Assessments: A Case Study of Teacher and Student Gains

Cathy Briand

Follow this and additional works at: https://digitalrepository.unm.edu/educ_llss_etds

Recommended Citation

Briand, Cathy. "Add+VantageMR® Assessments: A Case Study of Teacher and Student Gains." (2014).
https://digitalrepository.unm.edu/educ_llss_etds/5

This Dissertation is brought to you for free and open access by the Education ETDs at UNM Digital Repository. It has been accepted for inclusion in Language, Literacy, and Sociocultural Studies ETDs by an authorized administrator of UNM Digital Repository. For more information, please contact disc@unm.edu.

Cathy Briand

Candidate

Multicultural Teacher and Childhood Education

Department

This dissertation is approved, and it is acceptable in quality and form for publication:

Approved by the Dissertation Committee:

Richard Kitchen, Chairperson

Anne Madsen, Acting Chairperson

Jonathan Brinkerhoff

Kristin Umland

**Add+VantageMR® Assessments:
A Case Study of Teacher and Student Gains**

By

Cathy Briand

B.S., Mathematics/Secondary Education, University of Hartford, CT, 1973
M.A., CIMTE/Mathematics, University of New Mexico, 1988

DISSERTATION

Submitted in Partial Fulfillment of the
Requirements for the Degree of

**Doctor of Philosophy
Multicultural Teacher and Childhood Education**

The University of New Mexico
Albuquerque, New Mexico

December, 2013

Dedication

I dedicate this project to the people who mean the most to me: my mother, Helen, she was a wonderful mother, who gave generously of herself to all, and was both my mother and my best friend; my grandfather, Demetro Holowach, who grounded me through his wonderful stories of the old country, and made my childhood special; my daughter, Sarah Gosler, who has made me proud of all she has accomplished so early in her career, and for her continued love and support; and my good friend Rudy Sandoval, who had the most generous, giving heart, and was always there to help those in need, he brought out the best in all that were fortunate to know him.

A special dedication to my husband, Dan Briand. He is the best part of each day; has been my soul mate, inspiration, supportive friend, source of sound advice and constant encouragement, and best of all, he makes me laugh every day, a special gift in my life.

Acknowledgements

I would like to thank Dr. Richard Kitchen, my dissertation advisor, and Kennedy Endowed Chair in Urban Education, Denver University, for his support, guidance, encouragement, and friendship throughout this process. I am deeply grateful, and appreciate all I have learned from his example. I would also like to thank my committee members, Dr. Anne Madsen, for stepping in to become my official chair, and for sharing her expertise in mathematics education, as well as her organizational support in writing this dissertation; Dr. Jonathan Brinkerhoff for his expertise in writing, his extensive knowledge regarding APA format, and his willingness to painstakingly review my writing; and Dr. Kristin Umland for sharing her knowledge of current reforms in mathematics education and her expertise as a researcher. I appreciate all their efforts to assist me throughout this process.

Add+VantageMR® Assessments:
A Case Study of Teacher and Student Gains

BY

CATHY BRIAND

B.S., Mathematics/Secondary Education, University of Hartford, CT, 1973
M.A., CIMTE/Mathematics, University of New Mexico, 1988
Ph.D., Multicultural Teacher and Childhood Education, University of New Mexico, 2013

ABSTRACT

This case study analyzes the effect of the Add+VantageMR® (AVMR) program on a teacher's pedagogy and on her students' progress in mathematics. AVMR, a professional development program in early mathematics, trains teachers to assess their students' progress and apply those insights to their teaching pedagogy. The AVMR assessment uses a progressive interview approach to determine a student's current level of mathematical proficiency as well as the student's level of sophistication in solving problems. The study centers on an elementary school teacher and three of her students at an Aspen County, New Mexico, elementary school over a nine-month period. For the study, the participating teacher was interviewed and observed over the course of one academic year. Additional data included participating students' Everyday Math Journals, data derived from consultations with the participating teacher's AVMR Mathematics Coach, and participating students' AVMR pre and post-tests. Moreover, qualitative data were obtained through videotaping the

teacher's classroom protocol, including how the teacher used AVMR principles and strategies interactively to customize her instruction to meet individual student's needs.

The case study results suggest why the three students improved their mathematics skills albeit to differing levels. The results of this study suggest that AVMR mathematical strategies and activities helped to reinforce and build student understanding for the three participating students.

The study findings provide evidence that early professional development in mathematics, specifically in AVMR, supported the development of the participating teacher's pedagogy and improved the mathematical achievement of the three participating students. Even though the teacher missed some opportunities, on occasion, to use AVMR techniques, the study strongly suggests using AVMR assessments to reveal what each student knows mathematically, improved the participating students' learning and understanding of mathematical concepts. Furthermore, AVMR benefited the participating teacher by providing a resource that correlates with support areas in the school's mathematics curriculum.

Table of Contents

List of Tables	xii
Chapter 1: Project Overview	1
Vision of Math Proficiency in the NCTM Standards	2
Evidence We Are Not There Yet	4
Evidence Teachers Do Not Have the Knowledge and Skills Needed	6
Implications for Mathematics Teaching	7
What Is Needed to Support Teachers in Developing Their Knowledge.....	8
Research Questions.....	14
Chapter 2: Review of Literature Related to the Research Questions	15
Section 1: Professional Development	15
Professional development defined.	16
Why professional development is necessary.	18
Problems with professional development.	19
Necessity of conducting research on effective professional development.	21
Characteristics of effective professional development.	22
Sustained professional development.....	28
Student learning and achievement.	28
Why research in mathematics professional development is necessary.....	28
The Add+VantageMR® (AVMR) program.	30
Section 2: Assessment.....	37
Definitions of assessment.	38
Origins of assessment.	38

Purpose of assessment.....	39
Effective use of assessment.	40
Formative assessment.	40
Assessing the different types of mathematical knowledge.....	41
Types of Assessment.....	43
Research on assessment methods.....	48
Section 3: Children’s Cognitive Development	48
Research in cognitive development.	49
Historical views on cognitive development.....	49
Mathematics in early childhood.....	51
Cognitively Guided Instruction (CGI) and children’s mathematical thinking.	53
Counting.....	57
What is number sense?.....	59
Developing number sense in the early years.....	62
Can number sense be taught?.....	65
Learning to calculate.....	67
What teachers must know about how children learn.	68
Chapter 3: Methodology.....	74
Research Design.....	76
Location.	79
School district selection.	79
Teacher selection.	81

Teacher background.....	82
Student selection.....	84
Case Study Research.....	85
Data Collection	86
Research Question 1.	86
Research Question 2.	86
Research Question 3.	87
Teacher interviews.....	88
Teacher observation protocol.....	89
Student observation protocol.	91
How the students were assessed.	92
Student produced artifacts.....	94
Data Analysis	94
Researcher as instrument.	95
Chapter 4: Results and Findings of the Study.....	100
Results and Findings for Determining Holly’s AVMR Implementation Level.....	101
Teacher participant.....	109
Student Selection, Description, and Academic Overview	111
Study participants.....	112
Influence of AVMR on Holly’s Teaching and Interactions with Jasmine, Johanna, and Josh.....	136
Holly’s questioning of Jasmine, Johanna and Josh.....	137
Tasks and Skills Targeted in Holly’s Instruction.....	143

Missing Addend Problems.....	143
Summary	148
Responses to Research Questions Posed	150
Chapter 5: Discussion of Findings and Implications.....	162
Significance of the Study	162
Reflecting on the Study Findings vis-à-vis the Research Literature.....	164
Question 1.	164
Question 2.	166
Question 3.	170
Limitations of the Study.....	174
Recommendations.....	176
Study Conclusion	180
References.....	182
Appendices.....	211
Appendix A: Learning Framework in Number.....	212
Appendix B: Classroom Instructional Framework for Early Number	213
Appendix C: Summary of Alternative Assessments.....	214
Appendix D: AVMR and Common Core Standards.....	219
Appendix E: Teacher Interview Questions	221
Appendix F: Teacher Classroom Observation Protocols.....	223
Appendix G: Student Classroom Observation Protocols.....	227
Appendix H: AVMR Assessment.....	230
Appendix I: Construct Levels	230

Appendix J: Learning Levels	239
Appendix K: Mathematics Instructional Coach data	243
Appendix L: AVMR Guiding Principles for Classroom Teaching	244

List of Tables

Table 2.1 Pre- and Post-TerraNova scores for first graders in three groups—Mathematics Recovery, alternative intervention program, and comparison first graders.....	35
Table 3.1 Outline of Study	78
Table 4.1 AVMR Implementation Level Rubric	107
Table 4.2 Number Words and Numerals for Jasmine.....	116
Table 4.3 Structuring Numbers—Jasmine.....	117
Table 4.4 Addition and Subtraction—Jasmine	118
Table 4.5 Relational Thinking—Jasmine	119
Table 4.6 Number Words and Numerals—Josh	123
Table 4.7 Structuring Numbers—Josh.....	124
Table 4.8 Addition and Subtraction—Josh.....	125
Table 4.9 Relational Thinking—Josh	126
Table 4.10 Number Words and Numerals—Johanna	130
Table 4.11 Structuring Numbers—Johanna.....	131
Table 4.12 Addition and Subtraction—Johanna.....	132
Table 4.13 Relational Thinking—Johanna	133
Table 4.14 Teacher Interaction Level with Her Students	159

Chapter 1: Project Overview

To perform a reasonable analysis of the quality of mathematics teaching requires an understanding not only of the essence of mathematics but also of current research about how students learn mathematical ideas. Without extensive knowledge of both, judgments made about what mathematics should be taught to schoolchildren and how it should be taught are necessarily naïve and almost always wrong. (Battista, 1999, p. 25)

My interest in why some children “just don’t seem to get it” when it comes to mathematics began long ago when I was teaching junior high mathematics. As I went on to teach both high school and college mathematics, I continued to notice some students seemed trapped in a world of mathematical confusion. They learned mathematics was something to be memorized, it really did not make sense, there was no reasoning involved, and memorization was the only way to handle getting through it. My mathematical journey led me to ask why this happens for some students and others seem to just “get it” from the earliest grades on. Since all children should have an equal opportunity to learn mathematics in a way that is meaningful, useful, and the result of a logical progression in reasoning, I wondered why, for so many children, this was obviously not the case.

The reason for undertaking this study was to better understand why some children do not “get it” and what can be done to help more children attain a strong, sound, workable use of mathematics. Since it became evident to me that problems in mathematics for children developed in the early grades, this study focuses on mathematics instruction in the early grades. According to the National Research Council (2009), ensuring educational success and attainment must begin in the earliest years of schooling. In particular, I chose to look at

teacher instruction in early mathematics and how the use of assessment could help them provide student-appropriate instruction. To be effective, instruction has to be targeted to the student's learning level. In order for this to occur, teachers must have access to tools that will give them information about how children learn, how to measure what they know, and how to effectively guide them to the next level of sophistication in their learning (National Research Council, 2009; Wright, Martland, & Stafford, 2006)

Conducting research that addresses these topics is necessary as the educational community addresses the problem of low mathematics achievement for U.S. students (National Research Council, 2009; Stevenson, Lee, & Stigler, 1986). Too many students often struggle with mathematics and see mathematics as an obstacle in terms of realizing their educational potential, becoming, in essence, a gatekeeper (Stinson, 2004). To better understand this dilemma, and perhaps offer some further insight into addressing what has become a national problem, first a framework is provided in which to situate the study that examines the following areas: 1) the vision of mathematics proficiency in the National Council of Teachers of Mathematics (NCTM) standards, 2) evidence that we are not yet there, 3) evidence that many teachers do not have the knowledge and skills needed to effectively teach elementary mathematics, 4) the implications for mathematics teaching, and 5) what is needed to support teachers in developing their knowledge.

Vision of Math Proficiency in the NCTM Standards

Traditional mathematics teaching focuses on computational skills in which the teacher demonstrates several examples of how to solve a problem, and then has students practice this method in class and for homework. The movement to reform how mathematics was taught began in mid-1980 in response to the failure of traditional methods of teaching

mathematics, the widespread availability of computing devices, and to a major paradigm shift in the scientific study of mathematics learning (Battista, 1999). The publications, *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), and its companions *Professional Standards for Teaching Mathematics* (NCTM, 1991) and *Assessment Standards for School Mathematics* (NCTM, 1995) fostered a growing national awareness which focused on how to stimulate change in mathematics teaching and learning (NCTM, 2008). These documents promote a vision of mathematics teaching in which students engage in purposeful activities that grow out of problem situations which require both reasoning and creative thinking (Cwikla, 2002; Thompson, 1992).

Non-traditional teaching, according to the NCTM vision, is more concept-focused, centers on curriculum that uses student-constructed experiences to enhance concept development, and uses various types of assessment focusing on conceptual understandings (Madsen & Lanier, 1995). That is not to diminish the value of accurate computation; it is necessary. Calculation fluency, the ability to quickly and accurately solve mathematical problems, is critical for advanced arithmetic ability (Kilpatrick, Swafford, & Findell, 2001; Smith-Chant, 2010). However, the conceptual understanding needed to make sense of those computations is critical. In the mathematics curriculum recommended by the NCTM, the emphasis on pencil and paper calculations has been moderated, and increased attention is given to mathematical reasoning and problem solving (Battista, 1999; Silver, Smith, & Nelson, 1995).

Reform-based efforts promote a vision of school mathematics emphasizing thinking, reasoning, problem solving, and communication rather than memorization and repetition. Students are expected to collect and apply information, discover solutions, invent strategies,

communicate ideas, and test those ideas through critical reflection and mathematical discourse (Thompson, 1992). Change is necessary in mathematics teaching and learning to improve student understanding of mathematics and to increase student achievement in mathematics (Bender, 2005; Boaler, 2008; NCTM, 1989; National Research Council, 2009; O'Connell, 2005).

Evidence We Are Not There Yet

As a nation that must compete in a global marketplace and be evaluated in global terms, improving the mathematical achievement of all students is essential. The economic costs of mathematical mis-education are quite staggering when we consider 60% of college mathematics enrollments are in courses typically taught in high school, and the business sector spends as much on remedial mathematics education for its employees as is spent on mathematics education in schools, colleges and universities combined (Battista, 1999). Our nation has witnessed an emerging emphasis on mathematics instruction, and there have been national reports showing that over the past two decades achievement in mathematics among students in the United States has been increasing (Bender, 2005; Harniss, Carnine, Silbert, & Dixon, 2002; National Research Council, 2001a; Wright, 2000, 2003). According to Strauss (2003), mathematics achievement for U.S. students on several national indicators has increased. However, this is not widely accepted, as there is still a considerable deficit between the mathematics scores of students in the United States and the scores of students in other modern nations of the world, and this deficit causes great concern among educators (Bender, 2005).

In the U.S., as well as in nations throughout the world, an emphasis has been placed on raising student achievement in mathematics (National Research Council, 2009; Wright,

2000; 2003). The scores and rankings of 12 industrialized countries participating on the 2003 International Mathematics Assessments for TIMSS (Trends in International Mathematics and Science Study) Grades 4 and 8 and PISA Age 15 (Program for International Student Assessment) rank the USA in 8th place for 4th grade. By 8th grade the USA drops to the bottom third in international comparisons (Ginsburg, Lee, & Boyd, 2008).

Not much has changed since national and international assessments of students' mathematics achievement reported the same disturbing results: U.S. students fail to demonstrate the depth of knowledge, ability to reason and problem solve, and the skill and mastery expected of mathematically proficient students (Dossey, Mullis, & Jones, 1993; Schmidt, McKnight & Raizen, 1997; Silver, 1998).

Student mathematical performance is consistently lower than acceptable, and the tendency for mathematics learning gaps to grow throughout the K-12 experience is all too common for many students (National Research Council, 1989). The results of the 2005 National Assessment of Educational Progress (NAEP) mathematics test of 9,300 twelfth grade students revealed that more than 39 % fell below the proficient level in basic mathematics skills (Sousa, 2008). The Executive Summary from *Reassessing U.S. International Mathematics Performance: New findings from the TIMSS and PISA* states: "Efforts to improve U.S. international mathematics performance should include a component that strengthens U.S. mathematics education in the early elementary grades, because, generally speaking, a country's initial performance is correlated with its later performance" (p. 10). Thus, countries seeking to improve their mathematics performance should start by building a strong mathematics foundation in the early grades (Ginsburg, et al., 2008).

Evidence Teachers Do Not Have the Knowledge and Skills Needed

If we focus on the mathematics foundation of students in the early grades, perhaps the cause for such disparity in performance between the U.S. and countries that are more successful mathematically, such as Singapore and Japan, is due to the way children are taught mathematics from the earliest grades on in the United States. When we compare the ways teachers are trained and the nature of the teaching profession in American and Asian societies, it quickly becomes clear that American teachers are inadequately trained in the necessary skills teachers must know to effectively guide children as they learn mathematics (Stevenson & Stigler, 1992).

Ginsburg et al. (2008) remind us that young children can learn mathematical concepts, but also that they could learn much more if we supported their learning. Some look at teachers as a possible cause for lower than acceptable student achievement, and argue that little is learned in schools of education about teaching mathematics effectively (Ball, Hill, & Bass, 2005). From my experience, I would affirm that many university elementary education majors do not receive adequate training in the early number concepts necessary to provide their future students with a solid foundation in mathematics. For the most part their course work at the university does not include sufficient preparation in the specialized knowledge necessary for teaching early mathematics, and historically there has been a lack of available pedagogy on early mathematics teaching and learning to remedy this situation (Ginsburg et al., 2008). The National Research Council (1989) reported that few teachers, probably no more than 10% of the nation's elementary school teachers, meet contemporary standards for their mathematics teaching responsibilities. This situation has not improved significantly when compared with recent findings of Ginsburg et al., (2008) which show there are a

significant number of teachers who are poorly trained to teach mathematics, in some cases are afraid of it, feel it is not important to teach, and typically teach it badly or not at all (Ginsburg et al., 2008). The undergraduate degree for many teachers, even with a major in early childhood education, is not a good predictor of how effective the teacher will be in the classroom or the student's academic outcome (Early et al., 2007).

Implications for Mathematics Teaching

Teachers engaged in mathematics reform are searching for instructional ideas and methods that will help them increase the cognitive participation of all students in their classes (Bender, 2005). Teachers who are involved in implementing the current reform curriculum are expected to focus on student understanding, considering and debating alternative solutions, mathematical discourse, and developing mathematical connections and meanings (Elmore, 2002; Thompson, 1992). It is clear that professional development opportunities must include information that addresses these areas as well as early childhood learning and the mathematics children acquire in the early grades, including the knowledge they acquire before they begin formal schooling. It appears that how children are taught mathematics early on has a significant impact on their subsequent learning of mathematics. Effective early childhood education has been shown to provide a foundation for later academic success (Bowman, Donovan, & Burns, 2001; Campbell, Pungello, Miller-Johnson, Burchinal, & Ramey, 2001; Reynolds & Ou, 2003). Gormley (2007) states this is especially true in the short term, and Ludwig and Phillips (2007) state there are studies to show this benefit continues in the years thereafter. Professional development, such as Add+VantageMR® (AVMR) can be a potential resource for teachers as they seek to understand how children

learn mathematics and subsequently plan appropriate mathematics instruction for their students.

What Is Needed to Support Teachers in Developing Their Knowledge

Professional development that is well researched and clearly articulates what teachers need to know, provides information on how to use their curriculum and resources to improve the learning of *all* students, and uses reform-based strategies and techniques is needed for teachers to develop and grow their knowledge (NCTM, 2008). Professional development can also provide teachers reform-based methods for assessing students, as student assessment is critical to inform teachers about what their students know and do not know.

There are vast arrays of professional development programs to choose from and information is needed to help school districts, administrators, and teachers make appropriate choices as to the program of professional development that best meets their needs. As new information regarding what constitutes successful professional development is made available to the education community, school administrators and teachers must be knowledgeable in choosing the best possible professional development programs to meet the specific needs of their students.

This study provides additional research-based information to address a need expressed by Ball et al. (2005) which calls for more research that links teachers' mathematical preparation and knowledge to the achievement capabilities of their students. This is significant; as it appears teacher learning is a critical factor in student learning. "Improving teachers' subject matter knowledge and improving students' mathematics education are thus interwoven and interdependent processes that must occur simultaneously" (Ma, 1999, p. 147). Teachers who are knowledgeable and informed in sound instructional

practices and receive training in how students learn are much more likely to produce students who are more capable learners.

Many elementary education students have not received adequate training in the development of children's early number acquisition and mathematics. I have observed this firsthand as one who has taught courses in *Mathematics for Elementary Teachers* for education majors for many years. As a result of my experience with Add+VantageMR® (AVMR) professional development, I believe there are many teachers who are not familiar with the early mathematical learning of children, assessing a student to determine the student's learning level for specific tasks, and the subsequent training so that, as teachers, they can set goals for advancing the student to their next level.

It became apparent to many governments and school systems in several countries in the 1990s that attention needed to be focused on how well their teachers taught mathematics and subsequently how well their children were achieving in mathematics (Wright, Martland, et al., 2006). There already was significant evidence to show there are differences in the numerical knowledge of children when they enter school, and that these differences only increased as they progressed throughout their school years (Aubrey, 1993; Wright, 1991, 1994; Young-Loveridge, 1989, 1991). The Mathematical Sciences Education Board of the Center for Education at the National Research Council established the Committee on Early Childhood Mathematics which was charged with examining existing research to develop insights related to curriculum, instruction and teacher education to address the problems associated with the lower level of mathematics achievement for far too many students (National Research Council, 2009).

One of the products of this focus was an international comparison of the following components related to mathematics education, and more specifically of early mathematics education. The components considered were student achievement, curriculum content, and teaching methods (Wright, Stanger, et al., 2006). Overall, it was determined that children continue to experience difficulty in mathematics, and these difficulties can be caused by the individual characteristics of the child, inadequate or inappropriate teaching, frequent student absence, and the lack of pre-school or home experience in mathematics (DfES, 2004; Dowker, 2003; 2004; Wright, Stanger, et al., 2006). In addition to the aforementioned conditions is also the category of children with learning difficulties.

To address these difficulties, Mathematics Recovery, among other mathematics curricular programs for children, was created. My study specifically focuses on Add+VantageMR®, (AVMR) which is the professional development component of Mathematics Recovery. In order to fully understand the guiding principles of AVMR, it is necessary to familiarize the reader with Mathematics Recovery.

Mathematics Recovery specifically focuses on early intervention and asserts it is necessary to fill in the knowledge gaps for low-achieving students for them to succeed in mathematics. Mathematics Recovery was specifically created to address the great concern and emphasis placed on raising student achievement in mathematics throughout the world. Robert Wright developed the Mathematics Recovery Program at Southern Cross University in Australia over a period of three years (1992-95) to provide teachers with the necessary training and guidance to intervene on behalf of low achieving students (Wright, Martland, et al., 2006). Development of the program drew substantially on the constructivist teacher experiment research that Leslie Steffe at the University of Georgia conducted (Steffe &

Cobb, 1988). Mathematics Recovery involves first identifying the lowest achievers at the first grade level, then providing a program of intensive individual intervention in order to advance them to a level where they are likely to be successful in a regular classroom.

AVMR applies to whole class situations and thus the one-on-one intensive student interventions are not a part of AVMR, as they are in Math Recovery.

AVMR assists teachers by training them to assess their students, and providing the necessary tools, strategies, and activities to assist teachers in developing their young students mathematically. AVMR trains teachers to identify children's learning levels through formative assessment, and provides tools and strategies to help the teacher assist the child in reaching the next level of progression, much the same as Math Recovery. The assessment approach is quite distinctive in that it is interview-based rather than pencil and paper-based. The assessment involves presenting numerical tasks and engaging with the child to determine the extent of the child's knowledge as well as the sophistication level of the child's strategies.

I believe it is the mathematical assessment of students, and the information we have gained through student assessment, that is in large part responsible for the changes taking place in the field of mathematics education as evidenced by the Mathematics Reform movement which centers around conceptual understanding of mathematics, and problem solving. The use of assessment that uncovers student thinking has informed educators of how to better meet student learning needs. We have had to rethink what mathematical behaviors and skills we value in our students. Are we assessing what we value? Do our assessments accurately convey what we want our students to be able to do? Do our assessments enhance student learning? Does the assessment contribute valuable information about what the student knows?

According to the National Research Council (1989) we must ensure that tests measure what is of value, what we consider to be important skills, and not just what is easy to test. If we want students to investigate, explore, and discover, assessment must address and ultimately measure these skills (National Research Council, 1989). As we have changed our expectations of what students “should be able to do” mathematically, this change in focus must also be accompanied by a subsequent change in assessment. The development and use of assessment practices that align with mathematics reform instructional strategies is not an easy task (Stiggins, 1991). Many times, even though teacher beliefs have changed concerning their instructional practice, the change in ideology is not evident in their assessment tools.

An effort to improve the mathematical education of U.S. students has fostered a reform movement that has impacted mathematics instruction, what is expected of students, and teacher professional development. However, it has also made a significant impact on how students should be assessed (Kulm, 1994). Subsequently, assessment practices used in mathematics are currently being studied, researched and evaluated. It appears that teacher understanding of how children develop mathematically is essential if we are to utilize the information we receive about students from effective assessment.

Understanding how children develop mathematically should be factored into improving teacher instruction in mathematics in addition to assessment. Because of advances in neuro-science, we have the ability to know much more about children’s cognitive development. Lakoff & Nunez (2000), both of whom are cognitive scientists, propose it is up to cognitive scientists and neuroscience to apply the science of the mind to human mathematical ideas. Mathematics is deep, fundamental, and essential to the human

experience; as such it is necessary that each student understand mathematics (Lakoff & Nunez, 2000). As we understand more of the ancestral brain, the adaptability of the brain, and its function in processing mathematics, the more we can apply this knowledge to helping teachers help children learn. One thing seems certain: students who are poor in mathematics in their early years remain poor in mathematics in their later years (Sousa, 2008). Studies also show that a three-year difference in mathematical ability the early years of school becomes a seven-year difference for low attaining children after about ten years of school (Wright, Martland, et al., 2006).

It is essential that all children have an equal opportunity to learn mathematics. To do so, information must be available to, and utilized by, teachers in order to meet the needs of every child. The necessity of conducting research that will provide information regarding how the brain processes mathematics, how children develop number sense, and what teachers can do to promote student achievement in mathematics are topics of vital interest to all in the mathematics community. “There is concern about the chronically low mathematics performance of economically disadvantaged students; particularly alarming is that these disparities exist in the earliest years of schooling and even before the child starts school” (National Research Council, 2009, p.1).

Most students leave school without sufficient preparation in mathematics to cope with either on-the-job demands for problem solving or the necessary skills to prepare them for college expectations for mathematical literacy (National Research Council, 1989). More recently, longitudinal studies have shown that mathematical concepts, such as knowledge of numbers and the concept of ordinality, at school entry are the strongest predictors of later achievement, even stronger than early literacy skills (Duncan et al., 2007).

The confluence of the three topics: 1. Professional Development, 2. Assessment, and 3. Children's Cognitive Acquisition of Mathematics has led me to formulate the following research questions.

Research Questions

1. At what level in the AVMR classification scheme is the participating teacher after attending initial AVMR professional development training, and what is the impact of the training on a teacher's instruction?
2. How does a teacher who receives Add+VantageMR® (AVMR) professional development training utilize the AVMR assessment results to inform instruction for a small group of students?
3. Does the performance of this group of students improve when the teacher modifies her teaching by utilizing their assessment results?

A case study was conducted to observe how AVMR training initially impacted a teacher and how she used the AVMR student assessments to adjust instruction for a small group of students based on their assessment results. The study also focused on the mathematical growth of the students. I systematically analyzed each of the three participating students' emerging mathematical understanding vis-à-vis the AVMR assessments and student produced artifacts. How students' understanding of concepts such as their facility to make combinations of 10 was examined over the course of the five months of data collected in the form of student artifacts, classroom observations, and videotaping. In addition, the three students' achievements on the pre and post-AVMR assessments were investigated with the aid of a Two-Sample T-Test.

Chapter 2: Review of Literature Related to the Research Questions

This chapter contains a review of the literature that pertains to the study's three research questions. The chapter is divided into three parts. The first part provides a broad review of professional development for teachers and professional development in early mathematics; it then leads into the specifics related to Add+VantageMR® (AVMR), the professional development program used to train the subject teacher of this study. The second part reviews information on assessment and turns to focus on AVMR professional development for teachers, which is defined by its use of student assessment to guide teacher instruction. The third part provides information on children's cognitive development with a particular focus on how children develop mathematically, as the study is concerned with increasing students' mathematical understanding and subsequent mathematical achievement.

Section 1: Professional Development

Professional development is a resource that can provide teachers the necessary information and tools to assist them as they help children develop and build upon their mathematics skills. From my professional experiences working with elementary teachers during professional development institutes, as well as through the information I have gained through my investigations, I see a tremendous need for providing teachers with current research information, including on-going support, to help them develop their young students' mathematical skills. To this end, this review focuses on teacher professional development in the lower elementary grades, as this is a critical time in establishing the foundation for children's future success in mathematics. Teacher training and building teacher confidence with respect to teaching mathematics from the earliest grades on fosters student growth and achievement in mathematics. I believe that, for too many children, this valuable opportunity

is not realized because many teachers are not aware of the strategies and methods that should be used to foster mathematical growth in young children. Much is at stake, and developing children mathematically has far-reaching ramifications. For instance, longitudinal studies have shown that math concepts, such as knowledge of numbers and ordinality, at school entry are the strongest predictors of later achievement, even stronger than early literacy skills (Duncan et al., 2007).

Regardless of whether the subject of focus is mathematics, science, writing, or reading, professional development is a resource that increases teacher knowledge and improves instruction. The next level of progression in U.S. education requires that schools be continuously involved in the practice of school improvement with the focus of improving student achievement; however, we are not yet there as few school districts treat professional development as part of an overall strategy for school improvement (Elmore, 2002).

Professional development should be used to inform teachers of results from both research and best practices to facilitate student learning and provide teachers a means to concentrate on their teaching practice.

A vast array of literature is currently available that highlights the components necessary for what is inherently referred to as “effective” professional development. To give the reader a better sense of how the AVMR professional development program specifically aligns with what the research community has proposed as “effective” professional development in mathematics, this review begins with background information on professional development in general.

Professional development defined. I will first provide key definitions of professional development as stated in the literature, address why professional development in

mathematics is necessary, examine the findings from research on the characteristics of effective professional development, and then focus on professional development in mathematics for teachers in the early grades.

The National Research Council (2009) refers to professional development as an umbrella term referring to both formal education (i.e., the amount of credit-bearing coursework from an accredited institution) and training (i.e., the educational activities that take place outside the realm of formal education such as mentoring and workshops). Elmore (2002) claims professional development is at the center of the practice of improvement because “it is the process by which we organize the development and use of new knowledge in the service of improvement” (p. 30). I agree with both definitions of professional development given by the National Research Council and Elmore.

Professional development covers an array of activities from work with teachers around specific topics and teaching practices through short workshops designed to familiarize teachers and administrators with new ideas, new rules, and requirements to complete off-site courses and workshops designed to provide content and academic credit for teachers and administrators (Elmore, 2002). I see professional development as an opportunity for: educating teachers in specific content material, enhancing teachers’ understanding about student learning, providing a forum for mathematical discourse, allowing teachers to share best practices, providing a network of support from specialists and collegial teams, providing a means for teachers to exist in community with other teachers who have the same needs and concerns, allowing teachers the opportunity to discuss pedagogy and modeling good teaching.

Why professional development is necessary. In more fortunate school circumstances, teachers may have the opportunity to interact with colleagues and share good lessons or practices. Many times, however, this ideal is not realized. How, then, do we support teachers engaging in formal discourse about student learning? How do we make available to all teachers tools that can assist them in improving their teaching practice? For teachers who have not received training in the knowledge and skills they need to teach Standards-Based mathematics, the mathematics education community has responded by updating both pre-service teacher preparation programs and in-service teacher professional development programs (Heck, Banilower, Weiss, & Rosenberg, 2008). These efforts are sorely needed as research suggests the U.S. mathematics teaching force is not well prepared for the challenges involved in Standards-Based teaching (NCTM Research Committee, 2008). Thus, the necessity of investing in teacher education and professional development is well supported (Darling-Hammond & Sykes, 1999; Elmore, 2002; NCTM Research Committee, 2008; National Research Council, 1989, 2001b; Wu, 1999).

Professional development for teachers provides a venue for both pedagogical improvement and mathematical discourse to take place. If we are seeking to improve our nation's schools, we must realize the professional development of teachers is a key ingredient (Darling-Hammond & Sykes, 1999). Without investing in teachers, our nation's investment in education will come to naught (Wu, 1999). Changes should be supported in schools to make professional development an integral component of a teacher's job (Conference Board of the Mathematical Sciences, 2001). Elmore (2002) finds few portals open through which teachers and administrators can assimilate, adapt, and polish new ideas and practices about teaching and learning. Unlike the teaching profession in Asian communities where

professional development and time for teacher collaboration is built into the teachers' school day, teaching in the United States remains a relatively isolated profession.

The reasons for providing high quality professional development are obvious. Perhaps the most profound reason to study and provide professional development comes from Kitchen, DePree, Celedon-Pattichis, and Brinkerhoff (2007); their research affirms that teachers with access to professional development improved mathematics instruction in schools where, based on demographics, students would not be expected to perform well. This is an accomplishment that should be mirrored in all schools. One of the major themes to emerge from Kitchen et al.'s study of highly effective schools that serve the poor was the importance of teacher access to high quality professional development opportunities. Teachers in Kitchen et al.'s report attended professional development sessions to enhance their instructional strategies and then met together to share ideas and to support one another in the improvement of instruction.

Problems with professional development. Many professional development programs are hampered because school districts do not use the programs to meet intended purposes. Perhaps, in part, this situation arises because many professional development experiences are not positive and deemed a waste not only of teacher time but also of limited school fiscal resources. Further, it is not certain a given professional development will successfully impact teachers' practice for the better. In fact, some educators believe professional development as we now know it, involving after-school or one-day regional workshops (Zigarmi, Betz, & Jensen, 1977) will not transform teachers' knowledge, beliefs, understanding, and ultimately habits of practice (Smith, 2007). In some cases the connection between professional development, as currently practiced, and improved instruction and

student performance is practically nonexistent (Feiman-Nemser, 1983). Elmore (2002) argues that often “spending more money on existing professional development activities, as most are presently designed, is unlikely to have any significant effect on either the knowledge or skill of educators or on the performance of students” (p. 6). Too often administrators’ concept of professional development focuses narrowly on changing teaching behaviors (e.g. on helping teachers learn a new manipulative or piece of technology) and not on the impact such materials can have on what students already know and can presently do (Smith, 2007).

Cohen and Hill (2000) found that fragmented and unfocused professional development did little to provide teachers with new learning opportunities. Ball (1996) believes professional development is an uncertain practice and that professional educators should take a closer look at what our efforts to develop mathematics instructors into more reform-minded educators are accomplishing. To avoid the expense, lack of teacher benefit, and, ultimately, the loss of time in being able to assist students, informational research must be available that will assist school districts and teachers in making wise, informed, decisions regarding professional development programs.

Research that focuses on one attempt out of many to link professional development to student achievement has mixed results. For example, Yoon and his colleagues examined nine studies for the effect professional development has on student achievement (Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). Their study found that students in the classes of teachers who had, on average, 49 hours of professional development, had greater achievement levels than students in the classes of teachers who either had no professional development, or participated in less than 49 hours. Unfortunately, the research in the area of

professional development connecting teacher and student learning is limited (National Research Council, 2001b; Sykes, 1999; Wilson & Berne, 1999). On-going evaluation is necessary to determine the effect of professional development on student achievement as well as to provide updates on the effectiveness of the original research (Elmore, 2002; Begle & Gibb, 1980).

Certainly, for administrators who choose to implement a professional development program in their school districts, they must proceed with much consideration. This caution leads naturally to the following questions: how do we determine what constitutes effective professional development? How do we guarantee we are not investing in a program that is ineffective and wasteful of both time and resources?

Necessity of conducting research on effective professional development. To be effective, professional development must incorporate and reflect the best available research about teaching and learning (Corcoran, 1995; Office of Educational Research and Improvement, 1999). Research regarding professional development has informed the educational community of the necessary characteristics of effective professional development such as placing teachers' needs at the center of professional development, analyzing student thinking, and evaluating the impact of teachers' professional development on their students' learning (Gore & Ladwig, 2006; Supovitz & Turner, 2000; Van Driel, Beijaard, & Verloop, 2001). Isolated activities teachers may receive in some professional development programs do not help them build upon students' previous knowledge and understanding and are, therefore, not as likely to be of benefit in the long term.

To ensure professional development programs are selected that meet intended purposes; research on student learning, mathematics pedagogy, and professional

development in mathematics for teachers must be on-going. Research can ensure that the knowledge regarding how to support best practices for teachers is continually generated and improved upon. While educators generally agree on which features align with quality professional development, empirical bases for making such claims are limited. Therefore, additional research on the effectiveness of professional development is warranted (Desimone, Porter, Birman, Garet, & Yoon, 2002; Hill & Ball, 2004; Wilson & Berne, 1999). What is needed is more quantitative research investigating the links among teacher knowledge, the implementation of curriculum, and student learning (NCTM Research Committee, 2008). Additionally, research is needed that further identifies the characteristics of quality professional development but, most importantly, addresses the issue of teacher implementation of the professional development. The ultimate goal for such research is to produce professional development schema that improves student understanding, comprehension, and ultimately, academic performance (Elmore, 2002).

Characteristics of effective professional development. Many educators/researchers have offered their views as to what constitutes “effective” professional development. The key is in providing professional development that will educate teachers about student learning in mathematics, provide the teacher with resources that will improve their practice, assist the teacher in diagnosing students’ weaknesses, and, ultimately, increase students’ mathematical abilities. Common threads and themes run through the views of researchers. The following are representative of these themes.

Use of student assessment. Elmore (2007) groups together various viewpoints, one of which he identifies as the Consensus View. This viewpoint draws heavily on the original standards for professional development adopted by the National Staff Development Council

in 1995 (Sparks, 1995; Sparks & Hirsch, 1997). Active monitoring of student learning is emphasized during both assessment and evaluation. When student learning is assessed, teachers can then capitalize on teachable moments that would otherwise be overlooked (Dodge, Colker, & Heroman, 2002). Teachers come to recognize that, when a child is ready to learn, they can exploit that moment to help the child undertake further learning (National Research Council, 2009)

Collaboration. Elmore (2002) further argues that “professional development should be designed to develop the capacity of teachers to work collectively on problems of practice, within their own schools and with practitioners in other settings, as much as to support the knowledge and skill development of individual educators” (p.8). Likewise, Clarke (1994), advocates involvement of groups of teachers rather than individuals from a number of schools. This view is echoed by Cwikla (2002) and Little (1982) who maintain teacher isolation is an obstacle to setting learning goals and that professional development has the greatest influence when it occurs in a collegial environment where teachers believe they can learn from one another. Encouraging teachers to share ideas and conduct structured learning lessons in their classrooms could be a valuable source of support for teacher learning (Schifter & Fosnot, 1993).

Continuous improvement over time. Focus on continuous improvement over time, along with a philosophy that develops, reinforces, and sustains group work, is central to professional development. Overall, educators must realize that change is often a gradual, difficult, and, at times, a painful process; they must allow for opportunities in which on-going support is provided from both peers and knowledgeable others (Clarke, 1994). Research studies suggest that instructional improvement is facilitated by small steady change

and should be intensely focused (Franke, Fennema, Carpenter, Ansell, & Behrend, 1998; Schifter & Fosnot, 1993). Teachers must have time to understand new ideas about instruction and time to convince themselves, mainly through their own experiences and classroom practice, that instructional improvement is necessary, worthwhile, and feasible (Cwilka, 2002). In addition to allowing ample time for change to occur, professional development programs must incorporate the topics in which teachers have great interest.

Address topics of interest to teachers and impediments to teacher growth. Clarke (1994) affirms that the research literature provides key principles for effective professional development; specifically, professional development should include issues the teachers themselves indicate are topics of interest and concern. Effective practices center instruction on participants' teaching interests, using hands-on activities and projects with end products that are shared with the whole group, is ongoing, and teachers are paid and treated as professionals (Brinkerhoff, 2006). Professional developers realize that effective professional development must consider the teacher and the needs of the teacher first and foremost (Gore & Ladwig, 2006; Supovitz & Turner, 2000; Van Driel et al., 2001). Researchers, therefore, must realize that participants in teacher education programs come from a wide variety of backgrounds. For example, some may be in their first year of teaching or 30th year, and thus may be at different developmental stages of mathematical pedagogy development and have different needs for assistance; these factors might have an effect on their reactions to particular professional development programs and activities (Brown & Borko, 1992; Cwilka, 2002). This reality must be taken into account when constructing and seeking professional development opportunities.

Professional development should respond to individual participant's background, experiences, and current position in life (National Research Council, 2009). An important characteristic of a successful professional development program has been articulated as working with, rather than doing to, teachers (Loughran & Gunstone, 1997). Failure to place teachers at the center of any plans for reforming practice, incorporating sustained activities, or implementing innovations can only lead to disappointment in the achievement of positive outcomes (Gore & Ladwig, 2006; Supovitz & Turner, 2000; Van Driel et al., 2001).

In *The Professional Development Design Framework*, Loucks-Horsley, Stiles, Mundry, Love, and Hewson (1998) propose a process by which professional development programs in mathematics are created, implemented, and modified. Central to this process is the view of teachers as both "agents and objects of their professional growth" (p. 260) and the recognition that teachers need to be supported throughout a teaching cycle that includes building knowledge, translating knowledge into practice, teaching, and reflecting on teaching (Smith, 2007).

Models classroom practice. Much like Elmore, Clarke (1994) affirms the importance of using teachers as participants in classroom activities as well as students in real-life situations, where desired classroom approaches are modeled during in-service sessions to illustrate a clearer vision of the proposed changes. These researchers and others recognize that teachers' beliefs about teaching and learning are derived largely from classroom practice. Professional development programs must allow time and opportunities for teachers to plan, reflect, and provide feedback on their classroom practice in order to report successes and failures to the group, to share "the wisdom of practice," and to discuss problems and solutions regarding individual students and new teaching approaches (Clarke, 1994).

Mastery of content material. Another essential component adding to the effectiveness of teacher professional development, strongly advocated by Wu (1999), is teacher mastery of content material. He maintains positive student outcomes are much more likely to occur when the students are taught by professionals who have a deep understanding of the mathematics they are teaching because an educator cannot teach what he or she doesn't know. This is one of the key findings of the landmark 1983 document, *A Nation at Risk*. However, Wu (1999) also believes too many of our mathematics teachers may be doing exactly that: teaching what they do not adequately know.

Teacher support. Wu (1999) states that teachers should be paid for participating in professional development, which, in turn, provides schools leverage to ask for their commitment to learn and fully participate in the professional development activities and assignments. Year-round follow-up programs should monitor the teachers' progress and provide support. Much like Wu, researchers Black and Wiliam (2004) have identified a four-point scheme for professional development that includes teachers receiving support allowing them to work together; teachers incorporating ideas into classroom practice; teachers creating a balance between what the requirements of the curriculum demand and meaningful learning, and teachers receiving feedback from peer/external review of their practice and work. Although the Add+VantageMR® (AVMR) program provides support through the training provided in Course One and Course Two, depending on the area of the country, the program offers limited on-going support.

Focus on student thinking. Professional development must be directly connected to teachers' work with their students (Ball & Cohen, 1999; Darling-Hammond & McLaughlin, 1996). Several investigators argue that a focus on students' mathematical thinking provides

opportunities that lead to changes in classroom practice and improvement in student achievement (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996; Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Many teachers report their need to understand student thinking as well as to be able to assess student learning in mathematics (Weiss, Banilower, McMahon, & Smith, 2001). Importantly, teachers need activities that allow them to distinguish between and among different children's thinking (Whitenack, Knipping, Novinger, Coutts, & Standifer, 2000). As part of this process, a development program must provide sufficient support for teachers as they attempt to analyze the thinking of children, particularly children they do not know well, or about whom they have no other background information (Whitenack et al., 2000).

This process aligns with the major points of Cognitively Guided Instruction (CGI) and Add+VantageMR® teacher professional development in that AVMR seeks to uncover student thinking through the use of assessment before helping teachers design instruction modules at the cutting-edge of children's knowledge which will advance student understanding. Teachers find a professional development experience beneficial when they come away from it with the ability to learn from the thinking of their students; these teachers were able to continue learning and improving their practice as well as develop a broader range of pedagogical strategies from which they can draw even after the formal professional development support ended (Franke, Carpenter, Levi, & Fennema, 2001; NCTM, 1989, 1991, 2000).

Teacher professional development often plays a central role in Standards-Based reform efforts because these reforms rely heavily on teachers to develop mathematical understanding through classroom discourse (O'Day, Goertz, & Floden, 1995). When

professional development activities focus on students' thinking, the effect on teachers' instructional practices is positive and their potential for continued growth is supported (Cwikla, 2002; Franke et al., 1998; Simon & Schifter, 1991).

Sustained professional development. The information provided by Heck et al. (2008) suggests that some (or any) professional development can have a positive impact; however, to obtain the greatest impact on teacher behavior and practice, sustained teacher development is preferred. Professional development is most effective when it is ongoing, focused on strengthening both mathematical and pedagogical understanding, grounded in teacher practice, and tied to the school district goals and standards (Ball & Cohen 1999; Thompson & Zeuli, 1999).

Student learning and achievement. Ultimately, the measure of effectiveness or success of teacher professional development lies in student performance. When the needs and concerns of teachers have been addressed, each professional development program must be assessed in terms of its impact on student learning. Teacher learning should be driven by gaps identified between goals for student learning and actual student performance (Hawley & Valli, 1999). Changes in student outcomes, which reflect growth in their understanding and achievement, are the ultimate measures of professional development's success (Clarke, 1994; Clarke & Clarke, 2009).

Why research in mathematics professional development is necessary. The 2000 National Survey of Science and Mathematics Education indicated that more than half of the elementary, middle, and high school mathematics teachers nationally recognize a need for professional training in how to use teaching strategies requiring students to investigate and formulate questions to support learning (NCTM, 2008). Teacher professional development is

of paramount importance in the effective teaching of mathematics and subsequent learning of mathematics. Studying math professional development is critical because of a traditional/historical lack of math pedagogy for pre-service teachers (Elmore, 2002). Research must be used to improve, assess, and further the field of mathematics education (Begle, & Gibb, 1980; Desimone et al., 2002; Hill & Ball, 2004; Yoon et al., 2007).

The purpose of research in mathematics education is to find out what works whether it be in teacher learning, student learning, classroom practice, or any combination of these components. When these concerns are addressed through research, answers that significantly impact the mathematics education community are made available. Research must expand upon, and probe more deeply into, the components of knowledge teachers need to teach National Science Foundation supported mathematics curricula effectively (NCTM Research Committee, 2008).

Professional educators must consider research as a necessary component to assist in improving mathematics education for all. As researchers examine the effects of the National Council of Teachers of Mathematics (NCTM) recommendations on curriculum, teaching, and professional development, many opportunities arise for mathematics education researchers to conduct studies that have an impact on practice and policy (NCTM Research Committee, 2008). Professional development is needed to enrich and support existing teaching because schools face a growing shortage of properly trained teachers who are prepared to teach reform-based mathematics (Cwilka, 2002). Thompson (1992) cautions teacher educators (professional developers):

We should not take lightly the task of helping teachers change their practices and conceptions. Attempts to increase teachers' knowledge by demonstrating and

presenting information about pedagogical techniques have not produced the desired results. The research suggests that teacher's conceptions of mathematics, of how it should be taught, and how children learn it are deeply rooted. Research would caution us against underestimating the robustness of those conceptions and practices. The tendency of teachers to interpret new ideas through old mindsets--even when the ideas have been enthusiastically embraced should alert us against measuring the fruitfulness of our work in superficial ways. (Thompson, 1992, p. 143).

Thompson further states that we should look at change as a long-term process. Continual change is a natural and vital essential characteristic of mathematics education and, by necessity, implies continuous scrutiny (National Research Council, 1989). Research is the means to accomplish this task. Critical research is still needed to uncover process variables that will promote changes in teachers' knowledge and practices (Sheridan, Edwards, Marvin & Knoche, 2009). In order to determine the best methods of implementing reform-based curricula, research must focus on the knowledge and skill of educators, their characteristics such as years of teaching, style of teaching (i.e., whole class or small group instruction, etc.) and ultimately on the performance of their students (Elmore, 2002).

The Add+VantageMR® (AVMR) program. I believe Add+VantageMR® to be a program of teacher professional development that may significantly empower teachers to develop students mathematically and improve student learning. I chose this program for analysis because the AVMR program of teacher professional development incorporates all the features previously identified as essential for quality professional development and empowers teachers to improve mathematics instruction by putting into practice new research in the areas of assessing, teaching, and learning numbers. AVMR centers on student

assessment; teachers are taught how to use the AVMR assessment component, are expected to use the assessment component to assess each of their students, and then, based on assessment results, plan appropriate instruction for their students. The AVMR program and series of three books—*Early Numeracy: Assessment for Teaching and Intervention*, *Teaching Number: Advancing children’s skills and strategies*, and *Teaching Number in the Classroom with 4-8 year olds*—draw on a substantial body of recent theoretical research in mathematics teaching and learning (Wright, Martland, et al., 2006).

AVMR is one of the four components of Mathematics Recovery, the parent organization for AVMR. Mathematics Recovery was specifically created to address the great concern and emphasis placed on raising student achievement in mathematics throughout the world. Robert Wright developed the Mathematics Recovery Program at Southern Cross University in Australia over a period of three years (1992-95) to provide teachers with the necessary training and guidance to intervene on behalf of low achieving students (Wright, Martland, et al., 2006). Development of the program drew substantially on the constructivist teacher experiment research that Leslie Steffe conducted at the University of Georgia (Steffe & Cobb, 1988).

The central focus of Mathematics Recovery is on-going programs of professional development and teacher support in the form of intensive interventions for low attaining students in mathematics. This effort has been based on research studies showing specifically targeted interventions in numeracy can have a significant impact on children’s mathematics performance and self-confidence (Wright, Stanger, et al., 2006). The Department for Education and Skills (DfES, 2005) cites the Mathematics Recovery Program as one of only two examples of successful intervention for low-attaining 6-7-year old children. To date, the

Mathematics Recovery Program has been widely implemented to include 21 states in the U.S., 21 educational authorities in Britain and Ireland, the Bahamas, and the province of Manitoba, Canada.

In recent years, English-speaking countries such as Britain, America, Canada, Australia, and New Zealand have focused on mathematics with particular attention devoted to numeracy in both educational policy and practice (Wright, Stanger, et al., 2006). As Wright and his colleagues state with respect to student learning of mathematics, “initially gains were made in general levels of mathematics, but the gains either did not reach the desired levels or were not maintained” (2006, p. 1). Children continue to experience difficulty in mathematics, and these difficulties can arise from the individual characteristics of the child, inadequate or inappropriate teaching, frequent student absence, or the lack of pre-school or home experience in mathematics (DfES, 2004; Dowker, 2003; 2004; Wright, Stanger, et al., 2006).

Mathematics Recovery was developed as a systemic response to the problem of chronic failure of children in mathematics and seeks to address the specific difficulties experienced by children by providing teachers with suitable tools for assessing young children’s mathematics skills and knowledge (Wright, Stanger, et al., 2006). The program involves identifying the lowest achievers in the first grade before providing them with a program of intensive individual teaching in order to advance them to a level where they are likely to be successful in a regular classroom. The assessment approach used for both Mathematics Recovery and AVMR is distinctive in that it is interview-based rather than pencil and paper-based. The assessment involves a teacher presenting various numerical

tasks and then engaging with the child to determine the extent of the child's knowledge as well as the sophistication level of the child's strategies.

Using the interview method, the teacher can see, for instance, if a child must rely on always beginning the count with "one" or whether the child can "count on" among other strategies. For example in adding $4 + 3$, the teacher would be able to see whether the child must say "1, 2, 3, 4" then "1, 2, 3," then "1, 2, 3, 4, 5, 6, 7," which would be characteristic of a child who begins the count with one, or whether the child could use a more advanced strategy such as "counting on." With "counting on" the child would respond to $4 + 3$ saying "5, 6, 7"; the answer is "7." The assessment results in a profile of the child's knowledge across many aspects of early number and mathematical skills that is useful for documenting the child's progress over a period of time.

After they have assessed their children's early number strategies and knowledge, teachers then develop specific instructional approaches and targeted instructional activities that can be applied to individuals in small groups or whole class situations. Instruction uses assessment results as a platform on which to base teaching on a range of topics in early number. The detailed programs for developing instruction and providing guidance in teaching are the Math Recovery Learning Framework in Number (LFIN), which outlines how students move from using naïve strategies to increasingly sophisticated strategies to solve number problems and the Math Recovery Instructional Framework for Early Number (IFEN) which provides a plan for teaching. (See Appendices A and B respectively). Teaching is specifically focused just beyond the "cutting edge" of a child's current knowledge, as the teacher deliberately engages the child in tasks requiring the child to develop more sophisticated number strategies (Wright, Martland, et al., 2006).

In 2003 the U.S. Mathematics Recovery Council (USMRC) was established to ensure the quality of Mathematics Recovery in the U.S.; to provide oversight of training, materials, and intellectual property; to validate courses, and to certify leaders and teachers. The USMRC makes AVMR staff development available. Since there is no rigorous research on the impact of AVMR on teacher practice and student learning (Jenny Cobb, former president of the USMRC, personal correspondence, May 23, 2011), related research on Mathematics Recovery is provided.

During the 2006-2007 school year, as part of an initiative in which the Kentucky State Department of Education focused on early mathematics intervention, Mathematics Recovery was implemented in 13 elementary schools. One teacher from each of the 13 schools undertook the yearlong professional development program to become Mathematics Recovery Intervention Specialists. This program included having the teachers administer cycles of intensive, one-on-one instruction to no more than six low-attaining first graders during a single school year. Participants' progress was gauged using several assessments including the nationally normed TerraNova Assessment (CTB/McGraw-Hill, 2011) that generated pre and post-national percentile scores for each student. An analysis of student progress was produced by the Evaluation Services Center at the University of Cincinnati. In the report of the analysis, the outcomes relevant to Mathematics Recovery, an alternate intervention program, and comparison first graders were made. The results are shown in Table 2.1.

Table 2.1

Pre- and Post-TerraNova scores for first graders in three groups—Mathematics Recovery, alternative intervention program, and comparison first graders. (Cullivan, 2008)

Program	Average Percentile	Average Percentile
	On Pre-Assessment (Fall)	On Post-Assessment (Spring)
Math Recovery (n=66)	9	70
Alternative Intervention Program (n=159)	5	34
Comparison 1 st graders (n=252)	14	38

The report documents the progress of the Mathematics Recovery participants, progressing from an average percentile of nine on the pre-assessment to an average of 70 on the post-assessment.

In another study, MacLean (2003) described the effects that involvement in the Math Recovery and the Count Me In Too programs had on the achievement of first grade students who had been identified as low performing in mathematics with respect to the Forward Number Word Sequence, Backward Number Word Sequence, Numeral Identification, Spatial Patterning, and Tens and Ones Strategies. Three different professional development models on low-achieving first-graders in a large urban school district were used.

The first model consisted of a full Mathematics Recovery implementation in which intensive one-on-one tutorial intervention was provided to selected low-achieving first grade children and whose teachers had received professional development training in the Count Me In Too program. In the second model, the students did not participate in the Math Recovery intervention program, however, their teachers did receive professional development training in the Count Me In Too program. The third model involved students who did not participate in the Math Recovery intervention program, and whose teachers did not receive professional development training in the Count Me In Too program. Count Me In Too is a numeracy project operating throughout New South Wales in their primary schools. It is designed to assist teachers in broadening their knowledge of how children learn mathematics by focusing on the strategies children use to solve numerical tasks (New South Wales Department of Education & Communities 1999-2011).

MacLean (2003) found the Math Recovery implementation model for students along with teacher professional development in Count Me In Too significantly out-performed both the on-going professional development only model and the control model where students did not participate in Math Recovery implementation model and their teachers did not receive professional development in Count Me In Too. MacLean's findings replicate similar findings by other researchers (e.g., Phillips, Leonard, Horton, Wright, & Stafford, 2003). Williams (2001) found that Mathematics Recovery training significantly changed teacher practice in the classroom in that teachers participating in Mathematics Recovery became more reform oriented in their teaching.

Munter's study (2010) that evaluated Mathematics Recovery found that the Math Recovery program could reduce some of the pre-K mathematics achievement gap. This

improvement suggests the cost of the program per student might be justified although further work is needed to understand why initial gains made by participants appear to diminish after tutoring ends. The study also suggests the forms of arithmetic reasoning that Math Recovery develops need to be further supported in the regular classroom to see the full benefit of this form of tutoring (Munter, 2010). Longitudinal studies tracking Mathematics Recovery students and their initially higher performing peers until the end of elementary school are needed to address this question adequately.

Section 2: Assessment

Assessment assumed an increasingly important role in education and, specifically, in mathematics education. With the advent of the No Child Left Behind Act in 2002, assessment has increased in importance to both schools and teachers and thus has received a great deal of national attention (Webb, 2007). The concern over how well American students are doing in mathematics has generated national efforts to improve standards, instruction, and assessment (Kulm, 1994). Sato and her colleagues conclude, “classroom assessment may be a particularly productive, if generally underused, lever for transforming practice in ways that support student learning” (Sato, Wei, & Darling-Hammond, 2008, p. 1). The objective of this reform is to improve the quality of instruction, assessment, and ultimately the learning of mathematics for all students.

In the current literature, assessment has been defined in a variety of ways. I first provide several contemporary definitions of assessment and follow with summaries on the origins of assessment, the purpose of assessment, effective use of assessment, formative assessment, types of assessment, and the role of assessment in classroom instruction. I then present an analysis of the impact of assessing young students’ understanding by exploring

various types of assessments focusing on alternative assessment strategies aligned with the constructivist paradigm. Next, I note the problems found in many traditional assessment instruments. Finally, I affirm the need for supporting continued research in the field of mathematics assessment.

Definitions of assessment. Some definitions for assessment align with a more traditional, behaviorist approach, and others with a more reform minded, constructivist approach. Overall, assessment is a process for making inferences about what students know and can do related to a content domain (Webb, 2007). Assessment also includes obtaining information about the skills an individual already demonstrates as well as their potential skills (Gardner, 1992). Further, assessment is the primary tool used to gauge how students gain in academic achievement or how much value has been added to the youngsters by their schooling (Kulm, Wilson, & Kitchen, 2005). Assessment is an essential tool for teachers because it allows them to diagnose, monitor, and evaluate student progress (Kulm, 1994). Kulm's definition aligns best with all the components of assessment addressed in this study, as the diagnostic component of assessment is an integral part of my research.

Origins of assessment. Looking at the history of assessment and the types of assessment used in the United States over the years clarifies our current use of, and practices in, assessing students' skills. Knowing where traditional views of testing came from, and understanding how closely they are connected with past models of testing, are important, as many times new theories are defined, understood, and sometimes contrasted, to prior theories (Shepard, 2001). A historical analysis of the development of assessment in the United States traces the use of written examinations back to schools in Boston, Massachusetts, in 1845 (Webb, 1992). Horace Mann championed these first examinations and proclaimed school

examinations as a superior way to examine all pupils (Wilson, 2007). In 1864 Reverend George Fisher used some of the first objective measures of achievement. His measures compared student work to a set of standards called “standard specimens” (Haertel & Herman, 2005, pp. 2-3).

Many of our current assessment practices are deeply rooted in past practices, such as true/false tests, multiple choice tests, and short answer tests where the one “right” answer is expected. If educators look at assessment in terms of uncovering student thinking, they must analyze the role assessment plays not only as an indicator of the level of achievement or mastery of a subject, but also the role it plays in the learning process. In essence, we as professional educators need to determine the purpose of assessment.

Purpose of assessment. Mathematical assessment is used for a wide range of purposes– from providing information to help a teacher work with a student to plotting a national strategy that will have broad implications for improving mathematics education for the nation (Webb, 1992). Wiliam (2007) found that, even when less formal assessments are utilized, the purpose is far more likely to be about making a determination of a student’s current state of knowledge. According to Kulm (1994), the primary purposes of assessment are for the improvement of instruction and learning, and for the evaluation of student achievement and progress. Assessment also includes providing feedback for the students, which will help them in seeing inappropriate strategies, thinking, or habits (Webb, 1992; Kulm, 1994). Clearly, both Webb and Kulm view assessment as more than an instrument that produces a few numbers or grades.

The purpose of assessment varies depending upon whom you ask. From some teachers’ standpoint, assessment’s main purpose should be to find out what is known, what is

not known, and what should be known about our students (Kulm, 1994). Shepard (2000) echoes those thoughts but argues that the fundamental purpose of assessment in classrooms must be changed so it is used to improve instruction rather than being used only to rank students or to show mastery of subject matter.

Effective use of assessment. Wiliam (2007) suggests effective use of assessment for learning includes sharing the reasons for learning and the criteria for success, promoting effective classroom discussions that elicit evidence of learning, providing feedback that moves learners forward, encouraging students to use one another as educational resources, and inspiring students to be the owners of their own learning. Fontana and Fernandes (1994) also stress the power of getting students to take ownership of their own work and, thus, of their own learning.

In addition, assessment should be an integral part of teaching. Teachers must critically examine their classroom assessment practices and the familiar methods of testing, scoring, and grading—practices that have been traditionally used to monitor student mastery of skills and procedures (Webb, Meyer, Gamoran, & Fu, 2004). If assessment is aligned with good instruction, it can support learning in many ways, such as when students continue the learning process by actually learning something while completing an assessment (Sternberg & Williams, 1998). If they find and develop strategies to assess problem solving and conceptual understanding, teachers can change the way they teach and, fundamentally, the way students learn mathematics (Kulm, 1994).

Formative assessment. When we think of classroom assessment procedures providing the necessary information to guide instruction, we look to assessment that is formative rather than rigid because, as the National Research Council (2009) concludes,

formative assessment helps teachers effectively guide their instruction to best help children learn mathematics. Formative assessment is the process of gaining insight into children's learning and thinking in the classroom and using that information to guide instruction (Black & William, 1998b) and improve instruction (Black & William, 2004). Assessment of this type gives teachers the necessary information to provide instruction for students that adjusts instruction to advance students' knowledge. The regular use of formative assessment improves students' learning (National Research Council, 2009) and is especially valuable if teachers have additional guidance on using the assessment results to design and individualize instruction (National Research Council, 2009; National Mathematics Advisory Panel, 2008; William, 2007).

Assessing the different types of mathematical knowledge. In mathematics, educators need to determine what type of mathematical knowledge their assessments are measuring. Those in the education community have come to realize assessment must be diverse and broad enough to assess varied categories of mathematical knowledge. Many state assessments and commercial tests are already emphasizing performance and problem solving, so this task may not be as difficult as it has been in the past. In fact, teachers may find their classroom evaluation approaches will need to change in order to catch up to the standards and expectations of some state assessments (Kulm, 1994). As educators consider designing assessment approaches, they need to develop a clear picture of the characteristics of the different types of mathematical knowledge being assessed (Kulm, 1994).

Procedural knowledge. Procedural knowledge is often identified with mathematical skills. Bell, Costello, and Kuchermann (1983) believe the term *knowledge* cannot be restricted to computational procedures of arithmetic and algebra but also must include any

multi-step procedure that may involve symbolic expressions, geometrical figures, or other mathematical representations. The *Principles and Standards for School Mathematics* states that the assessment of procedural knowledge should include whether or not students can recognize the correct procedures and invent or adapt them to fit situations (NCTM, 2000).

Conceptual knowledge is typically measured verbally and by a variety of tasks, whereas procedural knowledge is measured in terms of whether the answer is correct or not (Maciejewski, Mgombelo, & Savard, 2011). Traditional thought maintains that procedural knowledge is perhaps the easiest and quickest to observe and test (Kulm, 1994). Because of this belief, traditional testing has always focused on procedural knowledge and has included testing on the vocabulary of concepts. Many times children's understanding of the concepts behind the procedures is vague at best. Without the connection to conceptual knowledge, children acquire flawed or often memorized procedural knowledge that cannot be extended and applied to future problems (Maciejewski et al., 2011; Kulm, 1994). Therefore, in addition to assessing procedural knowledge, we must have an interest in assessing conceptual knowledge as well.

Conceptual knowledge. Carpenter (1986) defines conceptual knowledge as a type of knowledge and understanding fostering a rich network of relationships between pieces of information and that permits flexibility in accessing and using the information. Traditional approaches to assessing conceptual knowledge have included whether students can define a concept, show or choose an example of a concept, or be able to distinguish between concepts (Kulm, 1994). To demonstrate understanding of a concept requires much more insight than this rote memorization. To display evidence of conceptual knowledge, i.e. knowledge rich in relationships, students should be able to generate original examples and be able to translate

from written to symbolic form (Maciejewski et al., 2011; Kulm, 1994). To be able to communicate mathematics effectively, students must possess such a deep conceptual understanding.

When conceptual knowledge is paired with procedural knowledge, children are more likely to be able to strategize and problem solve. As we become more aware of the need to assess both procedural and conceptual knowledge, we must carefully analyze and modify, when necessary, our assessment tools. A review of the various types of assessments, both those in the traditional and alternative models, follows.

Types of Assessment. Assessment is typically categorized as being traditional or alternative.

Traditional assessment. For many years, educators thought of assessment in terms of tests. Assessment in traditional programs is characterized by end of the week and unit tests in which information is merely transferred from the student to the test (Ross, McDougall, Hogaboam-Gray, & LeSage, 2003). This is often thought of as summative assessment. Many times these assessments are constructed to evaluate how well students can reproduce information they received in the past. This view sees mathematics as an accumulation of facts, rules, and skills to be used to achieve some external end, such as a passing grade (Ernest, 1989). Mathematics is thought of as a set of unrelated but useful rules and facts. The “big picture” philosophy, where the brain’s search for patterns, broad concepts, or organizing principles to foster understanding, is not emphasized in this type of assessment (Fuson & Wearne, 1997).

Kulm (1994) affirms that many times traditional tests only furnish information on the procedures students are able to do correctly. Even more specifically, this approach made the

“right answer” the primary focus of assessment. Clearly, the focus of mathematics education has changed. Computation has become a minor goal, giving way to estimation, problem solving, use of computers, representation with manipulatives, and many other performance related outcomes (Kulm, 1994). However, there is a recognition of the importance of both procedural and conceptual knowledge with the goal of determining how to best instill both in our students (Maciejewski et al., 2011). Students should know more important ideas about procedures than simply how to perform them. That being said...

The NCTM *Principles and Standards*, for example, suggest students should know how to recognize when to use a particular procedure or strategy. Only when we consider the wide range in variation of individual abilities and the ways of thinking can we take time to reflect on whether traditional approaches are effective for most students (Kulm, 1994). Educators have become more aware of the need for assessment that gives greater insight into students’ understanding, and they value more highly the learning process and the students’ acquisition of conceptual skills than students’ computational skills. This shift in focus is the embodiment of alternative assessment. Testing focused narrowly on mathematical skills and procedures are only assessing one part of the knowledge necessary to use mathematics effectively. Without deep conceptual understanding and knowledge of strategies for solving problems, these skills are useless in real situations (Kulm, 1994). Large-scale standardized tests are prone to error that also adds to the need for other sources of assessment such as alternative assessments that provide added insight into student learning (Wilson, 2007).

Alternative assessment. Alternative assessment, measured through a variety of means, provides an opportunity to gain viable insight into students’ broad knowledge and understanding of mathematics, not just their skills and procedures (Hiebert & Carpenter,

1992; Kulm, 1994). Students taught by teachers who have developed the practice of using assessments for learning outscored comparable students in the same schools by approximately 0.3 standard deviations on both teacher-produced and state-mandated tests (Wiliam, Lee, Harrison, & Black, 2004). From the Trends in Mathematics and Science Study (TIMSS), one year's growth is measured as 0.36 standard deviations (Rodriguez, 2004), a measure that implies the effect of this practice can be seen to almost double the rate of student learning. The implementation of reform mathematics, in which assessment is also used for learning, requires developing assessments providing insight into problem solving, mathematical understanding, mathematical expression, and computational skills (National Research Council, 1989). To address these concerns, the nation is prepared to invest in assessment as a critical component of good teaching (Kulm, 1994).

To use alternative assessments, teachers must learn the skills to implement appropriate assessments. Such a shift is supported by researchers who advocate revising assessment practices to bring about changes in instruction based on how children learn (O'Day & Smith, 1993). As assessments are changing, many positive outcomes are surfacing. Greater attention is being paid to the individual student versus the masses with the use of alternate assessment, and evidence is appearing about the subsequent benefits of such a focus. A major goal of alternative assessment is to reveal individual strengths as well as areas in need of further development (Kulm, 1994). In addition, alternative assessment approaches and the use of multiple assessment formats require students to communicate their thinking in a variety of ways (Wiggins, 1993).

When properly used, alternative assessment eventually becomes an integral part of instruction and is no longer set off from the rest of classroom activity (Kulm, 1994). One of

the most exciting aspects of alternative assessment strategies is that a large group of students who have been excluded from further work in mathematics-related fields now can be successful (Kulm, 1994). Windows into student thinking, which were not observable in a traditional assessment model, are now providing insight into students' learning and processing. Alternative assessment creates the possibility, whether formally or informally, for teachers to see a wide range of mastery, abilities, and skills in many different forms (Corcoran, Dershimer, & Tichenor, 2004; Kulm, 1994). If more traditional tests are used exclusively, many characteristics of students can go unnoticed and unappreciated.

In order to be more equitable, alternative assessments have now come to encompass a whole new array of forms. A broad variety of assessment instruments can be used to provide feedback to teachers and students so active and interconnected mathematical knowledge can be constructed (Kulm, 1994). Only broad-based assessments can reflect the important, higher-order objectives integral to the reform mathematics curricula (National Research Council, 1989). Many types of alternative assessment tools can be used to uncover student thinking and evaluate student thinking in greater depth. Some of the most frequently used methods of alternative assessment include open-ended questions, investigations, experiments, student journals, observations, student portfolios, and group assessment.

When alternative assessment approaches include open-ended questions and presentations of solutions in both written and oral form, substantively different messages are sent to students about what is important in mathematics learning (Kulm, 1994). We can no longer tell students problem solving and creativity are important but assess them with traditional, correct answer tests that do not provide students the opportunity to have their thinking and strategizing be part of the evaluation process (Kulm, 1994). The individual

differences in the ways in which students display their mathematical knowledge are the strongest arguments for the use of multiple assessments (Kulm, 1994). A summary for each type of alternative assessment can be found in the Appendix C.

Problems with assessment. Traditional tests assess only a narrow portion of students' capability (Kulm, 1994). Further, the misuse of assessments can and often does result in dire consequences for the individual student. Unfortunately, the results from a traditional assessment tool can undermine a student's educational opportunities. Since tests can, and often do, have a profound influence on students and their lives, educators must have assurances the tests we use are fair. But how can we be sure our tests and the grades we use to measure student learning are fair? Is it fair to determine a student's educational fate based on the results of a single test? Unfair testing practices can have dire consequences on students' performance and progress because these "tests serve as 'gate keepers' allowing those who achieve best according to a prescribed norm to pass and failing those who might have creative or reflective approaches to addressing problems" (Kulm, 1994, p. 3).

In addition to tests that do not necessarily provide detailed information about student thinking, tests sometimes provide misleading data. In fact, research has shown many students who have mastered the symbolic manipulations—and may produce a correct answer—have a fragile or perhaps no conceptual understanding of the concepts involved (Kulm, 1994). Assessment is a valuable, powerful tool that can serve a multitude of purposes yet, in many instances, is flawed and needs to be improved. Accountability must be associated with the assessments we use. Clearly, assessments must be evaluated on a continual basis, especially when we consider the power assessment can yield.

Research on assessment methods. More than ever before, both students and teachers feel the pressure being exerted on them from state and district assessments to achieve high levels of performance (Webb, 1992). Some studies have investigated whether formative assessment improves student achievement (e.g., Black & Wiliam, 1998a, 1998b; Heritage, Kim, & Vendlinski, 2008), but more are needed. It is imperative that research and development efforts in the areas of assessment and evaluation be ongoing and expanded (Ginsburg et al., 2008). Because assessment results have assumed such a powerful role in not only determining the future course of a student's life but also in the funding allocated to school districts, more research must be done to ensure assessments are appropriate for what they intend to measure.

Prominent scholars in education are calling for intensified efforts to conduct research in the field of assessment to ensure these instruments are valid and fair for all students (Wilson, 2007; National Research Council, 1990, 2009). There is a continuing need to address how to align assessment with current teaching models and to evaluate how well assessment matches what we value and promote in student performance. However, in addition, research is necessary to ensure assessment is not only effective but also utilized appropriately. Teachers of mathematics must become effective assessors as well as teachers (Cain, Kenney, & Schloemer, 1994).

Section 3: Children's Cognitive Development

Research into children's cognitive development has advanced in diverse areas. The following discussion centers on studies of cognitive development and its impact on instruction and on historical views of children's cognitive development and, in particular, mathematics in early childhood. Next, I turn to constructivism, the basis for the AVMR

professional development program in this study, and allied topics including Cognitively Guided Instruction (CGI) and children's mathematical thinking, counting, and number sense. Of importance in this discussion are how children develop number sense, learn to calculate, and what teachers must know. Finally, my discussion affirms the need for supporting continued research in the field of children's cognitive development to improve instruction.

Research in cognitive development. The main foci of research in cognitive development are to describe the growth of children's basic concepts over time and to explain the processes by which these concepts are acquired and applied (Carpenter, 1980). Because of advances in the field of neurocognition, psychologists turned their attention to the significant data about the abilities young children possess, rather than privileging previous studies focused on what children lacked (National Research Council, 2000). Research-based information regarding the cognitive development of children, beginning as early as infancy, can improve how students are taught and assessed (National Research Council, 2000). Only recently has emerging bio-medical research (i.e., brain-compatible research) progressed enough to inform teachers concerning effective instructional strategies for the math curriculum (Fuson & Wearne, 1997; Geller & Smith, 2002; Gersten, Chard, Baker, & Lee, 2002; Sousa, 2001).

Historical views on cognitive development. For much of the twentieth century, most of the psychologists accepted the traditional belief, codified in the seventeenth century by John Locke, that a newborn's mind is a blank slate. Beginning in the 1920s, Jean Piaget moved away from the concept of a newborn's mind as a *tabula rasa* or blank slate. By closely observing infants and the responses from the careful questioning of children, Piaget concluded that cognitive development proceeds through certain stages, each involving

different cognitive schemes (National Research Council, 2000). He observed infants seek environmental stimulation, and the environmental stimulation, in turn, energized their intellectual development. However, he thought their initial representations of objects, space, time, causality, and self were constructed only gradually during the first two years (National Research Council, 2000).

Piaget also believed children did not possess number sense or the concept of number conservation until about five years of age (Devlin, 2000). Number conservation involves an understanding that, when items in a collection are rearranged, the arrangement does not change their number (Copeland, 1974). Piaget and his fellow constructivists suggested that children do not develop a conceptual understanding of arithmetic until they are seven or eight years of age (Sousa, 2008). Constructivists see the learning process as an active one in which students are encouraged to invent their own mental and problem solving models (Gentner & Stevens, 1983; McKeown & Beck, 1999; Robinson, 2003). The learner constructs new ideas and concepts based upon his or her current and past knowledge. Because each individual forges knowledge on his or her own, this approach is called constructivism (Steffe, 1988; von Glaserfeld, 1985).

Sousa (2008) states, “contemporary research on number sense dramatically undermines Jean Piaget’s constructivist views of 50 years ago” (p. 12). Nonetheless, Piaget’s influence can be found not only in our current beliefs about the way children learn but also in our educational systems (Devlin, 2000). Many educators, for instance, interpreted Piaget’s findings to mean a child is not ready for arithmetic until the age of six or seven. This belief, in turn, led to a practice of not teaching children mathematics earlier than first grade because educators assumed children would learn distorted number concepts and feel frustrated.

According to Sousa (2008), Piaget felt learning even simple arithmetic operations too soon would only generate feelings of anxiety about mathematics.

Piaget felt it was better to start teaching logic and the ordering of sets early in a child's development because these ideas are essential in helping students acquire the concept of number. Many pre-schools still operate on the principles of Piaget's theories. However, today educators are proposing theories of children's learning of number that are in direct conflict with the theories of Piaget. Researchers today recognize that many of Piaget's experimental procedures with children were flawed, thus leading to erroneous conclusions (National Research Council, 2009; Sousa, 2008).

Some studies, for example, show birds and rats recognize a certain number of objects and they also have an awareness of the spatial configuration of objects (Koehler, 1951; Mechner & Guevrekian, 1962). The question posed by Sousa (2008) is, "Why, then, would human children have to wait until the age of four or five to gain the same arithmetic capabilities of other animals?" (p. 13). The proposed answer is that they don't have to wait. Human infants are at least as capable as animals in arithmetic, and their ability to acquire number concepts grows rapidly within the first year of life (Sousa, 2008). In fact, at three or four days old, a baby can discriminate between collections of two and three items (Antell & Keating, 1983). In the first few months of life, babies notice the constancy of objects and detect differences in their numerical quantities (Wynn, 1998; Sousa, 2008).

Mathematics in early childhood. Numerous reasons point toward the recent surge of attention given to mathematics in early childhood. Researchers have moved away from the position that young children have either little or no knowledge or capability to learn mathematics (e.g., Piaget & Szeminska, 1952; Piaget, Inhelder, & Szeminska, 1960;

Thorndike, 1922) to the belief that mathematical competencies are either innate or develop in the first few years of life (Baroody, Lai, & Mix, 2006; Clements, Sarama, & DiBiase, 2004; Dehaene, 1997; Gelman & Gallistel, 1978; Perry & Dockett, 2002). A consensus emerged that considered children to be active learners who responded to their environments (National Research Council, 2000). From birth to age five, young children develop an informal, “everyday” mathematics, including such concepts as more and less, shape, size, taking away, location, and, sometimes surprisingly, broad and complex patterns (Ginsburg et al., 2008; National Research Council, 2009).

Vygotsky (1978) emphasized the active role of learners, and his theory concerning the Zone of Proximal Development remains an important contribution to the field of developmental psychology. The Zone of Proximal Development is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978). Vygotsky felt that what children can do with the assistance of others is more indicative of their mental development than what they can do alone. Vygotsky’s theory has tremendous implications for educating children.

Children, for the most part, receive significant instruction prior to entering school when it comes to number and, specifically, counting. Teachers who understand the development of number and counting in children can better assist their students’ mathematical growth. The emphasis on brain-compatible instruction can inform teachers about what specific instructional tactics may be more useful (Bender, 2002; Tomlinson, 1999). Cognitively Guided Instruction (CGI) with its emphasis on observing the strategies

children use to problem solve can be used to increase teacher understanding of the development of children's mathematical thought.

Cognitively Guided Instruction (CGI) and children's mathematical thinking.

Cognitively Guided Instruction (CGI), based on an integrated program of research, focuses on the development of students' mathematical thinking, on the instruction they receive that influences their thinking, on teachers' knowledge and beliefs that impact their instructional practice, and on the way in which students' instruction is impacted by their teachers' understanding of the students' mathematical thinking (Carpenter, Fennema, Franke, Levi, & Empson, 1999). CGI proposes that students construct knowledge rather than simply assimilate parts of what they are taught (Cobb, 1994; Davis, Maher, & Noddings, 1990). Understanding student thinking can provide coherence to teachers' pedagogical content knowledge and their knowledge of subject matter (Carpenter, Fennema, & Franke, 1996). Knowledge of students' pre-conceptions, conceptions, and misconceptions can, consequently, assist teachers in helping students learn new subjects (Grossman, 1990; Shulman, 1986; Wilson, Shulman, & Richert, 1987).

Significant adaptations in practice depend on teachers fundamentally altering their epistemological perspectives so they appreciate that students construct their own knowledge (Carpenter et al., 1996). Children are capable of learning when teachers comprehend how children think and, in turn, provide children an opportunity to build upon their own thinking (Carpenter et al., 1999). CGI builds upon the knowledge children develop before formal schooling begins and, thus, can have a great impact on how teachers help their students develop mathematics skills.

Research indicates that gaps in children's mathematical knowledge appear in large part because of the lack of connection between children's informal (pre-school) or intuitive knowledge (Ginsburg & Russell, 1981; Hiebert, 1986) and school mathematics (Clements & Sarama, 2007). Young children naturally think pre-mathematically and then mathematically in the same way they perceive language before producing it (Clements & Sarama, 2007). As Carpenter and his colleagues assert, "We have not clearly recognized how much young children understand about basic number ideas, and instruction in early mathematics too often has not capitalized on their rich store of informal knowledge" (Carpenter et al., 1999, p. xiv). Because many teachers have not recognized this knowledge, many children find the mathematics they have been taught in school is often disconnected from the ways in which they thought about mathematics and have used mathematics to solve problems in everyday life (Carpenter et al., 1999).

Carpenter et al. (1999, p. 3) provide a revealing illustration of children's mathematical thinking based on the following illustration.

Elizabeth had 8 cookies. She ate 3 of them. How many cookies does Elizabeth have left?

Elizabeth had 3 dollars to buy cookies. How many more dollars does she need to earn to have 8 dollars?

Elizabeth has 3 dollars. Tom has 8 dollars. How many more dollars does Tom have than Elizabeth?

Most adults would solve the three problems in much the same manner, by subtracting 3 from 8. Young children, however, see these as three different problems. According to Carpenter et al. (1999), a first grader was observed solving the first problem by putting out

eight counters, removing three of them, and then counting the ones that remained. For the second problem, the child started with three counters and added more until there were eight counters. She found she had to add five more counters to get eight counters, so her answer was five. For the third problem, she made two sets. One set had five counters, and the other had three counters. She lined up both sets so the set of three matched up with the set of eight. Next she counted the unmatched counters to determine how many more counters she needed. The child in each case directly modeled the problem according to how it was worded. This process came naturally to the child; she did not have to be taught a particular strategy goes with a particular type of problem.

In an environment that encourages children to problem solve with methods that are meaningful to them, they will construct strategies for themselves (Carpenter et al., 1999). The major premise for CGI is that children enter school with a great deal of intuitive knowledge that can serve as the basis for developing an understanding of mathematics (Carpenter et al., 1999). Without formal or direct instruction, children can construct solutions to a variety of problems, and perhaps their journey to being problem solvers begins with counting, then acquiring the number facts and the basic operations of addition, subtraction, multiplication, and division (Carpenter et al., 1999).

To summarize, young children have the ability to engage in significant mathematical thinking and learning that extend beyond that which is introduced in most school and pre-school programs (Aubrey, 1997; Clements, 1984; Geary, 1994; Griffin & Case, 1997; Klein & Starkey, 2004). By the time children begin school, most have learned to count and demonstrate remarkable insight about how to use their emerging counting to solve problems (Carpenter et al., 1999). More often than not, these insights and self-taught skills are not

exploited in formal instruction. Initially, young children have quite different conceptions of addition, subtraction, multiplication, and division; however, that realization does not mean their conceptions are wrong or misguided (Carpenter et al., 1999).

Human beings are born with some remarkable capabilities; one of these is language acquisition, and the other is an inborn sense of number (Sousa, 2008; Dehaene, 1997). This number sense includes the ability to determine the number of objects in a small collection, to count, and to perform simple addition and subtraction without any direct instruction. From this innate ability with math, many times children by the age of 10 are saying, “I can’t do math.” Rarely, if ever, do children say, “I can’t do language.” What causes mathematics to be such an obstacle? Too often when children reach elementary school, they are presented with complicated notions and mathematical procedures for which their brains are not ready. Both our culture and society have changed significantly in the last 5,000 years; however, the human brain, designed for survival, has not changed at all, and this constancy can pose a problem.

How does the brain cope with tasks for which it was not really prepared? More specifically, how does the brain cope with learning mathematics? Thanks to modern imaging devices that can look inside the human brain, we have the capability to see which cerebral circuits are called into play when the brain tackles a task for which it has limited innate capabilities (Sousa, 2008). The advances in cognitive research can provide substantial information concerning how the brain adapts to its environment and learns from the environment both informally and formally. This capability can help us gain insight into the age-old question, “Why is learning mathematics so hard?” With information from both the

field of cognitive psychology and the field of education, educators can now explore how children develop numerically beginning with counting.

Counting. When a person is performing basic arithmetic, the greatest brain activity is in the left parietal lobe and the region of the motor cortex that controls the fingers (Dehaene, Molko, Cohen & Wilson, 2004). The Latin word *digit* means both *numeral* and *finger*. It is quite suggestive the region of the brain used for counting is the same part used to control our fingers. Evidence from brain scans lends further support for this number-to-finger connection (Sousa, 2008). Neuroscience laboratories have provided evidence that has been used by clinical psychologists to verify a connection between finger control and numerical ability (Devlin, 2005). In fact, patients who sustain damage to the left parietal lobe often exhibit a condition known as Gerstmann's syndrome in which sufferers lack awareness of their individual fingers (Quine, 2006; Butterworth, 1999). Perhaps if this association had been understood in the education community, teachers would not have pushed children away from using their fingers when doing mathematics.

Wynn (1990) was among the first researchers to examine how young children conceptualize the how and why of counting. For the young mind, counting is a process using a one-to-one principle that assigns a number word to each object being counted (Sousa, 2008). Eventually, children will develop the cardinal principle where they associate the last number in the counting sequence with the total number of objects in the collection (National Research Council, 2009). Children who do not attain the cardinal principle will be delayed in their ability to add and subtract with meaning; that is, they may be able to perform the operation but do not have the related conceptual understanding (Sousa, 2008). These students will recognize addition as an increasing operation but will not start from the last

number counted. As a result, they will always recount each number when adding (Sousa, 2008).

Another research finding that has implications for understanding how number sense is developed is that of the internal number line. According to Gallistel and Gelman (2000) and Sousa (2008), humans possess a mental number line where numbers are envisioned as points on a line with 1 on the left, 2 to its right, then 3, and so on. When determining which of two numbers is larger, we view the digits on our internal number line; more specifically, we determine which number is on the right. However, unlike the traditional number line seen in elementary school, our mental number line does not have the numbers equally spaced out; the further to the right one goes, the closer the numbers appear to be, making it more difficult to distinguish between the larger of a pair of numbers as their value gets greater (Sousa, 2008, Gallistel & Gelman, 2000).

This finding tells educators that our internal number line may offer a limited degree of intuition about numbers (Sousa, 2008). Our ancestors only dealt with positive numbers, as there were no negative numbers in their environment. This historical fact helps to explain why some people have no intuition regarding other numbers that modern mathematicians use, such as negative numbers, fractions, or irrational numbers (Sousa, 2008). These numbers can be difficult for the average person today because they do not correspond to any natural category in the brain.

Researchers now have some deeper insights into how the cognitive aspects of number are generated within the mind; this knowledge must be used to improve children's acquisition of early arithmetic. Sousa (2008) affirms that number sense, while considered the innate beginnings of mathematical intelligence, must be nurtured. He states, "The extent to

which it [number sense] becomes an individual's major talent still rests with the type and strength of the genetic input and the environment in which the individual grows and learns" (pp. 5-6). Elementary school teachers should establish an environment that facilitates the development of number sense by drawing on information provided by cognitive neuroscientists as well as mathematics educators if they are to best serve their students. Certainly encouraging children to use their fingers to help them in structuring numbers to five and ten, spending time reinforcing the quantitative, verbal, and symbolic aspects of number, and providing children multiple resources to foster counting and the recognition of number relationships are all instrumental in establishing an environment conducive to developing number sense.

When teachers build on individual students' innate number sense, they are better able to facilitate the growth of number sense. Our intuition about numbers and their structure and properties can be referred to as number sense. Since the term *number sense* repeatedly surfaces in terms of children and their ability to do well mathematically, the term must be formally defined.

What is number sense? Tobias Danzig (1967) introduced the term *number sense* in 1954, describing it as a person's ability to recognize something has changed in a small collection when, without that person's knowledge, an object has been added or removed from the collection. Devlin (2000), a mathematician, refined the definition by suggesting that number sense consisted of two important components: the ability to compare the sizes of two collections shown at once, and the ability to remember numbers of objects presented successively in time. Bender (2009) states, "Number sense may best be understood as a student's conceptual understanding of basic number and numeration concepts such as

counting, or recognizing how many objects are present in a set, and how a number may be used to represent that set of objects.” Gerstan and Chard (1999) define number sense as “fluidity and flexibility” with numbers. Reys (1991) states that number sense refers to an intuitive feel for numbers, as well as for their various uses and interpretations.

In essence, children with number sense can relate real-world situations to appropriate corresponding numbers. They can express numbers in several different ways. For instance, 8 is $3 + 5$ as well as $4 + 4$. Students with number sense can also recognize the relative size of numbers. They know that 8 is more than 3 although they may not know what the actual difference is between the two numbers.

In contrast, children without number sense may be able to count, recognize the figure that symbolizes the number, and write or point to the numeral “8,” but they do not comprehend the actual meaning of the number (Gerstan & Chard, 1999). They cannot tell for instance whether 8 is more than 6. Many children without number sense do not have the concept of what numbers mean or the fact that numbers may be used to represent objects in a set.

As children progress to middle school, those without a well-developed number sense may not realize that $\frac{5}{8}$ and $\frac{2}{3}$ are larger than $\frac{1}{2}$ because of the numerator/denominator relationship. Whenever the numerator is larger than half the value of the denominator or when the numerator is almost as large as the denominator, the fraction will be greater than $\frac{1}{2}$. Number sense begins with whole numbers, yet the basic sense of number relationships carries through to thought beyond whole numbers as in this case. The value of having number sense is clearly seen, as well as the negative future impact on children who don’t have a well-established number sense.

How then do educators view number sense? What do teachers expect of their students in terms of their number sense? When Berch (2005) reviewed the literature on cognitive development, mathematics cognition, and mathematics education, he found mathematics educators considered number sense to be more complex and multifaceted in nature than did cognitive neuroscientists. According to Berch's review, mathematics educators expand number sense to include the ability to recognize the change in a collection of objects when, without direct knowledge, an object has been added to or removed from a collection. Educators also include in their definition of number sense the capability to have elementary abilities or intuitions about numbers and arithmetic, an ability to make mental magnitude comparisons, and an ability to break numbers apart (e.g., $5 = 1 + 4$, and $5 = 2 + 3$). In addition, number sense includes an ability to develop useful strategies for solving complex problems, an ability to use arithmetic operations and understand the base-ten number system, and an ability to use numbers and quantitative methods to communicate, process, and interpret information.

According to educators, those with number sense have an awareness of levels of accuracy for calculations, a desire to make sense of numerical situations by making associations linking new information and previously acquired knowledge, a knowledge of the effects of operations on numbers, a fluency and flexibility with numbers, and an understanding of number meanings. Number sense also presumes a recognition of gross numerical errors, an understanding of numbers as tools to measure things in the real world, and the capability to invent procedures for conducting numerical operations and the ability to think or talk in a sensible way about the general properties of a numerical problem or expression without doing any precise calculations (Berch, 2005).

Portions of this more expansive view of number sense already appear as one of the five content standards of the National Council of Teachers of Mathematics' *Principles and Standards for Mathematics* (NCTM, 2000). According to Sousa (2008), parts of number sense are also found in mathematics textbooks and as a distinct set of test items included in the mathematics portion of the National Assessment of Educational Progress (NAEP), the Trends in International Mathematics and Science Study (TIMSS), and the Program for International Student Assessment (PISA).

With a general view of what number sense entails from both a cognitive neuroscientist's perspective as well as that of the mathematics' educator, natural questions arise: When does number sense appear, and how does number sense develop in a child's life? In fact, do all of us have number sense, and, if so, do some individuals have more of it than others? It is important at this point to examine how number sense develops from infancy and why number sense can, for some children, be shut down after participating in formal schooling.

Developing number sense in the early years. Griffin (2002) created a model suggesting the development of number sense goes through three major phases. First, an individual's visual processing system recognizes the objects in a collection. With small collections, we have an innate capacity to subitize (from the Latin word for *sudden*) or to see the amount instantly. During the first half of the twentieth century, researchers believed subitizing rather than counting implied a true understanding of number sense (Douglass, 1925; Clements & Sarama, 2007). Baroody (1987) posits, "subitizing is a fundamental skill in the development of students' understanding of number" (p. 115). Subitizing can be seen as a different process than estimating or counting in that, when the number in a collection

exceeds an amount that can be subitized, counting becomes necessary. Many educators saw the role of subitizing as a developmental prerequisite to counting (Clements & Sarama, 2007). When quantities grow larger, Griffin suggests people move into the second phase where they create number words to communicate an exact amount. According to Griffin, the third phase occurs when an individual realizes writing number words for large quantities is tedious and, therefore, creates symbols and operational signs to make expressing numbers more efficient.

Another way humans perform better mathematically than all other species is by being able to move beyond the basic one, two, three, ... to handle much larger numbers; other species are not able to account for numbers that are larger. A reminder that counting is a skill humans acquire, and not an ability they are born with, comes from studies of so-called primitive societies that do not have counting beyond two (Devlin, 2005). In order to expand beyond simple numbers, humans adopt a different method based on counting that uses different mental abilities located in a different region of the brain from number sense (Devlin, 2005). Closely connected to counting is the human use of arbitrary symbols to denote numbers and to manipulate numbers by the manipulation of those symbols (Devlin, 2005). These two human attributes enable us to take the first step from an innate number sense to the vast and powerful world of mathematics (Devlin, 2005). Counting is not merely saying how many members are in a collection. The number of members in a collection is simply a fact about the collection. Counting the members of a collection, on the other hand, is a process involving ordering the collection in some fashion, then going through the collection in that order, and counting off the members one by one (Devlin, 2005). Very young children see counting and number as unconnected (Devlin, 2005). If you ask a young

child to count his toys, he may say the counting correctly: “one, two, three, four, five.” He may even point to each toy as he counts. But, if you then ask him how many toys he has, he may very well say any number that pops into his head. Around four years of age, children begin to realize counting is a means to discover “how many” (Devlin, 2005). Children at this point begin to realize the order in which you count a collection does not matter; the number you finally reach is always the same.

Children become capable of demonstrating a wide range of math competencies such as numerical estimation, comparison, simple addition and subtraction; these competencies all emerge spontaneously without much explicit instruction (Griffin, 2003). A major reorganization in children’s thinking occurs around the age of five years old when the cognitive structures created in earlier years, such as global quantity and counting, integrate (Griffin, 2002). Global quantity relies on subitizing to determine, for example, which of two stacks of chips is larger; counting is used to count a small number of objects, mainly through one-to-one correspondence with fingers. Griffin (2002) explains this process as one in which the neural connections between the area of the brain responsible for global quantity and the area of the brain responsible for counting become stronger and allow for the formation of a larger structure representing the mental number line. Children’s mathematics competence, and the cognitive and neurological structures supporting it, is flourishing and fairly well developed before they start formal schooling at age six (Griffin, 2003). By the time children begin school, most have accumulated considerable relevant knowledge about arithmetic (National Research Council, 2000). Children also begin to realize that each number word occurs in a fixed sequence and that each number word can be assigned to only one object in a collection. Children also understand that the last number word said indicates the size of the

collection. Most children can count to five and some can count to ten. However, most children still rely on subitizing to make a quantity determination.

As we consider subitizing and then counting, we really are interested in the ability of children to make sense of numbers. How and when does this phenomenon happen? Aspects of number sense seem to be chronologically revealed, an observation leading researchers to ponder why some children lack certain capabilities regarding number sense. If number sense is innate, as educators, what can be done to stimulate and strengthen children's number sense?

Can number sense be taught? Those who view number sense as an intrinsic ability will argue that the fundamental components for number sense are genetically programmed, have a long evolutionary history, and develop spontaneously without explicit instruction as a young child interacts with the environment (Sousa, 2008). However, these researchers do not see number sense as a fixed entity. Rather, they suggest the neurocognitive systems supporting these elementary numerical abilities provide the foundational structure needed for acquiring more advanced abilities cited by mathematics educators (Sousa, 2008).

Researchers recognize both formal and informal instruction can enhance number sense development prior to entering school (Wright, 1996; Sousa, 2008). This perception supports Vygotsky's theory of the Zone of Proximal Development.

Developing an understanding of number, of how to manipulate, operate, and represent numbers, is one of the fundamental and important mathematical tasks for children during the early childhood years (National Research Council, 2009). Further, just as phonemic awareness is prerequisite to learning phonics and becoming a successful reader, developing number sense is a prerequisite for succeeding in mathematics. In addition, Gersten and

Chard (1999) propose that number sense is the missing component in the learning of early arithmetic facts and might explain the reason rote drill and practice alone do not lead to significant improvement in mathematics ability.

Gersten and Chard (1999) believe number sense is critical to success in learning mathematics. They have identified five levels that allow a teacher to assess children's understanding of number sense. Children at Level 1 have not developed a number sense beyond their innate notions of numerosity or their perception of approximate numerical quantities (Gerstan & Chard, 1999). A child may perceive one collection is more than or less than another without assigning an exact number. However, the child has no sense of relative quantity and may not actually know the difference between "less than and "more than."

At Level 2, children start to acquire number sense (Gerstan & Chard, 1999). They begin to understand the concepts of "less than" and "greater than" and what "three" and "nine" mean, but they don't have basic computational skills. When children reach Level 3, they fully understand "less than" and "greater than" (Gerstan & Chard, 1999). They now have a concept of what it means to compute and may use their fingers or objects to apply the "count up from one" strategy to solve problems. When a child is calculating numbers higher than five, errors start to occur because this computation requires the child to use both hands.

At Level 4 children readily use the "count up" or "counting on" process instead of "counting all," which was done at earlier levels (Gerstan & Chard, 1999). They have a conceptual reality of numbers in that they do not have to count to five in order to know five exists. Children at this level are able to solve digit problems such as $5 + 3 = ?$ routinely. The final assessment level, Level 5, is one in which children can use retrieval strategies for solving problems (Gerstan & Chard, 1999). Retrieval strategies would account for children

solving problems such as: “I have eight pieces of candy and give three to my friend. How many pieces of candy to I have now?” A child would not have to model this problem but would automatically know $8 - 3 = 5$. At this phase, children have automated addition facts and are acquiring basic subtraction facts.

As we consider the question “Can number sense be taught?” I would offer that it may not be possible to *teach* number sense, but teachers can be taught to recognize the levels of mastery within the construct of number sense and certainly create environments that stimulate children to progress to the next level of number sense mastery. Number sense appears to be developmentally acquired; children have fundamental intuitions regarding number, and children can be put in learning situations that stimulate them to advance further in their capabilities regarding number. As children develop and progress mathematically, they move through the levels of understanding number. When they reach Levels 4 and 5, children are essentially beginning to calculate.

Learning to calculate. Children actually begin to calculate when they add two sets together using their fingers. Prior counting of small quantities seems to be easily acquired; sometimes this counting ability occurs rather spontaneously or can occur by watching others count repeatedly. Children gradually progress to adding without their fingers. By the age of five, children understand the principles of commutativity of addition: $3 + 4 = 4 + 3$, for example. As calculations become more difficult, the rate of errors increases, not only for children but adults as well. Sousa (2008) states, “One thing is certain, the human brain has problems with calculations” (p. 35). Even though humans are born with a capacity to approximate numerical quantities, dealing with exact symbolic calculations can be problematic.

No one completely understands how number structures develop in young children. Recent developments in cognitive neuroscience have given valuable clues about brain development and how number structures evolve. Griffin (2002) and her colleagues reviewed research and developed tests that assessed large groups of children between the ages of three and 11 concerning their knowledge of numbers, units of time, and money denominations. Based on student performance on these tests, Griffin describes the following generalizations about the development of conceptual structures related to numbers in children within this age group:

1. Major reorganization in children's thinking occurs around the age of five when cognitive structures created in earlier years are integrated into a hierarchy.
2. Important changes in cognitive structures occur about every two years during the development period. The typical changes occur between ages 3 to 5, 5 to 7, 7 to 9, and 9 to 11.
3. This developmental progression is typical for approximately 60% of children in a modern, developed culture. About 20% will develop at a faster rate and about 20% will develop at a slower rate (Griffin, 2002, pp.1-32).

Information from assessments such as those carried out by Griffin and other researchers, as well as information provided from the field of cognitive neuroscience, can significantly impact teacher practice. Teachers must receive and utilize research data to better understand children's stages of mathematical development.

What teachers must know about how children learn. The National Council of Teachers of Mathematics (NCTM, 1989; 2000) developed principles and standards for school

mathematics based on the assumption that, if teachers have a clearer idea of the knowledge they are expected to have to teach at each grade level and the manner in which this knowledge develops, they will have an easier time teaching it and will achieve greater success in the process (Royer, 2003). The Common Core State Standards (CCSS) for Mathematics provides a useful guideline for teachers to utilize in planning instruction by providing the mathematical expectancies for students at each grade level.

Add+VantageMR® provides teachers a framework that can be used to assess students in K-5, and materials and tools to facilitate growth and mastery of the mathematical standards for the elementary grades as designated by the CCSS (see Appendix D). Teachers can use the emerging research base about teaching and learning to help them construct lessons promoting learning with understanding. This research base describes how students themselves construct meaning for mathematical concepts and processes and how classrooms can support that kind of learning (Carpenter & Lehrer, 1999). When they have knowledge of the complexity of mathematical thinking, teachers can adapt lessons to build connections that facilitate children's learning of mathematics.

Mathematical thinking is a highly complex process involving a number of areas within the human brain, including, at a minimum, the frontal lobe, the parietal lobe, the visual cortex, the angular gyrus, Wernicke's area, and Broca's area (Bender, 2005). The frontal and parietal lobes of the cerebrum are responsible for higher order thinking skills; the visual cortex is responsible for helping children visualize math problems; the angular gyrus decodes sounds and processes language; Wernicke's area is involved with language comprehension, and Broca's area searches for meaning in the context of the numeral and its relation to other numerals presented in a problem.

Perhaps one of the most significant findings for teachers to know is related to Broca's area of the brain. In short, teachers must take the time to show the importance of mathematics in students' daily lives in order for early learning to be successful (Bender, 2005). Teachers should not only be aware of, and knowledgeable about, students' mathematical learning they should comprehend how such awareness and knowledge significantly contribute to various aspects of the practice of teaching (Even & Tirosh, 2002).

Another critical factor for teachers to consider is that of affect. Based on research completed within the past twenty years or so, educators have realized that emotion and emotional intent play a much larger role in learning than previously thought (Bender, 2002; Sousa, 2001). Much of the information taken into the brain by the senses is first processed in the midbrain or "emotional brain." The emotional brain often serves as a filter through which stimuli must pass before being "considered" by the cerebrum or "thinking" areas of the brain (Bender, 2005). A negative emotional response to a particular task can, in and of itself, result in a lack of higher brain function involvement with the problem (Bender, 2005). Research has frequently shown that many students perceive math negatively or even fear math (Montague, 1997). In light of this research, it is critical that teachers take into account a student's attitude towards math and the student's motivation to learn math. Both factors are critical components in teaching students mathematics.

To address students' attitudes and motivation, teachers should both find ways to use "math-play" activities to make mathematics less threatening and to scaffold students' work to assist students in their mathematics learning (Bender, 2005). These types of teacher interventions may, over time, offset the negative feelings many students have toward math. Teachers have several effective ways to move past students' negative math emotions.

According to Bender (2005), novelty in teaching, color-coding, or novel presentations of new information can greatly assist students in focusing on the mathematical content to be mastered. Novelty in teaching could include a variety of practical multi-sensory teaching strategies such as the use of chants, music, and movement-based activities such as touching the head, or shaking the fingers.

Teacher's presentation of material is extremely important. One brain function that allows the brain to filter information involves the brain's search for patterns, broad concepts, or organizing principles (Fuson & Wearne, 1997). Broad concepts allow the brain to categorize and classify knowledge. This capacity allows the brain to store knowledge in a somewhat organized "file cabinet" based on some of the big concepts or broad ideas. These would include ideas such as the base ten number system and concepts based on that system such as place value, expanded notation, and the commutative, associative, and distributive properties (Harniss et al., 2002; Wiggins & McTighe, 1998). In order to get past the filtering function of children's brains, teachers should repeatedly address the "big ideas" in each math unit and help children make connections among these big ideas. With appropriate instruction based on these critical concepts, almost all learners can master the basic mathematics curriculum content in the elementary grades (Bender, 2005).

The reform movement in mathematics has several guidelines for appropriate instruction. A clear direction for reform, one that is grounded in research on how children think and learn, has been available since 1989 (NCTM, 1989, 2000). Since number sense provides a window into children's thinking and is critical to success in learning mathematics, teachers find that incorporating number sense at all grade levels is important. Gurganus (2004) agrees that number sense is analogous to phonemic awareness; but, unlike phonemic

awareness, number sense develops throughout students' mathematics education. She has developed a set of suggestions for teachers to promote number sense across the grade levels. Some of the activities Gurganus encourages teachers to incorporate into their work with students are

1. pair numbers with meaningful objects,
2. use language to gradually match numbers with objects and symbols,
3. incorporate counting activities in the classroom. For example, a teacher can ask younger students to count to ten and back. Older students can be challenged to count by 2s, 5s, 10s, or even by 3s, 4s, 7s, or 8s,
4. provide students experience with number lines,
5. plan meaningful estimation experiences, provide their students with objects that cannot be measured precisely to allow for estimation practice,
6. have students measure and then make measurement estimates,
7. introduce materials that involve numbers or number representations,
8. have students examine items such as rulers, clocks, dice, dominoes, playing cards, and coins,
9. read literature that involves numbers,
10. have children create magic number squares,
11. manipulate different representations of the same quantity (Examples would include modeling moving back and forth between fractions, decimals, and percent as well as modeling the same length with different units such as millimeters, centimeters and meters.),
12. explore very large numbers and their representation,

13. collect and chart data,
14. compare number presentations in other cultures,
15. solve problems and compare the reasonableness of the solution,
16. find everyday functional uses for numbers (Examples can be tracking a company on the stock market, calculating sale prices at the store, and following favorite sports team's averages.),
17. explore unusual numbers, (Fibonacci numbers, the golden ratio, abundant numbers, palindromes and perfect numbers can all be used to add fascination for the student.), or
18. model the enjoyment of numbers and number patterns.

Research has shown repeatedly that the teacher is the most critical factor for establishing a climate of curiosity and enjoyment of mathematics. The list of suggestions for promoting number sense and helping children grow mathematically offered by Gurganus (2004) are among a group of methods or learning theories that can be classified as constructivist in theory.

Chapter 3: Methodology

This chapter describes the research design of my study, the protocols used to select participants, and the methodology used to collect and analyze the data.

My research questions are:

1. At what level in the Add+VantageMR® (AVMR) classification scheme was the participating teacher after attending initial AVMR professional development training, and what was the impact of the training on the teacher's instruction?
2. How does a teacher who receives AVMR professional development training utilize the AVMR assessment results to inform instruction for a small group of students?
3. Does the performance of this group of students improve when the teacher modifies her teaching by applying their assessment results?

The topic for the research was chosen because, as Jenny Cobb, past president of the U.S. Math Recovery Council, has noted, no studies have been published on the effect of AVMR training on teacher instruction and development, or on how that training may impact student achievement. Several research studies that have been done on the effectiveness of Mathematics Recovery (Smith et al., 2007; Willey, Holliday, & Martland, 2007; Phillips et al., 2003; MacLean, 2003) involve using the same assessments, strategies, tools, and concepts as AVMR (J. Cobb, personal communication, May 28, 2011). In particular, as teachers seek opportunities to increase students' understanding and achievement through various teacher professional development programs in early mathematics, studies such as this

one seek viable means for continually evaluating these programs to ensure they meet current and future needs.

AVMR is intended for use with all students in the classroom and, as such, can reach many more students than Mathematics Recovery at a much lower cost. Mathematics Recovery is an intense one-on-one intervention for individual students rather than a program that can be applied to whole class instruction. Unlike AVMR, Mathematics Recovery can only reach a few students per year, and as an intervention can only be given by a Mathematics Recovery Intervention Specialist (MRIS). The cost for training an intervention specialist is high, approximately \$5500 per individual.

For my case study, I focused on a first grade teacher, Holly, (fictitious name) in the Aspen School District (fictitious name). This study examines Holly's AVMR implementation level after initial training, the impact of the AVMR training on her instruction, and the performance of the three children participating in the study. Rather than attempt large, complex data analyses typical of quantitative methodology, I used qualitative research methods because such methods allow an investigator to study selected issues in greater depth and detail. According to Shulman (1992) case histories, including specific stories about classroom experience, can enrich our collective "wisdom of practice." Qualitative research methods are useful tools that allow for the exploration of areas about which little information is known (Gay & Airasian, 2003; Patton, 1990) because these methods favor smaller groups of subjects; however, qualitative research methods may reduce generalizability. That is, qualitative research methodology favors a focus on the detail, the depth, and the triangulation capability found in smaller sample populations—the questions of *why* and *how* rather than those of *where*, *when*, or *how many*. I added quantitative research

methods to my observations, field notes, and teacher interviews by comparing pre and post-test results to those of a normative group previously investigated (Briand, 2012). Thus my study can be characterized as a characteristically mixed methods research project.

Research Design

The methodologies used for this study permitted me to determine how one teacher, Holly, used the AVMR procedures and assessment information with three first grade students—Jasmine, Josh, and Johanna (fictitious names)—to design student-appropriate instruction for each of them, and the impact Holly’s instruction had on the mathematical performance of these three students. To answer my first question, I used participant observation and field notes to determine the implementation level—high, medium or low—that best categorized how Holly used AVMR techniques, strategies, and tools in her classroom. To classify her level of implementation, and the impact of AVMR on her instruction, I observed her classroom activities at least once a week from August 2011 through December 2011 and continued observing on a monthly basis from January 2012 through April 2012 for a total of 18 observations. I compared Holly’s level of implementation to that of three other teachers studied in an earlier research project (Briand, 2012) that examined the impact of AVMR on teachers’ instruction with respect to teaching early mathematics and the subsequent impact of teacher’s AVMR training on students’ learning in the Aspen School District (ASD), New Mexico.

To address my second research question, I documented Holly’s use of the AVMR student assessments to inform classroom instruction. To do so, I made 14 classroom visits between August 2011 and December 2011 and recorded as field notes how Holly applied AVMR tools, strategies, and activities in the classroom to address Jasmine’s, Josh’s, and

Johanna's specific learning concerns. Quantitative methods, including each child's AVMR pre and post-test assessment, were also used to classify each student's learning level. To answer my third question, I investigated the mathematical understanding and gains of this small group of students by collecting and comparing the three students' worksheets from their Everyday Math Journals and their scores on the AVMR pre and post-tests for the eight-month period, August 2011 through April 2012. For this comparison I used a Two-Sample T-Test to determine whether the means of my case study and of the sample population in the earlier research project (Briand, 2012) differed.

The purpose of the study was to further understand the impact of the AVMR student assessments on teacher instruction and its potential impact on student achievement. Perhaps the most defining feature of AVMR professional development is its interview-based assessment component. The assessments provide information on what students know rather than only on what they don't know. Using student assessment information is critical to guiding effective teacher instruction, as assessment serves to inform teachers as they create instructional models and assist students. Information gained from this study may be used to inform administrators and teachers about the possible use of AVMR as a potential professional development program for their school districts.

Even though my sample was small, three students, my research question posits these students will exhibit greater understanding and achievement in the early number categories of Forward Number Word Sequence (FNWS), Number Word After (NWA), Numeral Identification, Backward Number Word Sequence (BNWS), Number Word Before (NWB), Numeral Sequences, Structuring Numbers which includes (Spatial or Dot Patterns, Finger Patterns, Combinations to 5 and 10, Combinations to 20) and Addition and Subtraction.

However, since there is no control group, I cannot infer AVMR was the cause for any change. The study followed the timeline provided in Table 3.1.

Table 3.1

Outline of Study

Order of Events	Description of Events
I AVMR Course One Training June 6 – 9, 2011	Teachers received four days of Course One training to include the early number categories of Forward Number Word Sequence (FNWS), Number Word After (NWA), Numeral Identification, Backward Number Word Sequence (BNWS), Number Word Before (NWB), Numeral Sequences, Structuring Numbers (Spatial Patterns, Finger Patterns, Combinations to 5 and 10, Combinations to 20) and Addition and Subtraction. I attended the AVMR training to meet Holly, the teacher-subject of the case study, and documented the AVMR concepts, strategies, activities, and assessment strategies presented.
II Classroom Observations began on a weekly basis on August 16, 2011	I observed Holly’s class, starting the first week of school (August 16, 2011), every Tuesday and Thursday until the end of the term (December 2011), thereafter once a month until April 2012. Audio and videotapes were recorded in addition to the Teacher and Student Observation Protocols. Interviews were conducted with Holly using an audiotape.
III AVMR Pre-test Administered to Students Mid-August 2011	A Mathematics Recovery Intervention Specialist (MRIS) administered the AVMR assessments to the three selected students. Their assessment results were used to indicate the AVMR tasks students were not able to perform and those that they were as a guideline for my observations.
IV Student Clinical Interviews	I informally evaluated student progress by talking to and observing each of the three subject-students as well as other students in the class asking them questions aligned with AVMR tasks.
V AVMR Post-test Administered to students in early December, 2011& April 2012	A Mathematics Recovery Intervention Specialist (MRIS) administered the post AVMR assessments to the three selected students. I studied the student assessments; student produced artifacts, and analyzed the data to determine student gains.

Below, I outline the selected school district and review that district's background information, characteristics, and reasons for its selection.

Location. This research study took place in the rural community of Aspen, New Mexico. The town is located along historic Route 66, one of the original highways connecting the Midwest to the West. The City of Aspen covers approximately 4.8 square miles with a population of 1910 (U.S. Census, 2010) and a median household income of \$25,150. Approximately 68% of the population is Anglo, 44% is Hispanic, and the remaining citizenry being American Indian, Black, or Asian (CityData.com, 2012). The area historically had been a ranching and farming community.

Data from the U.S. Census Bureau indicated that in 2011 New Mexico had 30% of children living at or below poverty level compared to the national figure of 21.6%. Only Mississippi had more children living at or below the poverty level, at 32%. The Aspen School District, had 33.6% of children living at or below the poverty level. The Aspen School District is a joint school district serving the towns of Aspen, and Oak. The district is comprised of five elementary schools with 2,150 students, two middle schools with 767 students, and one high school with 1,044 students. The student population is 43.7% Caucasian, 52% Hispanic, 3% American Indian, and 1% African American (NMPED 2009-2010 Accountability Report).

School district selection. I chose the Aspen School District (ASD) to conduct the study for several reasons. I began working with the Aspen School District in 2008 when funding was obtained from the New Mexico Public Education Department (NMPED) Math and Science Bureau to provide AVMR training for 16 schools located in a large school New Mexico school district, five schools located in a smaller school district, and five Aspen

Schools. Because of this funding, I had the opportunity to work with three school districts and meet many of their teachers and Mathematics Instructional Coaches. In particular, I had spent a significant amount of time with the ASD teachers and their District Mathematics Coach, establishing a good working relationship with them.

An important factor in my choosing the ASD to conduct this study is the district had already received three years of AVMR training; and, in that time period, they embraced AVMR. With the latest cohort of 11 teachers trained during the summer of 2011, there were now a total of 53 AVMR trained teachers in the Aspen School District.

Furthermore, the Mathematics Instructional Coach achieved an impressive record of accomplishments. She completed AVMR Course One and Two training, became a Mathematics Recovery Intervention Specialist, trained to become a Mathematics Recovery Champion, and created a three-year plan outlining the educational goals for the ASD. Like other Mathematics Recovery Champions, she is now certified to provide AVMR Course One and Two training for teachers in their district at a reduced cost. The ASD Mathematics Instructional Coach achieved these goals, and provided AVMR training for all K-2nd grade teachers, as well as other interested teachers, both for the elementary and middle school grades.

Because ASD and the Mathematics Instructional Coach actively supported AVMR, they created the necessary conditions for me to evaluate the impact of AVMR teacher training on the teacher's instruction, the teacher's use of the AVMR assessment component to guide instruction, and, ultimately, the impact the AVMR enriched instruction had on student achievement. In order to determine the effectiveness of an intervention, teachers must fully utilize it. Because AVMR was, and still is, so strongly supported in the district

both in terms of its acceptance and the follow-up support given to the teachers by the Mathematics Instructional Coach, I believed Holly would exploit what she had learned in AVMR training in her classroom. Another factor in my selection of the ASD was the strong support provided by district administrators. The Associate Superintendent welcomed and strongly supported my dissertation study and an additional study on the impact of AVMR on student achievement in their district based on a sample of approximately 54 students beginning in August 2011 (Briand, 2012).

The funding received in 2008 and 2009 from the NMPED Math and Science Bureau was, unfortunately, terminated in 2010 because of severe state budget cutbacks. Fortunately, in March 2011, funding from Sandia National Laboratories, Albuquerque, New Mexico, was granted and provided AVMR Course One training for ASD teachers during the summer of 2011, AVMR Course Two training in the fall of 2011, training for two previously trained AVMR teachers to become Mathematics Recovery Intervention Specialists, and supported a mixed methods study on the achievement outcomes for students whose teachers had been AVMR trained. This study complemented the Sandia National Labs study; however, my study focused more on the specifics of how the teacher used assessment data to inform her teaching and how students grew in their mathematical knowledge vis-a`-vis AVMR than it did on the student achievement comparison between students in the classes of teachers who were AVMR trained and teachers who were not.

Teacher selection. The Mathematics Instructional Coach for ASD selected a first grade teacher whom she felt would be an excellent candidate to participate in the study because she had noticed the teacher was enthusiastic about the prospect of being AVMR trained. The in-depth case study focused on this teacher, who is referred to as Holly. I

closely observed her class to document her use of AVMR student assessment results to inform instruction.

Teacher background. Holly is a Hispanic female who was in her mid- to late-twenties. Her family is from a town neighboring Aspen, and her mother, as well as brother and sister, are educators. She graduated from the University of New Mexico, and, coincidentally, she was a student of mine while at the University. She had been teaching first grade for two years at Aspen Elementary School at the time of this study. The initial teacher interview revealed that Holly had loved mathematics her entire life, a subject in which she always excelled. She strongly believes children are hands-on learners especially in math. In her two years of teaching, she found students learn mathematics concepts more easily and more quickly with manipulatives and through mathematics games. She teaches her students in small groups that rotate through mathematics centers each day. During her first year of teaching, she taught in a whole group setting the majority of the time; in her second year of teaching, she transitioned to a system in which she used small groups approximately 30% of the time. Her goal for her third year of teaching, the year of this study, was to use small groups to teach mathematics 100% of the time.

Prior to receiving training in AVMR Course One during the summer of 2011, Holly had only received specialized training in mathematics at a one-day Everyday Mathematics Institute. Holly shared with me that, although she was able to recognize if students were having problems in mathematics, she struggled to pinpoint exactly what the skill deficit was and needed information on how to target skill deficits and move students on to the next level.

Holly characterized her school as a failing school with a high percentage of students who lived in poverty conditions and lack home support. Aspen Elementary School had not met Adequate Yearly Progress (AYP) in either mathematics or reading for school years 2007-2008, 2008-2009, or 2009-2010. It was currently designated as an SI-2 school, meaning the school had not made AYP for three consecutive years. Supplemental educational services were made available to the school as well as technical assistance. If AYP was not met the following year, 2010-2011, the school faced corrective action that could include replacing school staff relevant to the failure or appointing outside experts to advise the school. Budget cuts reducing support staff at her school made meeting each child's need difficult if not impossible.

Holly believed students learn mathematics concepts best in small groups using manipulatives, hands-on activities, and mathematics games. She wanted her students to understand concepts and be able to explain how they got their answers. She used daily assessments from the Everyday Mathematics Program to plan and guide her mathematics interventions used to target skill deficits. She wanted to learn how to better target skill deficits in children and advance these students' math skills; she was hopeful her AVMR training would provide her help in this area. Holly collaborated with a kindergarten teacher but not with the other first grade teachers at her school because, she noted, these teachers did not teach mathematics the way she did. She felt they had "old school" teaching styles and did not play the mathematics games she played with her students.

I observed Holly's class at least once a week to learn how she taught first grade mathematics after her initial AVMR Course One training.

Student selection. From a pool of 18 students in the class, Holly chose three students to participate in the study based, primarily, on Holly's ability to make contact with parents and on the parents' willingness to have their child participate in the study. Another factor was Holly's perception of the students; we wanted a higher, medium, and lower achieving student for placement in the study, and she felt the selected students were representative of these varying achievement levels. This determination was made, mainly, from her knowledge of the students' work in class and, partly, from the AVMR student assessment results. The assessments were interview-based and were administered by a Mathematics Recovery Intervention Specialist (MRIS). Actually, two Mathematics Recovery Intervention Specialists assessed not only the three selected children, but the entire class. To ensure consistency between the two raters, multiple calibration meetings were held especially at first, to ensure an agreement as to the ratings given to students. Later, more focus was spent on students whose results may have been questionable.

I only chose three students for this study so I would have the opportunity to become knowledgeable with their mathematical strengths and weaknesses. The group was small enough that I was able to observe Holly's instruction and quickly determine which child's deficits were being met in the lesson. For example, I knew one of the students was an individual who always "counts-from-one" and her next cognitive level would be one where she begins to "count-on" rather than counting from one. When Holly had students doing activities that promoted counting-on, I knew she was addressing the needs of this particular student among possible others. The goal was for the teacher to implement strategies, activities, and questioning to help the child "discover" it would be easier to "count-on" rather

than stick with the “count-from-one” strategy. Holly knew I was observing her lessons to focus on how she met the learning needs of each of these students.

Case Study Research

I decided to use a case study approach to determine the extent of AVMR influence on teacher practice. A case study is described as an “intensive, holistic description and analysis of a bounded phenomenon such as a program, an institution, a person, a process, or a social unit” (Merriam, 1991, p. xiv). In this case study, the phenomenon was the influence of AVMR training on the pedagogical practices of a first grade teacher in the Aspen School District (ASD) during the class’s mathematics instruction time. Because I conducted an intense study of a single teacher, I found the basic characteristics of a case study provided a strong framework to answer my initial Research Question: At what level in the Add+VantageMR® (AVMR) classification scheme was the participating teacher after she attended the initial AVMR professional development training, and what was the impact of the training on a teacher’s instruction?”

Various qualitative research methods, in addition to the case study, could have been considered when preparing to do research. While several methods of conducting qualitative research are available as detailed in Gay and Airasian (2003) including ethnographic research, action research specifically tailored for education to find and solve educator’s problems in the classrooms and institutions, and historical research seeking to understand past events, I determined the case study method was the most appropriate approach to answer my first two research questions.

A case study is a form of qualitative research involving systematically gathering information about a particular person, group, event, or social setting (Berg, 2004). Detailed

information is gathered using a variety of qualitative strategies such as life histories, oral histories, documents, in-depth interviews, and participant observation (Hagan, 2002; Yin, 1994) over a sustained period of time (Creswell, 2003; Merriam, 2001; Yin, 1994; Hamel, Dufour, & Fortin, 1993). Berg (2004) refers to the methods of a case study as involving the systematic gathering of enough information about a particular person, event, or group to allow the researcher to understand how the person operates or functions; he suggests that, when conducting an individual case study, a single lengthy interview, or multiple interviews supplemented by field notes, be used.

Data Collection

Research Question 1. At what level in the Add+VantageMR® (AVMR) classification scheme was the participating teacher after attending initial AVMR professional development training, and what was the impact of the training on the teacher's instruction?

Data for Question 1 were collected in the form of weekly classroom observations to determine implementation level, an initial teacher interview, additional informal teacher interviews, the teacher observation protocols, videotaped sessions, and e-mail correspondences with Holly. The combination of these tools provided the necessary information to categorize Holly's level of implementation of AVMR strategies and activities, to document how she used AVMR assessment information to develop her students mathematically, and to estimate how her pedagogy was impacted.

Research Question 2. How does a teacher who receives AVMR professional development training utilize the AVMR assessment results to inform instruction for a small group of students?

Data for Question 2 were collected primarily through weekly classroom observations, informal teacher interviews, the teacher observation protocols, videotaped sessions, student-produced artifacts, and e-mail correspondences with Holly.

Research Question 3. Did the performance of this group of students improve when the teacher modified her teaching by utilizing their assessment results?

Data for Question 3 were collected primarily through AVMR pre- and post-test results and student produced artifacts.

The depth and quality of data gathered were dependent on my skills in observing the participants. Thus my observation skills factored into the portrait of Holly and her use of AVMR assessments, her use of AVMR strategies and activities, and my descriptions and evaluations of her students' perceived achievements. The guidelines associated with researcher skills established by Yin (1998) were my blueprint for the behaviors I modeled while carrying out the research. Yin (1998) identifies researcher skills associated with conducting case studies she considers pivotal for successful data collection. These skills include having an inquiring mind and the willingness to ask questions, having the ability to listen, developing a sense for the situation, and assimilating large amounts of data without bias. The researcher must also be adaptable and flexible enough to handle unanticipated events, and to change data collection strategies or sources if those being used do not seem effective or compelling. The researcher must have a thorough understanding of the issue being studied in order that she does not merely record data but interprets, reacts, and reports without bias to data once collected.

Following Yin's guidelines, I hoped to discover Holly's level of AVMR usage, how AVMR training impacted her instruction, and how she used student assessments to guide her

instruction. I was interested in observing how she incorporated the AVMR tools, strategies, and activities to help her facilitate mathematics growth in her students. Observing students moving from primitive strategies to more sophisticated strategies to solve more challenging mathematics problems would confirm student growth.

The observation protocol I used was formatted to contain comments or field notes about what I observed happening in the classroom both in terms of teacher action and behavior, and of her students during math instructional time. For a portion of each classroom observation, I videotaped the classroom math lessons and activities to allow me to study the interactions between the teacher and her students as well as student-student interactions. Following are descriptions and examples from the data collection instruments I used.

Teacher interviews. I interviewed Holly initially to learn more about her views on mathematics, teaching children math, diagnosing their math problems, perceptions of student thinking, and her AVMR training. (See Appendix E.) Some of the questions asked in the initial teacher interview included, but were not limited to:

1. “What are your feelings/perceptions regarding mathematics?” I wanted to know if the teacher considers herself to be comfortable, confident, and successful in mathematics or not.
2. “Under what conditions do children learn mathematics best? Do you believe these conditions hold true for all children?” I wanted to uncover the teacher’s perceptions about children and their learning of mathematics, particularly what methods she used to teach math and why she favored those strategies. Also, I wanted to learn whether she had alternate methods for teaching math to accommodate the different learning styles of her students.

3. “How skilled are you at helping students overcome difficulties in math?” I was interested in how secure the teacher was in her knowledge and use of available resources to help her students successfully navigate through mathematical problem solving.

Holly’s math core block was from 11:30 am—12:45 pm. Her math intervention was from 1:00 pm–1:30 pm. Her intervention time was primarily used to continue the daily math lesson rather than to conduct interventions. At times I was able to ask Holly a few questions during this intervention period; but, more often than not, we were interrupted or Holly needed to complete other tasks. As a result, we agreed to communicate via e-mail when she would have time to think about the answers to my questions. In addition to other questions based on the events of the week, additional questions included:

4. “I noticed you stressed ‘counting-on’ this week; can you tell me why?”
5. “It seems (student) is having trouble with making combinations for 5. What are your ideas to help this student be more facile for making combinations for 5?”
6. “After this week’s lessons, how will you plan to address student needs next week based on your observations?”

Teacher observation protocol. The teacher observation protocol was used to record specific ways in which the teacher used AVMR strategies, concepts, and activities during math instructional time (see Appendix F). The protocol was set up so each item learned in the Course One AVMR training appeared on the observation checklist. Some, but not all, of the overall behaviors, principles, and individual activities or strategies I recorded include:

- “The teacher encourages behaviors and activities that facilitate student transition from primitive to more sophisticated problem solving strategies.”
- “The teacher uses the Learning Framework in Number to determine what instruction should come next based on what the students have revealed from assessment,”
- “The class lesson involves working on the Forward Number Word Sequence.”
- “The class lesson involves working on spatial patterns.”
- “The teacher’s use of questioning to elicit students’ mathematical thinking.”

Videotapes of classroom interactions provided suggestions for future questioning such as why the teacher chose to use certain methods in assisting a student, why she chose to use certain activities, or if she had concerns about a student or the lesson. The videotapes were propitious as I was able to review what Holly taught in class and each child’s interactions with her. The digital reminders also provided a way for me to code the data according to AVMR topics included in the lesson such as skip counting or structuring numbers to 10 and to record Holly’s interactions with each student.

Instead of setting up a video camera in a back corner of the classroom as originally planned, I manually operated the camera so I had more control over what was being recorded and could closely record student work. I would videotape part of the time I was in the classroom, and, after capturing the content I needed, put the camera away and worked in the classroom helping the children. Very quickly, my presence in the classroom became normal and natural for all the students in Holly’s class. I simply became another person who could help them when they had a question, show them how they could solve a problem, or could

listen as they told me about their dog, baby brother, or what they had done the night before. Interacting with these students became an experience I looked forward to each day.

Student observation protocol. The student observation protocol was used to record AVMR tasks and behaviors displayed by the students in an efficient format. (See Appendix G). The AVMR tasks, activities, and concepts were listed in the Student Observation Protocol as specified behaviors. I used the student protocol at least once a week so I could record the AVMR skills and strategies used by the students.

The protocol focused on the mathematical activity vis-à-vis the AVMR assessment tool, so all my observations would be directly related to the mathematical skills and dispositions as delineated in the AVMR assessment component. One of the items listed in the Student Observation Protocol was the capability to witness change with regards to student use of number strategies. I documented the strategies students used from week to week, making note when they progressed to using more sophisticated strategies to perform AVMR tasks. The tasks included the following:

- “The student can say the Forward Number Word Sequence.”
- “The student can make combinations for 5, i.e. $1 + 4$, $2 + 3$.”
- “The student can make combinations for 10, i.e. $1 + 9$, $2 + 8$, $3 + 7$, $4 + 6$, $5 + 5$.”

Through the forms of data collection illustrated above, I gathered and could retell the story of the AVMR training, its impact on the teacher, her use of assessment to guide teaching, and the subsequent student achievement. These instruments were used along with the AVMR Assessments. (See Appendix H)

How the students were assessed. The observation skills of an assessor are critical in accurately portraying and recording case study data. An assessor must be able to discern subtle behaviors on the part of the student during the assessment. For example, a student may represent counters hidden under a screen with her fingers or some other object in the room. Someone who isn't experienced in giving the AVMR assessment may not notice particular behaviors that could provide further insight into the student's skill or learning level. These behaviors are recorded on the assessment itself, as they provide the teacher important cues about the student's strategy for solving the problem. Because experience in assessing students was extremely important, I decided two Mathematics Recovery Intervention Specialists would assess the students.

During the first week of school, each student received assent forms, and their parents received consent forms explaining the details of the study and a request for their consent to be part of the study was made. The students had one week to return the forms. Three students who were representative of a higher, medium, and lower achieving student were selected and formed the population of the study. The AVMR assessments were designed to specifically adapt to the learning level of the child. For example, in Task Group 5: Number Word Before (NWB), the child would first be asked, "Say the number that comes right before [i.e., such as numbers in the range between 11 and 30] 24." If the student's response was correct, the teacher would then ask "Say the number word before [i.e., such as numbers in the range from 31-100] 53." If the child were incorrect in saying the NWB 24, the teacher would drop back to determine whether the child could say the NWB in the range 0–10, such as "say the number word before 8."

Two Mathematics Recovery Intervention Specialists, who together assessed the students from the four classes in the Sandia Laboratory's study (Briand, 2012), consulted with each other as they used the individual assessment information to assign a construct level to each student. The three students in this case study were also part of a group of 54 students in four classes that were assessed earlier (Briand, 2012). The Assessments included the following early number categories of:

Forward Number Word Sequence (FNWS), Number Word After (NWA), Numeral Identification, Backward Number Word Sequence (BNWS), Number Word Before (NWB), Numeral Sequences, Structuring Numbers (Spatial Patterns, Finger Patterns, Combinations to 5 and 10, Combinations to 20) and Addition and Subtraction.

The following illustration is provided to familiarize the reader with the assessment process I used to assign a Construct Level for each of these categories. For example, the test format for a question would be, "I will tell you a number story and you tell me the answer. I have _____, I wish I had _____. How many do I need?" The test provides a space for the assessor to record the child's response and a separate space to indicate the strategy used by the child. For each task, the student's response is evaluated according to a Table of Learning/Construct Levels (see Appendices I and J) developed for each task. The learning/construct level most closely matching the level of sophistication or strategy demonstrated by the student would be recorded as the student's learning/construct level for that task.

The strategy choices include: "Knows immediately - (K); Counts to solve - (C); Uses fingers - (F) as well as any other observation the assessor notices. This model may seem straightforward but in some cases, proved restrictive. A child may, for instance, hide his

fingers to count, or may count by using a perceptual counter (e.g., a replacement counter, such as dots on the ceiling) that may not be noticed by an untrained or unaware assessor. Moreover, if the assessor were testing the child in numeral identification and the child could not identify some or all of the numerals in the range “0” to “10,” that child would be classified as a Level 0. If the student could identify all the numerals in the range “0” to “10,” that child would be classified as a Level 1.

Student produced artifacts. I collected student-produced artifacts mainly from their Everyday Math Journal. I also made notes about the work students did that was activity-based rather than paper and pencil-based. To analyze their activity-based work, I videotaped their activities related to AVMR such as the game “Count Around.” These methods provided resources I then used to document the change in the level of their work or their “learning/construct level.”

Data Analysis

To address Research Question 1, the Aspen School District Mathematics Instructional Coach and I met to assign an AVMR implementation level for Holly’s use of AVMR strategies and activities in the classroom. To determine the impact AVMR instruction had on Holly’s classroom procedures, I transcribed the video recordings made in her class and color coded the results to indicate where any related to an AVMR strategy or activity. I also referred to the Teacher Observation Protocols used on each visit and made note once again of any AVMR strategies or activities that occurred.

To address Research Question 2, I reviewed the students’ pre-assessments to become familiar with areas of weakness for each student. This awareness allowed me to note when

Holly's instruction covered topics matching areas in which one or more of those students were assigned a lower learning/construct level.

To address Research Question 3, I systematically analyzed each of the three participating students' emerging mathematical understanding vis-à-vis the AVMR assessments and student produced artifacts. How students' understanding of concepts such as their facility to make combinations of 10, for example, was examined over the course of the initial four months of data collected in the form of student artifacts, classroom observations, and videotaping. In addition, the three students' achievements on the pre- and post-AVMR assessments were investigated with the aid of a Two-Sample T-Test.

Researcher as instrument. Research Question 3 is directly focused on the AVMR pre and post-tests. If an instrument, such as the AVMR pre and post-tests, measures what it states it will measure, and is administered according to prescribed guidelines, then the research is considered reliable and valid (Williams, 2001). In contrast, the validity of qualitative research relies, to a great degree, upon the person conducting the research (Williams, 2001). Therefore, my background and knowledge are factors impacting this study. As Patton (1990) states, "the researcher is the instrument" (p. 14) and affects all aspects of a qualitative study. Guba and Lincoln (1981) state the naturalistic inquirer,

is himself the instrument, changes resulting from fatigue, shifts in knowledge, and cooperation, as well as variations resulting from differences in training, skill, and experience among different 'instruments,' easily occur. But this loss in rigor is more than offset by the flexibility, insight, and ability to build on tacit knowledge that is the particular province of the human instrument. (p. 113).

To meet these standards of researcher as instrument, I summarize my professional background relevant to this study. I have taught mathematics for over 30 years; my first three years were as a 7th, 8th, and 9th grade general math and algebra teacher. The next four years I taught Consumer Math, Algebra I, Geometry and Algebra II in three New Mexico high schools. For the last 23+ years, I have taught at institutions of higher learning. I taught Mathematics for Teachers and College Algebra at Anne Arundel Community College in Severn, Maryland; but my longest tenure teaching college has been in the Department of Mathematics & Statistics at the University of New Mexico (UNM), Albuquerque, New Mexico. In this capacity I teach the four-sequence program of mathematics courses for potential educators: Math 111: Mathematics for Elementary & Middle School Teachers I, Math 112: Mathematics for Elementary & Middle School Teachers II, Math 215: Mathematics for Elementary and Middle School Teachers III, and Math 339: Topics in Mathematics for Elementary & Middle School Teachers.

Four years prior to this study, I went to Nashville, Tennessee, to be trained in AVMR at the U.S. Headquarters for Mathematics Recovery and have attended or assisted Mathematics Recovery Specialists in the training of teachers in one of the larger school districts in New Mexico, a relatively smaller school district, and the Aspen School District in AVMR Course One and Course Two. These experiences solidified my credentials not only to observe, and but also interpret the observations and data as suggested by Yin (1998). Also, by using a variety of means to collect data such as the observations, questionnaires, assessment results, and videotaping, I have fulfilled Denzin's (1970, 1978) recommendation to triangulate during research. Fielding and Fielding (1986) suggest the "important feature of triangulation is not merely the simple combination of different kinds of data but the attempt

to relate them so as to offset the threats to validity identified in each” (p. 31). In this research study, I analyzed data from observations, survey questions, and interviews to determine consistencies as well as conflicting accounts with regards to the three data collecting methods.

Furthermore, I have spent the last three years presenting information at National Mathematics Recovery Conferences about our experiences in New Mexico with AVMR and writing grants that have provided AVMR training to over 200 New Mexico teachers, two Mathematics Recovery Intervention Specialists, and two Mathematics Recovery Champions. Other grants I’ve written have provided funding to enable several teachers to attend National Mathematics Recovery Council Conferences. Recently, I was the closing keynote speaker at the national USMRC Convention 2013 in Denver, Colorado. This presentation afforded me the unique opportunity to forge bonds with teachers and Mathematics Instructional Coaches in communities in New Mexico as well as throughout the nation. My work in early numeracy led to my election to a five-year term on the U. S. Mathematics Recovery Council (USMRC) Board of Directors where I currently chair the Board Development Committee.

Perhaps the biggest impact AVMR has made on me professionally has been to increase my knowledge concerning how to develop young children’s mathematical thinking through the use of AVMR assessments, tools, and activities. In my prior years of teaching university students who plan careers in teaching elementary school, I had minimal knowledge of how to effectively promote number sense in young students. Now I have more to offer my students, because I now include much of what I have learned from AVMR and CGI in my courses.

My students receive an important sense of what it means to teach young students mathematics. My work has involved establishing a program for pre-service teachers at the University of New Mexico (UNM) where students attend a half-day seminar covering the fundamentals of AVMR, and then are placed in the classrooms of AVMR trained teachers to assist them throughout the semester. This program puts UNM students into teachers' classes throughout the city and its surrounding communities. The university students are privileged to experience working with individual students, or small groups of students, or presenting whole-class lessons while being mentored by AVMR trained teachers. Oftentimes they lead class activities affording the teacher time to assess students in the class. In addition, I meet monthly with UNM interns in a working seminar where we discuss methods to meet the challenges faced by students with whom they are working. We also make AVMR tools and discuss strategies they can then use with their students in the classroom.

I am impressed by AVMR, and recognize this is a bias. I have experienced the benefits received from increasing my knowledge of children's early number development through AVMR, and now the additional insights developed from my training to become a Math Recovery Intervention Specialist. Both have provided me with resources and insights into developing children's mathematical thinking that I am privileged to share with my university students. However, the true benefit realized by AVMR appears when student improvement occurs as a result of teachers' utilizing knowledge gained from AVMR training in their classrooms, and they see student improvement. Preliminary findings from data collected by the Mathematics Instructional Coach from ASD indicate student achievement gains for students whose teachers have been AVMR trained. (See Appendix K.) My goal for this study was to provide insight to the questions posed throughout this study. I believe the

Aspen School District and its teacher, Holly, afforded me the opportunity to answer these questions. And I believe the results from this study bolster my bias about AVMR.

Chapter 4: Results and Findings of the Study

This chapter presents the results and findings of the study focused on the ways in which a teacher who received Add+VantageMR® (AVMR) professional development used AVMR instructional concepts, strategies, and assessments to improve student learning and achievement in her elementary school class. To provide a frame of reference for the results, the study emerged from the confluence of three topics discussed in earlier chapters: Professional Development, Assessment, and Children’s Cognitive Acquisition of Mathematics, which led to the following research questions.

1. At what level in the Add+VantageMR® (AVMR) classification scheme was the participating teacher after attending initial AVMR professional development training, and what was the impact of the training on a teacher’s instruction?
2. How does a teacher who receives AVMR professional development training in mathematics utilize the AVMR assessment results to inform instruction for a small group of students?
3. Does the performance of this group of students improve when the teacher modifies teaching by utilizing their assessment results?

As discussed in Chapter 3, the research method best suited to investigate these questions was the case study. This case study investigated the AVMR classification level of a teacher in the Aspen School District, named Holly (for this study) after initial AVMR training, how she incorporated her AVMR Teacher Professional Development training in the classroom, and how she used information she received from students’ AVMR assessments to improve her students’ learning and achievement. Specifically, the study focused on how she

met the instructional needs of three selected children in her first grade classroom based on their assessment results. The children are referred to as Jasmine, Johanna, and Josh. The study also sought to determine the mathematical growth of the three selected students through the use of AVMR pre and post-tests and the relationship of those results to the results of the other students in the classroom.

The results of the study are divided into four sections. The first section contains conclusions formed from observing Holly to determine the degree to which she used AVMR techniques, strategies, and tools in her classroom. To classify the impact of AVMR on her instruction, or the degree to which Holly used AVMR techniques, tools, and strategies, she was observed along with three other teachers from a simultaneous study (Briand, 2012) who provided a frame of reference for her implementation level. The second section describes how the students were selected, a description of each child, and an academic overview for each child including their individual assessment results on the AVMR pre-test and post-test. The third section focuses on the influence of the AVMR training on Holly's teaching practice with respect to how she interacted with and met the needs of Jasmine, Johanna, and Josh. Holly specifically covered some of the AVMR pre-test topics, in which one or more of the students scored lower, in class instruction. These topics are conveyed through vignettes linking the topics directly to areas on the AVMR pre-test. This section includes descriptions for when Holly recognized an opportunity to implement AVMR techniques and when she did not. The fourth section provides a summary of the findings.

Results and Findings for Determining Holly's AVMR Implementation Level

Beginning August 25, 2011, and continuing through December 18, 2012, I visited Holly's class on a weekly basis for a total of 14 visits during mathematics time. I continued

thereafter to visit once a month until April 2012, for a total of 18 visits. Her instruction and interaction with students was videotaped and recorded using the Teacher Observation Protocol form (Appendix F). The Teacher Observation Protocol contained a list of AVMR tasks and behaviors used to record and categorize classroom instruction. At times, a tape-recorder was used to capture my thoughts on what I observed in the classes. Holly was interviewed for background information, her views on how children learn mathematics, her teaching methods, how she used the assessment information about her students, and the goals she had for her students.

The level to which Holly implemented AVMR strategies and tools in her classroom would affect the results of the study. If her usage were minimal to non-existent, student progress would be independent of her AVMR professional development training. If her usage were medium to high, recording the AVMR topics she concentrated on during teacher/student instruction and interaction and comparing her students' progress in those areas to the frequency of her instruction could be indicators of the effect of the AVMR training on student progress. To provide a gauge for Holly's AVMR implementation level, I thought it best to compare her usage of AVMR topics, tools, and strategies to three of her peers who, along with Holly, were part of another AVMR-centered study as well. Therefore, I compared information from this Ph.D. study to that of a second study (Briand, 2012) I was conducting, simultaneously, on the AVMR implementation level of four teachers in the Aspen School District (two teachers from Aspen Elementary School and two teachers from Pine Elementary School). The name of the school district and both individual schools mentioned are fictitious.

The second study began at the same time as the case study in August 2011 and ended in May 2012. In the second study, both the Aspen District Mathematics Coach and I observed all four teachers over a period of nine months to determine the extent to which each teacher used AVMR topics, tools, and strategies in the classroom. To determine the AVMR implementation level for the four teachers, I attended the AVMR Course One training session to become familiar with what the teachers would be exposed to in their training. As I observed Holly and the other 13 teachers, as well as the five district principals who attended parts of the training, I could see their amazement with the program. First, they were shocked and dismayed after viewing the Math Recovery Kelsey video clip. Kelsey, a 7th grade student, successfully became an honor student, and registered for 8th grade pre-algebra, but was severely lacking in number sense. She could not, for example, name two numbers whose sum was 19. Even when the assessor provided her an additional scaffold by saying, “suppose one of the numbers is '18,' what would the other number be?” she could not correctly respond.

The discussion that followed focused on how to prevent other students from being mathematically under-served as Kelsey had been. The next four days of training centered on how to assess students to uncover what they knew and how to build upon it. The teachers were introduced to the Learning Framework in Number, which guides instruction based on what students currently know and provides the foundational piece on which future learning is built.

Holly and the other teachers were excited, inspired, and felt they now had the information needed to be effective in teaching math. As the school year began, I could see evidence of the impact of AVMR training in Holly’s classroom. She worked on practicing

the Forward Number Word Sequence (FNWS), Backward Number Word Sequence (BNWS), and counting forward as well as backward on the decade numbers such as 10, 20, 30, 40... and 60, 50, 40, 30 ...with her students. She told me she incorporated the math racks, dot cards, 10 frames, bingo games, the treasure hunt game, and the Great Race game into her classroom activities.

My classroom observations and information from initial teacher interviews for all four teachers showed a commonality among the teachers in that they all preferred a method of teaching mathematics that used “hands-on” materials, manipulatives, math games, and visuals. A second teacher, Mary (fictitious name), who taught at Holly’s school but who was not AVMR trained, used many audio-based learning strategies; that is, songs designed to help her students remember number facts such as the complementary numbers whose sum is ten. All teachers did some group work in their classes, however, Holly used group work almost exclusively in her classroom.

In Holly’s class, virtually all instruction was provided in small groups of approximately four or five students who would cycle around the classroom every 15-20 minutes. These small groups were not fixed, as Holly would assign students to groups on a weekly or daily basis. I noted the children in the groups changed from visit to visit. I was not able to confirm with Holly exactly how she chose her groups. Typically the classroom included four activity sites or centers. One site would be Holly’s instructional site; a second site would have a practice activity or game, which was typically led by a University of New Mexico (UNM) AVMR intern; a third site was typically student self-directed and might include a math worksheet or game, and a fourth site was the computer area. Two computers in the classroom had number adventure games children could play. When the students would

cycle to her center, Holly would introduce new concepts or work with students on concepts previously taught. Except for a few instances where she taught the whole class at once, typically when the class was reviewing for a test, Holly always taught her students in small groups of four to five children.

In addition to classifying the predominant method of instruction for each teacher, such as whole class or small group, I also wanted to know each teacher's thoughts on how children learn mathematics. When teachers were asked, "Under which conditions do children learn math best?" the difference in each of their responses was evident:

I find children are hands-on learners, especially in math, so I use manipulatives and math games (Holly [AVMR trained], Aspen Elementary School).

I find children learn best when they are focused and there is a quiet environment in which to learn new concepts (Mary [not AVMR trained], Aspen Elementary School).

Children learn math best when they are allowed to explore using math tools (manipulatives), then lots of practice with hands-on tasks, then moving to symbols and the abstract (Peggy [AVMR trained], Pine Elementary School).

Children learn math best under a learning environment that feels safe, and where they are free to express their way of seeing math (Elizabeth [not AVMR trained], Pine Elementary School).

The teacher quotations above are included to give insight into the conditions each teacher thought were necessary for optimal mathematics learning to take place. Interestingly, the AVMR-trained teachers concentrated on tactile learning environments where practice and

hands-on experiences were of importance. That is not to say the other teachers did not have similar opinions, but it was not their response to this particular interview question.

As we sought to determine the AVMR implementation level for each teacher, the Aspen District Mathematics Instructional Coach and I made numerous visits to the individual classrooms, basing our conclusions about each teacher's AVMR implementation level by using the AVMR Teacher Implementation Protocol that follows. We met at the end of the Ph.D. study to discuss the implementation level we assigned to each teacher and why we chose particular levels. We were in agreement concerning the final implementation levels we had chosen for each teacher. In the one case where the instructional coach and I differed in evaluating a particular behavior, both evaluations are shown in Table 4.1. The difference in evaluations involved Holly's on-going observations of the students and the subsequent fine-tuning of her teaching based on the student observations she made. Her District Mathematics Instructional Coach classified her as a low implementer while I classified her as a medium implementer. Since the Math Instructional Coach and I were not in the classroom observing every day, we were only able to determine a level assigned to each teacher based on the times we were able to visit. The following represents a classification level for each teacher based on the AVMR Implementation Level Rubric adapted from *Teaching Number in the Classroom* (Wright, Stanger, et al., 2006). I amended the rubric to include categories focused on number tasks and activities and the AVMR Guiding Principles for Classroom Teaching that are prevalent in an AVMR classroom.

Table 4.1

AVMR Implementation Level Rubric

Principle	High Implementation	Medium Implementation	Low Implementation
Teaching approach is problem based. Children routinely are engaged in thinking hard to solve numerical problems that for them are quite challenging.		Peggy- Attention to ‘how did you think about that.’ Seems comfortable in allowing kids to struggle with problems. Solicits a variety of solutions.	Holly, Mary, & Elizabeth Primarily engaged in answer-eliciting. Elizabeth- Many times providing the answer.
Teaching is informed by an initial, comprehensive assessment and ongoing assessment through teaching strategies, and continual revision of this understanding.	Peggy –Math Instructional Coach (MIC) had several conversations with her about how some kids were progressing based on AMC data and classroom performance.	Holly- Conversations with MIC revealed some surprises for teacher in student’s performance on AMC assessments.	
Teaching is focused just beyond the ‘cutting-edge’ of child’s current knowledge.		Holly & Peggy’s math centers took into account general needs of students. Not cutting edge, but some attention to general ‘trends’ in student needs.	Mary and Elizabeth’s observations were more geared to enforcing current knowledge.
Teachers exercise their professional judgment in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.		Peggy sought out help in identifying and creating a wide-range of settings and activities to forward her students learning.	Holly used a few AVMR and AMC settings last year. Math Instructional Coach- in my observations this year, I saw AMC settings, but not AVMR for Mary and Elizabeth.

Table 4.1 *AVMR Implementation Level Rubric* (continued)

Principle	High Implementation	Medium Implementation	Low Implementation
<p>The teacher understands children’s numerical strategies and deliberately engenders development of more sophisticated strategies.</p>	<p>Peggy was always pursuing other methods to solve problems by eliciting student strategies that were more sophisticated.</p>	<p>Holly encouraged sharing of solutions.</p>	<p>Mary and Elizabeth’s strategies focused more on procedural themes rather than on building conceptual knowledge.</p>
<p>The teacher provides the child with sufficient time to solve problems. Consequently the child is frequently engaged in episodes which involve sustained thinking, reflection on her or his thinking, and reflecting on the results of her or his thinking.</p>		<p>Holly and Peggy allowed children time to figure things out for themselves.</p>	
<p>Students gain intrinsic satisfaction from their problem solving, their realization that they are making progress, and from the verification methods they develop.</p>		<p>Holly, Peggy, Mary, and Elizabeth - all teachers tend to use lots of praise.</p>	
<p>Teacher verbally practiced Number Tasks such as Forward Number Word Sequence, Backward Number Word Sequence, Making Combinations of 10</p>	<p>Peggy and Mary</p>		

Table 4.1 *AVMR Implementation Level Rubric* (continued)

Principle	High Implementation	Medium Implementation	Low Implementation
Teacher selected activities for students to practice Number Tasks such as Forward Number Word Sequence, Backward Number Word Sequence, and Making Combinations of 10.	Peggy and Holly	Mary	Elizabeth
Teacher AVMR Implementation Level		Peggy ↓ High	Holly Mary ↓ ↓ Low

Teacher participant. Holly was classified as a Lower-Medium AVMR implementer as she used many activities focused on number but her questioning of students was primarily to elicit specific answers. She did engage in questioning such as “Why did you jump back 10?” to uncover student thinking; however, this was not the predominant type of questioning she used. The predominant questions she used included questions such as, “If your right hand has four pennies in it, how many pennies should you put in your left hand to make 10?” Or she frequently asked questions such as, “How much is a nickel worth?” that centered more on student recall.

Holly was able to recognize, support, and praise students’ accomplishments when the children displayed growth in number sense. On one occasion Holly and one of her students shared with me a particularly sophisticated number strategy the student used to solve the following word problem, “How much money would you need to buy a toothbrush which

costs \$.39 and toothpaste which costs \$.45?" The student said he got his answer by taking 4 tens and 3 tens to get 7 tens. Then he combined 9 and 5 to make 14. That is one more 10 added to the 7 tens. Now he said "I have 8 tens and 4 ones or 84 cents!" This type of classroom conversation and recognition of more sophisticated student strategies elevates Holly's evaluation over Mary's, but we do not consider Holly a full Medium implementer as her questioning was not consistently at that level.

In comparison, Peggy (fictitious name), another teacher from Pine Elementary School received AVMR training at the same time as Holly, and was classified as a higher-Medium AVMR implementer. Peggy employed many activities that concentrated on building number combinations and posed questions for students that required them to think and create solutions. She gave her students plenty of time to think about the problems and would have students come up to the board to show how they got their answers. She always solicited many students' varied responses by asking, "Did anyone solve the problem a different way?" On one particular occasion her class was doing the same number story problem that occurred in Holly's class. One student offered the same solution shared in Holly's class. As Peggy kept soliciting other ways to work the problem, another child said, "I started with 45 because it was bigger, then I hopped up ten, four times to add 40." This process gave him $45 + 40$ or 85. But since he only needed to add 39, he explained: "I hopped back one and got $85 - 1$ or 84 for the answer." Quite impressive, especially for first grade! Had she taken advantage of more questioning that required critical thinking on the part of her students, as demonstrated by Peggy, Holly's implementation level would have been rated higher.

In summary, Holly evidently did use AVMR teaching strategies in her classroom; however, she used the strategies at a lower level than Peggy. It should be noted that Peggy

was in her 23rd year of teaching as opposed to Holly, who was only in her second. Her years of experience could account for Peggy's greater use of the major tenets of AVMR's *Guiding Principles for Classroom Teaching* (Wright, Stanger, Stafford, & Martland, 2006). (See Appendix L, for a complete list of principles.)

Student Selection, Description, and Academic Overview

Holly selected three students of varying ability levels who could participate in this case study. Jasmine, Johanna, and Josh were not selected randomly but, rather, because their parents agreed to have them participate in the study and because they represented students who were classified as having higher, medium, and lower achievement levels in mathematics by their teacher. The children were observed from August 2011, through December 2011, on a weekly basis to make note of their initial performance in mathematics, their advancing mathematical abilities, and their resulting mathematical abilities. Thereafter the children were observed on a monthly basis thru April 2012. Assessment data for each child was also obtained in the form of the AVMR Student Assessment component administered in August 2011, February 2012, and April 2012, by a Mathematics Recovery Specialist. Part of the teacher's training in AVMR includes instruction and practice in administering the AVMR student assessment to children. For this study, and the Briand (2012) study conducted in the Aspen School District, all children were assessed by a Mathematics Recovery Intervention Specialist to support reliable ratings for each student on both the AVMR pre and post-tests. Josh was absent on two occasions when the Mathematics Recovery Intervention Specialist went to his school to test him and, therefore, was not assessed again until his post-test in April 2012. Each child's initial performance and progress through the period of the study was based on analysis of their performance through three main sources: 1) classroom

observation and videotapes, 2) their Everyday Math Journal, and 3) assessment results consisting of both the AVMR pre and post-test scores. A brief description of each child participant follows.

Study participants. *Jasmine.* Jasmine, a Hispanic female who was considered by her teacher as the high achiever in the group, was six years old at the time the study took place, having a February birthday. Holly described her as “superb at her math facts, counting forwards and backwards, money, and telling time.” She was helpful to others in the class and exuded a confidence not only in her mathematical abilities but in herself as well. Her confidence was appealing, as she never tried to dominate a group or act in a boastful manner. Her assurance in her abilities made it evident she enjoyed problem solving. Her ability to think and reason through problems was observed several times over the course of the study.

As first grade began in August, writing the numbers from one to ten in the Everyday Math Journal was one of the main objectives for all of the students. In this category, Jasmine was strong. Her hand control was excellent, and most of her numbers were perfectly formed except, at times, she would write the numbers “3” and “2” backwards when she did not have a model to follow. I noticed this anomaly in her Everyday Math Journal. Through September and October, some of the math tasks for the children required them to count up by tens from 20 to 120. This assignment posed no problem for Jasmine. However, when she had to count back by ten from 90, sometimes she would reverse the tens and ones place as she was writing her numbers; for example, in the following sequence, she wrote 60 as 06. Given 90, 80, 70, she would continue the sequence “06, 50, 40, 03, 20 and 10.” Again, she made these errors in her Everyday Math Journal. When subtracting, Jasmine would use the “count down from” strategy. For example, for $9 - 3$, she would say “8” holding up one

finger, then “7” holding up two fingers, then “6” holding up three fingers; the answer is 6. Her definite strengths were in writing her numbers, counting up, and addition.

By the time Jasmine took the AVMR post-test, her ability in counting backward rose from a Level 2 (i.e., student can produce the Backward Number Word Sequence (BNWS) from ten to one) on the pre-test to a Level 5 (i.e., student can produce the BNWS in the range from one to one hundred) on the post-test. On the pre-test she could not count back from 17 to 10; she had to stop at 13. On the post-test, she could successfully count back from 17 to 10. Previously in counting back from 72 to 66, she would omit the decade number of 70, saying “72, 71, 69, 68, 67, 66.” On the post-test, she could successfully count back from 72 to 66 with no omissions.

In the Number Words and Numerals portion of the assessment, her strength was in counting to 112; she was at the highest level (Level 5). In saying the Number Word After (NWA) from 31 – 100, she was slower on the NWA 59 and 99. On the post-test, she had no hesitations on NWA 59 or 99. For numeral identification, she was facile from 11 – 100 but did not know 168 and 354. Also, she called 205 twenty thousand five; the same error with 205 occurred on the post-test. With the numbers from 46-55, she could identify, sequence and read the numbers both on the pre and post-tests. In counting back from 72 to 66, she omitted the decade number (70). Jasmine did not omit the decade numbers when counting backward on the post-test.

The next category for evaluating Jasmine was that of “finger patterns.” Instructional strategies for combining and partitioning numbers in the range 1 to 10 often involve using finger patterns, which can support the development of more sophisticated arithmetical strategies (Wright, Stanger, et al., 2006). An example of using finger patterns would be to

ask the child to show how to make five using two hands. The child might then raise one finger on one hand and four on the other. Typically, the child would then be asked to show another way to make five using two hands. The child might then raise two fingers on one hand and three fingers on the other hand. Children can show finger patterns for combinations of five and ten.

Jasmine knew the finger patterns for five but did not know those for ten on the pre-test. On the post-test, Jasmine knew the finger patterns for five and ten, and the combinations for five, ten, and twenty. On screened addition tasks within and outside of finger range, Jasmine moved from counting by one to obtain an answer to counting on, which is the next highest level of sophistication. For example, in the problem $4 + 3$, counting by one looks like “1, 2, 3, 4” then “1, 2, 3” then “1, 2, 3, 4, 5, 6, 7” whereas in counting on she would say “4...5, 6, 7. The answer is 7.” For subtraction with screened collections, by the post-test Jasmine could do these tasks using a “count down from” strategy where she was unable to complete the tasks on the pre-test. The use of a “count down from” strategy on the problem $8 - 3$ would look like the following: 7 (is one), 6 (is two), 5 (is three) the answer is 5. The “count up from” strategy is less sophisticated. It involves the child saying, 4 (is one), 5 (is two), 6 (is three), 7 (is four), and 8 (is five). The answer is five.

An example illustrating the use of a screened collection for $13 - 5$ would involve showing the child 13 colored counters, then covering them with a screen or sheet of paper stock. The teacher would then partially lift the card stock, remove five counters, place the screen down again and ask the child how many counters are remaining under the screen. In the relational thinking category, Jasmine can now solve problems like $15 + 3$ and $18 - 3$; however, she solved each problem separately rather than using the first problem to help her

solve the second problem. Thus, she did not use relational thinking skills at this point.

Tables 4.2-4.5 display Jasmine's pre- and post- test results from the AVMR Assessment.

Table 4.2

Number Words and Numerals for Jasmine

Number Words & Numerals	<i>Forward Number Word Sequence (FNWS)</i>	<i>Number Word After (NWA)</i>	<i>Numeral Identification</i>	<i>Backward Number Word Sequence</i>	<i>Number Word Before (NWB)</i>	<i>Numeral Sequences</i>
Pre-test August 2011	Level 5 Counts to 112	Ok in 31-100 range slower on NWA 29, 59 and 99	Level 3- did not know 168, 354; in range 101-1000 could not identify 205 and 620. Could not identify numbers in range 1001-1,000,000.	Level 2 Omits decade number 30, 50, 60, 70 etc. on backward count	Fluent in range 11-30, does not know NWB 88	Can identify, read and sequence numbers in range 46-55
Post-test February 1, 2012	Level 5 Counts to 112	Fluent in 31-100 range	Fluent in range 101-1000 still could not identify 205. In range 1001-1,000,000 could identify 7,462 and 5,026	Level 5 in counting backward from 17-10, 38 to 27, 72-66	Fluent	Fluent

Table 4.3

Structuring Numbers—Jasmine

Structuring Numbers	<i>Spatial patterns</i>	<i>Finger patterns</i>	<i>Combinations to 5</i>	<i>Combinations to 10</i>	<i>Combinations to 20</i>
Pre-test August 2011 Level 0	Knows regular spatial patterns; must count on irregular spatial patterns for 3,4,5 & 6	For 5 & 7 on two hands counts from 1	Uses fingers or counts cubes in range 1 – 5	Did not do combos to 10.	Did not do combos to 10 or 20.
Post-test February 1, 2012 Level 1	Now knows irregular spatial pattern for 4 and 3	Knows all from 3 – 9	Knows all	Knows all	Fluent – uses fingers “I have 14, wish I had 20. How many do I need?”

Table 4.4

Addition and Subtraction—Jasmine

Addition & Subtraction Construct 2 Figurative Counter	<i>Addition Unscreened</i>	<i>Addition Screened within Finger Range</i>	<i>Addition Screened outside Finger Range</i>	<i>Missing Addend</i>	<i>Subtraction-Screened Collections</i>	<i>Addition & Subtraction – Bare Numbers</i>
Pre-test August 2011 Construct 2	Not necessary	Knows, uses fingers for 4 +2, 6+3	Knows 9 +5 , builds 9 counts out 5 more “14”	8 + _ = 11 Cannot do, guesses 6	Did not do-child not ready	Did not do-child not ready
Post-test February 1, 2012 Construct 3		Knows counts on for 4+2 & 6+3	Knows counts on 10,11,12,13,14	Can do correctly counts on from 8 using fingers to keep track	Knows 16-4 counts down from	Knows 8+4 counts on uses fingers to keep track, 17-6 good counts down from

Table 4.5

Relational Thinking—Jasmine

Relational Thinking	<i>Commutativity of Addition</i>	<i>Linking Addition & Subtraction</i>	<i>Related Subtraction tasks</i>
Pre-test August 2011	Did not do- child not ready	Did not do- child not ready	Did not do- child not ready
Post-test February 1, 2012	Knows: $4 + 12$, started at 12 and counted up	$15+3$ and $18-3$, solves each task separately	$21-4$ and $21 - 17$ solves each task separately

In summary, we can see Jasmine improved in the Number Word After (NWA) category, made significant improvement in the Backward Number Word Sequence (BNWS) category, now knew her combinations to five, to ten, and to twenty, and used the counting-on and counting down from strategies fluently in solving addition and subtraction tasks. She had also progressed in relational thinking to being able to do the tasks separately, but was not yet to the point of using one task to help her complete a second task.

Josh. Josh, a Hispanic male considered by his teacher to be the middle achiever in the group, was six years old at the time of the study, having a November birthday. He was the youngest of the three students by eight months. Holly described him in her December 9th e-mail to me as “struggling to count backwards and not having automaticity with his one digit math facts.” From his AVMR assessment, his strength was in counting backwards, however. In thinking about this strength, I wondered if he had practice in counting backwards because he liked racing, and, in racing, a participant often counts backwards and then says GO! Josh was a gregarious, active child who loved to talk, loved NASCAR racing, and was easily distracted. Many times he tried to complete his work quickly, without really understanding the directions first, only to have to erase and erase. On one occasion when rushing through an assignment, I noticed Holly say, “It’s okay...let’s just start again.” This comment calmed him down and got him on task again.

In the Number Words and Numerals category, Josh was a Level 3 on his pre-test. This ranking meant he could count in the two digit range, say the Number Word After (NWA) in the range up to 30, identify two digit numerals, produce the BNWS typically up to the 30 range (Josh, however, was higher here), and say the NWB in the range up to 30. At the time of his post-test, Josh had moved up to a Level 5 and could count to 112. He also had

improved in the Number Word After category, moving up to being able to say the NWA in the 11-30 range to the NWA in the 31-100 range. For Numeral Identification he moved up from Level 3, being able to identify one and two digit numerals to Level 4 where he could identify one, two and three digit numerals.

For the Backward Number Word Sequence, he moved up from Level 4 to Level 5, as he was facile in the range from one to 100 as opposed to the range from one to 30 as was indicated on his pre-test. Instead of counting from one in his finger patterns, he knew them all automatically. He knew the combinations of five and the combinations of 10, but still needed to use his fingers for 4 and 6, and 3 and 7. In the Addition and Subtraction task set, he could solve $9 + 5$ with a screened collection, could do the missing addend problem by counting from one, and do the addition and subtraction bare numbers problem by counting on, using groups, and counting down from moving from a Construct 1 to a Construct 3.

A Construct 1 classification indicates a child can count perceived items but cannot successfully count items in a concealed collection. To provide a frame of reference a Construct 0 child may not know the number words, may not be able to coordinate the number words with the visible items (i.e., one-to-one correspondence), or may not be able to tell the cardinality (how many) of a set. Josh was able to advance past a Construct 2 classification, indicating he could count a concealed collection using a re-presentation. For example, in the problem $6 + 5$, a Construct 2 child may re-present or hold up 6 fingers to stand for the 6 concealed counters. The child would then “count from one” saying “1, 2, 3, 4, 5, 6”, then “1, 2, 3, 4, 5”, and finally, “1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11.” The answer is “11.” Josh advanced past Construct 2 to a Construct 3 classification, indicating he could “count on” to solve addition or missing addend tasks. For the problem stated above, Josh would say, “6, 7,

8, 9, 10, 11.” The answer is “11.” In addition, a Construct 3 child uses the “count down” strategy to solve subtraction tasks. Tables 4.6-4.9 summarize Josh’s pre- and post- test results from the AVMR Assessment.

Table 4.6

Number Words and Numerals—Josh

Number Words & Numerals	<i>Forward Number Word Sequence (FNWS)</i>	<i>Number Word After (NWA)</i>	<i>Numeral Identification</i>	<i>Backward Number Word Sequence</i>	<i>Number Word Before (NWB)</i>	<i>Numeral Sequences</i>
Pre-test August 29, 2011	Level 3 Counts to 32	Ok in 11-30 range must count up to get NWA 14, 29, and 19	Level 3 Ok from 0 – 100. Difficulty in the 101-1000 range. 400 was called 104, 205 called 120	Level 4 on 38-31 good hesitated at 30 but finished well. 72 to 66 good but hesitates at 70	Good in range 31-100, does not know NWB 53	Can identify, read and sequence numbers in range 46-55 however reads 46 as 56
Post-test April 24 2012	Level 5 Counts to 112	Good can do NWA in 31-100 range	Level 4 Can do 101-1000 range	Level 5- counting backward can do all 17-10, 38 to 27, and 72-66.	Assessor did not do	Assessor did not do

Table 4.7

Structuring Numbers—Josh

Structuring Numbers	<i>Spatial patterns</i>	<i>Finger patterns</i>	<i>Combinations to 5</i>	<i>Combinations to 10</i>	<i>Combinations to 20</i>
Pre-test August 29, 2011 Level 0	Knows regular spatial patterns; must count on irregular spatial patterns knows only for 3	For 9 counts from one	Not able to complete successfully	Did not do combos to 10.	Did not do combos to 10 or 20.
Post-test April 24 2012 Level 2	Child knows regular & irregular	Child knows all	Knows all	Knows, but uses fingers for 4 & 6 and 3 & 7	Did not do

Table 4.8

Addition and Subtraction—Josh

Addition & Subtraction Construct 1 Perceptual counting	<i>Addition Unscreened</i>	<i>Addition Screened within Finger Range</i>	<i>Addition Screened outside Finger Range</i>	<i>Missing Addend</i>	<i>Subtraction-Screened Collections</i>	<i>Addition & Subtraction – Bare Numbers</i>
Pre-test August 29, 2011 Construct 1	Not necessary child knows	Good, uses fingers for 4 +2, child cannot do 6+3	Cannot do 9 + 5, builds 9 counts out 5 more answers “15”	8 + _ = 11 Cannot do, guesses 6	Did not do- child not ready	Did not do- child not ready
Post-test April 24, 2012 Construct 3	Not necessary child knows	Can do 6+3	Can do, for 9 + 5 child counts on	8 + _ = 11 child can do, counts from 1	Child can do using both count down from & count up from	Child can do using counts on, using groups, using counts down from

Table 4.9

Relational Thinking—Josh

Relational Thinking	<i>Commutativity of Addition</i>	<i>Linking Addition & Subtraction</i>	<i>Related Subtraction tasks</i>
Pre-test August 29, 2011	Did not do- child not ready	Did not do- child not ready	Did not do- child not ready
Post-test April 24 2012	Assessor did not do	Assessor did not do	Assessor did not do

In summary, Josh clearly improved in all categories in which he was assessed. He made considerable gains from pre to post-test. He progressed on combinations of ten and was successful on all combinations of ten when he could use his fingers. He did not reach the point of making combinations to twenty. He was not assessed on the post-test for relational thinking, but the reason why is not known. The Math Recovery Specialist did not remember why that section was omitted. It could have been she felt he was not ready or it could have just been an oversight.

Johanna. Johanna, a Caucasian female considered by her teacher to be the lower-achiever in the group, was six years old at the time of the study, having a March birthday. Holly stated in a December 9th e-mail, “Johanna struggles to get started counting backwards, does not have automaticity with her math facts, and struggles with telling time.” Johanna was a soft-spoken child, shy, and was consequently hesitant to answer questions in class; she didn’t have the confidence level displayed by a child like Jasmine. She had a wonderful smile and, although quiet, was receptive and warm. With a child like Johanna, who is much quieter and more hesitant, an assessor has to watch for nonverbal clues in her communication patterns. Lapakko (1997) stresses nonverbal communication often provides much more meaning than people realize. Bodily movement, facial expression, the use of space, the use of time, touch, and vocal cues are all considered nonverbal codes. Holly was able to, at times, pick up on the puzzled look on Johanna’s face when she had trouble with understanding how to do a problem or play a game. From her appearance and responses in school, Johanna received less care and support at home than Jasmine or Josh enjoyed. As mentioned earlier, writing the numbers from one to ten in the Everyday Math Journal was one of the main objectives at the start of the school year for all of the students in first grade.

As Johanna would write her numbers, I noticed she would frequently write her fives backwards unless she had an example of a “5” to follow. Many times her nines and threes were written backwards as well. Her ability to count by one beginning with the number seven, count up by fives from seven to complete a sequence, and count up by five from 0 was excellent. She could also answer problems involving sums of pennies on two hands; the problem presented the child with one hand holding three pennies and asked the child to show how many pennies would be in the other hand. Johanna was proficient in saying the Number Word Before the numbers 19, 23, 31, and 36.

Johanna scored a Level 5, the highest level, on the pre-test for the Forward Number Word Sequence (FNWS), meaning she could produce the FNWS from one to one hundred. She had some trouble with numeral identification past 100; for example, she would call 117 one hundred seven. In the structuring numbers category, Johanna knew the regular spatial patterns (e.g., dot patterns as on dice) but could not identify the irregular spatial patterns for five or six. When showing the finger patterns for nine, she began the count from one; however, for showing the finger pattern for eight, she used a more advanced strategy counting on from six. She could not show the number combinations to 10 or to 20. For addition and subtraction, she was classified as a perceptual counter, which means she must “see” the objects in order to count them. Her addition skills within finger range were good as she was able to answer $6 + 3$ without using fingers or verbal assists. She was not able to do the missing addend problem, screened subtraction problem, or the bare number addition and subtraction problems. Bare number problems are problems in which no counters are used; only the bare numbers appear on a card. The child must read the number sentence and supply the missing number, such as $8 + 4 = ?$ Johanna was not ready for the relational

thinking section. Tables 4.10-4.13 contain Johanna's pre-test results from the AVMR Assessment. Johanna moved out of state in early December before the AVMR post-test was administered.

Table 4.10

Number Words and Numerals—Johanna

Number Words & Numerals	<i>Forward Number</i>			<i>Backward</i>		
	<i>Word sequence (FNWS)</i>	<i>Number Word After (NWA)</i>	<i>Numeral Identification</i>	<i>Number Word Sequence</i>	<i>Number Word Before (NWB)</i>	<i>Numeral Sequences</i>
Pre-test August 2011	Level 5 Counts to 112 slower after 110	Ok in 31-100 range slower on NWA 32,	Level 2- ok on 11-100, called 41 fourteen. Trouble past 100, called 117 (107), 168 (108)	Level 2 Counting backwards from 17 to 10, stopped at 12.	Ok on NWB 11-30 range. Had to think on NWB 3, with 17 had to count up	Some mistakes on identify, but self-corrected. Read 51 as (15)
No Post-test Child moved						

Table 4.11

Structuring Numbers—Johanna

Structuring Numbers Level 0 Emergent	<i>Spatial patterns</i>	<i>Finger patterns</i>	<i>Combinations to 5</i>	<i>Combinations to 10</i>	<i>Combinations to 20</i>
Pre-test August 2011	Knows regular spatial patterns; could not identify 6 or 5 on irregular spatial patterns	For 9 counts from 1, for 8 counts from 6	Uses fingers or counts cubes in range 1 – 5. Missed $2 + ? = 5$, answered 2	Did not do combinations to 10. Thought $3 + 6$ was 10	Did not do combinations to 20.
No Post-test Child moved					

Table 4.12

Addition and Subtraction—Johanna

Addition & Subtraction Construct 1	<i>Addition Unscreened</i>	<i>Addition Screened within Finger Range</i>	<i>Addition Screened outside Finger Range</i>	<i>Missing Addend</i>	<i>Subtraction-Screened Collections</i>	<i>Addition & Subtraction – Bare Numbers</i>
Pre-test August 2011 Construct 2	Not necessary	Knows, guessed for 4 +2, 6+3 no verbal or fingers	Knows 9 +5, quiet verbal counts “14”	8 + _ = 11 Cannot do	Cannot do	Did not do- child not ready
No Post-test						

Table 4.13

Relational Thinking—Johanna

Relational Thinking	<i>Commutativity of Addition</i>	<i>Linking Addition & Subtraction</i>	<i>Related Subtraction tasks</i>	<i>Backward Number Word Sequence</i>	<i>Number Word Before</i>	<i>Numeral Sequences</i>
Pre-test August 2011	Did not do- child not ready	Did not do- child not ready	Did not do- child not ready			
No Post-test						

Although Johanna moved early in December and a comparison between her pre and post-test AVMR scores was not possible, some information about her progress was obtained from her Everyday Math Journal. At the start of the school year she wrote her fives backwards beginning on page three in her Everyday Math Journal. This practice continued through page 45, but after page 45 until the time she left Aspen, she wrote the numeral correctly. She was not able to do combinations of ten on the pre-test for $3 + ? = 10$; however, on page 15 of her Journal, when given seven pennies in one hand, she was able to figure out that three pennies needed to be placed in the other hand to make ten. Overall, I saw Johanna gained confidence; she was always engaged in the lessons even if, at times, she took longer to respond than other students. Johanna liked being successful, and if she was pretty sure she knew an answer, she was an eager participant.

Student participants' summary. All three children showed improvement in their mathematical abilities during the time period from August, 2011 to the time of their post-test. The student scores were computed as follows: AVMR Score = Level for FNWS + Level for BNWS + Level for Numeral Id + Level for Structuring Numbers + 2 (Level for Addition & Subtraction Construct). Jasmine made an overall AVMR pre-test score of 14. Her AVMR post-test score was 22, a net gain of eight points. Johanna's overall AVMR pre-test score was an 11. In Johanna's case, she moved before the post-test was given, but I observed that her skill level had increased. Josh's overall AVMR pre-test score was a 12. His AVMR post-test score was a 22. My observation was Josh received additional teacher-time in the classroom, as he needed it more, perhaps resulting in a greater gain for Josh. The average pre-test score for the class was 10.47. Each child scored above average on the pre-test, with Jasmine being the highest followed by Josh and then Johanna. The average post-test score

for the class was 20. For the students in Holly's class, some of the improvement in their scores was likely due to the students' maturing over the course of the study, or perhaps outside influences such as parents practicing with their children at home. In addition, the Everyday Mathematics Curriculum students were exposed to in class targeted many of the tasks included in the AVMR assessments, thus reinforcing several AVMR related concepts and strategies. The improvement I observed could have occurred because of specific topics Holly chose based on initial assessments that were correlated to areas in need of improvement for all three children and were functions of both the Everyday Mathematics Curriculum and AVMR instructional techniques. That the children showed improvement is to be expected. Just exactly how much of their gain is due to the teacher's use of their assessment results? At the beginning of the semester, I asked Holly how she used assessment in her instruction. She responded, "Everyday Math has daily RSA's (Recognizing Student Achievement). I use these daily assessments to plan and guide my math intervention time. My math intervention is used to target skill deficits." Responses from Holly about her use of the AVMR student assessment results to guide instruction gave me additional insight. She did indicate, when questioned, that she reviewed each student's pre-test as a means to guide her instruction. With respect to Jasmine, Holly said, "Jasmine is one of my brightest students in math, so I was shocked to see she was omitting decades in her BNWS. I was not surprised with the rest of her assessment; however, I was surprised she did not score higher with counting backward." With respect to Josh, she said, "The information from Josh's AVMR assessment that was helpful was that he could count backwards higher (*perhaps she meant more fluently*) than he could count forwards. However, this helped me to have him focus more on his counting forward than on his backward counting." With respect to Johanna, she

said, “It had been an assumption on my part that if a child can count forward, they can count backward just fine. However, with Johanna, this was definitely not the case. She had FNWS to 100 but only had BNWS to 10. This information was helpful in being able to pinpoint math skills to work on with Johanna. She was also a level 0 for structuring numbers; however, this information did not surprise me. We worked on this skill as a whole class since the majority of my students struggled with this skill. However, the information on her BNWS was the most helpful and insightful.” Following are classroom observations illustrating Holly’s questioning techniques, levels of questioning, and choices of questions as matched to her instruction targeting student areas in need of improvement during the timeframe of the study.

Influence of AVMR on Holly’s Teaching and Interactions with Jasmine, Johanna, and Josh. During our initial interview, Holly shared the following with regard to the goals she had for each of her students: “I want my students to understand how numbers work together. I want them to be able to explain how they got their answers. I also want them to see how math is meaningful to their life.” Her goals are directly aligned with the AVMR Guiding Principles for Teaching. (See Appendix L.) I asked her, “What teaching methods work best for you?” She responded, “I teach my math lesson in a small group. My students rotate through math centers each day. One of the centers is the math lesson. I also use a lot of manipulatives and math games.”

This section examines Holly’s teaching practices, choices, interactions with her students, and how she uses AVMR to inform instruction. The following examples provide insight into Holly’s instruction and interactions with each student. I speculate the reason for a child’s improvement is related to Holly’s evolving instruction as a result of her

participation in AVMR training. From my observations, and from Holly's comments concerning the value of the AVMR training, I believe this training was a plausible reason for the observed improvement in the students' achievement. In situations where a child continued to repeat the same errors, I noted whether Holly addressed the errors during instruction or in interactions with the child. Again, it must be remembered that Holly was not observed every day, so lessons or interactions with each child that addressed the area of weakness may have occurred but were not observed.

Holly's interactions with the three students will be examined on a topical basis beginning first with the type and level of questions she asked, and then with the topics she stressed in her teaching. In the following section on Holly's questioning, I will also compare and contrast her questioning to typical questioning techniques associated with the AVMR Guiding Principles of Classroom Teaching.

Holly's questioning of Jasmine, Johanna and Josh. Holly did not spend as much time questioning Jasmine as she did Josh or Johanna; I can speculate that, perhaps, Jasmine seemed to perform at a higher level than either Josh, or Johanna, and needed less guidance. However, Holly did have some quality interactions with Jasmine involving the use of her questioning, particularly in giving Jasmine the opportunity to discover patterns and problem solve as in the following excerpt that occurred when Jasmine was one of five students at Holly's instructional table.

T: Now we are going to count by fives and put a red marker on each number we say.

C: (Children) Putting on a red marker as they say each number...5, 10, 15, 20, 25, 30, ...

T: What do you notice about this column (fives)? What do each of these numbers end in?

C: Five.

J: (Jasmine) This column (pointing to fives) ends in five...this column (tens) ends in zero.

Holly used the strategy from the AVMR Guiding Principles for Classroom Teaching that encourages teachers to use inquiry-based learning. She questioned the students to get them to notice patterns, and, here, Jasmine was able to figure out the pattern. As teachers become more proficient with the AVMR Guiding Principles for Classroom Teaching, they would be more likely to ask, “What do you notice about this column of numbers?” rather than asking “What do each of these numbers end in?” It would have been interesting to hear the other possible relationships the children noticed rather than directing them to the answer she was seeking. However, asking open-ended questions of this type gave Jasmine the opportunity to be recognized for her abilities.

It appeared more interactions occurred between Johanna and Holly than Jasmine and Holly, but perhaps this occurred because Johanna needed more help in learning her number sequences than did Jasmine. The following exchange illustrates an interaction between Holly and Johanna. On one occasion, the children were placing scrabble-like tiles on a 100-Chart as part of a whole class activity. The purpose of this game was to provide the children practice in identifying the numbers from one to 100 and practice in placing the numbers in their correct position on the 100-Chart. A group of four children were placing tiles on the 100-Chart; Johanna was one of the children. She placed the number 74 in the incorrect place. She actually placed 74 in the same row as 62. The following student/teacher vignette

illustrates that Holly used questioning involving recall with Johanna but also some higher level questioning that required Johanna to explain the reasoning behind some of her choices.

T: (Teacher) How did you know to put that there, Johanna?

J: (Johanna): I counted.

T: Show me how you counted.

J: (Pointed with her finger vertically down the column, but she must have begun counting with 10, rather than with one, thus being off by one row. She places 74 in the same row as 62.)

T: So does 74 go in the same row as 62?

J: No.

T: Where does 74 go?

Johanna puts 74 in the correct row but in the wrong position. Holly removes 74 but does not say anything and directs her attention to another child in the group who was also placing number tiles on the 100-Chart. Perhaps giving Johanna time to reflect on where she placed the 74 tile by asking her to confirm why it goes where she placed it, then working with the other child, then returning to Johanna later would have been a way to provide an additional learning opportunity for Johanna. It is hard to say what was going through Holly's mind at the time. She may have been in tune with Johanna becoming more timid when she made a mistake and chose not to say anything at that time. Also, many first graders seek the teacher's attention at the same time, and it could be Holly's attention was drawn to another child. At any rate, Johanna seemed to enjoy the 100-Chart and was engaged in the activity but was very hesitant to place a number on the chart. I observed Johanna watching some of the other children placing numbers correctly on the 100-Chart. Johanna was engaged in the

activity, but she did not like making mistakes. If Johanna was not sure what to do, she was more apt to hang back, especially if she placed a tile down and it was not placed correctly the first time.

Approximately one month later, I had the opportunity to observe Johanna playing the 100-Chart game again. I could see significant improvement in her ability to play. She had gained confidence and was placing a greater number of tiles on the 100-Chart correctly. I noticed Holly frequently used the 100-Chart as an activity with each group as they circulated to her teaching center and also at other centers in her classroom, whether under the supervision of the UNM student, or at student self-directed learning centers. Mostly, at first, Holly used the 100-Chart at her station when she was working with students to advance them to being able to count from one to 100. Then she would rotate the 100-Chart to another station where she could provide the children with additional time to practice identifying and correctly placing the numbers from one to 100 on the chart. They could do this activity on their own or with the help of other children. This practice activity was used for at least a month and was directly related to tasks students saw in both the AVMR pre-test and the Everyday Math curriculum.

Holly's method of having children work on the 100-Chart under her guidance first and then as a student-directed activity may have helped in addressing Johanna's number identification weakness. On the AVMR pre-test, Johanna was a Level 2 in Numeral Identification and was primarily successful; however, she would call the number 41, fourteen. This is a common error for many young children, whether they are English Language Learners (ELL) or not, predominately because of the English language. When a

child says fourteen, the “four” sound is made first, causing the child to write the “four” first and consequently the child writes 41 instead of 14 (Sousa, 2008).

When working with Josh, I noticed Holly’s questioning was used more to keep Josh on track and would involve questioning based on recall rather than probing questions such as, “What do you notice?” or “Do you see a pattern or relationship?” to illicit further thinking. Some of the questions I heard her ask Josh were, “How much is a penny worth?” and “How much is a nickel worth?” and “Can you count for me again?” Holly spent a great deal of time with Josh helping him keep up as evidenced by the following dialogue that occurred on September 27th related to counting money; the exchange centered on counting by one (pennies) and counting by five (nickels). The following vignette demonstrates the type of questioning Holly frequently used with Josh. For the following vignette, Josh was one of five students at Holly’s instructional table. Holly was having the students work with money and she put actual coins on a whiteboard. The children had to write the amount of money the coins represented on their white boards.

T: (Teacher) I am putting two nickels and six pennies on the white board. How much is this? Count to yourself.

T: Tanya, your “six” is backwards...Annie, you wrote 61. Take your six and your one and flip them. Go ahead and fix your numbers. We will count and double check.

C: (Children) (as teacher points to coins) 5, 10, 11, 12, 13, 14, 15, 16.

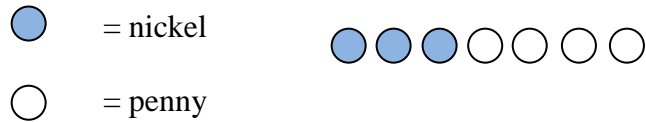
T: Very good! One more on the white board and then we are going to go to the journals. A very tricky one!! (Teacher puts out three nickels and four pennies and there is separation between nickels and pennies).

J: (Josh) Points to each coin and records an incorrect answer of 24 cents.

T: Josh, one more time. He counts again pointing to each coin.

J: 60.

T: Ok, can you count out loud for me?



J: 5, 10, 20, 30, 40, 50, 60 (points to each coin...ends up with 60)

(Once Josh says "10" he counts up by 10's.)

T: On your board you wrote 24. Do you remember how you got 24?

J: Yeah.

T: Ok. How much is a nickel worth?

J: Five.

T: How much is a penny worth?

J: Four. (There are four pennies on the white board.)

T: How much is one penny worth?

J: One.

T: So each penny is worth one cent. So when you count pennies you count by one. If nickels are worth five cents, so when you count nickels, what do you count by?

J: (Starts counting the coins...5, 10, 15 (self-corrected).

T: So, 5, 10, 15, teacher points to the penny after the 3rd nickel.

J: Says 10, 20...

T: Okay, count by fives; 5, 10, 15, with pennies count by one. You ended the nickels at 15...plus one more is?

J: 16, 17, 18, 19.

T: Very good. Okay, everyone, show me your boards...Annie, we are going to count yours too.

Holly spent a lot of time working with Josh to help him in adapting to counting by fives for nickels, and then counting by ones for pennies. I noticed several times Holly gave Josh more of her time to help him settle down and think through problems. She was attentive to the fact that he could easily be distracted and lose concentration or completely stop thinking about the problem he was working on. I believe she wanted him to know he could do the work successfully if he would only slow down and think the problem through.

Tasks and Skills Targeted in Holly's Instruction

Following are the AVMR categories Holly was observed targeting in her instruction.

Missing Addend Problems. Holly noted that all three children had varying degrees of difficulty with "missing addend" problems on the AVMR pre-test. Johanna and Josh were not assessed in this area as the Math Recovery assessor determined they were not yet ready. Jasmine was assessed on the "missing addend" problems on the AVMR pre-test, but was seen to have difficulties. When Jasmine was given the problem $8 + \underline{\quad} = 11$, she counted out eight fingers and then guessed "6." I observed that Holly frequently worked on addition and subtraction sentences with students at her instructional table. She would use red solo cups to model problems such as, "I baked three cookies in my first batch." Holly would place three cups on the table with the bottoms of the cups facing up. "Then I baked some more." Holly placed more cups on the table until there were seven cups on the table with the

bottoms facing up. “Now I have seven cookies. How many cookies did I bake in my second batch?” The students knew Holly placed three cups on the table to start, and then she placed some more cups on the table until there were seven cups on the table. The students had to count how many more cups she put on the table to get a total of seven cups.

Holly’s continued reinforcement of the missing addend problems through repetition may have helped both Jasmine and Josh as evidenced by their AVMR post-test assessments. By April, Josh could answer the missing addend problems by using a “count from one” strategy, and Jasmine could successfully answer the missing addend problems by using a “count on” strategy. For the problem $8 + \underline{\quad} = 11$, on the post-test, Jasmine was able to answer correctly by counting on 9, 10, 11, using her fingers to keep track and responds “3.” Josh would say, “1, 2, 3, 4, 5, 6, 7, 8, 9 (that’s one), 10 (that’s two), 11 (that’s three), the answer is 3.” Again, Johanna moved, but I was able to observe Johanna successfully solve “missing addend” tasks in class while working in her *Everyday Math Journal*.

Addition and subtraction problems. Holly told the children a number story. The children had to decide whether it was an addition or subtraction number story and transfer the information to a number line. Holly modeled the problem for them. “I start at four (puts a dot on four on her number line). I count forward three hops. Am I adding or subtracting? Do it with me on your number lines.” The children in the group including Johanna all got it. Holly also posed problems that involved subtraction such as, “I baked twelve cookies but was very hungry and ate four cookies. How many cookies do I have left?” Holly would put out twelve red solo cups on the table with the bottoms of the cups facing up. When she said she ate four, she would turn four cups down on their side. She would leave the cups in place until each student could write the corresponding number sentence on their whiteboards.

Holly did all the modeling for the students, but the children were responsible for writing the number sentence. The solution would be $12 - 4 = 8$. For this problem, I was able to observe that Jasmine was successful in writing both her number addition and number subtraction sentences.

On September 27, Holly posed the following problem: “I made 14 cookies. Then I ate six cookies (placed cups down sideways). How many do I have left? Write a number model to match the story.” Holly asked Johanna what the subtraction sign looked like. Johanna did not know but quickly caught on. Later the same day, Holly had the children write the problem and give the answer to $8 - 2 = \underline{\quad}$ and $12 - 7 = \underline{\quad}$. Johanna was able to produce the correct answers, but she wrote her respective answers “6” and “5” backwards. By October 13th as indicated in Johanna’s Everyday Math Journal, writing 5s and 6s backwards was still a problem for Johanna. The children had assignments involving practice writing their numbers during the month of August and September, but, for Johanna, additional practice would have been helpful.

Omitting decade numbers. Holly was able to address Jasmine’s weakness in skipping the decade numbers when counting backwards in the following vignette that occurred on October 25th. Holly gave the group of students (Jasmine was in this group) she was working with the following problem: 90, 80, 70, 60... Find the rule.

T: (Teacher) The numbers are getting smaller?

C: (Children) Smaller.

T: So are you adding or subtracting?

S: (Susie) Adding, no subtracting.

J: (Jasmine) When you go 90, 80, 70, you are subtracting ten, so (the rule) it is subtracting ten.

Jasmine also had some difficulties involving counting up by 2s. On two separate occasions, the first occurring on page 38 of the Everyday Math Journal, she had to count up by twos when 12, 14, 16 (were given) she answered 18, 20, 23, 25, 27, 29, 31, and 33. Days and possibly a week or more later, as recorded on page 45 of her journal, she was able to count by twos until 20 and then said 23. With a class of 18 students, it could have been difficult to keep track of individual student errors on a daily or weekly basis; but, had Holly seen this repeated error, she could possibly have corrected it with some additional practice for Jasmine.

Josh had similar difficulties with counting up by 3s. On page 32 in his Math Journal, where he had to color in a 100-Chart starting with the number 2 and then color in every second number, he colored in 2, 4, 6, and 7. Had he done this problem correctly, his 100-Chart would then have had all the even numbers, or multiples of 2, colored in. This realization would have enabled Josh to see the pattern formed when every other number is colored in. When he rushed and made a mistake, Holly responded, “It’s okay, Josh, slow down.” Josh says 6 then 7. Holly responds, “Look at your number grid if you are counting by twos...” Josh is always enthusiastic and sometimes goes too fast and makes an error.

On another occasion, Holly spent time working with Josh to help him learn the Forward Number Word Sequence (FNWS). She used the 100-Chart in which the numbers from one to 100 are represented on Scrabble-like tiles, and the chart has a place for each of the numbers from one to 100 to be arranged. All the tiles are spilled out on a table; and, much like putting a puzzle together; the children pick a number that they can place on the

100-Chart. They keep placing numbers on the chart as many times as they can with no restrictions as to turn taking.

J: (Josh) Puts in 39 in the incorrect spot. He places 39 where 29 should be. (At this time, there are few numbers on the board.)

T: (Teacher) Josh what comes before 80?

J: Aaaah.

T: 79

J: 79

The class is trying to figure out the tens column...10, 20, 30...

T: Where does 48 go?

J: Counts after the 41 tile...42, 43, 44, 45, 46, 47, 48! Yes!

T Look...1, 11, 21, 31, 41, 61, what did we forget??? We forgot the 50s...we will move everything in the 60s row down one.

This vignette illustrates Holly's objective in trying to get students to reflect on their work as she asked, "What did we forget?" Had she waited longer and actually had one of the children figure out the 50s were missing, she would have more effectively been promoting the Guiding Principles for Classroom Teaching, which is foundational in AVMR. Instead of telling the children, "We will move everything in the 60s row down one," had she provided the children time to think about how they could solve this problem, more of her students' thinking could have been uncovered.

Some of Holly's lessons and her interactions with Jasmine, Johanna, and Josh have been described. Now a performance summary for each child is provided, as well as Holly's possible influence on each child through her choice of lesson or activity.

Summary

Holly provided instruction that specifically addressed Jasmine's need in several cases. Her instruction and focus on the decade numbers through activities like the 100-Chart may have helped Jasmine move from a Level 2 to a Level 5 in the Backward Number Word Sequence. Another activity that helped Jasmine with remembering the decade numbers is called "What's my Rule?" in which Holly posed the problem involving the number sequence: 90, 80, 70, 60.... She then asked the children, "What's my rule?" The children had to figure out what rule they needed to use to get the next number in the sequence. This activity also helped in reinforcing the decade numbers, as this rule was "Count back 10."

Holly frequently did "Missing Addend" problems in class where the students used their whiteboards to find the missing addend. Holly's use of red solo cups was effective in having the children write their number sentences. Jasmine moved from a Construct 2 to a Construct 3 in Addition and Subtraction, which could indicate these activities helped Jasmine become more proficient in Addition and Subtraction tasks, including the missing addend task.

As indicated earlier, if Holly had noticed Jasmine's difficulty with counting by two's from the assessment data, Holly could have addressed this difficulty. However, this topic could have been taught in class at a time when I was not observing. Another remaining error from pre-test to post-test was Jasmine's trouble with the number 205. On both occasions, she called the number "twenty thousand five." Had Holly noticed this error from the pre-test, she would have had opportunities to work on place value to a point where Jasmine may have been successful on the post-test. Holly may have worked with Jasmine on this number identification problem at times when I wasn't in class to observe; yet Jasmine was still not

successful. Without being in the classroom each day, there was no way of knowing if Holly addressed these particular concerns.

Holly spent a lot of time helping Josh while he was at her teaching center. She was patient in waiting for him to respond to the questions she asked, encouraging, and provided numerous practice opportunities to address his specific needs. He progressed from Level 3 to Level 5 in the Forward Number Word Sequence category. Holly's use of the 100-Chart and the "Guess my Rule" activities could have contributed to the gain for Josh. He was initially proficient in the Backward Number Word Sequence category, but further improved in this category, moving from Level 4 to Level 5. As mentioned earlier, Holly was observed at least eight times during the months of October and November using the "Guess my Rule" activity to provide practice in counting backwards. Josh moved from not being able to do a "missing addend" problem to being able to do the problem $8 + \underline{\quad} = 11$ by counting from "one" as explained earlier. Both Jasmine and Johanna were able to "count-on" for this problem rather than "count from one."

Josh was able to match Jasmine's post-test score. This was a surprise for me, as I did not observe how much he had improved. His progress might be connected to the amount of time Holly spent in providing Josh additional practice, as well as the numerous times she assisted Josh in various tasks. Her interventions may have helped to improve Josh's performance on the AVMR post-test.

I believed Johanna was the middle-achiever strictly based on observation. When I reviewed the pre-tests again, I was surprised Josh actually scored one point higher than Johanna. Johanna did make significant progress from August through December in several categories. For example, she was now able to solve missing addend problems. Holly was

observed frequently working with the children on missing addend problems, and this additional attention could have made a significant impact on Johanna's improvement in this area. It took a long time for Johanna to stop writing many of her numbers backwards. Perhaps the classroom station that involved practicing how to write numbers could have been retained for a longer period of time to help students like Johanna.

Responses to Research Questions Posed

Question 1. At what level in the AVMR classification scheme was the participating teacher after attending initial AVMR professional development training, and what was the impact of the training on a teacher's instruction?

Over the course of the semester, Holly evinced using some AVMR concepts such as practicing the FNWS, BNWS, and counting forward, as well as backward, on the decade numbers such as 10, 20, 30, 40... and 60, 50, 40, 30.... When the Everyday Mathematics curriculum centered on money in September, I noticed Holly would have the children practice counting by 5s. With AVMR, counting by 2s, 3s, and 5s is frequently used to develop number facility. Activities that foster structuring numbers such as having the children recognize groups of 5 and 10 are also a prominent feature of AVMR in building five-wise and ten-wise number combinations. Having the students do activities such as "I have 7 pennies in my left hand; how many do I need in my right hand?" were often used in the classroom.

Holly used the 100-Chart in which the numbers from 1–100 are spilled on a table; and, much like a puzzle, children pick numbers they can place on the 100-Chart. Holly would begin by introducing the 100-Chart in her teaching center to help the children learn the activity and then rotate the 100-Chart to another learning center in the classroom. This game,

too, encourages number recognition, sequencing, and pattern recognition that are all included in the AVMR activities.

Holly told me she would use the Math Racks during small group time on Fridays to strengthen her students' ability to structure numbers. Structuring numbers in the range 1–10 involves the child's facility to combine and partition numbers without using the counting-by-ones strategy. The child using a Math Rack builds upon their emerging knowledge of doubles as well as using 5 and 10 as reference points. Unfortunately, I was never able to observe her on a Friday. This was the first year she was able to provide this method for her students.

I did see the beginnings of Holly's use of the Guiding Principles of Classroom Teaching (GPCT). She gave time for children to think through problems and then have them share their solutions with their group (GPCT 8), but on a limited basis. She also worked on the translation from verbal to written forms of arithmetic (GPCT 5); that is, she would have the children write a number sentence to match the story problems. Holly now also had a deeper understanding of children's numerical strategies and the knowledge necessary to create situations to advance their number strategies.

Holly did tell me that AVMR has helped her to identify what specific skills each child is struggling with, and, in turn, the training helped her to plan math activities targeting each child's skill deficit. AVMR also changed one of her previous assumptions that if a child could count forward they could count backwards as well. Overall, she conveyed to me she found it difficult to incorporate AVMR into the EDM program. However, she did say that she incorporated the math racks, dot cards, 10 frames, bingo games, treasure hunt game, and the Great Race games. She indicated the games have helped tremendously because the

school district's math program is lacking in the area of practice. She said, "My kids don't get enough practice with their basic facts, counting, or grouping numbers and these activities offer my students opportunity for extra practice".

Question 2. How does a teacher who receives AVMR professional development training in mathematics utilize the AVMR assessment results to inform instruction for a small group of students?

In one of our e-mail exchanges Holly told me she addressed deficits identified by the AVMR assessment results for each of the three children in the study. She seemed to target some topics more often such as the Forward Number Word Sequence, the Backward Number Word Sequence, and the Missing Addend Task. These were tasks each of the children needed help with to varying degrees. At times, she missed opportunities to provide student correction or extra practice for students, particularly in Jasmine's case, when it came to practicing the decade numbers or numeral identification.

Holly had a good start using the assessment data from multiple sources. She grouped students together for certain activities in a way that was consistent with their common need for extra practice with certain skills. She responded in a December 9th e-mail correspondence to my query about how she determined her students' mathematical ability level by writing, "I use their math journals, unit math assessments, and my observations during small group." Using formative assessment from these sources provided her the necessary information to group children together who were working on similar tasks. During several instances I observed her calling on specific children to work together to get more practice with a particular skill through an activity targeting that skill. With continued support in how to readily use the student assessment data and a method for managing the data, Holly could

become even more proficient at targeting student weaknesses, whether through instruction, or creating stations in her classroom to provide extra practice in specific areas to match student needs. She used initial information about her students; however, sustained support in effective ways to keep track of student gains and continued weak areas (i.e., managing the data) might have helped Holly as she worked with her students. This observation affirms the need for providing teachers on-going support as they work to incorporate new methodologies in both instruction and formative student assessment (Clarke, 1994; Wiliam, 2007; Wu, 1999).

Holly reviewed the AVMR Assessment pre-test data and indicated she used the initial AVMR assessments to target student weaknesses. In some cases, she told both the Aspen Mathematics Instructional Coach and me she was surprised by certain gaps in a specific student's knowledge. In one of our interview discussions that occurred primarily through e-mail, I asked Holly how specifically AVMR helped her as a teacher. In class, she had told me AVMR helped her, especially in working with the lower-achieving students; I wanted to know more about how it specifically helped her. She responded, "AVMR has helped me to pinpoint exactly where my students' understanding of math starts and ends. I know my low students struggle with math, but the AVMR has helped to identify what specific skills each child is struggling with and, in turn, helped me to plan math activities that target each child's skill deficit." When asked what tools she uses to determine her students' mathematical ability she responded, "I use their math journals, observations during small group, AMC testing (3 times a year), unit math assessments, and exit slips to assess their math abilities."

Holly did not mention keeping track of student progress, and I do not recall specifically asking her this question during the semester with respect to the pre-test. Had she

done so, she could have kept an on-going record of when each student mastered a particular skill. Such information might have resulted in Holly retaining a station in her classroom for a longer period of time. For instance, keeping a station where practice in writing numbers for students like Johanna or, in Jasmine's case, more practice with numeral identification past 100 could have been provided.

Holly did not use some AVMR-based tools during the times I was there to observe, such as the Bead String and Numeral Roll. These AVMR tools are exceptionally beneficial for uniting the verbal and quantitative aspects of number. The Bead Strings can help a child see groups of 5 and groups of 10 quite easily. The Bead String is composed of 5 red opaque beads, then 5 red translucent beads, followed by 5 blue opaque beads, and 5 blue translucent beads. This pattern continues until 100 beads complete the string. With the use of opaque and translucent beads with a color, 5 is easily distinguishable. With the use of two colors, red and blue, it becomes quite easy to see groups of ten beads within a color.

Numeral rolls are made of two colors that alternate between the decades. For example, the numbers 0-9 may be on yellow paper, and the numbers 10-19 follow on green paper, followed by 20-29 on yellow paper, and so on in the same fashion. The numeral rolls are exceptionally good for students as they lend themselves to discovering patterns in the number families, such as the ones, teens, twenties family and so on. They are easily kept in a student's desk as they roll up and are an excellent resource for the student.

Another activity I did not notice in Holly's class was a game called Treasure Hunt that helps students in ordering numbers. It is played very much like the game Concentration. Treasure Hunt can be customized to address any particular skill that needs reinforcement.

For example, if the students need practice with the teen numbers that are typically called the “Tricky Teens” because they do not look the same as they sound, Treasure Hunt is an excellent resource. A blank deck of cards is used to make a red, green, yellow, and blue row characterized by a large colored dot to designate each row. After the large red dot, the numbers from 10-20 are ordered in that row upon completion of the game. The same follows with the green, yellow, and blue rows. After placing the four dot cards on a table, the teacher shuffles the cards and places 11 cards in each row. The child turns over a card and must place it in the correct row by color and the correct position. The child is able to use the position of a correctly placed number in any row to help them place future cards down correctly.

After initial training in AVMR, as with any new tool or methodology, it takes teachers time to incorporate new ideas, activities, or concepts and to complete the curriculum they are required to follow. I would speculate that Holly would be able to begin bringing more of her AVMR training and activities into the classroom in the future, especially if her District Mathematics Coach, who was highly trained in AVMR principles and strategies, continued to support her.

Question 3. Does the performance of this group of students improve when the teacher modifies teaching by utilizing their assessment results?

All three children’s performances in mathematics improved over the course of this study. No doubt some of the improvement was due to their maturing over the course of the study. The Everyday Mathematics Curriculum they used in class targeted many of the tasks included in the AVMR assessments. The specific topics Holly chose that correlated to areas that needed improvement for all three children, and perhaps extra help at home from parents

or siblings contributed to student improvement. I noticed Jasmine had the potential to make greater gains than reflected in the AVMR post-test. Jasmine's strength was her ability to think through solutions. Providing her more opportunities to work on Relational Thinking may have solidified this concept for her to the point where she would have been proficient at using this skill on the AVMR post-test. Perhaps because Holly determined she was a good student, Holly did not spend as much time in trying to provide additional learning opportunities to further challenge her. The AVMR assessment data could be used to assist a teacher in providing more challenges to certain students based on their assessment outcomes and, thereby, prevent a ceiling effect on how much the child can advance. The first grade curriculum in Everyday Mathematics requires the teacher cover a fixed number of topics. Students who test high on the AVMR pre-test could be offered additional ways to stimulate mathematical growth based on a child's current level of understanding. For example, if a first-grade child were at the point of being facile with missing addends problems, providing the child experiences with relational thinking would be a way of diffusing the ceiling effect. This same approach would be used regardless of topic. Wherever the child's level in terms of mastery of a concept or skill, the next topic or a deeper experience in the current topic should be made available to the child.

This type of approach customizes the learning experience for each child based on the assessment data for that child. Of course, it can be quite a task for the teacher to keep up with a method for meeting the needs of 18 different children in the classroom. Making the child's education more of a collaborative effort between the teacher and child, where the child helps in setting the goals for mastery, can effectively help in managing this task for the teacher. For example, the teacher and child might set three goals the child works towards

accomplishing. These goals could be 1) saying the FNWS to 20, 2) saying the BNWS from 20, and 3) knowing all the combinations that make 20. The child could have their individual goals posted on a card that allows her to check off goals he or she accomplishes. When the child accomplishes a goal, a new goal is put in place and so on. This goal management program could be effective for customizing the learning of each child whether the child is currently a high, middle, or low achiever.

During the observational period, Jasmine was clearly perceived as the high achiever, as Holly identified her as one of her brightest students in response to one of my interview questions. I saw Jasmine as the high achiever primarily because I noticed her ability to reason mathematically and solve problems. Holly indicated Josh was her middle achiever; however, I perceived Johanna as the middle achiever as she had moments when it was clear she was readily processing new information, such as with the “missing addend” problems, even though she did struggle with writing her numbers backwards. I perceived Josh to be the lower achiever primarily because he often failed to follow directions or think through problems. However, his AVMR post-test results indicated he reached the same level as Jasmine in terms of a post-test score. This was a noteworthy outcome. I believe a combination of instruction on the AVMR number topics, topics covered in the Everyday Mathematics Curriculum, and the extra one-on-one time given to Josh by Holly all contributed to his gains on the post-test. Josh had the greatest gain; he rose ten points between pre and post-test, while Jasmine increased by eight points from pre to post-test.

The performance for all three children improved, yet one child in particular could have made greater gains. Jasmine scored higher than either Johanna or Josh on the pre-test, yet her overall gain was two points less on the post-test than Josh’s gain. I perceived Jasmine

as a child who had good problem-solving skills and, for the most part, could see mathematical patterns. She had a few areas that needed more attention, but overall she had the ability to move forward in several categories. Clearly, since Relational Thinking was Jasmine's most difficult task, more exposure to problems of this type would have provided an opportunity for Jasmine to advance in this category. Table 4.14 summarizes the categories and levels of interaction between Holly and her students Jasmine, Josh and Johanna.

Table 4.14

Teacher Interaction Level with Her Students

Student	Teacher's Level of Questioning	Teacher's Frequency of Interaction with the Child	Teacher's Intensity in Targeting Areas of Student Weakness	Student Progress
Jasmine	Medium – some recall, some open-ended to uncover student thinking	Low to Medium	Medium to High on Missing Addend, Low on Numeral Identification	Gained 8 points from pre to post test Pre-test – 14 Post-test – 22
Josh	Low – Predominately recall	High – significant time given to student to help discover mistakes	Medium to High on Missing Addend	Gained 10 points from pre to post test Pre-test – 12 Post-test – 22
Johanna	Medium – some recall, some open-ended to uncover student thinking	Medium	Medium to High on Missing Addend	Pre-test 11, no post-test as child moved Progress was seen in child's classroom performance

Using quantitative measures, a comparison of the two observed students, Jasmine and Josh, (Johanna had moved) to the rest of the students in the four classes using a standard Two-Sample T-Test (which does not consider any student and teacher characteristics and classroom and school intraclass correlation), the following was observed:

1. The overall comparison between the AVMR versus non-AVMR students was an average increase of three points. A comparison between an average of the two observed students' differences in the pre and post-test scores was not statistically different from the average difference in scores from the non-AVMR students (n=23). However, the two AVMR students had a two-point increase in score over the non-AVMR students
2. A comparison between an average of the two observed students' differences in the pre and post-test scores was not statistically different from the average difference in scores for the remaining AVMR students (n = 28). In fact, the two observed students had a lower increase in score than the other AVMR students, 9 versus 10.61, respectively.

A possible reason that a greater benefit was not associated with the students in the classes of AVMR trained teachers could be attributed to the gain Jasmine did not achieve because she received less teacher time and because her initial scores were higher, limiting the amount of gain she had the potential to realize.

With time and support, an AVMR trained teacher can more effectively exploit assessment data to support student learning whether the achievement level of the student is high, medium, or low. The high-achieving student can be further challenged and no longer have to face imposed ceilings on learning. An initially lower-achieving student can be

provided the opportunity to “fill in gaps” so he can successfully build on his knowledge. Perhaps rather than referring to a child as having a lower-achievement level in first grade, it would be more productive to classify the student as having a lower level of exposure to certain mathematical concepts. Another equally important factor for student achievement would be the maturity level of the student with regard to attention span, focus, and control.

Chapter 5: Discussion of Findings and Implications

This chapter summarizes the information provided in the preceding chapters and provides a path for the implementation of early mathematics instructional methods supporting national and state scholastic achievement efforts. First, the study's purpose is revisited within the context of the significance of the study. Next, salient points of the literature review and study approach are presented within the context of the study's results, followed by a concise summary of the findings and a discussion of how those findings complement the broader elements of implementing teacher professional development in early mathematics. Finally, the conclusions and limitations of the study are discussed followed by recommendations for future efforts.

Significance of the Study

This study concentrated on the following research questions:

1. At what level in the Add+VantageMR® (AVMR) classification scheme was the participating teacher after attending initial AVMR professional development training, and what was the impact of the training on a teacher's instruction?
2. How does a teacher who receives Add+VantageMR® professional development training utilize the AVMR assessment results to inform instruction for a small group of students?
3. Does the performance of this group of students change when the teacher modifies teaching by applying student assessment results?

This study was undertaken to examine a program of professional development in early mathematics designed to change how mathematics is taught in the early grades. By

looking closely at the impact of AVMR professional development on teacher instruction and by assessing the methods designed to assist teachers in providing student-appropriate instruction, researchers can contribute to the change needed to improve students' understanding of, and achievement in, mathematics. Enhancing students' mathematics skills begins in the earliest grades, and this preparation involves equipping teachers with knowledge that will assist them in developing their students mathematically. Since it is a professional development program in early mathematics, AVMR can remove elements of mathematics confusion and help students develop a conceptual framework for better understanding. Because of its strong assessment component, AVMR provides a platform for instruction informing teachers how to deliver instruction based on their student assessment outcomes, a relatively unique feature of this program.

Significantly, this study provides a first look at the impact AVMR-type methods and strategies may have on teacher improvement and student learning. In particular, as educators seek opportunities to increase student understanding and achievement through various teacher professional development programs in early mathematics, this study and future studies will answer the need for the continuous evaluation of professional development programs to ensure they meet current and, in particular, future needs.

The study centers, in part, on the initial impact of AVMR professional development on teacher instruction. In addition, this study investigates a teacher's use of AVMR student assessments to help guide instruction as a means to improve students' ability to understand mathematics. Using assessment tools effectively enables teachers to target student mathematical weakness, build upon a child's existing knowledge, and provide instruction commensurate with their students' learning level. This process may, in fact, provide

opportunities for equity in mathematics achievement. No longer is the student merely the recipient of a lesson, but the lesson is specifically calibrated to build upon the current understanding of the student, and advance the student forward, a process likely to increase the possibility for student achievement and success. As discussed earlier, assessment serves to inform teaching, and teaching, in turn, provides additional assessment information allowing a teacher to tailor classroom instruction to anticipate and meet individual students' instructional needs.

Reflecting on the Study Findings vis-à-vis the Research Literature

This section presents pertinent findings of the study and incorporates many of the specific results found in the previous chapter. These results are summarized with reference to the research questions:

Question 1. At what level in the AVMR classification scheme was the participating teacher after attending initial AVMR professional development training, and what was the impact of the training on a teacher's instruction?

Finding 1.1. Holly's Mathematics Instructional Coach and I met to discuss Holly's use of AVMR tasks and our observations of Holly during the school year. We agreed she would be classified as a Lower-Medium AVMR implementer because she used many activities focused on number, but her questioning of students was primarily to elicit specific answers. At times she would ask open-ended questions; however, her most frequent method of questioning posed questions such as, "I move forward (on the number line) three hops; am I adding or subtracting?"

As the school year began, I could see that Holly used some instructional practices that are consistent with the AVMR framework. She worked on practicing the Forward Number

Word Sequence (FNWS), Backward Number Word Sequence (BNWS), and counting forward as well as backward on the decade numbers such as 10, 20, 30, 40... and 60, 50, 40, 30 ...with her students. She told me she incorporated the math racks, dot cards, 10 frames, bingo games, the Treasure Hunt game, and the Great Race game into her classroom instruction and activity stations.

I also, at times, saw Holly use instructional practices consistent with the AVMR Guiding Principles of Classroom Teaching (GPCT). She gave her students time to think through problems, and then had them share their solutions with their group (GPCT 8) on a limited basis. She also worked on the translation from verbal to written forms of arithmetic (GPCT 5), as she had the children write number sentences to match story problems she read to them. Based on my observations, and those of her Mathematics Instructional Coach, we classified her as a lower-medium implementer as she did concentrate on number skills to some extent, but her use of the GPCT was limited.

Implication 1.1. Holly might have benefitted from additional, in-depth collegial support to sustain and strengthen her attempt to incorporate AVMR assessment information and AVMR principles in her classroom. To keep the initial teacher enthusiasm Holly had after attending the AVMR training, providing on-going collegial and coaching support could be helpful. Change is a process, and I believe providing teachers with support for implementing the AVMR activities into their lessons and learning centers should be on-going. It is also my belief that teachers should recognize that change is often a gradual, difficult, and, at times, painful process; they should pursue professional opportunities in which on-going support is provided from both peers and knowledgeable others (Clarke, 1994). One possible way to support teachers, based on the literature about teacher

professional development, is for teachers to share ways they have successfully or unsuccessfully used AVMR activities and techniques with colleagues, thus helping other teachers effectively incorporate AVMR into their existing curriculums as well. These collaborations can serve to keep the initial enthusiasm AVMR generates going. Wu (1999) suggests continuing support must be a component of professional development. This support could include year-round follow-up programs whereby the teachers' progress could be monitored, and additional assistance could be provided.

Finding 1.2. Overall, Holly said it was difficult to incorporate AVMR into the Everyday Mathematics program.

Implication 1.2. Teachers and their Mathematics Instructional Coach, if their district has one, can collaborate to pinpoint areas in their curriculum that could be supported by AVMR methods and activities. Teacher isolation can be an obstacle in setting learning goals. Moreover, enhancing curriculum with research-based ideas and materials requires teacher collaboration to improve instruction (Cwilka, 2002). As teachers begin to introduce new ways to provide students additional support through AVMR methods and activities, these methods can be shared with colleagues and be incorporated into the existing curriculum.

Question 2. How does a teacher who receives AVMR professional development training in mathematics utilize the AVMR assessment results to inform instruction for a small group of students?

Finding 2.1. Although this was her first year using AVMR assessments and tools, Holly demonstrated she used the AVMR assessment results for each of the three children in the study. She indicated to me, in one of our conversations, she reviewed the AVMR pre-test data and, consequently, in planning lessons, targeted some topics more often such as the

Forward Number Word Sequence, the Backward Number Word Sequence, and the Missing Addend Task. These were tasks each of the children needed help with to varying degrees. At times, she missed opportunities to provide student correction or extra practice for students, particularly in Jasmine's case when the child needed to practice the decade numbers or numeral identification. These omissions indicate that Holly did not address all of the deficits identified by the pre-test data in tracking student progress during the semester. The AVMR Course One handbook provides a Class Data Sheet for the purpose of tracking the students' learning levels by topic that could be helpful.

Implication 2.1. Finding 2.1 suggests the need for providing teachers on-going support and efficient methods for timely data collection as they work to incorporate new methodologies in both instruction and student formative assessment. AVMR Course One trainers could perhaps further stress to teachers the importance of using the Class Data Sheet. Keeping an on-going record of when each student mastered a particular skill would provide Holly the necessary information to guide her decisions concerning whether to stay on a topic or move on. By using the Class Data Sheet to plan instruction, she might have retained the practice of writing numbers until students, like Johanna, achieved greater proficiency. As the National Mathematics Advisory Panel (2008) reports, "Formative assessment improves student learning especially if teachers have additional guidance on using the assessment results to design and individualize instruction" (p. 47).

Black and Wiliam's (1998a) review of several hundred empirical articles on classroom formative assessment reports consistent learning gains for students when teachers use assessment practices that support learning. Beyond the initial assessment, teachers need

on-going assessment that supports continuous modification in the teaching focus and approach depending on the individual student and classroom needs.

Finding 2.2. For some unarticulated reason, it appears students were not made aware of their assessment results. Consequently, they did not have even a simple understanding of what skills they needed to work on. For example, had Jasmine known she was weak on the BNWS, she could have practiced the BNWS. Jasmine was quick to grasp new concepts, and, with minimal effort, she could have eliminated some of her areas of weakness. Another example illustrating the benefit of making students aware of areas needing improvement, was for Jasmine, her inability to identify the number 205 on both the AVMR pre-test and then, four months later, on the AVMR post-test. I believe Jasmine could have easily corrected this mis-identification of 205. Fontana and Fernandes (1994) stress the power of getting students to take ownership of their own work and thus of their own learning. Based on my observations of Jasmine, had she known the skill areas in need of improvement such as numeral identification and the BNWS, she might have been successful in these areas on the AVMR post-test.

Implication 2.2. Partnering a student's enthusiasm with the teacher's in creating a simple instructional plan for the student can assist the teacher in tracking the student's progress. Having students track their own progress is a hidden gem increasing interactions between teachers and students and providing them with clear guidance on how to enhance their own learning (Marzano, 2009/2010). As the teacher and student identify specific goals for the student to work toward, the student can monitor his or her own successes and move forward in accomplishing their goals. As the student reaches a goal, he or she can then inform the teacher they are ready to demonstrate his or her mastery. This cooperation, in

fact, places a portion of the responsibility for the student's education on the student. This, of course, is done within the limitations of young students. In a class of 20 or more students, this partnership for reaching goals between teacher and student can be instrumental in helping the teacher keep track of student mastery of goals, as well as areas of continued weakness. Having the student keep a record of his or her goals easily kept in their desk or brought home, allows the student to work with peers at school, or parents/guardians at home, to accomplish their learning goals.

Finding 2.3. A primary benefit to Holly's small group instruction was the opportunity to observe students individually as they were learning and using new concepts she had just taught them. By viewing students at close range as they worked at the same table, she was able to observe multiple nonverbal cues as well. Based on both verbal and nonverbal cues, she could ask students questions and redirect their thinking if they were making errors. Students in our classrooms often let us know they do not understand through nonverbal clues that may be as simple as a puzzled face or as dramatic as throwing one's hands up in the air whether in triumph or agony (Fischer & Frey, 2007). Johanna would often give nonverbal clues such as looking down or shyly smiling when she did not know how to begin. I observed these nonverbal clues specifically when Johanna was playing the 100-Chart game in which students were to place number tiles in the correct position on the empty 100-Chart. Another incident occurred when Holly asked if the children knew what the subtraction sign looked like; to this question, Johanna looked down and shook her head to mean "no." I noticed quieter children, like Johanna, giving more nonverbal clues than students who were more comfortable in speaking out.

However, I observed total reliance on small groups for instruction had a drawback as well. The AVMR Guiding Principles of Classroom Instruction stress giving students plenty of time to answer questions while soliciting numerous responses for how students solve a problem. Because of the time constraint associated with rotating small groups of students every 15-20 minutes, Holly had little time to ask children over-arching questions or to have the entire class participate in a discussion where students would have the opportunity to hear strategies and solutions from all classmates rather than from the limited number of students in their particular group.

Implication 2.3. This study supports the research literature suggesting using both small group instruction as well as incorporating whole class time to reflect on problems can advance students' learning, while also highlighting potential drawbacks of small group instruction. Slavin (1990) concluded that cooperative methods including both small group as well as whole class approaches were effective in improving student achievement. Small group time provides the opportunity for the teacher to observe individual students as they work on problems and to intervene and guide the student as necessary. Whole class time allows each student to hear solutions presented by members of the entire class, rather than just those from their smaller groups, and provides the teacher opportunities to stress the value of sharing multiple ways and varied methods to solve problems. Small group instruction limits students' exposure to various problem solving methods afforded by whole class discussions. AVMR, and other early mathematics instruction programs, may benefit from additional research in determining the best combination of large and small group instruction.

Question 3. Does the performance of this group of students improve when the teacher modifies her teaching by applying student assessment results?

Finding 3.1. The performance for all three children improved, which would be expected for any type of curriculum employed. The AVMR pre-test assessment scores of Jasmine, Josh, and Johanna were 14, 12, and 11, respectively, with the class average at the start of the year being 10.47. Jasmine and Josh both finished with a post-test score of 22; they both had a higher score than the class average of 20 for the post-test score. Since Johanna left the school at midyear, her improvement was determined directly through improvement in her Everyday Math Journal and through my observations of her class work. In addition, the Everyday Mathematics curriculum the three students were exposed to in class targeted many of the tasks included in the AVMR assessments, thus reinforcing several AVMR related concepts and strategies.

One of the Everyday Mathematics goals is to “count on” by 1s, 2s, 5s, and 10s past 100 and back by 1s from any number less than 100. These goals relate directly to the AVMR counting tasks designated as the Forward Number Word Sequence, Backward Number Word Sequence, and are found in such activities as Count Around Multiples, and Quick Draw Multiples. Another Everyday Mathematics goal is to demonstrate proficiency with adding doubles and finding number combinations that make 10. These skills are addressed in the section of AVMR known as Structuring Numbers, the component of AVMR that moves children from “counting by ones” strategies, to “counting on”, and to chunking numbers together to either “make a ten” or “make doubles” which fosters flexibility in adding and subtracting.

Some of the improvement I observed in the students could have occurred because of specific topics chosen by Holly based on the initial assessments, which could be correlated to

areas that needed improvement for all three children, and were functions of both the Everyday Mathematics Curriculum and AVMR instructional techniques.

To some extent, Holly made use of the initial AVMR assessment results for each of the three children in the study, although she seemed to target some topics more often such as the Forward Number Word Sequence, the Backward Number Word Sequence, and the Missing Addend Task. I believe Holly targeted these tasks more often as each of the children needed help with these tasks, albeit to varying degrees. She frequently had “Missing Addend” problems for the students to write and compute in class where the students used their whiteboards to find the missing addend. Holly’s use of red solo cups was particularly effective in having the children write their number sentences for the missing addend problems.

For example, when Jasmine was given the problem $8 + \underline{\quad} = 11$, she counted out eight fingers and then guessed “six.” To reduce errors such as this one, Holly frequently worked on addition and subtraction sentences with students at her instructional table. She would use red solo cups to model problems such as “I baked three cookies in my first batch.” Holly would place three cups on the table with the bottoms of the cups facing up. Holly continued, “Then I baked some more.” Holly placed more cups on the table until there were seven cups on the table with the bottoms facing up. She posed the problem to the children as “Now I have seven cookies. How many cookies did I bake in my second batch?” The students knew Holly placed three cups on the table to start, and then she placed some more cups on the table until she had seven cups on the table. The students had to count how many more cups she put on the table to get a total of seven cups.

Of the three children, Josh made the greatest gain from pre to post-test. He increased his score by ten points whereas Jasmine increased by eight. Josh's greater gain may have been based on the greater amount of time Holly spent with him using instruction that engaged specific AVMR principles. These principles included allowing the child greater time to think through situations, appropriately questioning the child to further elicit specific attention to areas of misunderstanding, and providing extra emphasis on number tasks. Holly was always very patient with Josh and gave him ample time to think and answer questions. Perhaps this extra time was given to Josh as he would frequently become distracted and get off task.

Implication 3.1. The AVMR assessments did help Holly target students' weaknesses and provided a path forward supported by the Learning Framework in Number to advance the student by bridging the students' knowledge gaps. Because the AVMR assessments are interview-based, they provide insight into what mathematics children know and pinpoint the exact areas where understanding breaks down.

Finding 3.2. The AVMR assessment provides information on both what students know and what they do not know. This information can be used to not only guide instruction to meet the needs of lower achieving students but can also be used to advance higher achieving students as well. For example, based on my observations of her ability to reason mathematically and solve problems, Jasmine had the potential to make greater gains than reflected in the AVMR post-test score. However, I did not observe Holly spending as much time with Jasmine in trying to help her overcome her areas of weakness, or in providing additional learning opportunities to further challenge her. This was particularly noticeable

with regard to the Backward Number Word Sequence category in Jasmine's case. This omission may have occurred because Holly perceived Jasmine as a good student.

Implication 3.2. The AVMR assessments could be used both to address areas of student weakness and provide information that can guide teachers to foster additional growth for high achieving students. Assessment data could be used to assist a teacher in providing more challenges for certain students based on students' assessment outcomes. Since formative assessment can improve student learning, especially if teachers have additional guidance on using the assessment results to design and individualize instruction, both lower and higher achieving students can benefit from its use (National Mathematics Advisory Panel, 2008).

However, limitations in both the student's curriculum and the AVMR assessments can hinder student advancement and subsequent measurement of advancement, respectively. For example, Jasmine had the highest possible score on the FNWS pre-test. For the most part, the first grade curriculum did not allow for additional advancement. Fractional parts were the last topic introduced in the final section of the Everyday Math text. The AVMR assessment did not assess fractional parts and would not be capable of reflecting a student's understanding or subsequent advancement, even if a student demonstrated understanding of fractional parts.

Limitations of the Study

This preliminary study concentrated, in detail, on a small sample. Only one teacher and her three students within one classroom were observed primarily over a four-month period although additional observations and AVMR assessment scores were accrued over a full academic year. Having such a small sample was beneficial because it availed itself to

efficiently tracking changes in student performance on a weekly basis in a detailed manner. Because I was only concentrating on three children, I could quickly identify whether the teacher was targeting areas of weakness for these students. The disadvantage of such a small sample was that comparisons between teachers based on criteria such as years teaching, years teaching using AVMR techniques, educational background, etc. could not be made and analyzed for impact on student learning which made the findings more difficult to generalize to a larger population.

Holly received AVMR training in June, just prior to the start of school in August. It would be beneficial to expand the study to include teachers who have had the AVMR training, and have been using it in the classroom for various time intervals, such as after one year, two years, and several years. Addressing such questions as to whether a teacher becomes more proficient in using the student assessment data to plan and deliver instruction or whether the effects of the training start to fade would provide necessary information about the degree to which follow-up support is required for the teacher. It takes time to fully implement new ideas and changes in a teacher's practice (Adelman & Walking-Eagle, 1997). Teachers need time to introduce and institutionalize new strategies into the on-going daily life of the school and the classroom as well as time to reflect on these changes. Knowing whether a teacher would become more proficient at targeting student areas in need of improvement, or how the teacher would track student progress and target new goals for the student would add to the existing research.

Another limitation of the study involves its duration. Although I continued to visit the classroom until the end of the year, I only visited the classroom weekly from August 2011 through December 2011. Beginning in January 2012, my visits were monthly. To fully

capture changes in practice, continuing a weekly visitation schedule would have allowed a better opportunity to observe how the teacher kept track of student achievement and subsequently modified her teaching over a longer period of time.

Recommendations

From Implication 1.1, a plan to build collegial teams could provide support for teachers as they incorporate what was learned in the initial AVMR training in their classroom. Collegial teams could help alleviate the feeling of isolation experienced by many teachers, especially when they are incorporating new methodologies and activities in their classroom. This implication mirrors that of Kitchen et al. (2007) who recommend teachers attend professional development sessions to enhance their instructional strategies and then continue meeting together to share ideas and to support one another in the improvement of instruction.

In areas where teachers have few opportunities to meet with AVMR trained colleagues, perhaps a type of on-line chat room sponsored by the United States Mathematics Recovery Council (USMRC), could be incorporated into their web site allowing teachers the opportunity to have questions answered and share best practices. This view is echoed by Cwikla (2002) and Little (1982) who maintain teacher isolation is an obstacle to setting learning goals and professional development has the greatest influence when it occurs in a collegial environment where teachers believe they can learn from one another.

From Implication 1.2, a mechanism for providing teachers information on where to incorporate specific AVMR activities in their existing curriculum could make using specific AVMR activities easier for teachers and could, subsequently, provide additional resources to promote student learning within the guidelines of the curriculum. Such support could be

made available on the USMRC website. Incorporating new ideas and lessons into the curriculum is suggested by Schifter and Fosnot (1993) as they recommend encouraging teachers to share ideas and create learning lessons in their classrooms to share as a valuable resource to support teacher learning.

From Implication 2.1, a method to provide continued teacher support that helps teachers monitor student progress beyond the initial AVMR assessment through on-going formative assessment could increase teacher capability to either correct on-going student errors or to move an individual child forward by focusing on the next area in need of improvement. The purpose of the AVMR assessment is to inform teachers' instruction so they can target areas of student weakness, thereby filling in gaps in student knowledge and increasing the opportunities for future mathematics learning to occur. Unfortunately, not all assessments are constructed to promote learning. For example, in direct contrast, Wiliam (2007) found that, even when less formal assessments are used, the purpose is far more likely to be about making a determination of a student's current state of knowledge. Clearly, many of our current assessment practices are deeply rooted in past practice.

The assessments highlighted by Wiliam (2007) suggest an effective use of assessment for learning includes sharing reasons for learning and criteria for success, promoting effective classroom discussions eliciting evidence of learning, providing feedback that moves learners forward, encouraging students to use one another as educational resources, and inspiring students to be the owners of their own learning. These components for assessment align directly with the tenets of the AVMR assessments. Coffrey, Hammer, Levin, and Grant (2011) and Bennett (2011) hypothesize that, until teachers are consistently given feedback

that unambiguously addresses their formative assessment practices, improving their skill in assessing students may be a low priority.

Additional research centered on how to best support teachers in this area would benefit teachers, students, and the educational community. Kulm (1994) asserts that little attention has been given to the effects of assessment on teachers, their instructional practices, and their students' learning when they use new evaluation approaches. One of the challenges of many professional development programs is to sustain support so new methodologies are fully integrated into teacher practice, thereby supporting their utilization. According to Wylie and Lyon (2012a, 2012b), professional development should help teachers gain an understanding of how to collect, analyze, and interpret evidence of student learning, how to make strategic adjustments in instruction, and how to provide students with feedback to support this learning.

From Implication 2.2 involving students to a greater degree in keeping track of their progress and setting new learning goals, commensurate with their level of maturity, could greatly increase a teacher's awareness of areas still needing remediation and areas in which the child should be advanced. A recommendation for future research would be to study the impact of increasing student ownership for remediating weak areas and facilitating the setting of new learning goals.

From Implication 2.3, AVMR-based instruction, strategies, and activities can offer insight into whether whole class instruction, small group instruction, or a mixture of both methods is optimal. Further, research involving the benefit associated with each of these instructional methods could be of value. Grouws and Cebulla (2002) state whole-class instruction following individual and group work improves student achievement. They go on

to say whole class discussions can be quite effective when used for sharing and explaining a variety of solutions by which individual students have solved problems. On the other hand, Davidson (1985) reviewed studies comparing student achievement in small group settings with traditional whole-class instruction. In more than 40% of these studies, he found students in classes using small group approaches significantly outscored control students on measures of student performance. In only two of the 79 studies Davidson conducted did control group students perform better than small group students.

From my observations of the four teachers in the Aspen School District, I saw benefits to using both methods. Small group instruction provided teachers the opportunity to carefully gauge a student's understanding and immediately adjust instruction as student errors occur; on the other hand, whole class discussion provided the opportunity for students to practice mathematical discourse, think about the solutions of peers, and be exposed to more strategies for solving problems.

From Implication 3.1, initial data from the three students indicated the AVMR assessments could help the teacher target student weaknesses. Additional research should be conducted to determine if this capability holds for a larger student population. Including observations of additional AVMR-trained teachers and their students would expand upon this study, increasing the data, so as to better inform conclusions drawn from initial data. I will be adding to this research by including data gathered in a neighboring community to the Aspen School District during the fall 2013 semester. Once again, two first-grade teachers, one AVMR trained and one not trained, will be studied and their students assessed to build on this initial study.

From Implication 3.2, the AVMR assessments can provide information on higher achieving students as well as their lower achieving counterparts. This information should be used to advance the higher achieving students as well as students who are considered lower achieving. Further research on the ceiling effect experienced by higher achieving students should be conducted to provide information regarding how teachers can avoid imposing barriers to greater student learning and achievement. When the abilities of higher achieving students are not being revealed due to the constraints of the given assessment, an alternate component or battery of assessments should be made available, enabling instruction for the high achieving student based on the student's ability level and future capability.

Another opportunity for additional research would be to study teachers who are in various stages of their teaching career to determine the impact years of teaching experience has on implementing professional development training. Observing teachers in the classroom who have been AVMR trained immediately after initial training, after one year, and after two or several years would provide indicators of how more experienced teachers take advantage of AVMR assessment data to improve mathematics instruction.

Study Conclusion

This study investigated the impact of AVMR professional development in early mathematics on a teacher's instruction and on her use of student assessment to inform her teaching with the intent of developing insights to improve early mathematics instruction. Since research suggests the U.S. mathematics teaching force is not well prepared for the challenges involved in Standards-based teaching (NCTM Research Committee, 2008), the study has provided information on how to better prepare teachers to meet the challenges involved in Standards-based teaching. Within the realm of assessment, issues surface such as

how to best use assessment to inform teaching, how to track student performance on an on-going basis to provide appropriate instruction, how to sustain teacher support to maximize the impact of professional development on teacher practice, and how to involve students in setting educational goals. AVMR addresses some of these issues specifically, such as how to best use assessment to inform teaching, track student progress, and sustain teacher support.

AVMR has the capability to incorporate methods to target student involvement in setting and attaining their educational goals. Extending these constructs to a broader level would involve using insights gained from AVMR principles and practice to guide instruction for all elementary school teacher professional development regardless of the subject matter. The principles found to be of greatest benefit should be incorporated into all teacher professional development and should be an integral part of a pre-service teachers' education to better prepare teachers to meet the challenges in Standards-based teaching. Principles, such as those provided in the Guiding Principles of Classroom Teaching, are universal and can be applied to any subject area. When teachers take the role of facilitator, rather than strict disseminator of knowledge, students are provided opportunities and time to think and share their thoughts. Professional development centered on such principles can be transformative. Unlike the findings of Smith (2007) who states that some believe professional development, as we now know it, will not transform teachers' knowledge, beliefs, understanding, and ultimately habits of practice, my observation has been that AVMR professional development can transform one teacher's knowledge, beliefs, and understanding. The challenge is to provide all teachers in-depth support to assist them as they change habits of practice to allow students opportunities to think, problem solve, create, and share regardless of their curriculum.

References

- Adelman, N.E., & Walking-Eagle, K.P. (1997). Teachers, time and school reform. In A. Hargreaves (Ed.), *Rethinking educational change with heart and mind: 1997 ASCD Handbook* (pp. 92-110). Alexandria, VA: Association for Supervision and Curriculum Development.
- Antell, S.E., & Keating, D.P. (1983). Perception of numerical invariance in neonates. *Child Development, 54*, 695-701.
- Aubrey, C. (1993). An investigation of the mathematical knowledge and competencies which young children bring into school. *British Educational Research Journal, 19*(1), 27-41
- Aubrey, C. (1997). Children's early learning of number in school and out. In I. Thompson (Ed.), *Teaching and learning early number* (pp. 20-29). Philadelphia, PA: Open University.
- Ball, D.L. (1996). Teacher learning and the mathematics reforms. *Phi Delta Kappan 77*(7), 500-509.
- Ball, D.L., & Cohen, D.K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice*. San Francisco: Jossey-Bass.
- Ball, D.L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(1), 14-46.

- Baroody, A. J. (1987). *Children's mathematical thinking: A developmental framework for preschool, primary, and special education teachers*. New York, NY: Teachers College.
- Baroody, A.J., Lai, M.-L., & Mix, K.S. (2006). The development of young children's number and operation sense and its implications for early childhood education. In B. Spodek & O.N. Saracho (Eds.), *Handbook of research on the education of young children* (pp. 187-221). Mahwah, NJ: Erlbaum.
- Battista, M.R. (1999). The mathematical miseducation of America's youth: Ignoring research and scientific study in education. *Phi Delta Kappan*, 80(6), 424-433.
- Begle, E.G. & Gibb, E.G. (1980). Why do research? In R. J. Shumways (Ed.), *Research in mathematics education* (pp. 3-19). Reston, VA: National Council of Teachers of Mathematics.
- Bell, A.W., Costello, J., & Kuchermann, P. (1983) *Review of research in mathematical education* (Pt. A). Berkshire, England: Nfer-Nelson.
- Bender, W. N. (2002). *Differentiated instruction for students with learning disabilities. Second edition*. Thousand Oaks, CA: Corwin Press.
- Bender, W. N. (2005). *Differentiating math instruction: Strategies that work for K-8 classrooms!* Thousand Oaks, CA: Corwin Press.
- Bender, W. N. (2009). *Beyond the RTI Pyramid: Solutions for the First Years of Implementation*. Bloomington, IN: Solution Tree Press.
- Bennett, R. E. (2011) Formative assessment: A critical review. *Assessment in Education: Principles, Policy, & Practice*, 18, 5-25.

- Berch, D.B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38*(4), 333-9.
- Berg, B.L. (2004). *Qualitative research methods: For the social sciences*. (5th ed.). Boston, MA: Pearson Education.
- Black, P. & Wiliam, D. (1998a). Assessment and classroom learning. *Assessment in Education, 5*(1), 7-74.
- Black, P., & Wiliam, D. (1998b). Inside the black box: Raising standards through classroom assessment, *Phi Delta Kappan, 80*(2), 139-148.
- Black, P., & Wiliam, D. (2004). The formative purpose: Assessment must first promote learning. In M. Wilson (Ed.), *Towards coherence between classroom assessment and accountability*. Chicago: University of Chicago Press.
- Boaler, J. (2008). Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal, 34*(2), 167-194.
- Bowman, B.T., Donovan, M.S., & Burns, M.S. (Eds.). (2001). *Eager to learn: Educating our preschoolers*. Washington, DC: National Academy Press.
- Briand, C. (2012). *Measuring the effects of Add+VantageMR® teacher professional development on student learning in the Moriarty-Edgewood school district*. Unpublished manuscript. Sandia National Laboratories, Albuquerque, NM.
- Brinkerhoff, J. (2006). Effects of a long-duration professional development academy on technology skills, computer self-efficacy, and technology integration beliefs and practices. *Journal of Research on Technology in Education, 39*(1), 22-43.

- Brown, C.A., & Borko, H. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp.127-145). New York, NY: Macmillan.
- Butterworth, B. (1999). *What counts: How every brain is hardwired for math*. New York, NY: The Free Press.
- Cain, R.W., Kenney, P.A., & Schloemer, C.G. (1994). Teachers as assessors: A professional development challenge. In D. B. Aichele (Ed.), *Professional development for teachers of mathematics*, 1994 Yearbook of the National Council of Mathematics. (pp. 67-78). Reston, VA: NCTM.
- Campbell, F.A., Pungello, E.P., Miller-Johnson, S., Burchinal, M.R., & Ramey, C.T. (2001). The development of cognitive and academic abilities: Growth curves from an early childhood education experiment. *Developmental Psychology*, 37, 231-242.
- Carpenter, T. P. (1980). Research in cognitive development. In R. J. Shumway (Ed.), *Research in mathematics education*, (pp. 146–206). Reston, VA: The National Council of Teachers of Mathematics.
- Carpenter, T. P. (1986). Conceptual knowledge as a foundation for procedural knowledge. In J. Heibert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 113-132). Hillsdale, NJ: Erlbaum.
- Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T.A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19-32). Mahwah, NJ: Erlbaum.

- Carpenter, T. P., Fennema, E., & Franke, M.L. (1996). Cognitively Guided Instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, 97(1), 3–20.
- Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L. & Empson, S.B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.
- Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, C.P., & Loeff, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26, 499-531.
- City Data, Moriarty, New Mexico. (2012) Retrieved April 1, 2012, from <http://www.city-data.com/city/Moriarty-New-Mexico.html#ixzz204jnsspb>.
- Clarke, D.M. (1994) Ten key principles from research for the professional development of mathematics teachers. In D.B. Aichele & A.F. Croxford (Eds.), *Professional development for teachers of mathematics* (Yearbook of the National Council of Teachers of Mathematics, pp. 37-48). Reston, VA: NCTM.
- Clarke, D.M., & Clarke, B. (2009). Effective professional development for teachers of mathematics: Key principles from research and a program embodying these principles. http://stwww.weizmann.ac.il/G-ath/ICMI/clarke_doug_amp_barbara_ICMI15_prop.doc
- Clements, D.H. (1984). Training effects on the development and generalization of Piagetian logical operations and knowledge of number. *Journal of Educational Psychology*, 76, 766-776.

- Clements, D.H., & Sarama, J. (2007). Early childhood mathematics learning. In F.K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 461-556). Charlotte, NC: NCTM Information Age Publishing.
- Clements, D.H., Sarama, J., & DiBiase, A.-M. (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Erlbaum.
- Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23(7), 13-20.
- Coffey, J.E., Hammer, D., Levin, D.M., & Grant, T. (2011). The missing disciplinary substance of formative assessment. *Journal of Research in Science Teaching*, 48(10), 1109-1136.
- Cohen, D., & Hill, H. (2000). Instructional policy and classroom performance: the reform in California. *Teachers College Record*, 102(2), 294-343.
- Conference Board of the Mathematical Sciences, (2001). *The mathematical education of teachers*. Providence, RI & Washington DC: American Mathematical Society and Mathematical Association of America.
- Copeland, R.W. (1974). *How children learn mathematics: Teaching implications of Piaget's research* (2nd edition). New York, NY: MacMillan Publishing.
- Corcoran, C.A., Dershimer, E.L., & Tichenor, M.S. (2004). A teacher's guide to alternative assessment: Taking the first steps. *The Clearing House*, 77(5), 213-216.
- Corcoran, T. B. (1995). *Helping teachers teach well: Transforming professional development*. CPRE Policy Brief.

- Creswell, J.W. (2003). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: Sage.
- CTB McGraw-Hill. (2011). TerraNova Assessment (3rd ed.). Retrieved from <http://www.ctb.com/ctb.com/control/ctbProductViewAction?productFamilyId=449&productId=757&p=products>
- Cullivan, D. (2008). Research Overview. Retrieved from <https://sites.google.com/a/hpes.org/donna-cullivan-title-1-reading-spec/home/math-recovery-1/research-overview>
- Cwikla, J. (2002, April). An interview analysis of teachers' reactions to mathematics reform professional development. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.
- Danzig, T. (1967). *Number. The Language of Science* (4th ed.). Windsor, CT: Yankee Clipper Books.
- Darling-Hammond, L., & McLaughlin, M.W. (1996). Policies that support professional development in an era of reform. In M.W. McLaughlin & I. Oberman (Eds.), *Teacher learning: New policies, new practices* (pp. 202-218). New York, NY: Teachers College Press.
- Darling-Hammond, L., & Sykes, G. (Eds.). (1999). *Teaching as the learning profession: Handbook of policy and practice*, San Francisco: Jossey-Bass.
- Davidson, N. (1985). Small group cooperative learning in mathematics: A selective view of the research. In R. Slavin (Ed.), *Learning to cooperate: Cooperating to learn*, (pp. 211-30). New York, NY: Plenum.

- Davis, R.B., Maher, C.A., & Noddings, N. (Eds.). (1990). *Constructivist views on the teaching and learning of mathematics. Journal for Research in Mathematics Education Monograph, 4.*
- Dehaene, S. (1997). *The number sense.* New York: Oxford University Press.
- Dehaene, S., Molko, N., Cohen, L. & Wilson, A.J. (2004). Arithmetic and the brain. *Current Opinion in Neurobiology, 14*, 218-224.
- Denzin, N. K. (1970). *The research act in sociology.* Chicago: Aldine.
- Denzin, N. K. (1978). *Sociological methods: A sourcebook* (2nd ed). New York, NY: McGraw Hill.
- Desimone, L., Porter, A.C., Birman, B.F., Garet, M.S., & Yoon, K.S. (2002). How do district management and implementation strategies relate to the quality of the professional development that districts provide to teachers? *Teachers College Record, 104*, 1265-1312.
- Department for Education and Skills (DfES) (2004). *What works for children with mathematical difficulties*, Research Brief RB554. London: DfES.
- Department for Education and Skills (DfES) (2005). *Targeting support: Implementing interventions for children with significant difficulties in mathematics*, Primary National Strategy, 1083-2005. London: DfES.
- Devlin, K. (2000). *The Math Gene: How Mathematical Thinking Evolved And Why Numbers Are Like Gossip*, Basic Books.
- Devlin, K. (2005). *The math instinct: Why you're a mathematical genius (Along with lobsters, birds, cats and dogs).* NY: Thunder's Mouth Press.

- Dodge, D.T., Colker, L., & Heroman, C. (2002). *The creative curriculum for preschool* (4th ed.). Washington, DC: Teaching Strategies.
- Dossey, J. A., Mullis, I. V. S., & Jones, C. O. (1993). *Can students do mathematical problem solving?* Washington, DC: US Department of Education.
- Douglass, H.R. (1925). The development of number concept in children of preschool and kindergarten ages. *Journal of Experimental Psychology*, 8, 443-470.
- Dowker, A.D. (2003). Interventions in numeracy: Individualized approaches. In I. Thompson (Ed.), *Enhancing Primary Mathematics Teaching* (pp. 127-135). Maidenhead, UK: Open University Press.
- Dowker, A.D. (2004). *Children with difficulties in mathematics: What works?* London: DfES.
- Duncan, G.J., Dowsett, C.J., Claessens, A., Magnuson, K., Huston, A.C., Klebanov, P., ...Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, 43: 1428-1446.
- Early, D.M., Maxwell, K.L., Burchinal, M., Alva, S., Bender, R.H., Bryant, D., ...Zill, N. (2007). Teachers' education, classroom quality, and young children's academic skills: Results from seven studies of preschool programs. *Child Development*, 78(2), 558-580.
- Elmore, R.F. (2002). *Bridging the gap between standards and achievement: The imperative for professional development in education*. Washington, DC: Albert Shanker Institute.
- Elmore, R. F. (2007). *School reform from the inside out: Policy, practice, and performance*. Cambridge, MA: Harvard Education Press.

- Ernest, P. (1992). The Nature of Mathematics: Towards a Social Constructivist Account. *Science and Education*, 1 (1), 89-100.
- Ernest, P. (1989). *Recent Developments in Primary Mathematics*. University of Exeter School of Education (Unit 1).
- Even, R., & Tirosh, D. (2002). Teacher knowledge and understanding of students' mathematical learning. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 219-240). Mahwah, NJ: Laurence Erlbaum.
- Feiman-Nemser, S. (1983). Learning to teach. In L. S. Shulman & G. Sykes (Eds.), *Handbook of teaching and policy*, (pp. 150-170). New York, NY: Longman Publishers.
- Fennema, E., Carpenter, T., Franke, M., Levi, L., Jacobs, V., & Empson, S. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, 27(4):403–434.
- Fielding, N.G., & Fielding, J.L. (1986). *Linking data*. Newbury Park, CA: Sage.
- Fisher, D., & Frey, N. (2007). *Checking for understanding: Formative assessment techniques for your classroom*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Fontana, D., & Fernandes, M. (1994). Improvements in mathematics performance as a consequence of self-assessment in Portugese primary school pupils. *British Journal of Educational Psychology*, 64, 407-417.
- Franke, M.L., Carpenter, T.P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal*, 38(3), 653-690.

- Franke, M.L., Fennema, E., Carpenter, T.P., Ansell, E., & Behrend, J. (1998). Understanding teachers' self-sustaining change in the context of professional development. *Teaching and Teacher Education, 14* (1), 67-80.
- Fuson, K.C. & Wearne, D. (1997). Children's conceptual structures for multi digit numbers and methods of multi digit addition and subtraction. *Journal of Research in Mathematics Education, 28*(2), 130-163.
- Gallistel, C.R. & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences, 4*, 59-65.
- Gardener, H. (1992) Assessment in context: The alternative to standardized testing. In B. Gifford & M. O'Connor (Eds.), *Changing Assessments: Alternative views of aptitude, achievement, and instruction*, (pp. 77-120). Boston, MA: Kluwer.
- Gay, L. R., & Airasian, P. (2003). *Educational research: Competencies for analysis and applications* (7th ed.). Upper Saddle River, NJ: Pearson Education.
- Geary, D.C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association.
- Geller, C.H. & Smith, K.S. (2002, October). *Improving the teaching of math: From textbook concepts to real-world applications*. Paper presented at the annual meeting of the Council for Learning Disabilities, Denver, CO.
- Gelman, R. & Gallistel, C.R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.
- Gentner, D. & Stevens, A. (1983). *Mental Models*. New Jersey: Lawrence Erlbaum Associates.

- Gersten, R. & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *Journal of Special Education, 33*, 18-28.
- Gersten, R., Chard, D., Baker, S. & Lee, D. (2002) (manuscript under review) A meta-analysis of research on mathematics interventions for elementary students with learning disabilities.
- Ginsburg, H.P. (1997). *Entering the child's mind: The clinical interview in psychological research and practice*. NY: Cambridge University Press.
- Ginsburg, H.P., & Russell, R.L. (1981). Social class and racial influences on early mathematical thinking. *Monographs of the Society for Research in Child Development, 46*(6), 1-69.
- Ginsburg, H.P., Lee, J.S., & Boyd, J.S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report: Giving child and Youth Development Knowledge Away, 22*(1). Publication of the Society for Research in Child Development.
- Gore, J., & Ladwig, J. (2006, November) *Professional development for pedagogical impact*. Paper presented the Conference of The Australian Association for Research in Education, Adelaide, Australia.
- Gormley, W.T. (2007). Early childhood care and education: Lessons and puzzles. *Journal of Policy Analysis & Management, 26*(3), 633-671.
- Griffin, S. (2002). The development of math competence in preschool and early school years: Cognitive foundations and instructional strategies. In J.M. Rover (Ed.) *Mathematical Cognition: A volume on current perspectives on cognition, learning, and instruction* (pp 1-32). Greenwich, CT: Information Age Publishing.

- Griffin, S. (2003, February). Laying the foundation for computational fluency in early childhood. *Teaching Children Mathematics*, 9, 306-309.
- Griffin, S., & Case, R. (1997). Re-thinking the primary school math curriculum: An approach based on cognitive science. *Issues in Education*, 3(1), 1-49.
- Grossman, P. (1990). *The making of a teacher*. New York: Teachers College Press.
- Grouws, D.A., & Cebulla, K.J. (2002). *Improving student achievement in mathematics: Part 1: Research findings*. Washington, DC: Office of Educational Research and Improvement. (ERIC Document Reproduction Service No. ED 463 952)
- Guba, E. G., & Lincoln, Y.S. (1981). *Effective evaluation*. San Francisco: Jossey-Bass.
- Gurganus, S. (2004). Promote number sense. (20 Ways to...). *Intervention in School and Clinic*, 40, 55-58.
- Haertel, E., & Herman, J. (2005). A historical perspective on validity arguments for accountability testing, Center for the Study of Evaluation Report 654. Los Angeles: National Center for Research on Evaluation, Standards, and Student Testing, University of California, Los Angeles.
- Hagan, F.E. (2002). *Research Methods in Criminal Justice and Criminology*. Boston: Allyn and Bacon.
- Hamel, J., Dufour, D., & Fortin, D. (1993). *Case study methods*. Thousand Oaks, CA: Sage.
- Harniss, M.K., Carnine, D.W., Silbert, J., & Dixon, R.C. (2002). Effective strategies for teaching mathematics. In E.J. Kame'enui, D.W. Carnine, R.C. Dixon, D.C. Simmons, & M.D. Coyne (Eds.), *Effective Teaching strategies that accommodate diverse learners*. Upper Saddle River, NJ: Merrill/Prentice Hall.

- Hawley, W.D., & Valli, L. (1999). The essentials of effective professional development: A new consensus. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 127-150). San Francisco, CA: Jossey-Bass Publishers.
- Heck, D.J., Banilower, E.R., Weiss, I.R. & Rosenberg, S.L. (2008). Studying the effects of professional development: The case of the NSF's local systemic change through teacher enhancement initiative. *Journal for Research in Mathematics Education*, 39(2), 113-152.
- Heritage, M., Kim, J., & Vendlinski, T. (2008). From evidence to action: A seamless process in formative assessment? Paper presented at the American Educational Research Association Annual Meeting, New York.
- Hiebert, J. (1986, April). *The nature of students' symbol rule systems*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco.
- Hiebert, J., & Carpenter, T.P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). New York: Macmillan.
- Hill, H.C., & Ball, D.L. (2004). Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal for Research in Mathematics Education*, 35, 330-351.
- Jacobs, V.R., Franke, M.L., Carpenter, T.P. & Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for Research in Mathematics Education*, 38(3), 258-288.

- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: National Academy Press.
- Kitchen, R.S., DePree, J. Celedon-Pattichis, S. & Brinkerhoff, J. (2007). *Mathematics education at highly effective schools that serve the poor*. Mahwah, NJ: Lawrence Erlbaum.
- Klein, A., & Starkey, P. (2004). Fostering preschool mathematical development: Findings from the Berkley Math Readiness Project. In D.H. Clements, J. Sarama, & A.-M DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education*, (pp. 343-360). Mahwah, NJ: Erlbaum.
- Koehler, O. (1951). The ability of birds to count. *Bulletin of Animal Behavior*, 9, 41-45.
- Kulm, G. (1994). *Mathematics assessment: What works in the classroom*. San Francisco, CA: Jossey-Bass.
- Kulm, G., & Lockmandy, T. (1976). An alternative approach to evaluating math achievement. *Indiana Mathematics Newsletter*, 39, 1-4.
- Kulm, G., Wilson, L.D., & Kitchen, R. (2005). Alignment of Content and Effectiveness of Mathematics Assessment Items. *Educational Assessment*, 10(4), 173-196.
- Lakoff, G., & Nunez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.
- Lapakko, D. (1997). Three cheers for language: A closer examination of a widely cited study of nonverbal communication. *Communication Education*, 46, 63-67.
- Little, J.W. (1982). Norms of collegiality and experimentation: Work-place conditions of school success. *American Educational Research Journal*, 19, 325-340.

- Loucks-Horsley, S., Stiles, K.E., Mundry, S.E., Love, N.B., & Hewson, P.W. (1998). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Corwin.
- Loughran, J., & Gunstone, R. (1997). Professional development in residence: Developing reflection on science teaching and learning. *Journal of Education for Teaching*, 23(2), 159-178.
- Ludwig, J., & Phillips, D. (2007). The benefits and costs of Head Start. *Social Policy Report*, 21(3), 3-18.
- Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, N.J.: Erlbaum.
- Maciejewski, W., Mgombelo, J., & Savard, A. (2011). Meaningful procedural knowledge in mathematics learning. *Proceedings of the 2011 Annual Meeting of the Canadian Mathematics Education Study Group*. Pp. 37-46. Report of Working Group B. Vancouver, BC: CMESG/GCEDM.
- MacLean, H.E. (2003). The effects of early intervention on the mathematical achievement of low-performing first grade students.
- Madsen, A.L., & Lanier, P.E. (1995). Does conceptually oriented instruction enhance computational competence? *Focus on Learning Problems in Mathematics, Fall Edition*, 17(4), 42-64.
- Marzano, R. J. (2009/2010). When students track their progress. *Educational Leadership* 67(4), 86-87.
- McKeown, M.G., & Beck, I.L. (1999). Getting the discussion started. *Educational Leadership*, 57(3), 25-28.

- Mechner, F., & Guevrekian, L. (1962). Effects of deprivation upon counting and timing in rats. *Journal of the Experimental Analysis of Behavior*, 463-466.
- Merriam, S. B. (1991). *Case study research in education: A qualitative approach*. San Francisco, CA: Jossey-Bass.
- Merriam, S. B. (2001). Learning stages in the construction of the number sequence. In S. Imel, & S. B. Merriam (Eds.), *The new update on adult learning theory, vol. 89* (pp. 3-13). San Francisco: Jossey-Bass.
- Montague, M. (1997). Student perception, mathematical problem solving, and learning disabilities. *Remedial and Special Education*. 18(1), 46-53.
- Munter, C. (2010). *Evaluating math recovery: The impact of implementation fidelity on student outcomes*. (Unpublished doctoral dissertation). Vanderbilt University, Nashville, TN.
- National Council of Teachers of Mathematics (NCTM). (1989). *Principles and standards for school mathematics*. Reston, VA: Author
- National Council of Teachers of Mathematics (NCTM). (1991). *Professional standards for teaching mathematics*. Reston, VA: Author
- National Council of Teachers of Mathematics (NCTM). (1995). *Assessment standards for school mathematics*. Reston, VA: Author
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author
- National Council of Teachers of Mathematics (NCTM). (2008). Situating research on curricular change. *Journal for Research in Mathematics Education*, 39(2), 102-112

National Council of Teachers of Mathematics (NCTM) Research Committee. (2008, March).

Jere Confrey, Marilyn E. Strutchens, Michael T. Battista, Margaret Schwan Smith, Karen D. King, John T. Sutton, Timothy A. Boerst and Judith Reed. *Journal for Research in Mathematics Education*, 39(2), 102-112.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.

National Research Council. (1990). *Reshaping school mathematics: A philosophy and framework for curriculum*. Washington, DC: National Academy Press.

National Research Council. (2000). *How People Learn: Brain, Mind, Experience, and School: Expanded Edition*. Washington, DC: The National Academies Press.

National Research Council. (2001a). *Adding It Up: Helping Children Learn Mathematics*. Mathematics Learning Study Committee. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council. (2001b). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education*. Washington, DC: National Academy Press.

National Research Council. (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC, National Academies Press.

- New Mexico Public Education Department (NMPED). (2009). *2009-10 School Accountability Report*. Retrieved from http://webapp.ped.state.nm.us/aypd/docs/2009/046_114_ALAMOGORDO_PUBLIC_SCHOOLS_NORTH_ELEMENTAR_AYP0910_20091016_2035.pdf
- No Child Left Behind Act. 2002. Section 9101(23). Download from <http://www.ed.gov/legislation/ESEA02/pg107.html#sec9101>.
- O'Connell, S. (2005). *Now I get it: Strategies for building confident and competent mathematicians, K-6*. New York, NY: Heinemann.
- O'Day, J.A., & Smith, M.S. (1993). Systemic school reform and educational opportunity. In S.H. Fuhrman (Ed.), *Designing coherent educational policy: Improving the system* (pp. 250- 311), San Francisco: Jossey-Bass.
- O'Day, J.A., Goertz, M.E., & Floden, R.E. (1995). *Building capacity for education reform* (CPRE Research Briefs Series, RB-18-12/95). New Brunswick, NJ: Consortium for Policy Research in Education.
- Office of Educational Research and Improvement. (1999). *National awards program for model professional development*. Washington, DC: U.S. Department of Education.
- Patton, M.Q. (1990). *Qualitative evaluation and research methods* (2nd ed.). Newbury Park, CA: Sage.
- Perry, B., & Dockett, S. (2002). Young children's access to powerful mathematical ideas. In L.D. English (Ed.), *Handbook of international research in mathematics education* (pp. 81-111). Mahwah, NJ: Erlbaum.

- Phillips, V. J., Leonard, W. H., Horton, R. M., Wright, R. J., & Stafford, A. K. (2003). Can math recovery save children before they fail? *Teaching Children Mathematics*, 10(2), 107-111.
- Piaget, J. (1976). *The Child and Reality: Problems of Genetic Psychology*. Trans. Arnold Rosin. New York: Penguin Books.
- Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry*. London: Routledge & Kegan Paul.
- Piaget, J., & Szeminska, A. (1952). *The child's conception of number*. London: Routledge & Kegan Paul.
- Quine, S.E.E. (2006). Noci-Notes on 'The Math Instinct.' [Web log comment.] Retrieved from <http://nociceptor.blogspot.com/2006/07/noci-notes-on-math-instinct.html>
- Reynolds, A.J., & Ou, S.-R (2003). Promoting resilience through early childhood intervention. In S.S. Luthar (Ed.), *Resilience and vulnerability: Adaption in the context of childhood adversities* (pp. 436-459). New York, NY: Cambridge University Press.
- Reys, B. J. (1991). *Developing number sense: Addenda series, grades 5-8*. Reston, VA: National Council of Teachers of Mathematics.
- Robinson, M.R. (2003). *The effect of standards-based professional development participation on the teaching of mathematics at two-year colleges in New Mexico*. (Unpublished doctoral dissertation). University of New Mexico, Albuquerque, NM.
- Rodriguez, M.C., (2004). The role of classroom assessment in student performance on TIMSS. *Applied Measurement in Education*, 17(1), 1-24.

- Ross, J.A., McDougall, D., Hogaboam-Gray, A., & LeSage, A. (2003). A survey measuring elementary teachers' implementation of standards-based mathematics teaching. *Journal for Research in Mathematics Education*, 34(4), 344-363.
- Royer, J. M. (2003). *Mathematical cognition*. Charlotte, NC: Information Age.
- Sato, M., Wei, R.C., & Darling-Hammond, L. (2008). Improving teachers' assessment practices through professional development: The case of national board certification. *American Educational Research Journal*, 45 (3): 669-700.
- Schifter, D., & Fosnot, C.T. (1993) *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform*. New York, NY: Teachers College Press.
- Schmidt, W.H., McKnight, C.C., & Raizen, S.A., (1997). *A splintered vision: An investigation of U.S. science and mathematics education*. Boston, MA: Kluwer Academic Publishers.
- Shepard, L.A. (2000). The role of assessment in a learning culture. *Educational Researcher*, 29(7), 4-14.
- Shepard, L. (2001). The role of classroom assessment in teaching and learning. CSE Technical Report 517. In V. Richardson (Ed.), *Handbook of research on mathematics teaching and learning* (4th ed.). Washington, DC: American Educational Research Association.
- Sheridan, S.M., Edwards, C.P., Marvin, C.A., & Knoche, L.L. (2009). Professional development in early childhood programs: Process issues and research needs. *Early Education and Development*, 20(3), 377-401.
- Shulman, J. H. (1992). *Case methods in teacher education*. Teachers College Press, New York.

- Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
- Silver, E. A. (1998). *Improving mathematics in middle school: Lessons from TIMSS and related research*. Washington, DC: U.S. Government Printing Office.
- Silver, E.A., Smith, M. S., & Nelson, B. S., (1995) The QUASAR Project: Equity concerns meet mathematics education reform in the middle school. In W.G. Secada, E. Fennema & L.B. Adajian (Eds.), *New Directions for Equity in Mathematics Education* (pp. 9-56). New York, NY: Cambridge University Press.
- Simon, M.A., & Schifter, D. (1991). Towards a constructivist perspective: An intervention study of mathematics teacher development. *Educational Studies in Mathematics*, 22, 309-331.
- Slavin, R.E. (1990). Student team learning in mathematics. In N. Davidson (Ed.), *Cooperative learning in math: A handbook for teachers* (pp. 69-102). Boston: Allyn & Bacon.
- Smith-Chant, B. (2010). Fostering early numeracy in the home, preschool, and kindergarten: A commentary on Blevins-Knabe and Baroody. *Encyclopedia of Language and Literacy development* (pp. 1-5). London, ON: Canadian Language and Literacy Network.
- Smith, M.S. (2007). *Practice-based professional development for teachers of mathematics*. Reston, VA: NCTM Inc.
- Smith, T., Cobb, P. Earran, D., Cordray, D., Munter, C., & Dunn, A. (2007). *Evaluating Math Recovery: Evaluating the Causal Impact of Math Recovery on Student Achievement*. Vanderbilt University.

- Sousa, D.A. (2001). *How the brain learns*. Thousand Oaks, CA: Corwin Press.
- Sousa, D.A. (2008). *How the brain learns mathematics*. Thousand Oaks, CA: Corwin Press.
- Sparks, D. (1995). A paradigm shift in staff development. *The ERIC Review*, 3 (3), 5-11.
- Sparks, D., & Hirsh, S. (1997). A new vision for staff development. Paper co-published by the National Staff Development Council (NSDC) and the Association for Supervision and Curriculum Development (ASCD).
- Steffe, L.P. (1988). Children's construction of number sequences and multiplying schemes. In J. Hiebert and M. Behr (Eds.), *Number concepts and operations in the middle grades*. Reston, VA: National Council of Teachers of Mathematics.
- Steffe, L.P., & Cobb, P. (1988). *Construction of arithmetical meanings and strategies*. New York, NY: Springer-Verlag.
- Stenmark, J. K. (1989). *Assessment alternatives in mathematics: An overview of assessment techniques that promote learning*. Berkeley: University of California.
- Sternberg, R. J., & Williams, W. M. (Eds.) (1998). Intelligence, instruction, and assessment. Mahwah, New Jersey: Lawrence Erlbaum Associates. (Reviewed by R. S. Nickerson in *Contemporary Psychology*, 45(6), 698-700.)
- Stevenson, H.W., Lee, S-y., & Stigler, J.W. (1986, February). Mathematics Achievement of Chinese, Japanese, and American Children. *Science, New Series*, 231(4739), 693-699.
- Stevenson, H.W., & Stigler, J.W. (1992). *The learning gap*. New York, NY: Summit Books.
- Stiggins, R.J. (1991). Relevant classroom assessment training for teachers. *Educational Measurement: Issues and Practice*, 10(1), 7-12.

- Stinson, D.W. (2004). Mathematics as “gate-keeper”(?): Three theoretical perspectives that aim toward empowering all children with a key to the gate. *The Mathematics Educator*, Vol. 14, No. 1, 8–18.
- Strauss, V. (2003, Dec 1). Trying to figure out why math is so hard for some. *Washington Post*.
- Supovitz, J.A., & Turner, H.M. (2000). The effects of professional development on science teaching practices and classroom culture. *Journal of Research in Science Teaching*, 37(9), 963-980.
- Sykes, G. (1999). Teacher and student learning: Strengthening their connection. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* (pp. 151-179). San Francisco: Jossey-Bass.
- Thompson, A.G. (1992). Teachers’ beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp.127-145). New York, NY: Macmillan.
- Thompson, C.L., & Zeuli, J.S. (1999). The frame and the tapestry: Standards-based reform and professional development. In L. Darling-Hammond & G. Sykes (Eds), *Teaching as the learning profession: Handbook of policy and practice*. San Francisco: Jossey-Bass.
- Thorndike, E.L. (1922). *The psychology of arithmetic*. New York: Macmillan.
- Tomlinson, C.A. (1999). *The Differentiated classroom: Responding to the needs of all learners*. Alexandria, VA: Association for Supervision and Curriculum Development.

- U. S. Census Bureau. (2010). *Child poverty in the United States 2009 and 2010: Selected race groups and Hispanic origin ACSR 10/05*. Retrieved from <http://www.census.gov/prod/2011pubs/acsbr10-05.pdf>
- Van Dreil, J.H., Beijaard, D., & Verloop, N. (2001). Professional development and reform in science education: The role of teachers' practical knowledge. *Journal of Research in Science Teaching*, 38 (2), 137-158.
- von Glasersfeld, E. (1985). Reconstructing the concept of knowledge, *Archives de Psychologie*, 53, 91-101.
- Vygotsky, L.S. (1978). *Mind in Society: The Development of the Higher Psychological Processes*. Cambridge, MA: Harvard University Press.
- Webb, N. L. (1992). Assessment of students' knowledge of mathematics: Steps toward a theory. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (1992). Reston, VA: National Council of Teachers of Mathematics.
- Webb, N.L. (2007) Mathematics content specification in the age of assessment, In F.K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp.1281-1292. Charlotte, NC: NCTM Information Age Publishing.
- Webb, N. L., Meyer, R. H., Gamoran, A., & Fu, J. (2004). *Participation in the Student Achievement Guarantee in Education (SAGE) program and performance on state assessments at grade 3 and grade 4 for three cohorts of students-grade 1 students in 1996-97, 1997-98, and 1998-99*, Milwaukee: Wisconsin Center for Education Research.

- Weiss, I.R., Banilower, E.R., McMahon, K.C., & Smith, P.S. (2001). *Report of the 2000 national survey of science and mathematics education*. Chapel Hill, NC: Horizon Research.
- Whitenack, J.W., Knipping, N. Novinger, S., Coutts, L., & Standifer, S. (2000). Teachers' mini-case studies of children's mathematics. *Journal of Mathematics Teacher Education*, 3, 101-123.
- Wiggins, G.P. (1993). *Assessing student performance: Exploring the purpose and limits of testing*. San Francisco: Jossey-Bass.
- Wiggins, G.P., & McTighe, J. (1998). *Understanding by design*. Alexandria, VA: Association for Supervision and Curriculum Development.
- William, D. (2007). Keeping learning on track. In F.K. Lester, Jr. (Ed.) *Second handbook of research on mathematics teaching and learning (pp.1053-1098)*. Charlotte, NC: NCTM Information Age Publishing Inc.
- William, D., Lee, C., Harrison, C., & Black, P.J. (2004). Teachers developing assessment for learning: Impact on student achievement. *Assessment in Education: Principles Policy and Practice*, 11(1), 49-65.
- Willey, R., Holliday, A. & Martland, J. (2007). Achieving new heights in Cumbria: Raising standards in early numeracy through mathematics recovery. *Educational & Child Psychology*, 24(2), 108-118.
- Williams, L. A. G. (2001). *The influences of participation in a Mathematics Recovery program on classroom practices*. (Unpublished Doctor of Education dissertation.) University of Virginia, Charlottesville, VA.

- Wilson, L.D. (2007). High stakes testing in mathematics. In F.K. Lester, Jr. (Ed.) *Second handbook of research on mathematics teaching and learning* (pp.1099-1142).
Charlotte, NC: National Council of Teachers of Mathematics Information Age Publishing Inc.
- Wilson, S.M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iran-Nejad & P.D. Pearson (Eds.), *Review of Research in Education*, 24, 173-209.
- Wilson, S.M., Shulman, L. S. & Richert, A.E. (1987). “150 different ways of knowing: Representations of knowledge in teaching” In J. Calderhead (Ed.), *Exploring Teachers’ Thinking*. Sussex: Holt, Rhinehart & Winston.
- Wright, R.J. (1991). The role of counting in children’s numerical development. *Australian Journal of Early Childhood*, 16(2), 43-48.
- Wright, R.J. (1994). A study of the numerical development of 5-year-olds and 6-year olds, *Educational Studies in Mathematics*, 36, 35-44.
- Wright, R. J. (1996). Problem-centered mathematics in the first year of school. In J. Mulligan and M. Mitchemore, *Research in early numberlearning: An aAustralian Perspective*. Adelaide: AAMT. Pp. 35-54.
- Wright, R.J. (2000). Professional development in recovery education. In L.P. Steffe and P.W. Thompson (eds.) *Radical constructivism in action: Building on the pioneering work of Ernst von Glasersfeld*. London: Falmer. Pp134-151.

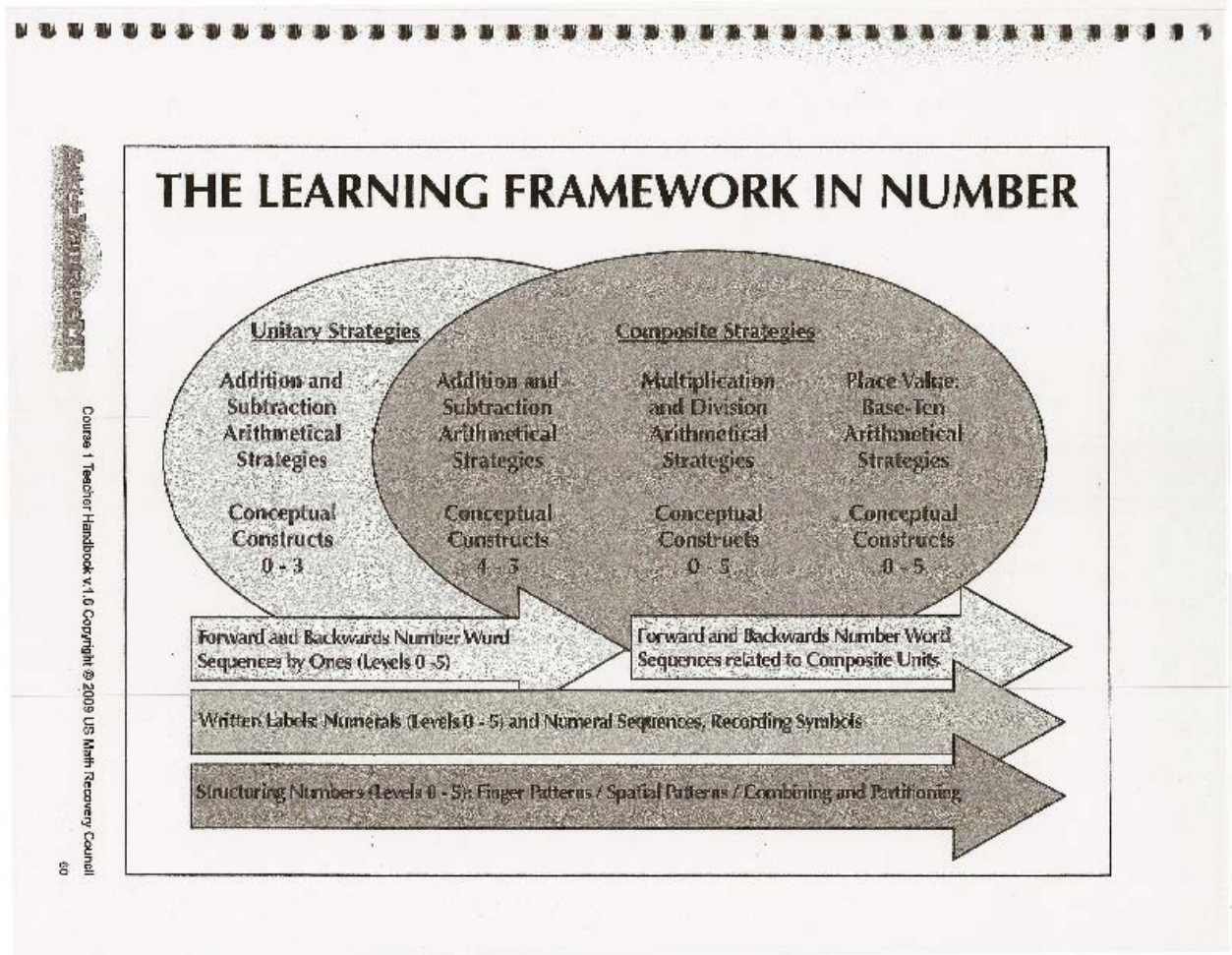
- Wright, R.J. (2003). A mathematics recovery: Program of intervention in early number learning. *Australian Journal of Learning Disabilities*, 8(4), 6-11. doi: 10.1080/19404150309546741
- Wright, R.J., Martland, J., & Stafford, A.K. (2006). *Early numeracy: Assessment for teaching and intervention*. London: Paul Chapman.
- Wright, R.J., Stanger, G., Stafford, A.K., & Martland, J. (2006). *Teaching number in the classroom with 4-8 year olds*. London: Paul Chapman.
- Wu, H. (1999). Professional development of mathematics teachers. *Notices of the AMS*, 46(5), 535-542.
- Wylie, E.C., & Lyon, C. (2012a, April). *Quality instruction and quality formative assessment: The same or different?* Paper presented at the annual meeting of the American Educational Research Association, Vancouver, Canada.
- Wylie, E. C., & Lyon, C. (2012b). Formative assessment—Supporting students' learning. *R & D Connections*, 19. Retrieved from http://www.ets.org/research/policy_research_reports/publications/periodical/2012/jjff
- Wynn, K. (1998). Psychological foundations of number: Numerical competence in human infants. *Trends in Cognitive Sciences*, 2, 296-303.
- Wynn, K. (1990). Children's understanding of counting. *Cognition*, 36, 155–193.
- Yin, R. K. (1994). *Case study research: Design and methods*. (2nd ed.). Newbury Park, CA: Sage.
- Yin, R. K. (1998). The abridged version of case study research: Design and method. In L. Bickman & D. J. Rog (Eds.), *Handbook of applied social research methods* (pp. 229-260). Thousand Oaks, CA: Sage Publications.

- Yoon, K.S., Duncan, T., Lee, S. W.-Y., Scarloss, B., & Shapley, K.L. (2007). *Reviewing the evidence on how teacher professional development affects student achievement* (Issues & Answers Report, REL 2007-No. 033). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southwest. Retrieved April 14, 2010 from <http://ies.ed.gov/ncee/edlabs>.
- Young-Loveridge, J. (1989). The development of children's number concepts: The first year of school. *New Zealand Journal of Educational Studies*, 24(1), 47-64.
- Young-Loveridge, J. M. (1991). *The development of children's number concepts from ages five to nine. Early mathematics learning project, Phase 2, Volumes 1 & 2*. Hamilton, NZ: University of Waikato.
- Zigarmi, P., Betz, L., & Jensen, D. (1977). Teachers preferences and perceptions of inservice education. *Educational Leadership*, 34(7), 545-551.

Appendices

Appendix A: Learning Framework in Number.....	212
Appendix B: Classroom Instructional Framework for Early Number.....	213
Appendix C: Summary of Alternative Assessments.....	214
Appendix D: AVMR and Common Core Standards.....	219
Appendix E: Teacher Interview Questions.....	221
Appendix F: Teacher Classroom Observation Protocols.....	223
Appendix G: Student Classroom Observation Protocols.....	227
Appendix H: AVMR Assessment.....	230
Appendix I: Construct Levels.....	230
Appendix J: Learning Levels.....	239
Appendix K: Mathematics Instructional Coach data.....	243
Appendix L: AVMR Guiding Principles for Classroom Teaching.....	244

Appendix A: Learning Framework in Number



Appendix B: Classroom Instructional Framework for Early Number

Classroom Instructional Framework For Early Number (CIFEN)

Number Words	Numerals	Addition and Subtraction	Structuring Numbers	Place Value	Multiplication and Division
Forward/Backward Number Word Sequences to 10	Numerals to 5	Emergent Counting	Early Spatial Patterns and Finger Patterns		
Forward/Backward Number Word Sequences to 30	Numerals to 10	Perceptual Counting	Facile Structures to 5		Initial Grouping
Forward/Backward Number Word Sequences to 100	Numerals to 20	Figurative Counting	Intermediate Structures to 10		
Forward/Backward Number Word Sequences for Composite Units	Numerals to 100	Initial Number Sequence (Counting-on and Counting-back)	Facile Structures to 10	Tens and Ones	Perceptual Counting in Multiples
Number Word Sequences to 1000 and Beyond	Numerals to 1000 and Beyond	Facile Number Sequence	Intermediate Structures to 20	Tens and Ones with Materials (Jump &/or Split)	Figurative Composite Grouping
			Facile Structures to 20	Tens and Ones without Materials (Jump &/or Split)	Repeated Abstract Composite Grouping
				Mental computation involving addition and subtraction with 2- and 3-digit numbers	Multiplication and Division as Operations

Course 1 Teacher Handbook v. 1.3 Copyright © 2008 US Math Recovery Council
 94

Appendix C: Summary of Alternative Assessments

Open – Ended Questions

Stenmark (1989) defines an open–ended question as one in which the student is given a problem or situation and is required to communicate a solution or response, usually in writing. The question may be as simple as asking a student to show the work done on a problem, or at the other extreme, it can involve complex situations that may require formulating hypotheses, explaining mathematical situations and solutions, writing directions, or making generalizations (Kulm, 1994). Alternative assessment approaches that include open – ended questions, presentations of solutions in both written and oral form, and other performances send very different messages to students about what is considered important or valued in mathematics learning (Kulm, 1994). This opportunity for student creativity yields many benefits in terms of assessment. In addition to the benefit of allowing for more opportunity to view student creativity, open-ended questions also have a place in uncovering student’s flawed thinking.

Following is an example of an open-ended question.

A friend says he is thinking of a number. When 100 is divided by the number, the answer is between 1 and 2. Give at least three statements that must be true of the number. Explain your reasoning (Stenmark, 1989, p. 16).

Investigations and Experiments

The goals set out by the NCTM Standards stress the importance of students making connections among mathematical topics and in linking mathematics with the sciences and other subject areas. Projects and investigations provide students the opportunity to make connections between not only topics within mathematics but also with other disciplines as

well (Kulm, 1994). Investigations and experiments incorporate various tools for learning mathematics and provide the opportunity for long term or self – directed work in which the student’s higher-order thinking can be viewed (Kulm, 1994).

Student Journals

A journal can be used to evaluate not only student knowledge but the ways students think (Kulm, 1994). As we take the time to reflect, learning occurs. Whether the reflection is in the form of taking time to think through problems and situations or more concretely to write out our thoughts, we essentially complete the learning cycle through this valuable process or assessment behavior. A journal can become a resource for self-reporting where a student is given the opportunity to assess their own abilities or reflect on their problem solving strategies (Kulm, 1994).

Student Interviews

Individual interviews can accomplish many objectives in assessment, from determining specific skill development, to just finding out about problems and interests, or how the students are doing (Kulm, 1994). To try to understand what is in children’s minds, teachers should engage in effective clinical interviewing (Ginsburg, 1997). There are many different ways to incorporate the interview into a student’s assessment program. Some teachers, as part of their final exam, may use an innovative approach by having individual interviews with each student (Kulm & Lockmandy, 1976). Ernest (1992) suggests that the advantages of formal interviews include the students’ opportunity to communicate orally and the teacher’s flexibility to adapt questions as the need arises during the interview.

Student Portfolios

The portfolio provides a means for helping students see a larger picture, for synthesizing their learning, and for reflecting on important ideas (Kulm, 1994). Perhaps the single most reason for this popularity is the wide range of information that is accessible through a portfolio. The portfolio provides an opportunity to feature multiple samples of student work. Portfolios have the great advantage of creating a record of student work that, at the same time, is diagnostic, formative and summative (Kulm, 1994). One of the important reasons for the use of portfolios is that they are a means of providing equity in mathematics. Portfolios have the ability to address and recognize different learning styles, making assessment less culture – dependent and less biased (Stenmark, 1989).

Most importantly portfolios include the student in his or her own evaluation process. This approach empowers students; it gives them a sense of control and responsibility for their own mathematical learning (Kulm, 1994). Kulm places great emphasis on the student in preparing the portfolio. The involvement of the student in selecting, structuring, reviewing, and reflecting on the portfolio is as important as the actual contents of the portfolio itself (Kulm, 1994). Another alternative tool is that of observation.

Observation

Observation is also an important alternative assessment tool as teachers can learn much from simply observing the child as they work to solve problems and perform mathematical calculations. Along with observation, we must also talk to the child. As Piaget (1976) pointed out many years ago, "...how many inexpressible thoughts must remain unknown so long as we restrict ourselves to observing the child without talking to him?" (Pp. 6-7). To learn what is hidden within children's minds, teachers must know how to engage in

effective interviewing (Ginsburg, 1997). According to Ginsburg et al. (2008) if we are to provide effective instruction, teachers need to understand what children know and don't know, and how they are learning. The field of early education has traditionally relied on and preferred observation as the primary method for understanding young children (Ginsburg et al., 2008). However, if teachers are to learn anything about children's mathematical knowledge, teachers must know what to look for as they observe children. We need research on how well teachers observe and interpret children's behavior, and we need methods to help teachers improve these skills (Ginsburg et al., 2008). Teachers may choose to observe students working individually, but much can be gained when the teacher also observes students working with their peers as a group. The group allows for observation of how individuals work collectively solving problems.

Group Mathematical Assessment

Group work is an essential component of the reform classroom. Since many educators are answering the call of the reform movement by allowing their students to learn together, there is a need to assess this type of activity (Kulm, 1994). The importance of students being able to work together is being understood more now than ever before. Effective group work not only helps to make students more productive citizens but it also helps them to learn mathematics by working with their peers (Kulm, 1994). In addition to the benefits that many students derive from learning in a group setting, business leaders place importance on collaborative work and are asking that students learn to work cooperatively on tasks (Kulm, 1994).

In assessing a group's performance some of the categories that can be assessed include: group participation, staying on the topic, involving others, communication and

responsibility (does fair share of the work, was reliable and well prepared). Types of learning appropriate for group assessment are problem solving, performance tasks such as measurement, computer work, investigations and projects (Kulm, 1994).

Appendix D: AVMR and Common Core Standards

Add+VantageMR
INFORMED ASSESSMENT FOR MATH TEACHING
A Math Recovery Product



Add+VantageMR and Common Core

Add+VantageMR: Giving teachers the tools to tackle Common Core Curriculum

Common Core standards define what students should understand and be able to do in their study of mathematics. However, meeting the standard of what a child understands means asking the instructor to assess the student's current mathematical knowledge. For many states, the need for implementing Common Core into core instruction is a priority.

Add+VantageMR gives teachers a roadmap to implementing expectations of Common Core standards in classrooms. The program provides tools to assess students K-5. In addition, Add+VantageMR offers curriculum independent activities that meet the Standards of Mathematics Practice outlined in the Common Core Curriculum.

US Math Recovery Council

205 Powell Place
Brentwood, TN 37027

Phone: 615-369-0700
Fax: 1-800-816-5631
E-mail: info@mathrecovery.org
www.mathrecovery.org

Add+VantageMR and Common Core

Add+VantageMR Professional development offers teachers, schools and entire districts the tools to meet the needs of all students.

Common Core Standards provide a clear and consistent understanding of what students are expected to learn. Teachers, specialists and administrators need the assessment and instructional tools to meet Common Core Standards.

Add+VantageMR is not a curriculum. Instead, the program teaches instructors how to use any curriculum package more effectively within the Core Curriculum Standards.

Add+VantageMR is professional learning for math teachers, giving them the knowledge and tools to assess students and offer each student appropriate tasks to facilitate math understanding.

The Add+VantageMR Road map...

1. Make sense of problem and persevere in solving them.

Add+VantageMR uses a wide range of teacher selected tasks, allowing students to actively engage in solving the mathematics. Instructors focus on mathematical problems, not rote learning, giving students the chance to make sense of the mathematics until they develop an understanding of the basic mathematic principle it illustrates.

2. Reason Abstractly and quantitatively.

Add+VantageMR gives teachers scaffolding tools to help students move from concrete representations into symbolic abstract thinking about mathematics.

3. Construct Viable arguments and critique the reasonings of others.

Students use and compare a variety of strategies when solving problems with others. Add+VantageMR also provides teachers with a specific lens for listening to student strategies.

4. Model with mathematics

Add+VantageMR activities use a wide range of models to explore number, addition and subtraction and multiplication and division. These modeling activities are designed for individual, small group and classroom use.

5. Use appropriate tools strategically

Add+VantageMR provides instructors with tools for learning specifically designed to support student mathematical connections and understanding. This gives them the appropriate tool to offer at appropriate times to meet the student at his or her level of understanding.

6. Attend to precision

Add+VantageMR instruction revolves around the instructor creating an environment where the student explains his actions using math language. As the student progresses in mathematical understanding, he learns to confidently express himself accurately and precisely.

7. Look for and make use of structure

Add+VantageMR identifies tasks that highlight the structure of early number including addition and subtraction, and multiplication and division. By reflecting on genuine problem solving experiences, students identify the structure of basic operations, preparing them for the more complex disciplines of fractions and algebra.

8. Look for express regularity in repeated reasoning.

Add+VantageMR trained instructors focus on the student's understanding of mathematical operations. The repeated actions give the student the time and ability to recognize and reason through regular mathematical operations.

US Math Recovery Council — A non-profit organization dedicated to early numeracy teacher professional development.

© US Math Recovery Council. All rights reserved.

Appendix E: Teacher Interview Questions

Date _____

Teacher _____

School _____

Grade _____

1. What are your feelings/perceptions regarding math?
2. What are your general goals for your students related to mathematics?
3. What methods of teaching mathematics work best for you?
4. How do you collaborate with other teachers when planning math lessons?
5. What resources do you use to teach math?
6. How do you use assessment in your instruction?
7. Have reforms in mathematics education influenced your pedagogy?
8. What are your strongest attributes when teaching math?
9. What areas in teaching math do you want help with?
10. How skilled are you at diagnosing student difficulties in math?

11. How do you help students overcome difficulties in math?
12. I noticed you focused on or utilized ... can you tell me why?
13. Once student difficulties in mathematics have been determined, how confident are you in planning lessons that will assist the child in moving forward?
14. What specific training(s) that you received in AVMR were most valuable to you?
15. In what ways, if any, has your confidence level in teaching your students mathematics changed?
16. Can you reflect on your AVMR training and share your thoughts and feelings regarding your AVMR training?
17. How has AVMR changed your thoughts or beliefs in teaching children mathematics?
18. How have you changed your views regarding assessment and it's purpose in the classroom?
19. Would you recommend AVMR training to other teachers? Why or why not?

Appendix F: Teacher Classroom Observation Protocols

Date _____ Week Number _____ Teacher _____

Teacher Action or Behavior	Observed	Comments
1. Encourages strategies that and activities that facilitate transition from (count- by- ones) strategies to (collection-based) strategies		
2. Uses activities to promote number knowledge		
3. Class lesson or activity involves numeral identification		
4. Class lesson or activity involves working on Forward Number Word Sequence (FNWS)		
5. Class lesson or activity involves working on Backward Number Word Sequence (BNWS)		
6. Class lesson or activity involves working on number word after (NWA)		
7. Class lesson or activity involves working on number word before (NWB)		

Teacher Action or Behavior	Observed	Comments
8. Class lesson or activity involves subitizing (instant recognition)		
9. Class lesson or activity involves sequencing numbers		
10. Class lesson or activity involves combinations of 5 & 10		
11. Class lesson or activity involves combinations of 20		
12. Class lesson or activity involves working on spatial patterns		
13. Class lesson or activities involving finger patterns		
14. Class lesson or activity involves working on five and ten frames		
15. Class lesson or activities using arithmetic strings or arithmetic racks		
16. Class lesson involves addition with unscreened collections		
17. Class lesson involves addition with unscreened collections		

Teacher Action or Behavior	Observed	Comments
18. Class lesson involves addition with screened collections		
19. Class lesson involves subtraction with unscreened collections		
20. Class lesson involves subtraction with screened collections		
21. Encourages the use of relational thinking (i.e. commutativity of addition, inverse relationship between addition & subtraction)		
22. Uses the Learning Framework in Number (LFIN) to inform what instruction should come next		
23. Class lesson involves the 3 Aspects of Number (quantitative, symbolic, and verbal)		
24. Uses informal assessment strategies		
25. Class lesson or activities directed at areas needing improvement or reinforcement for Student A		

Teacher Action or Behavior	Observed	Comments
26. Class lesson or activities directed at areas needing improvement or reinforcement for Student B		
27. Class lesson or activities directed at areas needing improvement or reinforcement for Student C		

Appendix G: Student Classroom Observation Protocols

Date _____ Week Number _____ Student _____

Student Action or Behavior	Observed	Comments
1. Can say the Forward Number Word Sequence to _____		
2. Can say the Number Word After (NWA) in the range 1-10, 11-30, 31-100		
3. Can recognize numbers from 0-10, 11-100, 101-1000, 1001-1,000,000		
4. Can count backwards from 10, from 17 – 10, from 38 – 27, from 72-66		
5. Can say the Number Word Before (NWB) in the range 0-10, 11-30, 31-100		
6. Can put the numbers from 1 – 10 in order and read them, the numbers from 46 – 55 in order and read them		
7. Can recognize spatial patterns on dot cards		
8. Can display finger patterns to show numbers in more than one way		

Student Action or Behavior	Observed	Comments
9. Can make combinations for 5		
10. Can make combinations for 10		
11. Can make combinations for 20		
12. Can count a collection of objects up to 13		
13. Can count a collection of objects up to 13 where some of the objects are shown briefly and then screened and the others are left unscreened		
14. Can count a collection of objects up to 13 where the objects are shown briefly and then screened		
14. Can solve problems involving the missing addend (i.e. here are 8 counters, briefly show and then screen, I have additional counters under a 2 nd screen, altogether I have 11 counters; how many are under the second screen?)		
15. Can solve problems involving subtraction with screens		

Student Action or Behavior	Observed	Comments
16. Can solve problems involving a missing subtrahend		
17. Can solve addition problems		
18. Can solve subtraction problems		
19. Can link addition and subtraction lesson involves subtraction with unscreened collections		
20. Uses the 5 frame		
21. Uses the 10 frame		
22. Uses the arithmetic string to count		
23. Can subitize regular patterns		
24. Can subitize irregular patterns		
25. Can flash finger patterns		
26. Demonstrates relational thinking in addition		
27. Demonstrates relational thinking involving addition and subtraction		
28. Appears confident/engaged or uncertain/distracted		

Appendix H: AVMR Assessment

Add+VantageMR

Number Words and Numerals - Assessment Schedule

Student _____	Assessor _____	Date _____
Grade _____	Date of Birth _____	Age _____ Classroom Teacher _____

Task Group 1: Forward Number Word Sequences (FNWS)

<p>→ "Start counting from _____ and I will tell you when to stop."</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%;">a) 1 (to 32)</td> </tr> <tr> <td>b) 38 (to 51)</td> </tr> <tr> <td>c) 76 (to 84)</td> </tr> <tr> <td>d) 83 (to 112)</td> </tr> </table>	a) 1 (to 32)	b) 38 (to 51)	c) 76 (to 84)	d) 83 (to 112)
a) 1 (to 32)					
b) 38 (to 51)					
c) 76 (to 84)					
d) 83 (to 112)					

Task Group 2: Number Word After (NWA)

<p>NWA in the range 0 to 10</p>	5 <input type="text"/> 9 <input type="text"/> 7 <input type="text"/> 3 <input type="text"/> 6 <input type="text"/>
<p>NWA in the range 11 to 30</p>	14 <input type="text"/> 20 <input type="text"/> 11 <input type="text"/> 29 <input type="text"/> 23 <input type="text"/> 12 <input type="text"/> 19 <input type="text"/>
<p>Introductory example: "Say the number that comes right after two." Ask each number in turn.</p>	
<p>NWA in the range 31 to 100</p>	59 <input type="text"/> 65 <input type="text"/> 32 <input type="text"/> 70 <input type="text"/> 99 <input type="text"/>

Task Group 3: Numeral Identification

<p>Numeral Recognition 0 to 10</p> <p>Place the numeral cards 0 to 10 on the table in random order. Ask in turn. "Which number is the ___?"</p>	<p>6 <input type="checkbox"/> 8 <input type="checkbox"/> 2 <input type="checkbox"/> 9 <input type="checkbox"/> 7 <input type="checkbox"/> 5 <input type="checkbox"/></p>
<p>Numeral Identification 0 to 10</p>	<p>4 <input type="checkbox"/> 2 <input type="checkbox"/> 9 <input type="checkbox"/> 6 <input type="checkbox"/> 0 <input type="checkbox"/> 8 <input type="checkbox"/></p> <p>10 <input type="checkbox"/> 3 <input type="checkbox"/> 1 <input type="checkbox"/> 7 <input type="checkbox"/> 5 <input type="checkbox"/></p>
<p>Numeral Identification 11 to 100</p> <p>Show each card in turn. "What number is this?"</p>	<p>19 <input type="checkbox"/> 34 <input type="checkbox"/> 15 <input type="checkbox"/> 90 <input type="checkbox"/> 41 <input type="checkbox"/></p> <p>12 <input type="checkbox"/> 17 <input type="checkbox"/> 20 <input type="checkbox"/> 66 <input type="checkbox"/> 25 <input type="checkbox"/></p>
<p>Numeral Identification 101 to 1,000</p>	<p>168 <input type="checkbox"/> 400 <input type="checkbox"/> 117 <input type="checkbox"/></p> <p>354 <input type="checkbox"/> 205 <input type="checkbox"/> 620 <input type="checkbox"/></p>
<p>Numeral Identification 1,001 to 1,000,000</p>	<p>7,462 <input type="checkbox"/> 5,026 <input type="checkbox"/> 46,803 <input type="checkbox"/></p> <p>90,380 <input type="checkbox"/> 247,641 <input type="checkbox"/> 700,090 <input type="checkbox"/></p>

Task Group 4: Backwards Number Word Sequences (BNWS)

<p>→ "Start counting backwards from _____ and I will tell you when to stop."</p>	a) 10 (to 1)
	b) 17 (to 10)
	c) 38 (to 27)
	d) 72 (to 66)

Task Group 5: Number Word Before (NWB)

NWB in the range 0 to 10	7 <input type="checkbox"/> 10 <input type="checkbox"/> 4 <input type="checkbox"/> 8 <input type="checkbox"/> 3 <input type="checkbox"/>
NWB in the range 11 to 30	24 <input type="checkbox"/> 17 <input type="checkbox"/> 20 <input type="checkbox"/> 11 <input type="checkbox"/> 14 <input type="checkbox"/> 21 <input type="checkbox"/> 30 <input type="checkbox"/>
Introductory example: "Say the number that comes right before two."	
Ask each number in turn.	
NWB in the range 31 to 100	53 <input type="checkbox"/> 70 <input type="checkbox"/> 88 <input type="checkbox"/> 41 <input type="checkbox"/> 96 <input type="checkbox"/>

Task Group 6: Numeral Sequences

Sequencing 1 - 10	Identify:
Present as with the previous task.	Sequence:
	Read:
Sequencing 46 - 55	Identify:
<p>→ "Tell me the numbers as I lay them down." Place the cards face up, one at a time, in random order. "Put these in order." When finished, ask the student to read the cards.</p>	Sequence:
	Read:

Add+VantageMR

Structuring Numbers - Assessment Schedule

Student _____	Assessor _____	Date _____
Grade _____	Date of Birth _____	Age _____ Classroom Teacher _____

Task Group 1: Spatial Patterns

<p>Regular Spatial Patterns</p> <p><i>"I'm going to show you some cards very quickly. Tell me how many dots are on each card."</i></p> <p>Flash (for 1/2 second) each card in turn.</p>	<p>Indicate response and strategy: Knows immediately - (K) Partitions the dots - (P) Counts dots by 1's - (C)</p> <p>4 <input type="checkbox"/> <input type="checkbox"/> 3 <input type="checkbox"/> <input type="checkbox"/> 6 <input type="checkbox"/> <input type="checkbox"/> 5 <input type="checkbox"/> <input type="checkbox"/></p>
<p>Irregular Spatial Patterns</p> <p>Present as above.</p>	<p>4 <input type="checkbox"/> <input type="checkbox"/> 6 <input type="checkbox"/> <input type="checkbox"/> 3 <input type="checkbox"/> <input type="checkbox"/> 5 <input type="checkbox"/> <input type="checkbox"/></p>

Task Group 2: Finger Patterns

<p>Displaying Finger Patterns</p> <p><i>"Show me four on your fingers."</i></p> <p>Repeat for other tasks.</p>	<p>Indicate response Indicate strategy: Knows - raises fingers simultaneously - (K) Counts sequentially - (CF1 or CF_)</p> <p>4 <input type="checkbox"/> <input type="checkbox"/> 3 <input type="checkbox"/> <input type="checkbox"/> 9 <input type="checkbox"/> <input type="checkbox"/> 6 <input type="checkbox"/> <input type="checkbox"/> 8 <input type="checkbox"/> <input type="checkbox"/></p>
<p>Displaying Finger Patterns In More Than One Way</p> <p><i>"Show me five on your fingers..."</i> <i>"Show me another way to show five on your fingers."</i></p> <p>May prompt - "Can you use 2 hands to show me '5'?"</p> <p>Repeat for other task.</p>	<p>Indicate combinations: (e.g. $\downarrow \downarrow$) Indicate strategy: Knows - raises fingers simultaneously - (K) Counts sequentially - (CF 1 or CF_)</p> <p>5 <input type="checkbox"/> <input type="checkbox"/> 5 <input type="checkbox"/> <input type="checkbox"/> 7 <input type="checkbox"/> <input type="checkbox"/> 7 <input type="checkbox"/> <input type="checkbox"/></p>

Task Group 3: Combinations to 5 and 10

<p>Combinations to Five</p> <p>Present a train of 5 linking cubes (all the same color) and ask, "How many?"</p> <p>Without the student seeing, remove and conceal cubes to display the indicated amount and ask, "How many are here?" "How many did I keep?"</p>	<p>Indicate responses and strategies: Knows immediately - (K) Counts - (CF1 or CF_) Uses fingers - (F)</p> <p>4 <input type="checkbox"/> <input type="checkbox"/> ? <input type="checkbox"/> <input type="checkbox"/> 2 <input type="checkbox"/> <input type="checkbox"/> ? <input type="checkbox"/> <input type="checkbox"/></p> <p>1 <input type="checkbox"/> <input type="checkbox"/> ? <input type="checkbox"/> <input type="checkbox"/> 3 <input type="checkbox"/> <input type="checkbox"/> ? <input type="checkbox"/> <input type="checkbox"/></p>
<p>Combinations to Ten</p> <p>Present a train of 10 linking cubes (in two groups of 5 using differing colors) and ask, "Now how many?"</p> <p>Without the student seeing, remove and conceal cubes (from one end of the train) to display the indicated amount and ask, "How many are here?" "How many did I keep?"</p>	<p>Indicate responses and strategies: Knows immediately - (K) Counts - (CF1 or CF_) Uses fingers - (F)</p> <p>8 <input type="checkbox"/> <input type="checkbox"/> ? <input type="checkbox"/> <input type="checkbox"/> 6 <input type="checkbox"/> <input type="checkbox"/> ? <input type="checkbox"/> <input type="checkbox"/></p> <p>3 <input type="checkbox"/> <input type="checkbox"/> ? <input type="checkbox"/> <input type="checkbox"/> 4 <input type="checkbox"/> <input type="checkbox"/> ? <input type="checkbox"/> <input type="checkbox"/></p>

Task Group 4: Combinations and Partitions to 20

<p>Combinations in the range of 1 to 20</p> <p>"I will tell you a number story and you tell me the answer." "I have __, I wish I had __." "How many do I need?"</p>	<p>Indicate response and strategy: Knows immediately - (K) Counts to solve - (C) Uses fingers - (F)</p> <p>7/10 <input type="checkbox"/> <input type="checkbox"/> 10/13 <input type="checkbox"/> <input type="checkbox"/></p> <p>10/18 <input type="checkbox"/> <input type="checkbox"/> 14/20 <input type="checkbox"/> <input type="checkbox"/></p>
<p>Partition in the range of 1 to 20</p> <p>"Tell me two numbers that go together to make __." "Tell me another two."</p>	<p>Indicate combinations given. Indicate strategy: Knows immediately - (K) Counts to solve - (C)</p> <p>9 <input type="checkbox"/> and <input type="checkbox"/> 9 <input type="checkbox"/> and <input type="checkbox"/></p> <p>14 <input type="checkbox"/> and <input type="checkbox"/> 14 <input type="checkbox"/> and <input type="checkbox"/></p> <p>17 <input type="checkbox"/> and <input type="checkbox"/> 17 <input type="checkbox"/> and <input type="checkbox"/></p>

Add+VantageMR

Addition and Subtraction - Assessment Schedule

Student _____	Assessor _____	Date _____
Grade _____	Date of Birth _____	Age _____ Classroom Teacher _____

Task Group 1: Addition - Unscreened and Screened Collections

Counting a Collection Place out a collection of 13 counters, all one color. <i>"How many counters are there?"</i>	13	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Pull-off <input type="checkbox"/> Touch each <input type="checkbox"/> Eye point <input type="checkbox"/> 1 to 1 corresp error <input type="checkbox"/> Num Word Seq error
Unscreened Collections Place out 8 red counters. <i>"Here are 8 red counters ..."</i> Place out 7 blue counters. <i>"... and here are 7 blue counters. How many counters are there altogether?"</i>	8 + 7	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Counts All <input type="checkbox"/> Counts On <input type="checkbox"/> Uses Groups <input type="checkbox"/> 1 to 1 corresp error <input type="checkbox"/> Num Word Seq error
Partially Screened Collections Briefly display and then screen 7 red counters. <i>"Here are 7 counters..."</i> Place out 2 blue counters. <i>"...and here are 2 blue counters. How many counters are there altogether?"</i>	7 + 2	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Counts From 1 <input type="checkbox"/> Counts On <input type="checkbox"/> Uses Groups <input type="checkbox"/> Finger use (describe) <input type="checkbox"/> Other (describe)
Totally Screened Collections (within finger range) Briefly display and then screen 4 red counters. <i>"Here are 4 red counters ..."</i> Briefly display and then screen 2 blue counters. <i>"... and here are 2 blue counters. How many counters are there altogether?"</i>	4 + 2	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Counts From 1 <input type="checkbox"/> Counts On <input type="checkbox"/> Uses Groups <input type="checkbox"/> Finger use (describe) <input type="checkbox"/> Other (describe)
Totally Screened Collections (within finger range) Present 6+3 as above.	6 + 3	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Counts From 1 <input type="checkbox"/> Counts On <input type="checkbox"/> Uses Groups <input type="checkbox"/> Finger use (describe) <input type="checkbox"/> Other (describe)



<p>Totally Screened Collections (outside finger range) Briefly display and then screen 9 red counters. "Here are 9 red counters..." Briefly display and then screen 5 blue counters. "... and here are 5 blue counters. How many counters are there altogether?"</p>	$9 + 5$	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Counts From 1 <input type="checkbox"/> Counts On <input type="checkbox"/> Uses Groups <input type="checkbox"/> Finger use (describe) <input type="checkbox"/> Other (describe)
<p>Missing Addend Briefly display and then screen 8 red counters. "Here are 8 red counters..." Without the child seeing, place 3 blue counters under a second screen. "...and without you seeing, I put some blue counters under here. Altogether there are 11 counters. How many blue counters are under here?" Indicate the second screen.</p>	$8 + \square = 11$	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Counts From 1 <input type="checkbox"/> Counts On <input type="checkbox"/> Uses Groups <input type="checkbox"/> Finger use (describe) <input type="checkbox"/> Other (describe)

Task Group 2: Subtraction - Screened Collections

<p>Removed Items (within the finger range) Briefly display and then screen 7 red counters. "Here are 7 counters." Remove 3 counters, briefly display and then screen with a second cover. "If I take away 3 counters, how many counters are left?"</p>	$7 - 3$	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Counts Down From <input type="checkbox"/> Counts Up From <input type="checkbox"/> Uses Groups <input type="checkbox"/> Finger use (describe) <input type="checkbox"/> Other (describe)
<p>Removed Items (outside the finger range) Briefly display and then screen 16 red counters. "Here are 16 counters." Remove 4 counters, briefly display and then screen with a second cover. "If I take away 4 counters, how many counters are left?"</p>	$16 - 4$	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Counts Down From <input type="checkbox"/> Counts Up From <input type="checkbox"/> Uses Groups <input type="checkbox"/> Finger use (describe) <input type="checkbox"/> Other (describe)
<p>Missing Subtrahend Briefly display and then screen 9 red counters. "Here are 9 counters." Without the child seeing, remove and screen 2 counters. "Without you seeing, I took some away. Now there are only 7 left." Briefly display, then screen the 7. "How many counters did I take away?"</p>	$9 - \square = 7$	<input type="checkbox"/> Correct <input type="checkbox"/> Incorrect <input type="checkbox"/> Counts Down From <input type="checkbox"/> Counts Up From <input type="checkbox"/> Uses Groups <input type="checkbox"/> Finger use (describe) <input type="checkbox"/> Other (describe)

Task Group 4: Relational Thinking

<p>Commutativity of addition</p> <p>Place out the card. <i>"Read this card. Work out the problem."</i> If necessary, <i>"How did you work it out?"</i> If necessary, <i>"Is there an easier way to work it out?"</i></p>	<p>4 + 12</p>	<p><input type="checkbox"/> Correct <input type="checkbox"/> Incorrect</p> <p><input type="checkbox"/> Counts On from 4 <input type="checkbox"/> Counts On from 12 <input type="checkbox"/> Uses Groups <input type="checkbox"/> Finger use (describe) <input type="checkbox"/> Other (describe)</p>
<p>Linking addition and subtraction</p> <p>Place out the card -- 15 + 3. <i>"Read this card. Work out the problem."</i> Place out the card -- 18 - 3 <i>"Read this card. Can you use this (indicate the 15 + 3), to work this out."</i></p>	<p>15 + 3</p> <p>18 - 3</p>	<p><input type="checkbox"/> Correct <input type="checkbox"/> Incorrect</p> <p><input type="checkbox"/> Solves each task separately <input type="checkbox"/> Uses inverse relationship <input type="checkbox"/> Other (describe)</p>
<p>Related subtraction tasks</p> <p>Place out the card -- 21 - 4 <i>"Read this card. Work out the problem."</i> Place out the card -- 21 - 17 <i>"Read this card. Can you use this (indicate the 21 - 4), to work this out."</i> <i>"Can you use this to tell me an addition sentence?"</i> <i>"Can you use this to tell me another addition sentence?"</i></p>	<p>21 - 4</p> <p>21 - 17</p>	<p><input type="checkbox"/> Correct <input type="checkbox"/> Incorrect</p> <p><input type="checkbox"/> Solves each task separately <input type="checkbox"/> Uses inverse relationship <input type="checkbox"/> Other (describe)</p>

Appendix I: Construct Levels

Addition and Subtraction Arithmetical Strategies	
Conceptual Construct	Brief Description
Unitary	Construct 0 <i>Emergent Counting</i> Cannot count visible items. The student either does not know the number words or cannot coordinate the number words with items or cannot use a cardinal number to quantify the collection.
	Construct 1 <i>Perceptual Counting</i> Can count perceived items but not those in concealed collections. This may involve seeing, hearing or feeling items.
	Construct 2 <i>Figurative Counting</i> Can count concealed items using a re-presentation, but counting typically includes what adults might regard as redundant activity. For example, when presented with two screened collections, told how many in each, and asked to find the total, the student will count from "one" instead of counting on.
	Construct 3 <i>Initial Number Sequence</i> The student <i>counts-on</i> rather than counting from "one", to solve addition or missing addend tasks. The student uses a <i>count-down-from</i> strategy to solve removed items tasks. (e.g. $17 - 3$ as 16, 15, 14; answer 14).
Composite	Construct 4 <i>Intermediate Number Sequence</i> The student <i>counts-down-to</i> to solve missing subtrahend tasks (e.g. $17 - 14$ as 16, 15, 14; answer 3). The student can choose the more efficient of <i>count-down-from</i> and <i>count-down-to</i> strategies.
	Construct 5 <i>Facile Number Sequence</i> The student uses a range of <i>non-count-by-one strategies</i> . For example, in additive and subtractive situations, the student uses strategies such as compensation, using a known result, adding to ten, commutativity, subtraction as the inverse of addition, and awareness of the "ten" in a teen number.



Appendix J: Learning Levels

Development of Forward Number Word Sequences (FNWS)	
Level	Brief Description
Level 0 <i>Emergent FNWS</i>	The student cannot produce the FNWS from "one" to "ten".
Level 1 <i>Initial FNWS to "ten"</i>	The student can produce the FNWS from "one" to "ten." The student cannot produce the number word just after a given number word in the range "one" to "ten." Note: Students at Levels 1, 2 and 3 may be able to produce FNWS beyond "ten".
Level 2 <i>Intermediate FNWS to "ten"</i>	The student can produce the FNWS from "one" to "ten". The student can produce the number word just after a given number word in the range "one" to "ten", but drops back to generate a running count when doing so.
Level 3 <i>Facile with FNWS to "ten"</i>	The student can produce the FNWS from "one" to "ten". The student can produce the number word just after a given number word in the range "one" to "ten" without dropping back. The student has difficulty producing the number word just after a given number word for numbers beyond "ten".
Level 4 <i>Facile with FNWS to "thirty"</i>	The student can produce the FNWS from "one" to "thirty". The student can produce the number word just after a given number word in the range "one" to "thirty" without dropping back. Note: Students at this level may be able to produce FNWS beyond "thirty".
Level 5 <i>Facile with FNWS to "one hundred"</i>	The student can produce FNWS in the range "one" to "one hundred". The student can produce the number word just after a given number word in the range "one" to "one hundred" without dropping back. Note: Students at this level may be able to produce FNWS beyond "one hundred".

Development of Backward Number Word Sequences (BNWS)

Level	Brief Description
Level 0 <i>Emergent BNWS</i>	The student cannot produce the BNWS from "ten" to "one".
Level 1 <i>Initial BNWS to "ten"</i>	The student can produce the BNWS from "ten" to "one". The student cannot produce the number word just before a given number word in the range "one" to "ten". Note: Students at Levels 1, 2, and 3 may be able to produce BNWS beyond "ten".
Level 2 <i>Intermediate BNWS to "ten"</i>	The student can produce the BNWS from "ten" to "one". The student can produce the number word just before a given number word in the range "one" to "ten", but drops back to generate a running count when doing so.
Level 3 <i>Facile with BNWS to "ten"</i>	The student can produce the BNWS from "ten" to "one". The student can produce the number word just before a given number word in the range "one" to "ten" without dropping back. The student has difficulty producing the number word just before a given number word for numbers beyond "ten".
Level 4 <i>Facile with BNWS to "thirty"</i>	The student can produce the BNWS from "thirty" to "one". The student can produce the number word just before a given number word in the range "one" to "thirty" without dropping back. Note: Students at this level may be able to produce BNWS beyond "thirty".
Level 5 <i>Facile with BNWS to "one hundred"</i>	The student can produce BNWS in the range "one" to "one hundred". The student can produce the number word just before a given number word in the range "one" to "one hundred" without dropping back. Note: Students at this level may be able to produce BNWS beyond "one hundred".



Development of Numeral Identification

Level	Brief Description
Level 0 <i>Emergent</i>	The student cannot identify some or all of the numerals in the range "0" to "10".
Level 1 <i>Numerals to "10"</i>	The student can identify numerals in the range "0" to "10".
Level 2 <i>Numerals to "20"</i>	The student can identify numerals in the range "0" to "20".
Level 3 <i>Numerals to "100"</i>	The student can identify one and two digit numerals.
Level 4 <i>Numerals to "1,000"</i>	The student can identify one, two, and three digit numerals.
Level 5 <i>Numerals to "1,000,000"</i>	The student can identify numerals through six digit numerals.

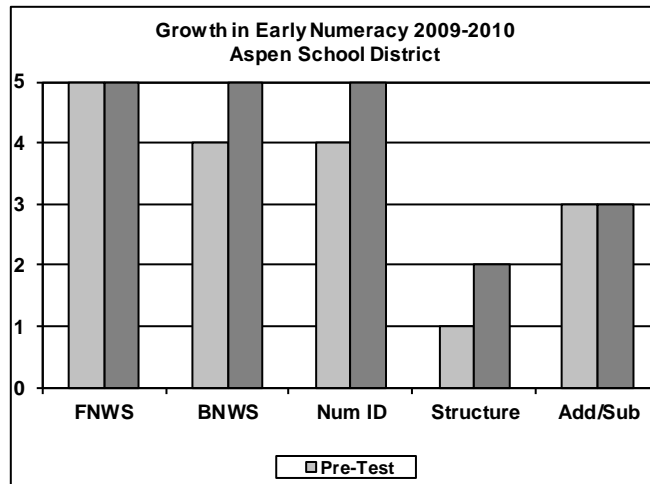


Development of Structuring Numbers

Levels	Brief Description
Level 0 <i>Emergent</i>	The student can subitize only small quantities (up to 3) and relies on counting to quantify larger groups. The student builds finger patterns by raising fingers sequentially.
Level 1 <i>Facile</i> Structures to 5	The student can subitize regular spatial patterns to 6 and irregular spatial patterns to 5. The student can create finger patterns in the range of 1 to 5 by raising fingers simultaneously. The student is able to combine and partition numbers in the range of 1 to 5 without counting.
Level 2 <i>Intermediate</i> Structures to 10	The student can recognize spatial patterns and create simultaneous finger patterns in the range of 6 to 10 using five-wise (5-plus) and pair-wise (doubles) structures. The student is able to combine and partition numbers in the range of 6 to 10 using 5-wise and pair-wise structures without counting. The student may be unable to generate other combinations and partitions in this range.
Level 3 <i>Facile</i> Structures to 10	The student is able to utilize reference numbers involving the sub-base of 5 and doubles to combine and partition numbers in the range of 1 to 10 without counting.
Level 4 <i>Intermediate</i> Structures to 20	The student is able to combine and partition numbers in the range of 11 to 20 using the 10-plus aspect of the "teens" and the doubles of 6 through 10. The student may be unable to generate other combinations and partitions in this range.
Level 5 <i>Facile</i> Structures to 20	The student is able to utilize reference numbers involving the base of 10, the sub-base of 5, and doubles to combine and partition numbers in the range of 1 to 20 without counting.

Appendix K: Mathematics Instructional Coach data

In conjunction with UNM, the Aspen School District has implemented two of a three-year AVMR Implementation Plan. Preliminary data from ASD shows promising results for children whose teachers have had the AVMR training. The graph shows growth in student levels for 19 students' skills specifically targeted by AVMR: Backward Number Sequence, Numeral Identification and Structuring Numbers.



Appendix L: AVMR Guiding Principles for Classroom Teaching

More than ten years ago, in research projects where we worked formally with teachers and school systems, [Robin Wright, Garry Stanger, Ann K. Stafford, and James Martland (2006, p. 6] developed the following set of nine guiding principles of teaching. In the last ten years these principles have been applied extensively to guide the teaching of number in the early years of school:

1. The teaching approach is inquiry based, that is, problem based. Children routinely are engaged in thinking hard to solve numerical problems that for them are quite challenging.
2. Teaching is informed by an initial, comprehensive assessment and ongoing assessment through teaching. The latter refers to the teacher's informal understanding of children's current knowledge and problem-solving strategies, and continual revision of this understanding.
3. Teaching is focused just beyond the 'cutting edge' of child's current knowledge.
4. Teachers exercise their professional judgment in the selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.
5. The teacher understands children's numerical strategies and deliberately engenders the development of more sophisticated strategies.
6. Teaching involves intensive, ongoing observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.

7. Teaching supports and builds on children's intuitive, verbally based strategies, and these are used as a basis for the development of written forms of arithmetic which accord with the child's verbally-based strategies.

8. The teacher provides the child with sufficient time to solve a given problem. Consequently the child is frequently engaged in episodes involving sustained thinking, reflection on her or his thinking and reflecting on the results of her or his thinking.

9. Students gain intrinsic satisfaction from their problem solving, their realization that they are making progress, and from the verification methods they develop.